# Extrapolation in games of coordination and dominance solvable games

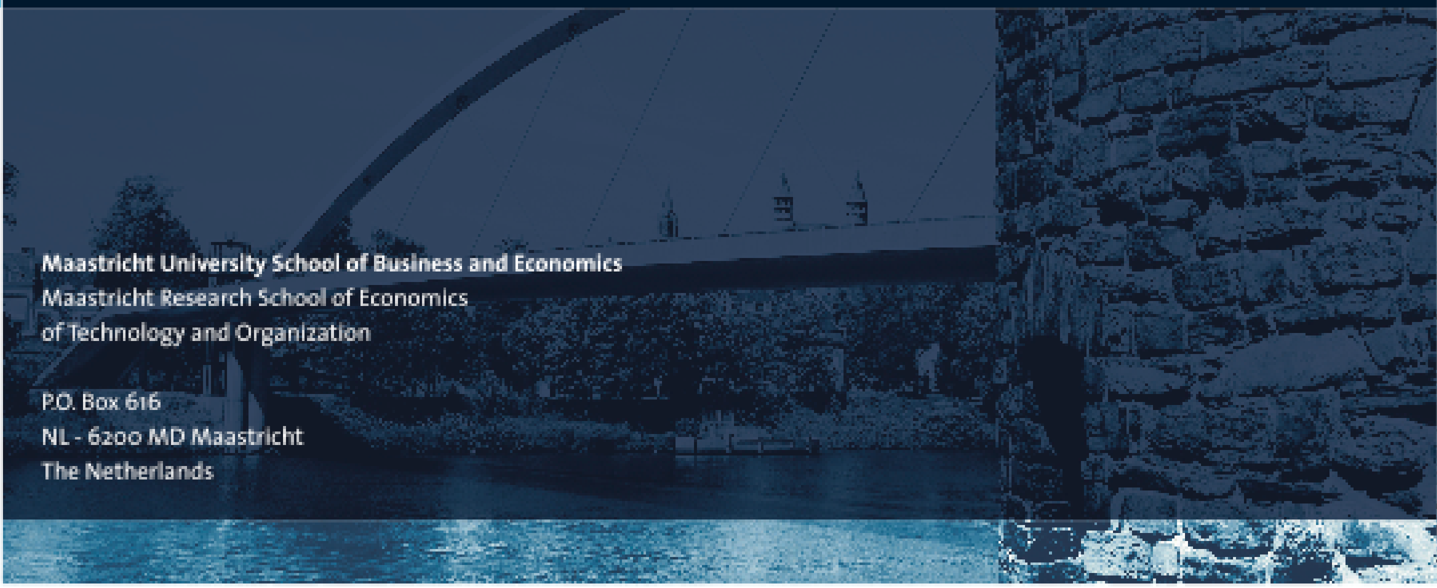
Maastricht University

Friederike Mengel, Emanuela Sciubba

# Extrapolation in Games of Coordination and Dominance Solvable Games

RM/10/034

METEOR



Maastricht University School of Business and Economics
Maastricht Research School of Economics
of Technology and Organization

P.O. Box 616
NL - 6200 MD Maastricht
The Netherlands

# Extrapolation in Games of Coordination and Dominance Solvable Games*

Friederike Mengel†
Maastricht University

Emanuela Sciubba‡
Birkbeck College London

July 12, 2010

**Abstract**

We study extrapolation between games in a laboratory experiment. Participants in our experiment first play either the dominance solvable guessing game or a Coordination version of the guessing game for five rounds. Afterwards they play a 3x3 normal form game for ten rounds with random matching which is either a game solvable through iterated elimination of dominated strategies (IEDS), a pure Coordination game or a Coordination game with pareto ranked equilibria. We find strong evidence that participants do extrapolate between games. Playing a strategically *different* game hurts compared to the control treatment where no guessing game is played before and in fact impedes convergence to Nash equilibrium in both the 3x3 IEDS and the Coordination games. Playing a strategically *similar* game before leads to faster convergence to Nash equilibrium in the second game. In the Coordination games some participants try to use the first game as a Coordination device. Our design and results allow us to conclude that participants do not only learn about the population and/or succesful actions, but that they are also able to learn structural properties of the games.

Keywords: Game Theory, Learning, Extrapolation
JEL-classification: C72, C91.

*We thank Larry Blume, Frank Heinemann, Willemien Kets, Dan Levin, Joshua Miller and Aldo Rustichini as well as seminar participants in Kopenhagen, Maastricht, Santa Fe and Toulouse for helpful comments. Friederike Mengel thanks the European Union for financial support (grant PIEF-2009-235973).

†Department of Economics (AE1), Maastricht University, PO Box 616, 6200 MD Maastricht, The Netherlands; Tel:+31-43-38 83891; Fax:+31-43-38 84878; E-mail: f.mengel@maastrichtuniversity.nl

‡Department of Economics, Mathematics and Statistics, Birkbeck, University of London, Malet Street, London WC1E 7HX. e.sciubba@bbk.ac.uk

# 1 Introduction

How people learn in any given game has received a lot of attention both by economic theorists and in experimental economics. In many cases of interest, though, decision-makers are faced with many different strategic situations, and the number of possibilities is so vast that a particular situation is practically never experienced twice. A tacit assumption of standard learning models is that players extrapolate their experience from previous interactions similar to the current one. How people transfer knowledge between seemingly quite different situations, though, has received much less attention in the literature.[1]

From an applied perspective understanding extrapolation and learning transfers across games is important for at least two reasons. First it can help us make predictions in situations where such effects can be expected. An example could be the introduction of a new law or institution where behavior under the new institution is likely to be affected by previous regulations. Understanding how people extrapolate between games can also enable the designer of a mechanism to structure interactions in such a way that "desired" extrapolation takes place. Think for example about the problem of organization design. Some given set of tasks need to be accomplished in the organization, but the designer can choose who interacts with who and in which types of strategic situations. Understanding extrapolation can help her design workflows in a more efficient way.

On a more fundamental level one could even argue that, unless we understand which knowledge economic agents extrapolate from one game to other games, we cannot understand what they have learned in the first place.[2] In particular we may distinguish three different types of learning and extrapolation.

There is a large class of learning models, where learning is *purely outcome based*. In those models people learn simply about succesful actions and choose more succesful actions more often in the future. Reinforcement learning rules are an example of this first category.[3] If this is all people learn in games, then in terms of extrapolation there can be at best "naive extrapolation" , i.e. participants may tend to choose succesful actions in one game with higher probability in different subsequent games.

In a second class of models people learn about the *population*. In these models players learn about the behaviour of other players in the population and then best respond to beliefs which they have formed through learning. Learning models such as fictitious play or myopic best responses fall in this category.[4] In terms of population learning models extrapolation could imply that participants transfer some knowledge about the population to other games. If this is the case than playing any game (irrespective of the strategic content) should improve convergence to equilibrium in games played subsequently within the same population.

All of these rules have in common, though, that players do not learn anything structural about the underlying game. For example all the rules mentioned before imply that if we play the same game again but e.g. relabel actions participants will have to start learning all over again. Hence some authors have enhanced standard models with the possibility of action-relabeling. (See e.g. Stahl and Haruvy, 2009).

However learning may also be *structural*, i.e. people may learn something about the underlying structure of strategic interactions. This may go well beyond the case where games are the same up to action-relabeling.

[1]For some theoretical work illustrating such effects see Jehiel (2005),Mengel (2007) or Steiner and Stewart (2008). Experimental literature will be discussed below in detail.

[2]Manski (2002) makes a somewhat related point showing that learning or decision rules cannot be identified from observing choice data in a single game.

[3]See e.g. Bush and Mosteller (1951), Roth and Erev (1995) or Mengel (2009).

[4]See e.g. the textbook by Fudenberg and Levine (1998). There are also hybrid models with elements of both outcome based learning and population learning. One example is experience weighted attraction learning (Camerer and Ho, 2004).

For example people may learn how to identify dominated strategies, or they may learn about how to avoid coordination cycles etc. In such a case, experience in games which are similar in the type of strategic interaction that is entailed (e.g. games which appear very different, but are all dominance solvable), will aid convergence towards equilibrium play in subsequent games of the same type. Such structural learning does obviously not exclude the possibility that people learn about the population or engage in outcome based learning in addition.

In this paper we attempt a systematic study of extrapolation between games. To this aim we design a simple experiment where participants interact in two different games and study (i) whether and (ii) which type of extrapolation (and hence learning) takes place between the two games.

Empirically extrapolation is most interesting if it occurs between two different games. In all our treatments the two games studied differ in the action set, the set of players, the number of players, the framing of the game (payoff matrix versus guessing game), the payoffs and the repeated nature of the game. The only dimension in which our games are similar is their 'strategic nature'.[5] In particular we consider two types of games. Firstly we study games which are solvable through deliberative reasoning (more precisely through iterated elimination of dominated strategies, henceforth IEDS). Secondly we study games which require some coordination among players, i.e. games where deductive equilibrium analysis based on common knowledge of rationality fails to determine a unique strategy profile.

To analyse extrapolation we run treatments where subjects are able to learn through strategically similar games (i.e. a dominance solvable game followed by another dominance solvable game, or a coordination game followed by another coordination game) and treatments where sujects are faced with a sequence of strategically dissimilar games (i.e. a dominance solvable game followed by a coordination game or a coordination game followed by a dominance solvable game).

In all our treatments the first game being played is a variant of the so called 'guessing game' (or 'beauty contest game'), which is played 5 times repeatedly in fixed groups of four (Nagel, 1995). In the first (standard) variant all participants have to simultaneously guess a number between 0 and 100. The participant closest to 70% of the average guess wins five Euros. This game is solvable through IEDS and the unique Nash equilibrium is that everyone guesses zero. In the second variant the participant closest to the average guess wins. This second variant is hence a coordination game, where every outcome in which all participants choose the same number is a Nash equilibrium. The second variant actually corresponds to Keynes's original 'Gedankenexperiment'. (Keynes, 1936).

As a second game participants play one of three possible 3x3 normal form games for ten rounds, being randomly rematched in each round. One of the two 3x3 games is a game solvable through IEDS with a unique Nash equilibrium and the other games are a pure coordination game and a coordination game with pareto ranked equilibria.

Our focus is on subjects' behavior in the second game. For each game - the IEDS and the coordination game - we have three treatments (hence a total of nine treatments). One where only the 3x3 game is played, one where the 3x3 game is preceded by the 70% version of the guessing game and one where it is preceded by the 'Keynes version' (100%). Hence for each of the two 3x3 games we have a treatment where subjects gain prior experience in a strategically similar game, a treatment where subjects gain prior experience in a strategically dissimilar game, and a baseline treatment where the 3x3 game is played on its own.

[5] Some other papers have studied learning transfer among similar games. Gneezy et al (2010) for example show that participants are able to 'learn' backward induction by playing similar games. See also Rapoport et al (2000). We discuss related literature in more detail below.

Our experimental findings as far as the guessing games are concerned are in line with the established experimental literature on these games. In the standard version of the guessing game (70% version), we find that guesses converge towards zero (with an average of about 9 in the last round). In the 100% version of the guessing game, they converge to about 40-50.

As far as extrapolation is concerned, we find solid evidence that learning across games does take place: for both the 3x3 coordination game and the 3x3 IEDS game behavior is significantly different across the three treatments. Hence subjects do behave differently when they have obtained previous experience in strategically similar or dissimilar games compared to when they play the 3x3 game on its own.

More in detail we find that having played a strategically *similar* game before leads to better (faster) learning in the second game. On the other hand playing a strategically *different* game hurts play, not only when compared to the situation where a *similar* game is played but also compared to the baseline, where no other game is played before. In particular it impedes convergence to Nash equilibrium in both dominance solvable and coordination games. One possible interpretation for this finding is that playing a coordination game where some intuition (or 'gut-feeling') is needed in order to anticipate what others will play and reach a Nash equilibrium, may bring participants into the wrong 'mode' of play when they afterwards face a game which is solvable through IEDS, and vice versa. Kuo et al. (2009) in fact find evidence that two different brain regions are involved when people play these different classes of games.

Our observed treatment differences cannot be explained by models of outcome-based or population learning alone. The fact that playing a strategically similar game speeds up convergence while playing a strategically different game hurts compared to the baseline provides evidence for structural learning. Subjects do seem to recognise structural properties of the game and succeed in transfering acquired knowledge across two games which are strategically similar.

We also find some evidence for 'naive extrapolation' (or learning about successful strategies, irrespective of the strategic context). In particular, we find that in the first round of the 3x3 game subjects are relatively more likely to play an action which has a similar label to the modal outcome of the guessing game that they have played before. This effect is weak, but statistically significant, in the 3x3 IEDS game. It is much stronger in the 3x3 coordination games, where clearly some participants attempt to establish the modal outcome of the guessing game as a coordination device. There is some literature trying to show how exogenous (and payoff irrelevant) statements may create "sunspot equlibria" and lead participants to play different equilibria in a coordination game.[6] In our experiment we show that previous experience in a different game can potentially play a similar role, since some participants try to use the modal behavior in the first game as a Coordination device.

Our experiment can also shed some light into the question of how we should think about similarity in games as opposed to decision problems. Gilboa and Schmeidler (1995) and Rubinstein (1988) both use distance in payoff space or probabilities as a similarity criterion for decision problems. In games it is much less clear when we should call two games "similar".[7] The reason is that while two payoff matrices may be very close in terms of distance in payoff space the games may still be very different strategically. Our experimental results suggest that such strategic similarity matters indeed for extrapolation in spite of the fact that games appear very different.

[6] See e.g. Fehr, Heinemann and Llorente-Saguer (2010).

[7] Steiner and Stewart (2009) use closeness in payoff space. Mengel (2009) shows that strategic similarity matters in an evolutionary model with endogenous equivalence classes of games.

In addition our results allow us to gain some insight into interpretations of the guessing game.[8] There has been some debate as to how to interpret the results in this game and whether people do learn about IEDS. Our results show that there is indeed some structural learning in this game. In particular it seems that participants do learn something about the process of IEDS in the guessing game, since they are subsequently able to extrapolate such knowledge to a different game, also solvable through IEDS. In line with Grosskopf and Nagel's (2009) explanation it seems to be mostly the quality (and speed) of learning which is positively affected by extrapolation in our experiment.

There are a few other papers in the experimental literature dealing with learning transfers, extrapolaton or categorization. Stahl and Haruvy (2009) study learning transfer between 'dissimilar' symmetric normal form games.[9] They find that a model of experience-weighted attraction learning augmented with action relabeling performs well in explaining the initial choices in each game. Grimm and Mengel (2009) study learning in a multiple games environment, where participants face different normal form games randomly drawn in each period. They find evidence for outcome based learning and extrapolation along action labels. Huck, Jehiel and Rutter (2007) also find evidence for categorical thinking based on population learning in an experiment. Grosskopf et al.(2010) find support for case based decision making as proposed by Gilboa and Schmeidler (1995).

Other than the papers mentioned above we are not aware of studies that systematically study extrapolation between different games. Several studies have found more or less explicit evidence that there is learning transfer between games. Examples are Weber (2003) in a study of 'feedback-less' learning, Rapoport et al. (2000), Cooper and Kagel (2003, 2008) or Chong et al. (2006) among others. Other studies have analysed learning across games which are very similar except for one or two parameters or payoffs. Camerer et al (1998) look at learning transfer between IEDS guessing games with different parameters. Selten et al. (2003) let subjects submit strategies for tournaments with many different $3 \times 3$ games and find that the induced fraction of pure strategy Nash equilibria increases over time.

The paper is organised as follows. In Section 2 we describe the experimental design. In Section 3 we present our hypotheses and in Section 4 we present the results. Section 5 concludes the paper.

## 2 Design

The experiment was conducted in February and June 2010 at the BEE-Lab at Maastricht University. 206 students participated in one of the following treatments:

**T1 (IEDS):** In IEDS participants were randomly rematched for ten rounds to play the game shown in Table 1.

**T2 (COR):** In COR participants were randomly rematched for ten rounds to play the game shown in Table 2.

**T2* (COR*):** In COR* participants were randomly rematched for ten rounds to play the game shown in Table 3.

**T3 (IEDS-IEDS):** In IEDS-IEDS participants were first matched in fixed groups of four players to play the 70% version of the guessing game during five rounds. In this game all group members have to

[8] See Nagel (1995), Stahl (1998) or Grosskopf and Nagel (2007,2009) among many others.

[9] For them the term dissimilar means that there is no re-labeling of actions which makes games monotonic transformations of each other. This is true for all our games considered even for those that we call 'strategically similar'. See also Rankin et al (2000).

| | H | M | L |
|---|---|---|---|
| H | 10, 10 | 8, 12 | 16, 8 |
| M | 12, 8 | 10, 10 | 6, 6 |
| L | 8, 16 | 6, 6 | 10, 10 |

Table 1: 3x3 IEDS game

| | H | M | L |
|---|---|---|---|
| H | 10, 10 | 8, 6 | 8, 8 |
| M | 6, 8 | 10, 10 | 6, 6 |
| L | 8, 8 | 6, 6 | 10, 10 |

Table 2: 3x3 pure Coordination game

simultaneously guess a number between 0 and 100. The participant closest to 70% of the average guess wins five Euros and ties are resolved randomly. Afterwards they were randomly matched for ten rounds to play the game shown in Table 1.

**T4 (COR-COR):** In COR-COR participants were first matched in fixed groups of four players to play the 100% version of the guessing game during five rounds. In this game all group members have to simultaneously guess a number between 0 and 100. The participant closest to the average guess wins five Euros and ties are resolved randomly. Afterwards they were randomly matched for ten rounds to play the game shown in Table 2.

**T4* (COR-COR*):** In COR-COR* participants first played the 100% version of the guessing game during five rounds. Afterwards they were randomly matched for ten rounds to play the game shown in Table 3.

**T5 (IEDS-COR):** In IEDS-COR participants first played the 70% version of the guessing game during five rounds. Afterwards they were randomly matched for ten rounds to play the game shown in Table 2.

**T5* (IEDS-COR*):** In IEDS-COR participants first played the 70% version of the guessing game during five rounds. Afterwards they were randomly matched for ten rounds to play the game shown in Table 5.

**T6 (COR-IEDS):** In COR-IEDS participants first played the 100% version of the guessing game during five rounds. Afterwards they were randomly matched for ten rounds to play the game shown in Table 1.

Written instructions were distributed at the beginning of each phase. In treatments T3-T6 participants knew at the start of phase 1 that there would be a second phase in the experiment but did not know what it would look like.[10] Matching groups were of size 8 in all treatments. Participants were informed that they were matched with the same group of participants in the first phase and that they would be randomly

[10]Sample Instructions for treatment IEDS-IEDS can be found in the Appendix.

| | H | M | L |
|---|---|---|---|
| H | 10, 10 | 8, 5 | 6, 3 |
| M | 5, 8 | 12, 12 | 5, 3 |
| L | 3, 6 | 3, 5 | 15, 15 |

Table 3: 3x3 Coordination game with tradeoff between risk dominance and efficiency

rematched in each period in the second phase. Moreover, in T3-T6 participants were told in the instructions for phase 2, that 'they will likely be paired up with participants who they have *not* played with in phase 1'. Actions H,M,L were labeled 'High', 'Medium' and 'Low' in the experimental instructions and were labeled as shown in Tables 1 and 2 on the decision screens during the experiment. This labeling was chosen in order to enable participants to draw a connection between the action sets in the two games, either because of "naive extrapolation" or in order to use behavior in the first game as a Coordination device.

At the end of each round of the guessing game participants were informed about the different guesses made in their group and whether they had won or not. At the end of each round of the 3x3 game participants were told their action choice and that of their match as well as their payoff.

In addition to a show up fee of 2 Euros, overall earnings were the sum of earnings from all rounds. Earnings from the first phase were directly given in Euros. Earnings from the second phase were given in ECU (experimental currency unit) and converted into Euros according to the exchange rate 1Euro=20ECU. The experiment lasted between 35 (T1, T2 and T2*) to 60 minutes (T3-T6) and participants earned on average 12,60 Euros with a minimum of 6,40 Euros and a maximum of 34,20 Euros.

## 3 Hypotheses

The most basic hypothesis we wish to test is Hypothesis 0 which claims that there is no extrapolation between games. This hypothesis amounts to saying that there should be no treatment differences in how the 3x3 game is played between IEDS, IEDS-IEDS and COR-IEDS, no differences between COR, COR-COR and IEDS-COR and no differences between COR*, COR-COR* and IEDS-COR*.

**Hypothesis 0** *There is no extrapolation between games. In particular, subjects' behaviour in the 3x3 game is not different across treatments IEDS, IEDS-IEDS and COR-IEDS, nor across COR, COR-COR and IEDS-COR, nor across COR*, COR-COR*, IEDS-COR*.*

Next we wish to test whether experienced subjects "learn faster" in the second game.

**Hypothesis 1** *Extrapolation always "improves" convergence to equilibrium. In particular convergence in the 3x3 game is faster in both IEDS-IEDS and COR-IEDS compared to IEDS, faster in both COR-COR and IEDS-COR compared to COR and faster in COR-COR* and IEDS-COR* compared to COR*.*

If we failed to reject Hypothesis 1, this would provide support for the conjecture that subjects learn about the population they are playing against. As an extreme case consider a fully rational decision maker who understands and distinguishes both games perfectly but who is unsure about the distribution of types in the population. If this were the case the decision maker could use the first game to reduce her uncertainty about the composition of the population (e.g. in terms of rationality of other players). If we can reject Hypothesis 1, then we can reject the idea that extrapolation is limited to learning about the population alone.

The following hypotheses 2a and 2b capture the idea that there is structural learning. If this is the case extrapolation will only improve convergence to equilibrium if the first game (from which participants extrapolate) is strategically similar to the second game.

**Hypothesis 2a** *Extrapolation "improves" play if a strategically similar game has been played before.*

**Hypothesis 2b** *Extrapolation "hurts" play if a strategically different game has been played before.*

Taken together, hypotheses 2a and 2b imply the following ranking: IEDS-IEDS $\geq$ IEDS $\geq$ COR-IEDS, where "$\geq$" here means faster convergence to equilibrium, COR-COR $\geq$ COR $\geq$ IEDS-COR and COR-COR* $\geq$ COR* $\geq$ IEDS-COR*. If we fail to reject Hypotheses 2a and 2b, this would be consistent with the conjecture that some structural learning occurs. If in addition IEDS $\geq$ COR-IEDS and COR(*) $\geq$ IEDS-COR(*), then we could conclude that structural learning matters at least as much as learning about the population in our context.

Our last hypothesis conjectures that extrapolation may be more effective in games which can be solved through deliberative reasoning (in particular games solvable through IEDS) as opposed to games which have to be solved through intuition. The conjecture is that since in the first class of games there is a unique way of solving the game, extrapolation may be more effective in such games.

**Hypothesis 3** *Extrapolation is more effective if games can be solved by deliberative reasoning (as opposed to games which have to be solved intuitively).*

# 4 Results

## 4.1 Behavior in the Guessing Games

Let us start by describing behavior in the guessing games. The following two graphs illustrate the distribution of guesses in the 70%-version and the 100%-version of the game. While in the 70%-version guesses are decreasing over time, there is no such time trend in the Keynes game (100% -version), where most guesses are concentrated around 40-50. Variance remains higher in the 100% version. Clearly the two games are played differently. In the 70%-version subjects' behavior seems to converge over time towards the unique Nash equilibrium where everyone guesses 0. In the Keynes guessing game the focal Nash equilibrium seems to be to bid 50, which is the amount where most initial guesses are concentrated. In fact in the Keynes version 65% of guesses are in the interval [40,60] already in Period 1 and the modal guess is 50. In the standard version the modal guess is 10. By Period 5 only one guess is higher than 30 and 85% of guesses are below 12.

A simple panel data OLS regression indicates that while there is a strong and significant time trend in the IEDS version of the guessing game there is only a much weaker and marginally significant time trend in the Coordination version of the guessing game. The regression table can be found in the Appendix. The following table illustrates the average guess in period 5 (i.e. the last round of the guessing game) in the different treatments.

| | IEDS-IEDS | IEDS-COR | IEDS-COR* | COR-COR | COR-IEDS | COR-COR* |
|---|---|---|---|---|---|---|
| average guess period 5 | 9.41 | 8.91 | 8.87 | 36.83 | 43.25 | 44.29 |
| (Standard Deviation) | (6.39) | (7.02) | (8.06) | (13.92) | (10.05) | (10.85) |

Table 4: Average Guess in last period of Guessing Game.

As expected average guesses in the 70 percent (IEDS)-version of the game are much lower than average guesses in the 100% (COR)-version of the game and those differences are statistically significant. (Mann-Whitney, $p < 0.0001$). Also, as expected, there are no treatment differences between IEDS-IEDS, IEDS-COR and IEDS-COR* or between COR-COR, COR-IEDS or COR-COR*.

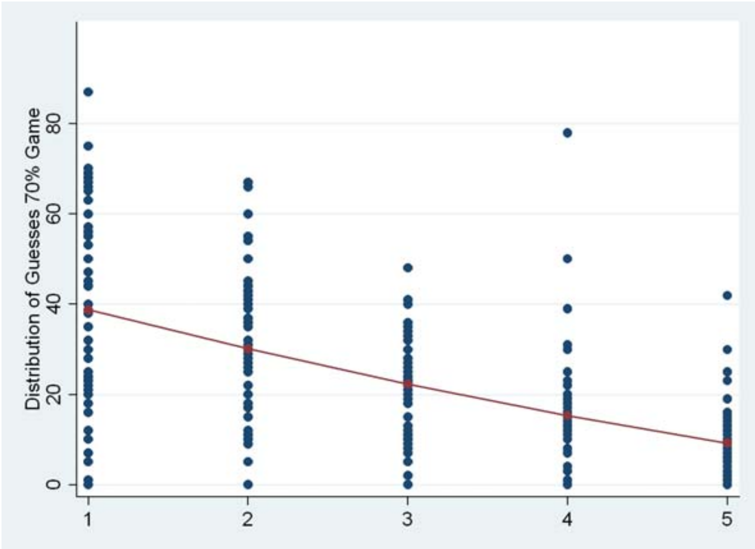



Figure 1: Distribution of Guesses 'IEDS guessing game' (70%-version).

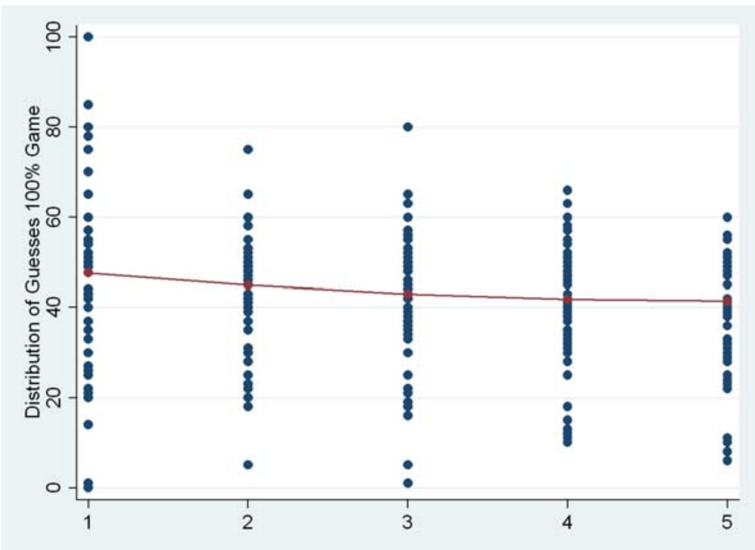



Figure 2: Distribution of Guesses 'Keynes guessing game' (100%-version).

## 4.2 Extrapolation in Games solvable through IEDS

Now we are ready to look at extrapolation. Let us start with the games which are solvable through deliberative reasoning, i.e. the games solvable through IEDS. The unique Nash equilibrium in the 3x3 game represented in Table 1 is (M,M). Table 5 illustrates the distribution of choices in the 3x3 IEDS game. There seem to be somewhat more $M-$ choices in IEDS-IEDS compared to the other treatments and somewhat less $H-$ choices. On average, though, there is not much difference.

| | H | M | L |
|---|---|---|---|
| IEDS | 0.29 | 0.70 | 0.01 |
| IEDS-IEDS | 0.21 | 0.75 | 0.04 |
| COR-IEDS | 0.24 | 0.70 | 0.06 |

Table 5: Average share of H/M/L choices in the 3x3 IEDS game.

To see treatment differences we have to look at the distribution of choices over time. Figure 3 illustrates the share of M-choices (i.e. Nash choices) over time in the three treatments.

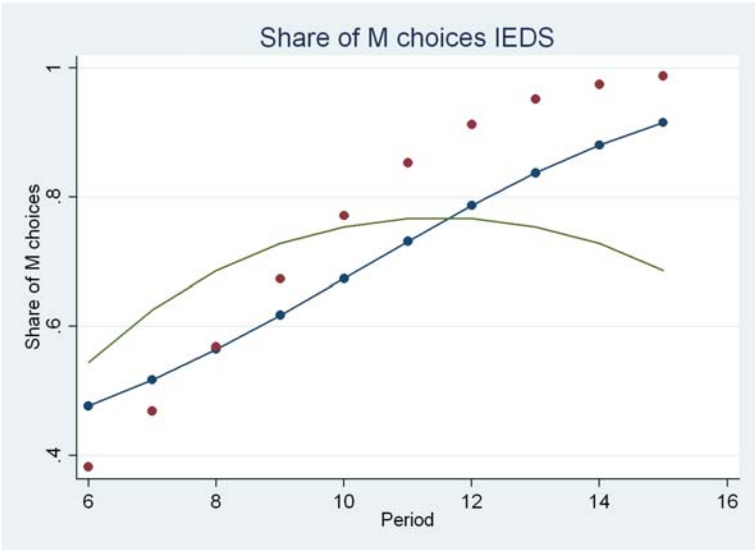



Figure 3: Share of M Choices over time in the 3x3 IEDS game. The dotted line displays treatment IEDS, the dots are IEDS-IEDS and the line without dots represents COR-IEDS.

The first thing to notice is that while participants seem to learn the Nash equilibrium in treatments IEDS and IEDS-IEDS, there is no convergence in treatment COR-IEDS. Also learning seems to be better in IEDS-IEDS compared to IEDS (especially faster as the Figure and the regression below illustrate) but it is substantially worse in COR-IEDS. Playing a game largely based on intuition seems to hurt effective learning in games based on reasoning. Table 6 shows that, indeed, there is very little learning in treatment COR-IEDS compared to other treatments. One possible explanation is that there is structural learning. Participants seem to be able to learn about the structural properties of the guessing games and then try to apply this knowledge afterwards, leading to better convergence in IEDS-IEDS and worse convergence in COR-IEDS. Playing a coordination game where intuition (or some 'gut-feeling') is needed to reach a Nash equilibrium, seems to bring participants into the wrong 'mode' of play when afterwards playing a game which is solvable through IEDS. In support of this explanation, Kuo et al. (2009) find evidence that two different parts of the brain may be used to play these different classes of games.

| Learning | |
|---|---|
| IEDS | 0.50 |
| IEDS-IEDS | 0.55 |
| COR-IEDS | 0.12 |

Table 6: Percentage of M choices. Difference between period 10(15) and period 1(6).

We also ran panel data logit regressions to explain the share of M choices in the IEDS game through the time period, treatment dummies as well as interaction terms. The period variable counts from 6, ...15 and time period in IEDS is normalised to this count. The baseline is treatment IEDS. The results are presented in Table 7.

| $M-$ Choices in IEDS Game | (1) | (2) |
|---|---|---|
| constant | $-0.6506^{**}$ | $-0.3768$ |
| | (0.1226) | (0.4207) |
| period | $0.3608^{***}$ | $0.3389^{***}$ |
| | (0.0682) | (0.0632) |
| IEDS-IEDS | $-4.3673^{***}$ | $-4.9408^{***}$ |
| | (1.1366) | (1.1033) |
| COR-IEDS | 0.8455 | |
| | (0.8986) | |
| periodX(IEDS-IEDS) | $0.3924^{***}$ | $0.3942^{***}$ |
| | (0.1247) | (0.1222) |
| periodX(COR-IEDS) | $-0.2423^{***}$ | $-0.1817^{***}$ |
| | (0.0905) | (0.0633) |
| $\rho$ | 0.4918 | 0.4917 |

Table 7: Panel Data Logit Regression $^{***}1\%,^{**}5\%,^{*}10\%$. $((\Pr > \mathcal{X}^2) < 0.0001)$

The results from the regression confirm our intuitions derived from the Figure. The positive coefficient on period indicates that over time people learn to play the equilibrium. There are significantly less $M-$ choices in IEDS-IEDS than in IEDS initially. The positive coefficient on periodX(IEDS-IEDS) shows that there is faster learning, though, in IEDS-IEDS compared to IEDS, implying that starting in period 10 approximately (i.e. the 5th period of the 3x3 IEDS game) there are more $M-$ choices in IEDS-IEDS compared to IEDS.[11] The coefficient on COR-IEDS is not significant, i.e. initially there are not significantly more $M-$ choices in COR-IEDS compared to IEDS. On the other hand the negative coefficient on periodX(COR-IEDS) shows that convergence to equilibrium is much worse in COR-IEDS compared to IEDS, and hence also in IEDS-IEDS. We also ran the same regression but including a square term period$^2$ as well as interaction terms period$^2$X(IEDS-IEDS) and period$^2$X(COR-IEDS) and found that we can jointly omit them from the regression ($\Pr > \mathcal{X}^2 = 0.1548$).

[11] Note that since the graph depicts the predictions for the quadratic rather than linear regression the intersection of the two curves there is already around Period 8.

The evidence rejects Hypothesis 0 that there is no extrapolation across games. Play in all three treatments is pairwise significantly different.

It also seems that playing a strategically similar game before, as it is the case in IEDS-IEDS helps convergence to equilibrium. There is a both a higher share of $M-$ choices in the last period in IEDS-IEDS compared to IEDS and COR-IEDS and learning is faster in IEDS-IEDS compared to the other treatments. Playing a strategically different game hurts. Learning is much worse in COR-IEDS compared to the other treatments. Hence we can reject Hypothesis 1. Learning (and extrapolation) is not limited to learning about the population. On the other hand the ranking of our treatments seems to suggest that there is structural learning. Participants are able to learn the structural properties of a game and extrapolate such knowledge when playing other games subsequently.

We ask next if we can detect any evidence of 'naive extrapolation', possibly in the first rounds of play for the 3x3 IEDS game. Given that in the 70%-version of the guessing game subjects' behaviour converges to low guesses, if naive extrapolation is taking place, we would expect a relatively higher share of L choices in IEDS-IEDS than in IEDS. Similarly, given that in the 100%-version of the guessing game subjects coordinated on medium guesses, we would expect a relatively higher share of M choices in COR-IEDS compared to IEDS.

Table 8 shows the percentages of M and L choices in the first period of the 3x3 game.

| | L in period 1 (6) | M in period 1 (6) |
|---|---|---|
| IEDS | 0 | 0.46 |
| IEDS-IEDS | 0.08 | 0.41 |
| COR-IEDS | 0.08 | 0.54 |

Table 8: Naive Extrapolation - M and L choices in the first period of the 3x3 IEDS game.

This table suggests that there is only a weak effect of naive extrapolation in the IEDS game. We do observe more L choices in IEDS-IEDS than in IEDS and more M choices in COR-IEDS than in IEDS. However, a Mann-Whitney test shows that the distribution of choices in the first round does not differ pairwise between any two treatments ($p > 0.1529$). Moreover the distribution of M choices between IEDS and COR-IEDS does not significantly differ (Mann-Whitney, $p = 0.2160$). We do however find significant differences in the distribution of L choices between IEDS and IEDS-IEDS if we focus on the first five rounds of play (Mann-Whitney, $p = 0.0269$). Overall the effect of naive extrapolation seems rather weak. The predominant effect of extrapolation affecting play in the 3x3 game seems to be a consequence of structural learning.

### 4.3 Extrapolation in Coordination Games

Let us now see whether and how extrapolation affects the coordination games, i.e. games which are not solvable through deliberative reasoning alone but which require subjects to use some intuition on what the others will play in order to reach an equilibrium. Table 9 shows the percentage of H/M/L choices across all periods.

Quite amazingly in COR participants coordinate immediately on the equilibrium (H,H). In both COR-COR and IEDS-COR coordination is worse with a significant share of participants choosing L. Interestingly in COR-COR where participants play the coordination version of the guessing game before many more participants choose the risk dominated action M. This is a strong indicator that some participants may try

| | H | M | L | | H | M | L |
|---|---|---|---|---|---|---|---|
| COR | 0.98 | 0.01 | 0.00 | COR* | 0.16 | 0.02 | 0.81 |
| COR-COR | 0.52 | **0.27** | 0.20 | COR-COR* | 0.19 | **0.10** | 0.71 |
| IEDS-COR | 0.78 | 0.01 | 0.19 | IEDS-COR* | 0.16 | 0.03 | 0.80 |

Table 9: Overall share of H/M/L choices in the Coordination games

to use the coordination version of the guessing game (where coordination occurred on Medium guesses) as a coordination device in the following game.

In the coordination game with pareto ranked equilibria we observe a qualitatively similar pattern. Now participants try to coordinate on the efficient equilibrium (L,L). Again, though, in COR-COR* there are more M-choices than in any of the other treatments suggesting that participants try indeed to use the modal behavior in the guessing game as a Coordination device. We will come back to this later in this section.

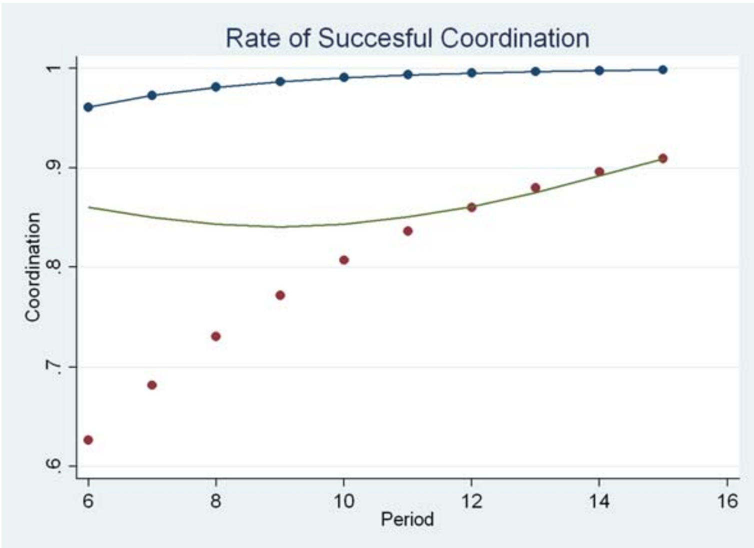



Figure 4: Rate of Succesful Coordination over time in the 3x3 Coordination Game. The dotted line shows treatment COR. The dots indicate Coordination in COR-COR. The line without dots represents treatment IEDS-COR.

Figure 4 shows the percentage of succesful Coordination (on any of the Nash equilibria) over time. In COR there is almost perfect coordination on (H,H) from the beginning. In COR-COR initially coordination is worst, but then there is a steep learning curve with more than 90% of succesful coordination eventually. In IEDS-COR there is more coordination initially than in COR-COR, although less than in COR, and there is almost no learning. Hence, as in the case of the IEDS games playing a strategically different game seems to distort learning significantly.

Figure 5 shows the analogous graph for the Coordination game with pareto ranked equilibria. Learning is much better in COR-COR* compared to COR* or IEDS-COR*. Hence again playing a strategically similar game before seems to improve convergence in the 3x3 game providing additional evidence for the importance of structural learning.

Again we ran panel data logit regressions to explain the rate of succesful coordination in the coordination game through the time period, treatment dummies, square as well as interaction terms. Table 10 shows the

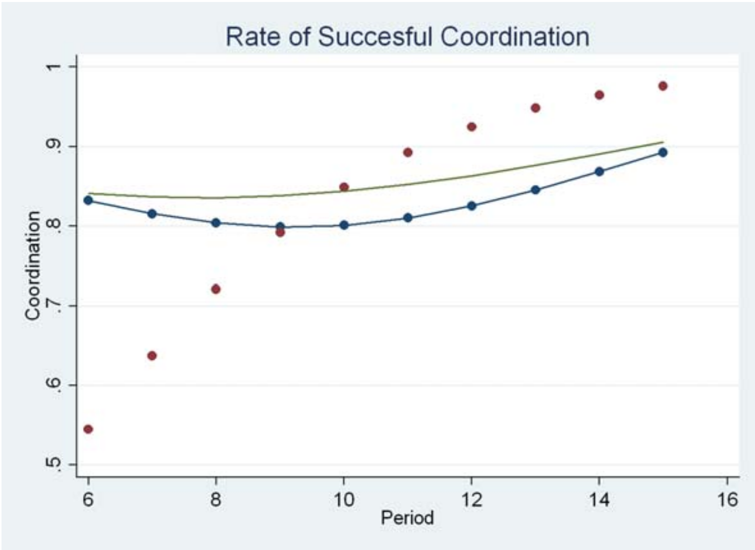



Figure 5: Rate of Succesful Coordination over time in the 3x3 Coordination Game given by . The dotted line shows treatment COR*. The dots indicate Coordination in COR-COR*. The line without dots represents treatment IEDS-COR*.

results of this regression. The baseline treatment is COR or COR* respectively.

The regression table 10 shows the results. Unlike in the case of IEDS games, this time we cannot omit the period$^2$ terms entirely from the regression ($(\Pr > \mathcal{X}^2) < 0.0001$), but we can omit the interaction terms periodXperiodX(COR-COR) and periodXperiodX(IEDS-COR) ($(\Pr > \mathcal{X}^2) = 0.4021$).

Again Hypothesis 0 has to be rejected, since behavior in all three treatments significantly differs. Hypotheses 1 is also clearly rejected, as it is not true that any type of previous experience aids equilibrium play and coordination and in particular playing a strategically different game as in IEDS-COR or IEDS-COR* does not help (and in IEDS-COR even hurts) convergence to equilibrium compared to the control treatment. On the other hand in COR-COR* convergence is much better compared to COR*. Hence we can again reject Hypothesis 1. Learning and hence extrapolation is not limited to learning about the population also for the Coordination games.

Again there is strong evidence for structural learning. Convergence to equilibrium is much better in COR-COR* compared to both COR* and IEDS-COR* and much better in COR-COR compared to IEDS-COR. However the rate of succesful coordination overall is still higher in COR compared to COR-COR and initially coordination is much worse also in COR-COR* compared to COR*. The reason is that now in the Coordination games there is a strong effect of what we have termed "naive extrapolation" before. We are a bit reluctant to call this type of extrapolation naive in the context of coordination games, since it may make sense to try and use the modal choice in the first game as a Coordination device for the second game.

Table 11 shows the percentage of M and L choices in the first period of the 3x3 coordination game.

Indeed participants seem to use the outcome of the guessing game as a coordination device for the coordination game in treatment COR-COR. There are more M choices in the first period of the game in COR-COR compared to COR (Mann-Whitney, $p = 0.0430$). This is inspite of the fact that action M is risk dominated. Also in COR-COR* there are more M-choices initially compared to COR* (Mann-Whitney, $p = 0.0046$). There are also more L choices in IEDS-COR compared to COR (Mann-Whitney, $p = 0.0387$) and in IEDS-COR* compared to COR* (Mann-Whitney, $p = 0.0478$).

| Succesful Coordination | (COR-1) | (COR-2) | (COR*) |
|---|---|---|---|
| const | 3.8889** | 2.3902** | 2.1827*** |
| | (1.8058) | (1.0487) | (0.7555) |
| period | 0.4339 | 1.5065*** | −0.2193 |
| | (0.9876) | (0.2574) | (0.2976) |
| periodXperiod | −0.0040 | −0.1154*** | 0.0261 |
| | (0.1081) | (0.0143) | (0.0269) |
| COR-COR(COR*) | −16.4971*** | −14.4743*** | −13.9374*** |
| | (2.6468) | (1.6394) | (2.0566) |
| IEDS-COR(COR*) | −17.1326*** | −15.8419*** | −4.1908 |
| | (2.6252) | (1.7421) | (1.8360) |
| periodX(COR-COR(COR*)) | 2.5340** | 1.1813*** | 2.5483*** |
| | (1.0694) | (0.2552) | (0.5172) |
| periodX(IEDS-COR(COR*)) | 1.1562** | 0.2795*** | 2.8992 |
| | (0.6656) | (0.0611) | (0.7799) |
| periodXperiodX(COR-COR(COR*)) | −0.1258 | | −0.1141*** |
| | (0.1100) | | (0.0346) |
| periodXperiodX(IEDS-COR(COR*)) | −0.1015 | | −0.1433* |
| | (0.1099) | | (0.0329) |
| $\rho$ | 0.4450 | 0.4432 | 0.3065 |

Table 10: Panel Data Logit Regression $^{***}$ 1%,$^{**}$ 5%,$^{*}$ 10%

| | L in period 1 (6) | M in period 1 (6) | | L in period 1 (6) | M in period 1 (6) |
|---|---|---|---|---|---|
| COR | 0 | 0.04 | COR* | 0.62 | 0.08 |
| COR-COR | 0.16 | **0.25** | COR-COR* | 0.55 | **0.29** |
| IEDS-COR | 0.13 | 0.08 | IEDS-COR* | 0.88 | 0.00 |

Table 11: M and L choices in the first period of the 3x3 Coordination games.

The fact that there are also more L-choices in COR-COR compared to COR is only weakly significant (Mann-Whitney, $p = 0.0767$) and could be due to the fact that one of the groups in COR-COR did coordinate on low guesses of around 10 in the guessing game. If we correlate a binary variable indicating whether participants chose M in the first period of the 3x3 game (in treatment COR-COR) with the last guess participants make in the guessing game, we find some support for the conjecture that some participants may try to use the guessing game as a Coordination device. (Spearman, $\rho = 0.3234^{**}$).

## 5 Discussion and Conclusions

We conduct an experiment to study whether and how people extrapolate between very different, but strategically similar games. We found clear evidence that extrapolation does occur between games. Playing a strategically *different* game hurts convergence to Nash equilibrium, while playing a strategically *similar* game before leads to better (faster) convergence in the second game.

Our analysis allows us to conclude that (apart from learning about the population and about succesful actions) participants are also able to learn about the structural properties of the underlying game. This structural learning is particularly relevant for extrapolation and seems to be the main type of knowledge people transfer to other games. This effect seems robust and is true for both IEDS-games as well as for Coordination games.

There is one fundamental difference, though, in the impact of extrapolation on equilibrium play for games which can be solved through deliberative reasoning, such as dominance solvable games, and games for which some intuition is also needed in order to converge to a Nash equilibrium, such as coordination games.

While structural learning is important for both, there is an additional strong effect of "naive extrapolation" of action labels in Coordination games. In these games players try to use the modal behavior in the first game as a Coordination device for the second game. (Of course this is only possible if action labels are the same or similar). Hence any previous experience acquired by the experimental subjects through the guessing game increases strategic uncertainty for the players by introducing competing focal points. This occurs irrespectively of whether the experience that subjects have acquired is in a (potentially useful) strategically similar game, or a dissimilar one.

Such consideration leads us to a more general discussion on what could ideally be learned and extrapolated in games. Within the class of dominance solvable game it is clear that the skill that subjects should ideally take from one game to the next is the ability to identify the (unique) equilibrium through the process of iterated deletion of dominated strategies. In the coordination games what seems more difficult is to successfully coordinate on one of the equilibria. Here any shared (or individual) experience can be used as a focal point. We conjecture that in larger games than those studied here extrapolation of action labels may be crucial to achieve succesful coordination.

These results can be very helpful to make predictions about outcomes or design mechanisms in applications. Some examples are organization design or the introduction of new laws and regulations in generally. Understanding how knowledge is transfered between games and how this depends on the type of strategic situation faced can also inform theoretical models of learning across games and categorization. Most of the current literature on categorization or learning across games focuses on what we have called 'naive extrapolation'.[12] Understanding structural learning and improved learning via extrapolation of strategic context seems harder to model and to understand. There is large scope for future research to understand these effects.

## References


[1] Bush, R.R. and F. Mosteller (1951): A mathematical model for simple learning, Psychological Review, 58, 313-323.

[2] Camerer, C., T. Ho and K. Weigelt (1998), Iterated Dominance and Iterated Best Response in Experimental "p-Beauty Contests", American Economic Review, 88(4), 947-969.

[3] Chong, J-K., C. Camerer and T. Ho (2006), A Learning-based Model of Repeated Games with Incomplete Information, Games and Economic Behavior, 55(2), 340-371

[4] Cooper, D. J. and J. H. Kagel (2003), Lessons Learned: Generalizing Learning across Games, American Economic Review, 93(2), 202-207

[5] Cooper, D. J. and J. H. Kagel (2008), Learning and Transfer in Signaling Games, Economic Theory, 34, 415-439

[6] Fudenberg, D. and D.K. Levine (1998), The Theory of Learning in Games, Cambridge: MIT-Press.

[7] Gilboa, I. and D. Schmeidler (1995), Case-Based Decision Theory, The Quarterly Journal of Economics, 110(3), 605-639.

[8] Gneezy, U., A. Rustichini and A. Vostroknutov (2010), Experience and Insight in the Race Game, Journal of Economic Behavior and Organization, forthcoming.

[9] Grimm, V. and F. Mengel (2009), An Experiment on Learning in a Multiple Games Environment, Meteor Research Memorandum Maastricht University.

[10] Grosskopf, B. and R. Nagel (2007), The Two-Person Beauty Contest, Games and Economic Behavior, 62 (1), 2008, pp. 93 - 99.

[11] Grosskopf, B. and R. Nagel (2009), Unraveling in Two-Person Guessing Games, working paper.

[12] Grosskopf, B., R. Sarin and E. Watson (2010), An Experiment on Case-Based Decision Theory, Games and Economic Behavior, forthcoming.

[13] Haruvy, E. and D. Stahl (2009), Learning Transference between dissimilar symmetric normal-form games, working paper.

[14] Huck, S., P. Jehiel and T. Rutter (2007), Learning Spillover and Analogy Based Expectations: A Multi Game Experiment, working paper UCL.

[15] Jehiel, P.(2005), Analogy-based expectation equilibrium, Journal of Economic Theory 123, 81-104.


[12] See e.g. Steiner and Stewart, 2008 or Mengel, 2007.

[16] Keynes, J.M. (1936), The general theory of interest, employment and money. London: Macmillan.

[17] Kuo, W.J., T. Sjoestrom, Y.P. Chen, Y.H. Wang and C.Y. Huang (2009), "Intuition and Deliberation: Two Systems for Strategizing in the Brain", Science 324, 519-522.

[18] Manski, C.F. (2002), Identification of decision rules in experiments on simple games of proposal and response, European Economic Review 46, 880-891.

[19] Mengel, F. (2007), Learning Across Games, IVIE working paper.

[20] Nagel, R. (1995), Unraveling in Guessing Games: An Experimental Study, American Economic Review 85 1313-1326.

[21] Rankin, F., J. B. van Huyck and R. Battalio (2000), Strategic Similarity and Emergent Conventions: Evidence from Similar Stag Hunt Games, Games and Economic Behavior 32, 315-337.

[22] Rapoport, A., A. Seale and E. Winter (2000), An experimental study of coordination and learning in iterated two-market entry games, Economic Theory 16 (3), 1432-1479.

[23] Roth, A. and I. Erev (1995): Learning in extensive-form games: Experimental data and simple dynamic models in the intermediate term, Games and Economic Behavior, 8, 164-212.

[24] Selten, R., K.Abbink, J.Buchta and A.Sadrieh (2003), How to play $3 \times 3$ games. A strategy method experiment, Games and Economic Behavior 45, 19-37.

[25] Steiner, J. and C. Stewart (2008), Contagion Through Learning, Theoretical Economics 3, 431-458.

[26] Weber, R.A. (2003), Learning and transfer of learning with no feedback: An experimental test across games, working paper Carnegie Mellon University.

# A Additional Regression Tables

| guess | 70% version | 100% versiom |
|---|---|---|
| constant | 48.283*** | 51.066*** |
| | (2.95) | (2.73) |
| period | −9.910*** | −3.820* |
| | (2.11) | (1.88) |
| period*period | 0.4176 | 0.3799 |
| | (0.3465) | (0.3055) |
| $\rho$ | 0.3673 | 0.5280 |

Table 12: Panel data OLS regression of Guesses.

# B Sample Instructions T3

Welcome and thank you for participating to this experiment.

This is an experiment in the economics of decision making. The instructions are simple. If you follow them closely and make appropriate decisions, you may make an appreciable amount of money that will be paid to you, in cash, at the end of the experiment. For your participation you will receive 2 Euros. Further winnings in the experiment depend on your decisions and that of others as well as random events.

There are two phases to this experiment. Detailed instructions for the second phase will be distributed to you after the first phase has been completed.

**Instructions for the first phase**

- In this first phase you will play a simple guessing game together with 3 other participants in the experiment.
- All participants simultaneously guess a number between 0 and 100 (inclusive).
- In order to win, try to guess the number closest to 70 percent of the average of all numbers guessed by the participants in your group (including yourself). The guess closest to this number, i.e. the guess closest to
$$0.7 \times \frac{\text{the summation of all guesses}}{4}$$
wins. If people tie for the closest guess, the winner will be selected randomly from amongst those people.
- The winner receives 5 euros.
- You will play this guessing game 5 times, always with the same participants.
- After each period we will inform you about the different guesses made and whether you won or not.

- The winnings for the first phase of this experiment are equal to the sum of the winnings you may have made in the 5 rounds of the guessing game.

If you have any questions about these instructions or the experiment, then please raise your hand.

Enjoy!

**Instructions for the second phase**

- In this second phase you will likely be paired up with participants who you have *not* played with in phase 1. There are ten rounds in this second phase and in each round you will be randomly rematched with a new participant.
- In this phase you have to make one out of three possible bids, labeled High, Medium and Low.
- Your winnings in each round depend both on your bid and on your partner's bid.
- The choices that both you and your partner make are blind: you will have to choose your bid without knowing what your partner is choosing; similarly, your partner will choose his bid without knowing what you are choosing.
- However both you and your partner are fully aware of the consequences in terms of winnings for both of each combination of bids. The winnings from each combination of bids by yourself and your partner are summarised in the table below. Winnings are stated in ECU (Experimental Currency Units).

| | your partner bids *High* | your partner bids *Medium* | your partner bids *Low* |
|---|---|---|---|
| you bid *High* | you win 10<br>your partner wins 10 | you win 8<br>your partner wins 12 | you win 16<br>your partner wins 8 |
| you bid *Medium* | you win 12<br>your partner wins 8 | you win 10<br>your partner wins 10 | you win 6<br>your partner wins 6 |
| you bid *Low* | you win 8<br>your partner wins 16 | you win 6<br>your partner wins 6 | you win 10<br>your partner wins 10 |

- After each period we will let you know which action your interaction partner in that period has chosen and how much money you earned.
- Your winnings are equal to the sum of your winnings and converted into Euros according to the exchange rate 1 Euro=20ECU.

If you have any questions about these instructions or the experiment, then please raise your hand.

Enjoy!