Incomplete contracts and the core

Summary

This thesis presents a study of the cooperative solution concept known as the core in several game–theoretic and economic contexts. Two principal topics covered in the thesis are: (I) Payoff–dependent balancedness and the core, and (II) Sequential core concepts. The thesis is correspondingly divided into two parts. Two parts of the thesis differ in the subject matter as well as in the methodology of research. Part I should be seen as a relatively technical part of the thesis. Part II is intended as an introduction to the dynamic cooperative game theory, a new discipline that receives increasing attention in the literature. The challenges in the second part have a conceptual rather than technical nature.

Chapter 2 of the thesis elaborates on the classical problem of the non–emptiness of the core in games with non–transferable utility. In the class of games with non–transferable utility sufficient conditions for the non–emptiness of the core, known as balancedness conditions, were given in Scarf [30] and in Billera [4]. These conditions are known to be excessively restrictive, in the sense that there exist games having a non–empty core that do not satisfy Scarf's, or even a weaker Billera's balancedness conditions.

Chapter 2 closes a gap between necessary and sufficient conditions for the non–emptiness of the core. We introduce a new condition for the non–emptiness of the core, the so–called Π–balancedness condition, as a natural generalization of the balancedness condition of Billera [4]. The main result of Chapter 2 states that Π–balancedness is a necessary and sufficient condition for the non–emptiness of the core in a general non–transferable utility game. Chapter 2 also contains a discussion of the concept of Ψ–balanced core introduced recently in Bonnisseau and Iehlé [8].

Chapter 3 presents an application of the idea of payoff–dependent balancedness. We give a new proof of the non–emptiness of the fuzzy core in a pure exchange economy with finitely many agents. The proof reveals and exploits an interesting connection between the fuzzy core of a pure exchange economy and the Ψ–balanced core of a game with non–transferable utility.

The traditional cooperative game theory considers the core along with other solution concepts in static and deterministic contexts such as games with non–transferable utility and one–period economies. The cooperation that takes place in dynamic and uncertain environments is a subject of a new line of research that attracts increasing attention of the scholars. Thus, a number of core concepts has recently been proposed in the literature for specific game–theoretic and economic situations where time and uncertainty play an

essential role. Part II of this thesis seeks to extend the concept of the core to the class of economies with time, uncertainty, and incomplete set of assets. We consider two concepts of the sequential core for dynamic economies: the strong sequential core and the weak sequential core. Chapters 4 and 5 treat the strong and the weak sequential core, respectively, in the context of a two–period economy with assets. Chapter 6 considers the strong sequential core in the framework of a stationary exchange economy. Chapter 7 analyzes the strong sequential core in a stationary cooperative game.

To define the strong and the weak sequential core in a two–period economy with assets, we distinguish between the coalitional deviations that occur in period $t = 0$ and in period $t = 1$. The strong sequential core is a refinement of the classical core. It selects only those classical core allocations that are robust to coalitional deviations in period $t = 1$, in the sense that there is a trade in assets such that the allocation at hand belongs to the core of the induced (static) subeconomy in each state. The central idea behind the concept of the weak sequential core is that of a credible deviation. A period $t = 0$ deviation by a certain coalition is considered credible if it is robust to deviations by subcoalitions in period $t = 1$. The weak sequential core consists of those allocations that are stable to deviations in period $t = 1$ and at the same time are robust against all credible deviations in period $t = 0$.

The formal problems addressed in Part II of the thesis are the following: (1) to assess the comparative statics of the strong and the weak sequential core with respect to the asset structure in the two–period economy, and (2) to investigate the non–emptiness properties of the strong and the weak sequential core.

The strong sequential core is weakly increasing in the number of assets. When the there is a strongly complete set of assets in the economy, the strong sequential core coincides with the classical core. Apart from the special case of a strongly complete asset structure, however, the non–emptiness of the strong sequential core in the two–period economy is *difficult to guarantee. In particular, we show that in the finance economy the strong* sequential core is generically empty, provided that the difference between the number of states and the number of assets is greater than one.

The weak sequential core may be empty even in the economy with no assets and three players all having identical state–independent elementary utility function. Furthermore, the comparative statics of the weak sequential core is complicated. Thus, we give an example of an economy where the weak sequential core is non–empty when no assets are present, and is empty when an appropriately specified asset is introduced into the economy.

Positive results on the non–emptiness of the strong sequential core are obtained in the framework of a stationary economy. We show that the strong sequential core of a stationary economy is non–empty, provided that the discount factor is sufficiently close to one. An analogous result can be obtained for a stationary cooperative game as well: the strong sequential core of a stationary cooperative game is non–empty, provided that (1) the instantaneous NTU–games in all states satisfy an additive π–balancedness condition, (2) at least one of these NTU–games satisfies a strong additive π–balancedness condition, and (3) the discount factor is sufficiently close to one.

The above findings indicate that neither the strong sequential core nor the weak se-

quential core is an entirely coherent solution concept. Nevertheless we believe that the analysis undertaken in this thesis is valuable as an important step towards a general and a fully consistent concept of the sequential core.