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ICT-Investment, Knowledge Accumulation and Endogenous Growth

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Abstract

In this paper we present an endogenous growth model based on Lucas (1988). We have extended the Lucas model by incorporating ICT-capital next to human capital. We take account of spillovers from ICT use in human capital formation to final output production. The effects on growth of these spillovers depend very much on whether they are external or completely internalised. We find that welfare is positively affected, the stronger these spillovers are, but also the more these spillovers are internalised. In addition, we find that in the case of limited internalised knowledge spillovers, we may face a multiple equilibria steady state growth situation, that has an inherent tendency to select the non-optimum (high growth) equilibrium in which all types of capital are 'over accumulated', including ICT-capital. This suggests that there is room for policy intervention here, because there exists an 'optimum' value of the knowledge-spillover parameter where both equilibria coincide and over accumulation does not happen.

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1. Introduction

This paper is concerned with investment in ICT, not so much as the 'ultimate' source of growth, but as a factor that, potentially at least, influences growth performance in a significant manner.[1] There are many ways in which ICT-investment may do that. According to Bartelsman and Hinloopen (2000), for instance, an increase in the efficiency of communication and data-processing leads to lower transaction costs as well as lower search costs, and hence to more efficient matching of demand and supply on various markets, including the labour market. With respect to the latter, Harris (1998) stresses the role of the Internet in increasing the (virtual) mobility of (skilled) labour, and so improving the efficiency of production at a more aggregate level. But perhaps more importantly, at least from the point of view of integrating ICT-investment in an endogenous growth model as we intend to do, Bartelsman and Hinloopen (2000) point to the possibility that the use of ICT-equipment may actually increase the marginal productivity of knowledge workers, and so extend the 'growth base' of the economy. They also claim that for the Netherlands, at the micro-level at least, the use of ICT-raises total factor productivity (Bartelsman et al. (1998)), while TFP measurements in the US (Oliner and Sichel (2000)) indicate that the contribution of ICT-investment to output growth has indeed increased especially during the second half of the Nineties. This contradicts Solow's 'deadly' observation that computers can be seen everywhere except in the productivity data, so that observation must either have been a problem of unrealistic a priori productivity expectations (which Oliner and Sichel (1994) claim to be caused by the fact that up to the early Nineties, ICT-equipment took up only a marginal proportion of total equipment, hence its contribution to TFP should be 'small' on that account[2]), the data themselves (i.e. TFP (mis-) measurement issues), or at least an adjustment problem (cf. Kiley (1999)), since one can hardly imagine that 'virtually' nothing would happen with the productivity data if computers disappeared from the economic scene altogether.[3] But measurement problems (in the sense of measurements not being 'theory-independent' and therefore 'soft') continue to exist, for Jorgenson and Stiroh (1999), also measure the contribution of ICT-investment to output growth in the US, and they claim that the relatively limited contribution (at least in comparison with Oliner and Sichel (2000)) is just a consequence of the substitution of relatively cheap computers for other factors of production, (thus lowering the 'input'-content of output, and so raising total factor productivity) rather than ICT-investment influencing total factor productivity growth directly through shifts of the aggregate production function. Oddly enough, Jorgenson and Stiroh also claim that '… The resolution of the Solow paradox is that computer-related gains, large returns to the production and use of computers, and network effects are fundamentally changing the U.S. economy. However, they

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[1] After all, since the advent of endogenous growth theory we 'know' that the sole source of growth is knowledge accumulation.

[2] Interestingly enough, this directly contradicts Solow's notion that computers are indeed everywhere.

[3] One just has to think of the recent breakdowns of a number of computers used to manage the Dutch railway system, and the problems this created for people trying to reach their workplace.

are not ushering in a period of faster growth of output and total factor productivity.' (Jorgenson and Stiroh (1999, p. 114)). It seems to us that it is at least debatable whether, despite those 'fundamental economic changes', the US aggregate production function would still not shift, so that their finding of a relatively low contribution of ICT-investment to growth might be caused first and foremost by an a priori assumption of the 'non-production function-shifting' character of ICT-investment (hence a 'soft mismeasurement'?)[4].

In this paper we set-up a growth model that allows for investment in ICT-capital as a separate factor of production in final output generation (in order to capture (in principle at least) substitution effects a la Jorgenson and Stiroh (1999)), but also in human capital formation (in order to capture the marginal labour productivity effects for 'knowledge workers' a la Bartelsman en Hinloopen (2000)). By making this choice, we neglect the growth effects of ICT-investment through its (potential) impact on the variety of products[5]. Nor do we explicitly consider the growth effects of more efficient communication that allows firms to concentrate on their core-business and so lower fixed costs (de Groot (2001)). Moreover, we do not pay attention to the scope of ICT as a general-purpose technology (Helpman (1998)) that is able to create new 'markets' and the corresponding products and services. Instead, given the ICT-related empirical doubts about Solow's paradox mentioned above, we would like to define a neo-classical endogenous growth setting based on Lucas (1988), that actually incorporates the computers that most of us (start) see(-ing) everywhere….

In the extended Lucas model that will be presented below, we will start from the notion that ICT-((including hardware and complementary software) can be used as a productive input in final output generation, but also in the generation of knowledge that, in turn, is of direct use for the production of final output. The *first* reason to include the latter is that these days, in the Netherlands at least, the government tries to improve the 'computer-literacy' of the population by stimulating the use of computers and modern modes of communication (the Internet in particular) at various school-levels, even in primary education. Apparently the Dutch government spends the corresponding resources in the firm believe that (eventually) computer literacy will turn up in the productivity data. The *second* reason is, of course, that access to information has improved considerably, for instance through the Internet, but communication possibilities themselves have grown too, thus potentially improving the overall productivity of the learning process as well as the economic transformation process in general.

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[4] Obviously, in this setting 'soft' mismeasurements relate to the use of models that are principally different from the neo-classical aggregate production function growth model used by Solow (1956) himself.

[5] If, for instance, in the context of the Romer (1990) model, the marginal productivity of knowledge workers in product innovation would be positively affected by ICT-investment, there would be a direct impact of ICT-investment on growth in a love-of-variety setting. Note, however, that ICT-itself also gives rise to new products directly. The latter aspects of ICT-could be integrated with a GPT approach, as described (in general terms) in Helpman (1998). We leave this for future research.

We have chosen the Lucas (1988) model[6] as a starting point for our analysis, rather than Romer (1990) or the AK-model as in Barro (1990), for four main reasons. *First*, in the Lucas model knowledge accumulation is still at the heart of the growth process, and so, ICT-investment, by altering the productivity of the knowledge accumulation process, could then directly but partly determine growth performance. *Secondly*, the Cob-Douglas production function used by Lucas, and containing just physical capital and human capital as arguments, can easily be extended by allowing (effective-) capital to consist of both 'ordinary' physical capital and ICT-capital. By doing so, ICT has two different uses, one that could generate proper growth, and one that accommodates growth. *Third*, Lucas has focussed in part on the growth-repercussions of knowledge spillovers, and we would like to do the same with respect to 'computer-literacy' spillovers that flow from knowledge accumulation to final output production. By allowing for such spillovers, that can either be completely external from the point of view of decision makers or that may be completely internalised by these decision makers, we may see whether these provide some scope for policy intervention by raising the 'effective computer literacy spillover potential' of formal education, for instance. *Fourth*, the Lucas model is a relatively straightforward extension of the Solow (1956) growth framework, that is essentially the same as the growth accounting framework in Solow (1957). But, as far as we know, an extension of the Lucas model to incorporate the growth effects of ICT-investment does as yet not exist.

The remainder of this paper is organised as follows. In section 2 we present the extended Lucas (1988) model, while section 3 is devoted to discussing the steady state growth results with and without knowledge spillovers. Finally, section 4 contains some concluding remarks.

2. The Extended Lucas Model

Consumer Utility

Total consumer utility of a representative consumer consists of the present value of an infinitely long stream of consumption of final outputs, as given by the standard *C*onstant *I*ntertemporal *E*lasticity of *S*ubstitution (*CIES*) utility function:

$$U = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta}\, dt \tag{1}$$

In equation (1), *U* represents total utility, *C(t)* is the flow of consumption at time t, while ρ is the rate of discount, and $1/\theta$ is the elasticity of substitution between flows of consumption at different points in time.

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[6] We also borrow elements from Rebelo (1991) further discussed in Barro and Sala-I-Martin (1995), in that we allow other factors than just human capital to be used in knowledge generation. Thus we arrive at a tow-sector,

Final Output Generation

Output *Y* is produced using three different inputs, i.e. physical capital K_y, ICT-capital K_{iy} and human capital services $h.L_y$ where L_y is labour measured in physical units and used in final output production. *h* is an index of the human capital content of a physical unit of labour, further referred to as knowledge per worker. Like Lucas, we use a linear homogeneous Cobb-Douglas production function:

$$Y = A(h.L_y)^{\alpha}(K_{iy})^{\beta_y}(K_y)^{1-\alpha-\beta_y} \tag{2.A}$$

It should be noted that *A* in equation (2.A) represents total factor productivity. α and β_y are constant parameters reflecting the partial output elasticities of labour (measured in efficiency units) and ICT-capital used in final output production.

Knowledge Accumulation

Like Lucas (1988), we assume that the growth rate of knowledge accumulation is proportional to the time spent on schooling:

$$\dot{h} = \delta_h' .\ h \cdot L_h / L \tag{3.A}$$

where a dot over a variable denotes the time-derivative of that variable. Moreover, *L* is the total labour force measured in man-years per year and $L_h = L - L_y$ is the total number of man-years per year spent accumulating knowledge. δ_h' is the productivity of the knowledge accumulation process. Equation (3.A) states that knowledge per person will grow with the relative amount of time spent per person on accumulating knowledge.

Linking ICT-and Productivity

There are two obvious spots in the model where ICT-investments can be linked directly to productivity developments, i.e. the total factor productivity parameter *A* in final output generation, and δ_h', i.e. the productivity of the knowledge accumulation process. In final output generation, the direct impact of ICT-investment is taken into account through $\beta_y > 0$. The potential knowledge spillovers are linked to the parameter *A*, which we reformulate as:

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multi-factor growth model.

$$A = (K_{ih} / (h.L_h))^{\sigma} \tag{4.A}$$

where K_{ih} is the amount of ICT-capital (both hardware and software) used in knowledge accumulation. σ is a constant spillover parameter. If σ =0 there are no spillovers from knowledge accumulation to the productivity of the final output sector. Equation (4.A) states that for $\sigma > 0$ the level of total factor productivity depends positively on the amount of ICT-capital services consumed per efficiency unit of labour time used in accumulating knowledge. One way to view this positive relation is that using ICT-capital while accumulating knowledge generates experience that can be used to handle the ICT-capital stock in the final output sector more productively. One could actually implement this more directly by postulating ICT-capital augmenting 'technical change', and in turn linking the associated 'augmentation factor' to ICT-capital use in knowledge accumulation. This notion of ICT-augmentation is easy to implement, because we use a Cobb-Douglas production function, and so we only need to change σ in (4.A) into $\sigma \cdot \beta_y$. This re-specification of *A* also has the advantage that if $\beta_y = 0$, spillovers have zero productivity effects in final output production, which seems to be a logical requirement, for, generally speaking, if one doesn't use computers, computer-literacy won't matter.

For δ_h' we postulate, very much as above:

$$\delta_h' = \delta_h \cdot (K_{ih} / (h.L_h))^{\beta h} = \delta_h \cdot \mu_{ih} \tag{4.B}$$

where δ_h and β_h are constant positive parameters. Moreover, μ_{ih} is a direct indicator of the ICT-capital intensity of the knowledge accumulation process, and it is implicitly defined by the equivalence between the right most part of (4.B) and its middle part. Substitution of (4.A) and (4.B) into (2.A) and (3.A), respectively, while taking the 'ICT-augmentation' approach outlined above, gives rise to equations (2.B) and (3.B):

$$Y = A(h.(L - L_h))^{\alpha} ([K_{ih} / (h.Lh)]^{\sigma} K_{iy})^{\beta_y} (K_y)^{1-\alpha-\beta_y} \tag{2.B}$$

$$\hat{h} = \delta_h (K_{ih} / (h.L_h))^{\beta_h} \cdot L_h / L \tag{3.B}$$

where a 'hat' over a variable name denotes its instantaneous proportional rate of growth.

The Macro-Economic Budget Constraint

For reasons of simplicity, we assume that the various types of capital are made up, one-for-one, out of consumption foregone.[7] Furthermore, we ignore depreciation in order to keep the model as simple as possible. We therefore have:

$$\dot{K}_y = v_y.(Y - C) \tag{5.A}$$

$$\dot{K}_{iy} = v_{iy}.(Y - C) \tag{5.B}$$

$$\dot{K}_{ih} = (1 - v_y - v_{iy}).(Y - C) \tag{5.C}$$

where v_y and v_{iy} are the volume shares of investment in physical capital and investment in ICT-capital by the final output sector in total investment (including investment in ICT-capital by the knowledge generating sector), and where $v_y, v_{iy} \geq 0$ and $v_y + v_{iy} \leq 1$. *Y*-*C* is simply total final output not used for consumption purposes, hence used for investment purposes.

3. Steady State Results

Introduction

The model so far, is too complicated to obtain a closed form solution, except for the cases of no knowledge spillovers, external knowledge spillovers and steady state growth. For the remainder of this paper we will focus on steady state growth situations, although we will cover both zero and non-zero knowledge spillovers. Furthermore, we will consider three different cases: the no-spillover case, the external-spillover case and the internalised spillover case. The difference between ‘external spillovers’ and ‘internalised spillovers’ is that with ‘internalised spillovers’ the beneficial effects of ICT-investment in knowledge accumulation for the final output sector is taken into account from the start, whereas these effects are completely ignored, at least with respect to making resource allocation decisions, in the case of external spill-overs.[8]

Following Lucas (1988), we will now turn to the central planner solution of the model described in section 2. In the following sub-paragraphs of this section, we will only cover the ‘external spillovers’ and ‘internalised spillovers’ cases explicitly, since the ‘no-spillovers’ case is a special case

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[7] By doing so, we neglect a further source of growth, namely productivity improvements in the ICT-producing sector itself. But in order to take this into account we would have to revert to a three-sector model, rather than the two-sector model we have here. At this exploratory stage, we are more interested in the direct growth effects of ICT-investment, than the ‘second order’ growth effects through induced ICT-substitution that Jorgenson and Stiroh (1999) focus on.

[8] We implement this by treating the ICT-capital intensity of knowledge accumulation as if it were an exogenously given number in deriving the first order conditions (further called FOCs) of the Hamiltonian problem. Then this number is replaced again by its definition in terms of the variables of the system, and the consequences of these revised FOCs for steady state growth are evaluated.

of both the ‘external spillovers’ and the ‘internalised spillovers’ cases for $\sigma = 0$. The mathematical results presented below mainly refer to ‘internalised spillovers’ case, but, where relevant, we indicate how the corresponding equations for the ‘external spillovers’ and ‘no spillovers’ cases can be obtained from their ‘internalised spillovers’ counterparts, since this involves relatively simple and straightforward operations.

The Hamiltonian Approach

The Hamiltonian function with state variables *h*, K_y, K_{iy} and K_{ih}, and control variables *C*, L_y, v_y, v_{iy} is now given by:

$$\begin{aligned} H = {} & e^{-\rho t}(C^{1-\theta}-1)/(1-\theta)+\lambda_h \cdot \dot{h}+\lambda_{Ky}.\dot{K}_y+\lambda_{Kiy}.\dot{K}_{iy}+\lambda_{Kih}.\dot{K}_{ih} = \\ & e^{-\rho t}(C^{1-\theta}-1)/(1-\theta)+\lambda_h(\delta_h.(K_{ih}/(h.(L-Ly))^{\beta h}.((L-Ly)/L).h+ \\ & \lambda_{Ky}.v_y.((K_{ih}/(h.(L-Ly))^{\beta_y\sigma}(h.L_y)^{\alpha}(K_{iy})^{\beta y}(K_y)^{1-\alpha-\beta y}-C)+ \\ & \lambda_{Kiy}.v_{iy}.((K_{ih}/(h.(L-Ly))^{\beta_y\sigma}(h.L_y)^{\alpha}(K_{iy})^{\beta y}(K_y)^{1-\alpha-\beta y}-C)+ \\ & \lambda_{Kih}.(1-v_y-v_{iy}).((K_{ih}/(h.(L-Ly))^{\beta_y\sigma}(h.L_y)^{\alpha}(K_{iy})^{\beta y}(K_y)^{1-\alpha-\beta y}-C) \end{aligned} \tag{6}$$

where $\lambda_{Ky}, \lambda_{Kiy}, \lambda_{ihy}$ and λ_h are the corresponding co-state variables, and where we have used equations (2.B), (3.B) and (5.A)-(5.C) to arrive at (6).

It is easy, although somewhat tedious, to show that by setting the derivatives of the Hamiltonian w.r.t. the control variables equal to zero, one obtains:

$$\lambda_{Ky} = e^{-\rho t}C^{-\theta} \tag{7.A}$$

$$\lambda_{Ky} = \lambda_{Kiy} = \lambda_{Kih} \tag{7.B}$$

$$\begin{aligned} \lambda_h = {} & \lambda_{Ky} h^{\alpha+\beta_h-\sigma\beta_y-1} K_{ih}{}^{\beta_y\sigma-\beta_h} \cdot K_{iy}{}^{\beta_y} \cdot K_y{}^{1-\alpha-\beta_y} \cdot L(L-L_y)^{\beta_h-1-\sigma\beta_y} \cdot L_y{}^{-1+\alpha} \cdot \\ & (L\alpha + L_y(\sigma\beta_y-\alpha))/((1-\beta_h)\delta_h) \end{aligned} \tag{7.C}$$

where (7.C) refers to the ‘internalised spillovers’ case. The corresponding equation for the ‘external spillovers’ case can easily be obtained by dropping the term $L_y\sigma\beta_y$ from the numerator of (7.C), without changing anything else.

The right hand side of equation (7.A) is equal to marginal utility, i.e. the utility that would be lost if one would allocate one unit of output to capital formation instead of consumption. So, equation (7.A) states that an optimum path can not be improved upon by shifting output from consumption to investment, or vice versa, because of the interpretation of the co-state variables as the shadow prices of the corresponding state-variables. Equation (7.B) states that a marginal unit of investment in the three different capital stocks should generate the same return; otherwise total utility could be improved by shifting capital from the low return investment opportunities to the high return investment opportunities. Equation (7.C) states that on an optimum path, the two uses of labour hours (i.e. production of final output, and production of (more) future final output through knowledge accumulation) should generate the same return, in terms of total utility.

The shares of investment in the three different capital stocks can now be obtained by combining the dynamic constraints:

$$\dot{\lambda}_j = -\frac{\partial H}{\partial K_j} \quad \forall\, j = y, iy, ih \tag{8}$$

with the time derivatives of equation (7.B). The latter simply imply that de derivatives of the Hamiltonian w.r.t. the various capital stocks should be the same. This results in:

$$K_{iy} = K_y \frac{\beta_y}{1-\alpha-\beta_y} \tag{9.A}$$

$$K_{ih} = K_y \frac{((L-L_y)\alpha\beta_h + L_y\beta_y\sigma)}{L_y(1-\alpha-\beta_y)(1-\beta_h)} \tag{9.B}$$

where (9.B) is associated with the 'internalised spillovers' case and the 'external spillovers' case can be obtained from (9.B) by dropping the term $L_y\sigma\beta_y$ from the numerator of (9.B), without changing anything else.

Equation (9.A) requires the efficient use of ICT-capital and physical capital in final output production. This equation is relevant for all spillover cases considered, since the capital stock ratio given by (9.A) is the ratio that maximises effective capital (i.e. $K^e = K_{iy}{}^{\beta_y} \cdot K_y{}^{1-\alpha-\beta_y}$) for a given budget in terms of consumption foregone. The other capital stock ratio given by (9.B) depends on the allocation of labour over its two different uses. This is because the capital stocks here have totally different functions: in knowledge generation, the IT-capital intensity of learning has a positive impact on the productivity of the learning process, hence on the growth rate of output for a given allocation of labour, while an increase in the capital intensity in final output production only has a level effect. Because changes in the allocation of labour also have a growth effect, it is not surprising that the allocation of labour turns up in the link between both types of capital stocks.[9] Note, moreover, that (9.B) for a given value of L_y implies that the ICT-capital intensity decreases relative to the capital intensity of production in the final output sector, if spillovers are not taken into account, i.e. if $\sigma = 0$.

Because the capital/effective labour ratios remain constant in a situation of steady state growth for a fixed allocation of labour (as it is the case in the Lucas (1988) model), it follows immediately from equations (9.A) and (9.B) that the growth rates of the various capital stocks are the same too.[10] This result can be used in combination with equations (5.A)-(5.C) to obtain the values of the volume shares of ICT- and non-ICT-investment, as given by equations (10.A) and (10.B) below:

$$v_{iy} = \frac{L_y(1-\beta_h)\beta_y}{L\alpha\beta_h + L_y(1-\alpha-\beta_h+\beta_y\sigma)} \tag{10.A}$$

$$v_y = \frac{L_y(1-\beta_h)(1-\alpha-\beta_y)}{L\alpha\beta_h + L_y(1-\alpha-\beta_h+\beta_y\sigma)} \tag{10.B}$$

Both equations (10.A) and (10.B) are evaluated for the 'internalised spillovers' case. The only difference with the 'external spillovers' case is that if spillovers are completely external, the term $\beta_y\sigma$ drops out of the respective denominators, indicating that for given L_y the investment share of ICT-capital in knowledge accumulation will fall, since the latter is the complement of the investment shares given by (10.A) and (10.B). In general, though, the distribution of investment over its three different uses depends on the actual allocation of labour, for the same reasons as before. Note that an increase in L_y by a given percentage, increases the numerators of (10.A) and (10.B) more than the denominators, hence the investment shares of capital used for final output production would rise, and that for knowledge production would fall, as one would expect, since an increase in L_y would raise the marginal productivity of the other factors used in final output production.

Steady state growth requires the marginal productivity of capital to remain constant for an increasing capital stock. Since we have used a linear homogeneous Cobb-Douglas production function, this requires output and capital to grow at the same rate. Together with the conclusion that the various stocks of capital should all grow at the same rate, this leads to the conclusion that output itself (hence the various capital stocks) should grow with the growth rate of *h*. However, a constant growth rate of output that is equal to the growth rate of the capital stock implies that the saving rate is constant too. Therefore, the propensity to consume, being equal to one minus the saving rate, must

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[9] Note that in the absence of spillovers (i.e. $\sigma = 0$), equation (9.B) requires the capital/effective labour ratios in both sectors to be proportional, with factor of proportion $\alpha\beta_h/((1-\alpha-\beta_y)(1-\beta_h))$.

[10] Note that the re-specification of the productivity parameters in equations (4.A) and (4.B) also allows these parameters to be constant in the steady state.

also be constant. But then the growth rate of consumption is equal to the growth rate of output. Consequently, on an equilibrium path the growth rate of consumption is given by equations (7.A) and (8), the latter for $j=y$, giving:

$$\hat{\lambda}_{Ky} = -\frac{\partial Y}{\partial K_y} = -r \tag{11.A}$$

$$\hat{C} = \hat{Y} = \hat{h} = (-\hat{\lambda}_{Ky} - \rho)/\theta = (r - \rho)/\theta \tag{11.B}$$

where r is the real rate of interest, i.e. the marginal productivity of physical (and ICT-) capital in the final output sector. (11.A) and (11.B) hold, irrespective of the spillover regime.

Equation (7.C) as well as the steady state growth requirement, i.e. the equality of the growth rates of output, the various capital stocks and human capital per head (as given by (11.B)), and equation (11.A) can be used to arrive at the conclusion that:

$$\hat{\lambda}_{Ky} = \hat{\lambda}_h = -r \tag{12}$$

where (12) holds irrespective of the spillover regime.

Using the definition $\hat{\lambda}_h = (\hat{\lambda}_h / \hat{h}) \cdot \hat{h}$, where the terms within the round brackets are substituted for by using equations (8) and (3.B), and substituting (12) we get:

$$\hat{h} = r\frac{(L - L_y)\alpha + L_y \beta_y \sigma}{\alpha L(1 - \beta_h)} \tag{13}$$

Equation (13) provides the second relation between $\hat{h}$ and the real interest rate r (equation (11.B) being the first together with the requirement that $\hat{C} = \hat{h}$). Note that for the ‘external spillovers’ case, the term containing the spillover parameter σ drops out of the numerator. By equating (13) and (11.B) one arrives at a relation between r and L_y:

$$r = \frac{L\alpha(1 - \beta_h)\rho}{L\alpha(1 - \beta_h - \theta) + L_y \theta(\alpha - \beta_y \sigma)} \tag{14}$$

For the ‘external spillovers’ case, the term containing σ vanishes, so that for given L_y , the steady state interest rate will be lower than in the ‘internalised spillovers’ case. Together with our findings regarding (13), this implies that, ceteris paribus, the steady state growth rate itself should be lower in the ‘external spillovers’- case than in the ‘internalised spillovers’ case.

Equation (14) can be substituted back into (13) to arrive at a relation between $\hat{h}$ and L_y , which, in combination with (3.B), allows us to arrive at the following relation between μ_{ih} and L_h (where we have substituted $L_h = L - L_y$), which we will use to derive the equilibrium growth rate in the steady state.

$$\mu_{ih} = \frac{L\rho(L_h\alpha + (L - L_h)\beta y\sigma)}{L_h\delta_h(L\alpha(1-\beta_h) - L_h\alpha\theta - (L - L_h)\beta_y\theta\sigma)} \qquad (15.A)$$

For the ‘external spillovers’ case, the terms containing σ all vanish, so that for given L_h the steady state ICT-capital intensity of knowledge accumulation activities will be higher in the ‘internalised spillovers’ case than in the ‘external spillovers’ case. It should be noted that for $\sigma = 0$, equation (15.A) is reduced to:

$$\mu_{ih} = \frac{L\rho}{\delta_h(L((1-\beta_h) - L_h\theta)} \qquad (15.B)$$

Equation (15.B) shows that the graph of μ_{ih} as a function of L_h is upward sloping. Changes in the parameters shift (15.B) around in the μ_{ih} , L_h –plane.

In order to finally obtain the equilibrium growth rate, we must first find an independent relation between the same variables. That relation can be obtained from equation (11.A), by substituting for K_y and K_{iy} in the marginal productivity of capital, while making use of equations (9.A) , (9.B) and (3.B). In that case we obtain a relation between μ_{ih} , r and L_h., which in combination with (14) results in an additional relation between μ_{ih} and L_h.:

$$\mu_{ih} = \left(\frac{L{L_h}^{\alpha}\alpha(1-\beta_h)^{1+\alpha}(1-\alpha-\beta_y)^{-(1-\alpha-\beta_y)}{\beta_y}^{-\beta_y}\rho)}{(L_h\alpha\beta_h + (L - Lh)\beta_y\sigma)^{\alpha}(L\alpha(1-\beta_h) - L_h\alpha\theta - (L - L_h)\beta_y\theta\sigma)} \right)^{\beta_h/(\beta_y\sigma - \alpha)} \qquad (16.A)$$

In order to obtain μ_{ih} for the ‘external spillovers’ case, we just have to drop the terms containing σ in the denominator of (16.A). Again, for $\sigma = 0$, (16.A) is reduced to:

$$\mu_{ih} = \left(\frac{L\alpha^{-\alpha}(1-\beta_h)^{1+\alpha}(1-\alpha-\beta_y)^{-(1-\alpha-\beta_y)}\beta_y{}^{-\beta_y}\rho)}{\beta_h{}^{\alpha}(L(1-\beta_h)-L_h\theta)} \right)^{-\beta_h/\alpha} \tag{16.B}$$

Note that (16.B) allows us to derive the 'external spillovers' case by replacing the exponent $-\beta_h/\alpha$ of the outer brackets in (16.B) by $-\beta_h/(\beta_y\sigma-\alpha)$, thus in fact raising the right hand side of (16.B) to the power $\alpha/(\alpha-\beta_y\sigma)$, which is positive and larger than one for values of $\beta_y,\sigma<\alpha$. The graph of (16.B) for the 'external spillovers' case will therefore be 'above' that associated with the 'no spillovers' case. In addition to this, it should be noted that equation (16.B) represents μ_{ih} as a concave downward sloping function of L_h in the μ_{ih}, L_h –plane, since presumably $\beta_h \le \alpha$. The point of intersection of the graphs given by (15.B) and (16.B) represents the unique equilibrium allocation of labour time in the absence of knowledge spillovers. Unfortunately, the graphs for $\sigma>0$ can not be obtained that easily, because the derivatives of (15.A) and (16.A) with respect to L_h depend in a non-linear fashion on the various system parameters. In addition to this, combining (15.A) and (16.A) does not provide a closed form solution for L_h in terms of the parameters of the growth system for the 'internalised spillovers' case, unless we put $\sigma=0$, although we can obtain a closed-form solution also for the 'external spillovers' case, as we will show below. For the 'internalised spillovers' case, however, we can still approximate (15.A) and (16.A) in the neighbourhood of $\sigma=0$, and so obtain an indication of the local behaviour of the steady state.[11] Equating (15.B) and (16.B) therefore, we get:

$$\overline{L}_h = \frac{L(1-\beta_h)}{\theta}(1-\rho(\alpha^{-\alpha}(1-\beta_h)^{\alpha(\beta_h-1)/\beta_h}\beta_h{}^{-\alpha}(1-\alpha-\beta_y)^{-(1-\alpha-\beta_y)}\beta_y{}^{-\beta_y}\delta_h{}^{-\alpha/\beta_h})^{\beta_h/(\alpha+\beta_h)}) \tag{17.A}$$

where a bar over a variable denotes its steady state value. The corresponding 'external spillovers'-allocation of labour time can again be obtained by replacing the exponent $\beta_h/(\alpha+\beta_h)$ 'closest to' the outer round brackets by $\beta_h/(\alpha+\beta_h-\sigma\beta_y)$, without changing anything else.

Equation (17.A) can be plugged back again into either (16.B) or (15.B), to obtain:

$$\overline{\mu}_{\mathrm{ih}} = (\alpha^{\alpha}(1-\beta_h)^{-(1+\alpha)}\beta_h{}^{\alpha}(1-\alpha-\beta_y)^{1-\alpha-\beta_y}\beta_y{}^{\beta_y}\delta_h{}^{-1})^{\beta_h/(\alpha+\beta_h)} \tag{17.B}$$

---

[11] Moreover, we will show for a priori values of the system parameters what (15.A) and (16.A) actually look like, for values of $\sigma>0$. Although these results are not generally valid, being representations of the characteristics of the steady state in a specific spot of the parameter-space, they do provide the basis for some interesting conclusions later on.

In order to get the steady state value $\overline{\mu}_{ih}$ for the ‘external spillovers’ case, we just have to replace the exponent $\beta_h/(\alpha+\beta_h)$ by $\beta_h/(\alpha+\beta_h-\sigma\beta_y)$, without changing anything else. This implies that for the same parameter values the exponent in the ‘external spillovers’ case is larger than that in the ‘no spillovers’ case, implying that the ICT-capital intensity of production in the external spillover case exceeds that of the ‘no spillovers’ case. Finally, using (17.A), (17.B) and (3.B), we arrive at:

$$\overline{\hat{h}} = ((\alpha^{\alpha}(1-\beta_h)^{\alpha/\beta_h-\alpha}\beta_h{}^{\alpha}(1-\alpha-\beta_y)^{1-\alpha-\beta_y}\beta_y{}^{\beta_y}\delta_h{}^{\alpha/\beta_h})^{\beta_h/(\alpha+\beta_h)}-\rho)/\theta \qquad (17.C)$$

where the steady state growth rate for the ‘external spillovers’ case is easily obtained from (17.C) by replacing the exponent of the term in the inner round brackets, i.e. $\beta_h/(\alpha+\beta_h)$, by $\beta_h/(\alpha+\beta_h-\sigma\beta_y)$ again. By doing that, we note that the steady state growth rate associated with the ‘external spillovers’ case should be larger than that of the ‘no spillovers’ case.

Equation (17.C) looks very much like the Lucas result without any knowledge spillovers. However, except for consumer preferences represented by ρ and θ, and the productivity of knowledge generation δ_h, all the other parameters of the production functions in the final output sector and in the knowledge generating sector now also determine the steady state growth rate[12], although the actual signs of their impact are hard to determine analytically. However, since we have to revert to numerical approximations to the system with $\sigma > 0$, we give an impression of the sensitivity of the solution to changes in the various parameters for a limited number of constellations of the parameters β_y and β_h, since these are the ones most directly associated with ICT-investment.

First, however, we show how the graphs referred to above can be used to obtain the growth rate of the system. The graph for (15.B) is a convex upward sloping function of L_h. It has a positive intercept at $\mu_{ih}=\rho/((1-\beta_h)\delta_h)$, and a vertical asymptote at $L_h=(1-\beta_h)/\theta$. The graph for (16.B) has a positive intercept, and reaches a value of zero also at $L_h=(1-\beta_h)/\theta$, as depicted in the 4-quadrant figure below.

In Figure 1, the Northwest and Southeast quadrants contain a 45-degree line. The Southwest quadrant contains an iso-growth field. Iso-growth lines further away from the origin denote higher rates of growth. An iso-growth line can be obtained by solving equation (3.B) for L_h in terms of μ_{ih}, for given values of $\hat{h}$. The iso-growth line corresponding to a growth rate $\hat{h}'$ would therefore be given by:

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[12] In fact, this result depends on capital (rather than just ICT-capital) being used in knowledge accumulation, as one can easily find out for oneself, by adding capital as an additional factor of production in knowledge accumulation in the original Lucas model.

$$L_h = \hat{h}'/(\delta_h \mu_{ih}) \qquad (18)$$

The point of intersection between (15.B) and (16.B) in the Northeast quadrant then translates into the choice of a specific iso-growth line, hence a corresponding growth-rate. The value of the corresponding growth rate can be obtained by calculating the value of L_h for $\mu_{ih} = 1/\delta_h$ in accordance with (18), in which case we would have $L_h = \hat{h}'$, as shown in Figure 1.

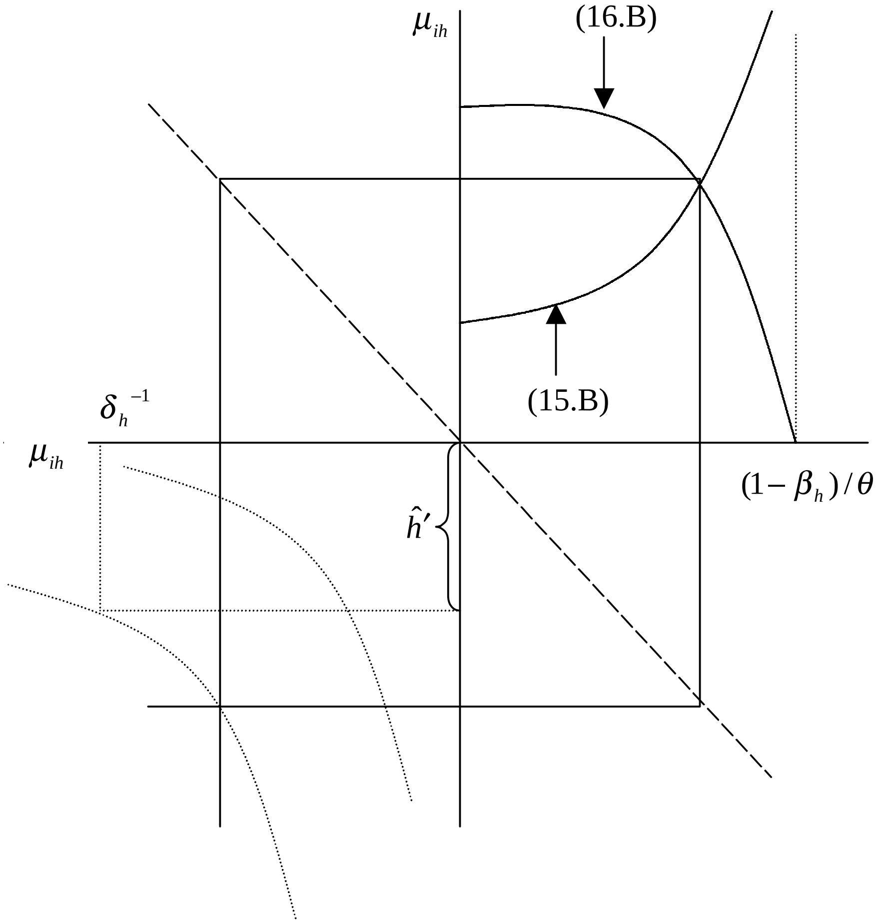

Figure 1. The steady state

From the discussion about the changes that need to be introduced into (15.A) and (16.A) to get from the 'internalised spillovers' case, to the 'external spillovers' case, it is clear that equation (15.B) (that is actually associated with the 'no spillovers' case is the same as the one for the 'external spillovers' case, whereas the graph of (16.B) for the 'external spillovers' case would be above the one depicted in Figure 1, while retaining the same general shape. This implies that the 'rectangle' that links the point of intersection in the Northeast quadrant in the 'external spillovers' case actually contains that of the 'no spillovers' case, and so the corresponding iso-growth curve must lie further from the origin than in the Figure above. Consequently, as we already stated above, both the ICT-capital intensity of knowledge accumulation, the labour time spent on knowledge accumulation and therefore the steady state growth rate[13] in the 'external spillovers' case exceed those in the 'no spillovers' case.

For the internalised spillover case, that does not allow a closed form solution, we notice that for $\sigma > 0$ both the numerator and the denominator of (15.A) rise relative the 'no-spillovers' case, so nothing definite can be said. This also holds regarding (16.A), as one can verify oneself. However, in order to develop a notion of the basic characteristics of the 'internalised spillovers' steady state, we will perform some numerical experiments. In addition to this, we provide some additional information, starting with the 'no-spillovers' case about the sensitivity of the outcomes for changes in the parameters of the model, in the following subparagraphs.

The Basic Parameter Set

The 'basic' parameter set we have chosen, is given in Table 1 below:

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
θ	0.5	β_h	0.2	ρ	0.05	L	1.0
α	0.7	β_y	0.2	δ_h	0.055	-	-

Table 1. The 'Basic' Parameter Set

The parameter values in Table 1 were chosen on a priori grounds, since little or no information is available with respect to these parameters. However, they are close to the values used by Lucas (1988), and so are the outcomes. An exception is the intertemporal elasticity of substitution, which is rather high in comparison with Lucas, but this value ensures the existence of a unique steady state.[14] In addition to this, the Lucas model does of course not contain any investments in ICT, hence the parameters β_h and β_y are chosen on a priori grounds, but in such a way that outcomes are still in line with Lucas' results and broadly in line with Oliner and Sichel (2000). The corresponding steady state values for $\sigma = 0$ are $\bar{\hat{h}} = 0.0188$, . $\bar{L}_h = 0.2534$, and $\bar{\mu}_{ih} = 1.3502$. Note that the allocation of time to knowledge accumulation is roughly in line with our 'back of the envelope' requirement that with an 'active' lifetime of roughly 60 years (divided into roughly 15 years of education, and 45 years of labour services), approximately 25% of a person's time should be spent on knowledge accumulation.

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[13] See equation (3.B).

[14] It ensures that the vertical asymptote in the Northeast quadrant of Figure 1 lies at a value of $L_h > 1$.

Parameter Sensitivity Results: the ‘No Spillovers’ Case

The sensitivity of the steady state outcomes for changes in the parameters in case $\sigma = 0$ can be obtained by calculating the shifts in the graphs of equations (15.A) and (16.B), and then observing what would happen to the allocation of labour time and growth itself. This is relatively straightforward for the ‘growth’ parameters δ_h, ρ, less so for θ, and still less for the other parameters. But we can get an impression of the local parameter sensitivity of the steady state by calculating the numerical values of the parameter-elasticities of the steady state values of $\bar{\hat{h}}$, $\bar{L}_h$ and $\bar{\mu}_{ih}$. These are provided in Table 2 below. We have calculated these elasticities for different values of β_h and β_y, as shown in the table, since first these parameters are the least well ‘known’, while secondly they are directly associated with the impact of ICT-investment itself.

In Table 2 we present the numerical values of the point elasticities of the variables in the second column with respect to the parameters listed in the other columns, for the ‘basic’ parameter set listed in Table 1. The first column of Table 2 indicates the constellation of the parameters β_h and β_y, where constellation 1 is given by $\beta_y = 0.2, \beta_h = 0.2$, constellation 2 is equal to $\beta_y = 0.2, \beta_h = 0.15$, and constellation 3 is given by $\beta_y = 0.15, \beta_h = 0.15$.

There are three main observations to be made from Table 2. *First*, the productivity parameter associated with the knowledge accumulation process has a relatively high growth elasticity, just like the parameters associated with consumer preferences. This is because these parameters determine growth directly, since knowledge accumulation is still the ultimate source of growth, and because growth (in future consumption possibilities) is the (required) reward for abstaining from consumption now to accumulate (ICT-) capital instead. *Secondly,* the growth elasticity with respect to β_y is about an order of magnitude smaller than that of β_h. Again, this reflects the fact that a change in β_h influences the productivity of the knowledge accumulation process directly, while a rise in β_y changes the return to ICT-capital used in final output production, which brings about a reallocation of not only ICT-capital, but also of labour between the final output sector and knowledge accumulation activities. Note however, that because of the linear homogeneity of the production function, a rise in β_y for a given value of the partial output elasticity of labour α, automatically implies a fall in the partial output elasticity of physical capital, which would be a further incentive to increase the ICT-capital content of the effective capital stock, as suggested by equation (9.A). *Third*, the growth elasticity of the partial output elasticity of labour, i.e. α, is high, and it gets higher if the partial output elasticities of ICT-capital decrease, while moving from constellation 1 through 2 to 3. The reason why a rise in α has a negative effect on growth is that for a given allocation of labour, the opportunity cost of shifting labour-time from final output production to knowledge generation, increases.[15] Hence, we will have higher levels of output, but lower growth rates if α increases.

Case	Var	SSV	θ	α	β_h	β_y	δ_h	ρ
1	L_h	0.2535	-0.9909	-0.7394	0.9887	0.1672	4.0957	-5.3127
1	$\hat{h}$	0.0188	-0.9909	-0.8774	1.4790	0.1987	4.9044	-5.3127
1	μ_{ih}	1.3503	0.0000	-0.1390	0.4855	0.0315	-0.2209	0.0000
2	L_h	0.1885	-0.9909	-0.9628	1.0439	0.2004	6.5431	-8.0176
2	$\hat{h}$	0.0125	-0.9909	-1.0816	1.3771	0.2254	7.4198	-8.0176
2	μ_{ih}	1.2029	0.0000	-0.1199	0.3298	0.0250	-0.1754	0.0000
3	L_h	0.1840	-0.9909	-1.3915	1.0576	0.0022	6.7247	-8.2401
3	$\hat{h}$	0.0121	-0.9909	-1.5573	1.3883	0.0024	7.6028	-8.2401
3	μ_{ih}	1.1993	0.0000	-0.1686	0.3273	0.0003	-0.1754	0.0000

Table 2. Parameter Elasticities

Parameter Sensitivity and Policy Relevance

As stated above, Table 2 provides only local results. In order therefore to get an impression of the sensitivity of the steady state outcomes for the parameters ‘further away’ from the ‘basic’-parameter set, we have plotted the value of the steady state growth rate against some parameter for fixed (basic-) values of the other parameters, as given in Table 1 above. The ‘more interesting results’ are provided in Figures 2.A-2.C below. These pertain to parameter elasticities that change sign, as opposed to the other parameters that do not, and that are therefore not shown here. We should stress here, as will become clearer through the results regarding the ‘internalised spillovers’ case, that the spillover parameter σ is also ‘interesting’. We will come back to this further below.

Neither of the ‘more interesting’ parameters shown in the Figures below are ‘policy’ instruments, so the fact that their growth elasticities may change sign, does not have direct policy implications. However, that is probably less the case with the spillover parameter σ. One can envisage a situation, where the ICT-schooling one receives, and the ICT-equipment used during the accumulation of knowledge, corresponds more or less closely to the type of ICT-schooling and -equipment that is used in final output production. In the case of a less than perfect match between

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[15] Also here, the partial output elasticity of physical capital is negatively affected, so, for a given allocation of resources, an increase in α raises the relative marginal product of labour, thus providing an incentive for more labour intensive final output production.

ICT-schooling and -equipment ‘demanded’ and ‘supplied’, it is easy to envisage that σ will be lower than in the case of a better match. Conversely, σ may be increased by actively matching ICT-schooling and -equipment ‘supplied’ and ‘demanded’, thus giving σ, in part at least, the character of a ‘policy instrument’. But rather than stating how many resources would have to be spent changing σ and incorporating that decision in the model, we will look at what a non-zero value of σ under different spillover regimes may mean for the steady state growth rate.

Non Local/Non Global Parameter Sensitivity Results: the ‘No Spillovers’ Case Again

In Figures 2.A-2.C *Gh* represents the growth rate of human capital, which is equal to the growth rate of output in the steady state. From Figures 2.A-C it is clear that the output elasticities of ICT-capital are ‘interesting’, as well as their complement, the partial output elasticity of labour. The reason why growth elasticities may change sign is that the partial output elasticities of ICT-capital, as well as the partial output elasticities of labour have a double role, since their complements measure the partial output elasticities of the other production factors entering the same linear homogeneous production function. A rise in the partial output elasticity of labour therefore, implies a fall in the partial output elasticity of capital. For low values of α, for instance, a rise in α raises the marginal productivity of labour in final output production by a larger proportion than the marginal productivity of physical capital would fall, and so one would expect a reallocation of labour time from knowledge generation to final output production, thus lowering the steady state growth rate. For high values of α a still further rise in α will influence the marginal productivity of labour only slightly but positively, whereas the marginal productivity of capital will fall by a relatively large proportion, leading to higher capital intensity of production in the knowledge generating sector, and so to a rise in labour productivity there and a corresponding inflow of labour, thus raising the growth rate, ceteris paribus.

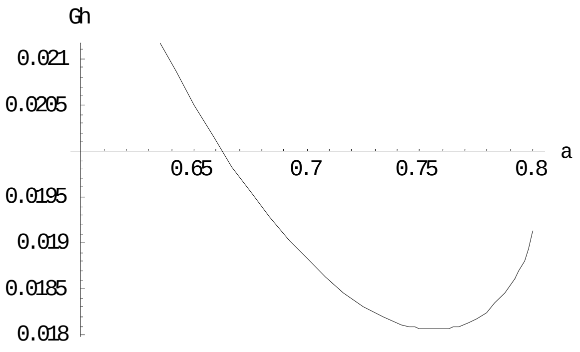

Figure 2.A $\bar{\hat{h}}(\alpha)$

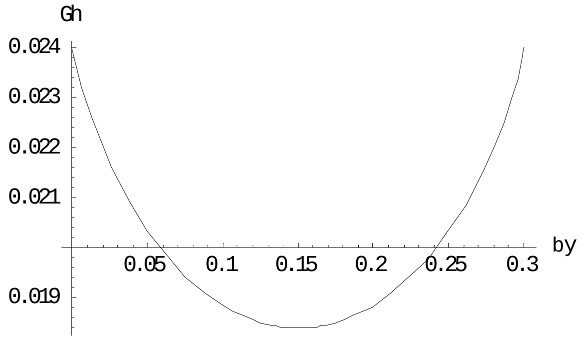

Figure 2.B $\bar{\hat{h}}(\beta_y)$

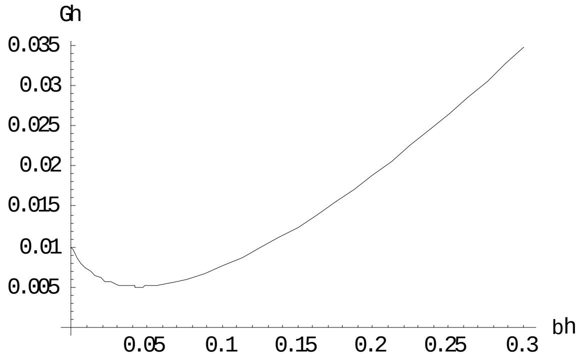

Figure 2.C $\bar{\hat{h}}(\beta_h)$

As stated above, the steady state growth rate depends in a non-linear way on the various partial output elasticities of the production factors concerned. This doesn't make the steady state growth rate variable, though, since one should realise that for constant values of these parameters and for $\sigma = 0$ the steady state growth rate will depend in a unique way on the parameters in question, as shown in Figure 1. For non-constant 'interesting parameters', the possibility of multiple equilibria exists.

Steady State Results: the 'External Spillovers' Case

In order to show how the steady state depends on the spillover parameter σ, we calculate (15.A) and (16.A) as functions of L_h, for different values of σ, and for just the 'basic' parameter set as given by Table 1. The results are plotted in Figures 3.A and 3.B. Figure 3.A contains the ratio of (15.A) and (16.A) as a function of L_h. A value of this ratio equal to 1, therefore ensures that (15.A) and (16.A) are equal, as we should have in the steady state.

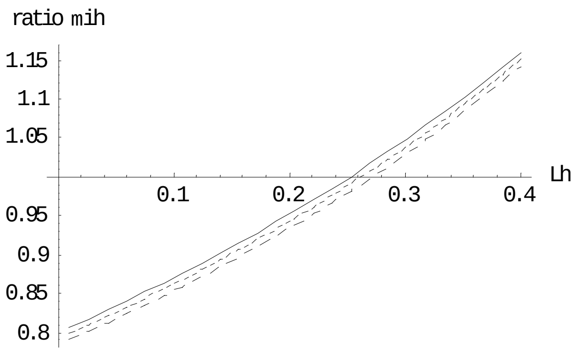

Figure 3.A

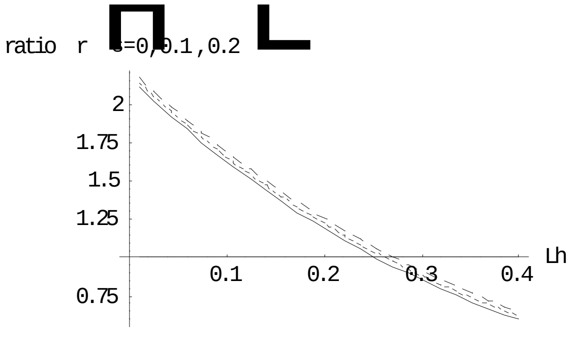

Figure 3.B

The solid lines in Figures 3.A and 3.B corresponds with $\sigma = 0$, while the dotted line corresponds with $\sigma = 0.1$, and the 'striped' line with $\sigma = 0.2$. The horizontal line in Figure 3.A corresponds to a value of the ratio of ICT-capital intensities equal to 1. The points of intersection of the three curves with this horizontal line therefore correspond to equilibrium allocations of labour time between final output generation and human capital generation. The solid curve in Figures 3.A and 3.B correspond with $\sigma = 0$, hence with the unique steady state equilibrium that is obtained for the 'no spillovers' case. Note that when σ increases to a value larger than zero, the allocation of labour time towards knowledge accumulation increases, so the existence of 'external spillovers' promote growth, relative to the 'no spillovers' case. Figure 3.A can be put into more intuitive terms perhaps, by calculating the ratio of the marginal productivity of capital, and the interest rate that is consistent with steady state growth. We have done this for different values of σ, as shown in Figure 3.B. If the 'interest ratio' exceeds the value of one, then the marginal productivity of capital is larger than the interest rate that is required to maximise utility at the given growth rate, hence the growth rate should increase, which implies a larger input of L_h as there is a positive relation between the size of L_h and the steady state growth rate. Note, however, that raising the ICT-capital intensity of knowledge accumulation can also bring about the increase in the growth rate. In both cases, however, the marginal productivity of capital in the final output sector would be negatively affected, thus forcing down the 'interest ratio' in Figure 3.B, while L_h increases. This suggests that the equilibria found in the 'external spillovers' case are 'stable' in the sense that they are consistent with the economic incentives provided by divergences between the actual rate of interest and the rate that is consistent with steady state growth.

Steady State Results: the 'Internalised Spillovers' Case

For the 'internalised spillovers' case we have done the same as for the 'external spillovers' case. The results are again presented in Figures 4.A and 4.B below .

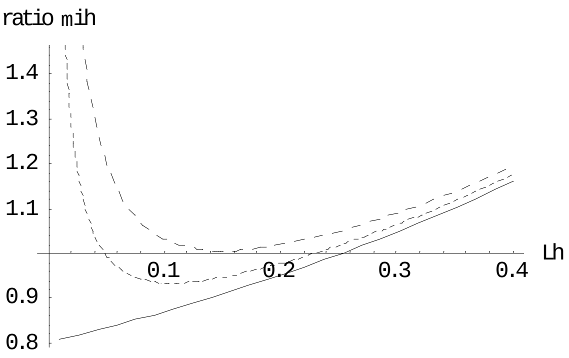

Figure 4.A

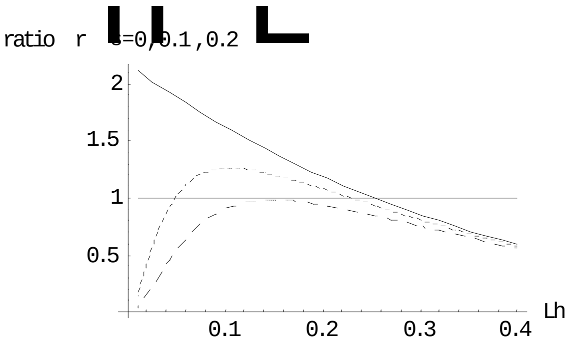

Figure 4.B

Comparing Figures 3.A and 4.A, it should be noted that an increase in σ to non-zero values now 'lowers' the equilibrium allocation of labour time between knowledge accumulation and final output production, because the internalisation of the spillovers favours employment in final output production in two different ways. *First*, because the perceived opportunity cost of learning has risen, and *secondly*, because the reallocation of labour time this entails, even strengthens the spillover effect, since the ICT-capital intensity of knowledge accumulation will rise, ceteris paribus. Moreover, the system now has ‘multiple’ equilibria, i.e. a high- and a low-growth allocation of labour time where the first order conditions of the Hamiltonian optimisation problem are satisfied. In fact, for $\sigma = 0.1$ the model allows for two different steady states, whereas for $\sigma = 0.2$, there is no steady state solution at all. Note furthermore that when σ increases to a value larger than zero, two things happen. *First,* the high growth allocation of labour towards final output generation increases, because of the rise in the opportunity cost of spending time on schooling, now that schooling activities have positive spillovers to final output production. *Secondly,* if σ increases, the two steady state solutions grow closer together.

Comparing Figures 3.B and 4.B, we notice that there may be two equilibria in the ‘internalised spillovers’ case. However, by the same reasoning as above in the ‘external spillovers’ case, also in the ‘internalised spillovers’ case, the high-growth equilibrium would be selected by the market. The question that automatically arises, of course, is whether the high-growth equilibrium is also the highest ‘present value of utility’ equilibrium. In order to answer this question, we provide the numerical results regarding total (‘net’) utility we have calculated for the various spillover regimes in the following subparagraph.

Steady State Welfare Results

In order to calculate welfare results, we disregard transitional dynamics, but simply compare the two steady states under consideration, assuming that all capital stocks per efficiency unit of labour (i.e. *K/(h.L)),* have their optimum values from the start of the steady state, and that the stock of knowledge per person starts from a value equal to 1. However, one should realise that different parameter values imply different steady state values for the quasi state-variable *K/(h.L),* which (at time t=0) is the initial capital endowment of an economy, assuming that the steady state actually begins at time t=0. In the steady state, the amount of capital per efficiency unit of labour will remain constant, since both *K* and *h* will grow at the same rate.[16] Note that because capital can be consumed (see the macro economic budget constraints, but also the first order constraint (7.A) that is valid in all spillover cases considered here), the initial endowment of capital per worker represents a present value of utility equal to $K^{1-\theta}/(1-\theta)$ [17].

Under the assumptions above, the *CIES* function can be rewritten as:

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[16] That’s why it is a quasi-state variable, since it depends on the original state variables, but is itself not variable, at least in the steady state.

[17] We actually have assumed that the intertemporal elasticity of substitution is not equal to 1, hence we can drop the part $-e^{-\rho t}/(1-\theta)$ from the standard CIES function. That part is there only to define the CIES function (through l’Hopitals rule) if $\theta = 1$. Moreover, we have used *h=1, L=1* at *t=0.*

$$U = \int_0^\infty e^{(\hat{h}-\rho)t} \, \frac{((1-s).(Y/h))^{1-\theta}}{1-\theta} \, dt \qquad (19.A)$$

where s is the proportional rate of saving, and where we have used $h=1$ at $t=0$, and $L=1$.

In the steady state, output per efficiency unit of labour will be constant, just like the saving rate, since in the steady state s in turn is given by:

$$s = \dot{K}/Y = \hat{K}.K/Y = \hat{h}.K/Y \qquad (19.B)$$

where $K=K_y+K_{iy}+K_{ih}$ is the total capital stock. K itself can be related to K_{ih} and L_h, through the substitution of equations (9.A) and (9.B) in the definition of K. Equations (9.A) and (9.B) also allow Y (through the production function) to be rewritten entirely in terms of K_{ih} and L_h , and therefore the ratio K/Y can also be expressed in those terms. But K_{ih} is directly linked to μ_{ih}, through (4.B). Finally, μ_{ih} depends entirely on L_h and the parameters of the system (see (15.A) and (16.A)), so that (19.B) and then (19.A) can also be solved entirely in terms of L_h and the parameters of the system. In the steady state all the terms in (19.A) are constant, except for the antilog, and (19.A) can therefore easily be integrated and evaluated for the equilibrium values of L_h.[18]

In order to be able to compare two different steady states, we can simply subtract the present utility value of the initial capital stock per efficiency unit of labour from the present value of the infinite consumption stream obtained by means of the procedure described above, and so obtain the 'net present value of the steady state'. However, instead of presenting the analytical calculations and corresponding results that are quite 'ugly' and therefore difficult (if not impossible) to interpret, we just provide the corresponding numerical results in Table 3.

In Table 3, the labels '*NSP*', '*XSP*' and '*ISP*' denote the 'no spillovers', 'external spillovers' and the 'internalised spillovers' cases described above. The entries '*NPVu*', correspond with the 'net present value of the steady state' that are calculated in accordance with the procedure outlined directly above, while G_h and L_h correspond to the steady state growth rate, and to the amount of labour time used in knowledge accumulation, respectively. The postfixes *'low'* and *'high'* refer to the low- and high-growth equilibria associated with the 'internalised spillovers' case. For the other spillover cases the results are listed only under '*high'*, since the associated equilibria are unique.

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[18] Note that out of equilibrium, the actual value of the present value of total utility, as given by (19.A), depends on which equation one uses to substitute for μ_{ih} . Since in equilibrium both (15.A) and (16.A) generate the same value, the choice of either (15.A) or (16.A) in evaluating (19.A) does not matter.

	NSP	*XSP*	*ISP*	*XSP*	*ISP*	*XSP*	*ISP*
σ	0.00	0.05	0.05	0.10	0.10	0.15	0.15
Lh-low	-	-	0.0224	-	0.0487	-	0.0829
Lh-high	0.2535	0.2580	0.2444	0.2626	0.2292	0.2673	0.2048
Gh-low	-	-	0.0023	-	0.0050	-	0.0083
Gh-high	0.0188	0.0192	0.0190	0.0196	0.0186	0.0201	0.0176
NPVu-low	-	-	45.999	-	47.014	-	47.950
NPVu-high	44.2257	44.715	44.949	45.223	45.831	45.748	46.924

Table 3. Comparative Steady State Results

Table 3 allows a number of conclusions to be drawn. *First*, the welfare results in the 'no spillovers' case are lower than that of the 'externalised spillovers' case, that in turn are lower than that of the 'internalised spillovers' case. Spillovers contribute therefore positively to welfare, but the more they are indeed taken into account, the higher the (minimum) welfare effects will be. *Secondly,* as regards the 'internalised spillovers' case, we furthermore conclude that as σ increases, the corresponding values of N*PVu* increase as well, both for the low growth equilibria and the high growth ones. *Third*, and perhaps contrary to our a priori notion that higher growth represents higher utility, we notice that for a given value of the spillover parameter σ, the low-growth equilibrium is associated with the highest value of *NPVu*. *Fourth*, this suggests that, from a welfare point of view and if left on its own, the economy described in this paper will tend to 'over accumulate'. *Fifth*[19], given this tendency to over accumulate, we could perhaps pose the question whether we have discovered, albeit very implicitly at this stage, the reason why computers don't show up (as much as we would 'expect' on a priori grounds) in the productivity[20] data? *Sixth*, from a policy perspective it seems that there exists an 'optimum' value of the knowledge-spillover parameter σ, i.e. the one where both equilibria coincide, so that over accumulation does not occur.

4. Summary and Conclusion

In this paper we have presented an endogenous growth model based on Lucas (1988). We have extended the Lucas model by incorporating the notion that there may be a positive link between the ICT-capital intensity of production and total factor productivity, but also the efficiency of knowledge accumulation as suggested in the literature. Especially the link between ICT-investment

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[19] And somewhat 'tongue in cheek'.

[20] In this context it would actually have to be 'utility data', of course.

and the efficiency of the learning process provides a direct entry-point for ICT-investment to influence growth performance. In this paper then we have investigated whether endogenous growth (perhaps not so much caused but at least strengthened by) ICT-investment was possible at all, but also how the various parameters that link ICT-investment to total factor productivity and to the productivity of the learning process influence the steady state growth performance. Numerical calculations show that for the same ICT-capital elasticities, the growth elasticity of a change in the learning elasticity of ICT-capital is an order of magnitude larger than that of an equal change in the partial output elasticity of ICT-capital in the final output sector. Generally speaking then, we do indeed find the expected positive links.

We study these links under three different spillover regimes: one without any spillovers from knowledge accumulation to final output production, and two others where these spillovers do exist, but are completely external, or internalised, respectively. We find that welfare is positively affected, the stronger these spillovers are, but also the more these spillovers are internalised. In addition we find that in the case of limited internalised knowledge spillovers, we may face a multiple equilibria steady state growth situation, that has an inherent tendency to select the non-optimum (high growth) equilibrium in which all types of capital are ‘over accumulated’, including ICT-capital. This suggests that there is room for policy intervention here, because there exists an ‘optimum’ value of the knowledge-spillover parameter σ, where both equilibria coincide and over accumulation does not happen.

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