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in Complex Technology Spaces*

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A Percolation Model of Innovation in Complex Technology Spaces

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Abstract

Innovations are known to arrive more highly clustered than if they were purely random, and their rate of arrival has been increasing nearly exponentially for several centuries. Their distribution of importance is highly skewed and appears to obey a power law or lognormal distribution. Technological change has been seen by many scholars as following technological trajectories and being subject to 'paradigm' shifts from time to time.

To address these empirical observations, we introduce a complex technology space based on percolation theory. This space is searched randomly in local neighborhoods of the current best-practice frontier. Numerical simulations demonstrate that with increasing radius of search, the probability of becoming deadlocked declines and the mean rate of innovation increases until a plateau is reached. The distribution of innovation sizes is highly skewed and heavy tailed. For percolation probabilities near the critical value, it seems to resemble an infinite-variance Pareto distribution in the tails. For higher values, the lognormal appears to be preferred.

Keywords: innovation, percolation, search, technological change, R&D

JEL codes: C15, C63, D83, O31

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1. Introduction

While we like to think of innovations as distinct, easily identifiable entities, closer inspection reveals that they are anything but: they can be resolved into smaller sub-steps, making the definition somewhat arbitrary. Nevertheless, when the minimum number of essential subunits comes together, one does have the feeling that the innovation ‘pops out’ and becomes a recognizable Gestalt. Thus a seemingly simple innovation such as the bicycle is a concatenation of many sub-innovations spread out over time:

In 1818, K.V. Drais de Sauerborn presented his Draisine, a kind of walk-drive bicycle (Lauftrad). In 1839 Mannilau demonstrated how wheels can be driven by pedals, and in 1861 at the latest pedals were built into the Draisine. In 1867 they were used on the front wheel by Michaux, and during the next few years the bicycle industry in France grew rapidly. A model of the bicycle approaching the one we are accustomed to today was constructed by Lawson in 1879, but a commercially successful ‘safety bike’ was not introduced by Starley until 1885. If we take 1818, 1839 or 1861 alternatively as years of invention, and 1867, 1879 or 1885 alternatively as years of basic innovation, we can obtain 9 different results for the time-span between invention and innovation. [Brockhoff (1972, p. 283), cited by Kleinknecht (1987, p. 61)]

Undoubtedly, numerous other examples could be found in the history of technology to reinforce this point. What we normally perceive as a unitary entity, a radical innovation, in reality is usually composed of a number of smaller steps dispersed in time, often involving borrowing from other fields or dependent on specific unrelated advances in order to make the final step possible. In the bicycle case we could add the availability of pneumatic tires and ball bearings (and thus precision machining, the precision grinding machine ...) as essential complementary innovations without which the bicycle boom of the 1890s would have been unthinkable. The bicycle is not one innovation but a succession of several smaller ones. In fact, our problem is not reducible à la Schumpeter to just radical vs. incremental innovations; rather innovations come in all sizes, suggesting a fractal structure to the process of innovation.

This ambiguity regarding the timing and definition of innovations is not merely a matter of historical curiosity. It can also be profitably exploited in a representation of technology as consisting of a multitude of elemental small inventive steps that must come together, much like the pieces of a mosaic, to form a coherent whole and constitute an innovation. The purpose of this paper is to present a model of the dynamics of this process making as few assumption about the nature of technology as possible except that it is in some sense complex and shrouded in uncertainty.

The paper is organized as follows. In Section 2 we briefly present some stylized facts about technical change and innovation and some empirical data highlighting a number of distinctive statistical patterns associated with the innovative process. Section 3 outlines the framework of the model, which is derived from Silverberg (2002). Section 4 presents the results of extensive numerical simulations. We propose more sophisticated search strategies in Section 5 and draw some conclusions.

2. Stylized Facts about Innovation and Technological Change

We briefly note some well-known stylized facts about technological change, some of which have been substantiated quantitatively while others are still only impressionistic hypotheses,

which either flow into the formulation of our model or serve as benchmarks for evaluating its output:

1. Technical change is cumulative: new technologies build on each other and often draw on advances in seemingly unrelated fields. Thus Edison's electric light presupposed both advances in the generation of electricity and improvements in vacuum pump technology.
2. Technical change follows relatively ordered pathways, as can be measured ex post in technology characteristics space (see the work of Sahal, Saviotti, Foray and Grübler, etc.). This has led to the positing of *natural trajectories* (Nelson 1977), *technological paradigms* (Dosi 1982), and *technological guideposts* (Sahal 1981). An example is provided by the long-term history of computing technologies, where Nordhaus (2001) has compiled an indicator of technological performance. The essentially static trajectory of manual and mechanical computation was replaced by an exponentially changing electronic trajectory (which in turn went through several generations of underlying component technologies) after the later 1930s (see Fig. 1).
3. The arrival of major innovations appears to be stochastic, but more highly clustered than Poisson, and increasing exponentially over time since the 18th century.¹
4. The 'size' of an innovation is drawn from a highly skewed, heavy-tailed distribution with a power-law character (linear on a log-log plot), as evidenced by citation and co-citation frequencies (e.g., Trajtenberg 1990 and van Raan 1990), and innovation returns studies (e.g., Scherer 1998, Harhoff, Narin, Scherer and Vopel 1999, Scherer, Harhoff and Kukies 2000). Fig. 3 reformulates the Trajtenberg CT-scanner patent data as a rank-order distribution. The power-law nature is apparent. The innovation-returns distributions seems to be situated somewhere between a lognormal and a true Pareto power law.
5. Technological trajectories bifurcate and also merge.
6. There appears to be a certain arbitrariness in the path actually chosen, which could be the result of small events (path dependence or neutral theory?) and cultural and institutional biases (social construction of technology?).
7. Incremental improvements tend to follow upon radical innovations according to rather regular laws (learning curves).

3. Technology Space as a Percolated Lattice and R&D as Stochastic Interface Growth

Consider a lattice, unbounded in the vertical dimension, anchored on a baseline (or space), with periodic boundary conditions, as in Fig. 4. The horizontal space represents the universe of technological niches, with neighboring sites being closely related. While the technology space is represented here and in the following as one-dimensional (with periodic boundary conditions, i.e., a circle), it can easily be generalized to higher dimensions or different

¹ Silverberg and Lehnert 1993 presents nonparametric tests to this effect. Silverberg and Verspagen 2000 applies Poisson-regression techniques to estimate the time trend and differentiate a nonclustering model (inhomogeneous Poisson) from a randomly clustering, overdispersed one (such as negative binomial). The latter is clearly preferred (see Fig. 2).

topologies. The vertical axis measures an indicator of performance intrinsic to that technology and could also be conceived as multidimensional. For simplicity we will restrict ourselves to a two dimensional lattice in the following.

A lattice site \mathbf{a}_{ij} can be in one of four states: 0 or technologically excluded by nature, 1 or possible but not yet discovered, 2 discovered but not yet viable, and 3, discovered and viable. A site moves from state 2 to 3, from discovered to viable, when there exists a contiguous path of discovered or viable sites connecting it to the baseline (see Fig.4). The neighborhood relation we shall use is the von Neumann one of the four sites top, bottom, right and left $\{\mathbf{a}_{i\pm 1,j}, \mathbf{a}_{i,j\pm 1}\}$, with periodic boundary conditions horizontally. The intuition here is that a discovered technology only becomes viable or operational when it can draw on an unbroken chain of supporting technologies already in use. Until such a chain is completed, the technology is still considered to be under development – it is still an invention, not an innovation (see Fig. 5). Impossible states 0 remain so forever. State 1 can progress to state 2 if it is uncovered by the R&D search process, and state 2 can possibly but not necessarily progress to state 3 if a connecting chain exists and all its links are discovered.

The lattice dynamics result from the interplay of natural law with the history of human-driven technological search. Two extreme views stake out the range of approaches now current in technology studies:

1. The *social construction of technology (SCOT)* perspective says that any site we try is valid technological knowledge that can potentially be incorporated into a viable technology. Thus in this case, a tried site will immediately become occupied and placed in state 2. The paths that result from innovative search will be pure accidents of history.
2. The alternative *technological determinism (TD)* perspective says that a tested site only represents true technological knowledge if it accords with the a priori underlying laws of nature. Thus when we ‘invent’ a site, we must first test whether it is technologically possible (in state 1). If it is, we raise it to state 2, if not, we leave it in state 0. This is a bit like playing the game minesweeper. The paths that result will be a selection from the technologically possible ones.

If we are willing to allow for natural law, we must first initialize the lattice at time 0 by assigning each site the state 0 or 1. To reflect our a priori ignorance of the laws of nature we regard this as a random process creating a percolation on the lattice with some probability q .² The essential property of percolation is the behavior of connected sets as a function of the (uniform and independent) probability of occupation of sites. On an infinite lattice (including the half plane) there exists a threshold probability p_c below which there is no infinite connected set and above which with probability one there is one (and only one) infinite connected set. The probability that any site will belong to the infinite connected set is obviously zero below p_c and increases continuously and monotonically above p_c .³ For bounded lattices such as in Figure 4, the interesting question is the probability of finding a connected path spanning the lattice from the bottom edge to the top one. This will increase rapidly and

² In this case we speak of site percolation, as opposed to working with the lines connecting nodes, known as *bond* percolation (see Grimmett 1989, Stauffer and Aharony 1994). For the purposes of this paper there is no obvious preference for one or the other (and bond percolation can always be reformulated as a site model). An early application of percolation theory to technological change can be found in Cohendet and Zuscovitch (1982). David and Foray (1994) applied a hybrid site and bond percolation model to the standardization and diffusion problem in electronic data interchange networks. Some recent applications of percolation theory to social science problems include Solomon et al. (1999), Goldenberg et al. (2000), Gupta and Stauffer (2000) and Huang (2000a).

³ For bond percolation on the unbounded plane it can be proven that p_c is exactly $\frac{1}{2}$. For site percolation it has been numerically established to be around 0.59.

nonlinearly in the neighborhood of p_c . A metaphor that may help to sharpen intuition is to regard rain falling on a yard as a percolation problem. After only a bit of rain the yard consists of islands of wetness surrounded by dry pavement. After more rain has fallen the yard suddenly flips to being islands of dryness surrounded by wetness. Regarding technology space as a percolation is of course only one way to generate a ‘complex’ problem setting. Other possibilities are the use of NK-landscapes (see e.g. Frenken 2001) or directed networks (Vega-Redondo 1994, although networks can also be used as the substrate for percolated structures).

If $q < p_c$, then there will only be finite connected sets (clusters) and technological change will eventually come to an end. If, however, nature is so bountiful that $q > p_c$, then there will a unique infinite cluster and thus potentially unbounded paths of innovation (see Fig. 6). And the larger q , the denser the network of potentially viable technologies will be. The social construction of technology case results from technological determinism in the limit $q \rightarrow 1$.

We now come to the R&D search half of the dynamics. At any point in time t a best-practice frontier can be defined consisting of the highest sites in state 3 for each baseline column (of which there are N_c):

$$BPF(t) = \{(i, j(i)), i = 1, N_c\}, \text{ where } j(i) = (\max j \mid a_{i,j} = 3).$$

(If there is no viable site in column i^* we set $j(i^*) = -1$.) The $BPF(t)$ is needed as the anchor for the R&D search process, which is characterized by a search radius m . Around each point $(i, j) \in BPF(t)$ with $j > -1$, i.e., around each occupied point on the frontier, we draw a (diamond-shaped) neighborhood of radius m containing all points at a distance of m or less (according to the ‘Manhattan’ metric induced by the neighborhood relation). We suppose R&D search to proceed within these local neighborhoods anchored around current best practice, and thus includes technology sites not only directly above the current best practice sites, but sites laterally related to it and even sites lying behind it. Search itself is viewed as uncertain and characterized by a uniform probability p_s of testing any one of the $2m(m+1)$ neighboring points (not counting the anchor point). If the total R&D ‘effort’ at the disposal of any point on the BPF is E , then

$$p_s = E / 2m(m+1).$$

If a site is tested and in state 0, i.e., it is intrinsically impossible, then it remains in this state. If it is in state 1 it is marked as ‘discovered’ and advanced to state 2. Sites already in state 2 or 3 remain unchanged. A site may be tested several times in a period if it is in the m -neighborhood of several sites on the BPF.

Figure 7 shows how connected paths may represent some relevant technological phenomena. First, any connected path beginning on the bottom line can be thought of as a natural trajectory. On the left we see two trajectories diverging from a common origin. In the middle we see technological convergence (e.g., the convergence of mechanical and electronic technologies to mechatronics, or optical and mechanical technologies to optonics). While the purely technological performance characteristics of an operational site are measured by its height above the baseline, its economically relevant *technological competitiveness* can be measured in different ways. The point of introducing a separate technological competitiveness is to reflect the ease of realization (related to cost) of a given level of technological performance and allow subsequent incremental innovations to operate. Additionally, we may want the extent of parallelism in the realization of a technology to be counted as an advantage. Thus we propose two separate measures of competitiveness, both based on path length (if L is a path then let $|L|$ be its length. The first measure is

$$c_1 = y^2 / |L_s|,$$

where L_s is the shortest path connecting the site to the baseline. If this path is simply a straight vertical line, then $c_1 = y$. The more indirect the path, the more the competitiveness is diminished. The second measure corresponds to the current that would be extracted at the site if we apply a one-volt potential difference between the site and the baseline and set the resistance of a single lattice nearest-neighbor link to one. If two paths L_1 and L_2 converge at a site, then

$$c_2 = y^2(1/|L_1| + 1/|L_2|).$$

For more complicated connections Kirchhoff's laws have to be applied.

A relevant technological analogy would be the different generations of microprocessors. While each generation represents a certain gain in performance, it usually comes at a certain price. However, over time that price declines as learning takes place in the production and design of the product. This can be captured in a natural way in our framework by allowing subsequent shortcuts (which we identify with *incremental* innovation) to reduce the length of the connecting base of a site (rightmost in Figure 7). Thus we will allow innovation to take place both ahead and behind of the current best practice frontier, so that radical and incremental innovation take place simultaneously.

Consistent with our 'blunderbuss' vision of the search process, we allow innovation to take place in a neighborhood of radius m centered around each point on the frontier. The union of these regions creates a band of innovative percolation extending ahead and behind of the frontier (Figure 8). Within this region new sites will be tested at random with some probability p . A discovered site of course need not connect immediately with the operational network. It is this fact that permits innovations of variable length (as measured by the jump in y they entail) to occur spontaneously. Thus we obtain a natural explanation of innovation clustering (but of the random kind), as shown in Figure 9. This happens when a disjoint extended network of discovered but not yet operational sites is finally connected to the technological frontier, and/or when an 'overhanging cliff' advances laterally, pulling up the BPF at neighboring sites by increments that can be much larger than m , the search radius.

4. Numerical Simulations

In the following we investigate the behavior of the system just described as a function of certain key exogenous parameters, in particular the search radius m and the lattice percolation probability q . We will focus solely on changes in pure technological performance, leaving economic performance, incremental innovation and learning curves for a later paper. We hold the number of columns N_c fixed at 100 and set the total search effort E_0 to 0.05. The vertical dimension can be allowed to grow over time without limit without exceeding the memory capacity of the computer by simply following a band on the lattice around the BPF whose height is greater than $\max_j BPF(t) - \min_j BPF(t) + 2m$. The system is set in motion after percolation by randomly seeding the baseline with 'discovered' sites, that is, turning baseline sites in state 1 into 2's with probability 0.5.

We begin by asking whether a more 'crafts' or a more 'scientific' search procedure will be more successful in maneuvering through the percolation maze. By crafts we mean a search for new techniques close in technology space to existing practice, i.e., small values of the search radius m . Scientific search by contrast looks farther a field and uses larger values of the search radius. To make this comparison fair, we hold the total R&D effort constant per

BPF site and thus let the search probability scale downwards with increasing search radius. To this end we perform a grid search over a range of values of m and q , the percolation probability of the lattice, reflecting the density of seeding of the space with potential technologies. To control for the statistical variability of the runs, we report the results of ten runs for each vector of parameter variables, differing only in the random seed used to initialize the random number generator.

Fig. 10 reports the results for the mean height of the frontier attained after 5000 time periods for $q=0.593$, the critical percolation probability for the full plane (and thus presumably near the critical value for our half cylinder). We see that this increases more or less linearly with search radius until a value of about 8, when it reaches a plateau. Thus gains exist for more farsighted R&D strategies, but they saturate. A measure of the unevenness of the BPF is its standard deviation, reflecting the variability of technical change across technology categories. This is shown in Fig. 11, where again values increase with search radius until about 8. Finally, Fig. 12 shows the number of runs per parameter setting which **deadlock**, i.e., reach a state in which no further advance is possible even in principle, before the 5000 periods are over. At a percolation probability of 0.593 the infinite cluster or at least a large finite one will have a high probability of existing. And given that the width of our lattice is finite, it may not even intersect the baseline, in which case it will be impossible to find with our search procedure. What is striking is that runs with low search radius have a high probability of becoming deadlocked, but that this probability declines rapidly with radius until the magic number 8 again. (It appears that we have indeed generated one lattice out of the ten, which is simply unsearchable). This result at first glance seems somewhat paradoxical, since even with search radius one but search probability $p=1$ our search procedure should track the infinite cluster with certainty. Increasing the search radius should not make any difference in this ability. However, for a search probability per BPF site and time period less than one, this is no longer the case for the following reason. Assume that a BPF site has reached a branching point on the infinite cluster, with one branch leading to a cul-de-sac while the other continues on the backbone of the cluster. If, for $m=1$ say, it chooses by chance the wrong branch, in the next time period the ‘right’ branch is already out of reach and can no longer be tested, regardless of how often we repeat the trial. For $p=1$ this problem disappears, since the system will always take both branches. For higher values of m the problem also declines in severity since even if the system takes the wrong branch, until it has pushed the BPF $m-1$ steps down the wrong branch, there is still a chance to discover the right branch before it is too late. Thus probabilistic search and shortsightedness introduce extreme path-dependence into the R&D process: the system can easily become trapped in a cul-de-sac even if a path of continuing technical progress exists.

In Fig. 13 we summarize the results of the grid search. As q approaches the critical percolation probability 0.593 from below the number of deadlocked runs declines precipitously. And for each value of q above this threshold, increasing the search radius improves performance markedly. Even at high values of q , however, very myopic search ($m=1$) can still lead to entrapment. Fig. 14 reports the mean height attained by the BPF after 5000 periods for the same grid search. This is an increasing function of q (strongly) and m (weakly), with an at first surprising trough at $m=2$ for high values of q .

The explanation of this trough may follow from the interaction of two competing forces. Larger search radii decrease the probability of becoming deadlocked, but for higher values of q this becomes less and less likely anyway, since the infinite cluster becomes an increasingly large proportion of all clusters (this follows from Figure 6) and indeed of the entire lattice. The amount of duplication of R&D search, however, increases with the search radius m . This can easily be seen by considering two adjacent points on the BPF that are also lattice neighbors (of course they need not be lattice neighbors at all, but on average will be close). The area of the intersection of their m -neighborhoods will be an increasing share of the

area they jointly cover (starting with 1/8 at $m=1$, 4/9 at $m=2$, and going to 1 as $m \rightarrow \infty$). This duplication of effort means that a given rate of investment in R&D search will produce less technical change, i.e., advancement of the BPF, than otherwise. On balance, therefore, in a rich technological environment (q high compared to the critical percolation probability), increasing duplication at first outweighs the benefits of foresighted search and thus lowers the average rate of technical change. In a sparse technological environment (q near the critical probability), in contrast, where local dead-ends abound, the long-term benefits of foresighted search strongly outweigh any losses from duplication for all values of the search radius.

The size distribution of innovations can be examined by defining an innovation as any change in the height of a BPF site, and its size as the number of vertical sites covered by the change in one time period. In Fig. 15 we present a raw innovation size distribution and the corresponding rank-order distribution of the same data. While it is clear that the distribution is highly skewed, the rank-order distribution does not quite conform to a power law. The plots in Fig. 16 show the rank-order distribution stepping through values of the search radius from one to ten. While linearity seems to hold over one or two decades on the right sides, the curves in general display clear convexity on the left (the fact that jumps greater than or equal to 100 are lumped together distorts the picture on occasion, however).

The nature of the innovation size statistics can be examined more rigorously by applying some measures developed for the study of heavy-tailed distributions (cf. Crovella, Taquu, and Bestavros 1998). Consider n observations of a random variable X_i , and denote by $X_{[i]}$ the order statistics $X_{[1]} \geq X_{[2]} \geq \dots \geq X_{[n]}$. Then the Hill estimator is defined by

$$H(k, n) = \frac{1}{k} \sum_{i=1}^k (\ln X_{[i]} - \ln X_{[k+1]}).$$

Plotting this estimator against k for small values of k (compared to n) will indicate if it converges to some value, which will then be an estimate for the downward slope of the double-log rank-order plots of Figures 15 and 16, or the inverse of the exponent α of the estimated Pareto distribution

$$\Pr(X \geq x) \propto x^{-\alpha}.$$

We calculate estimates for α using the Hill estimator in Figures 17, 18, and 19 for different values of q and $m=5$. It is apparent that for lower values of q there is an intermediate range of k where the estimate converges to a value between 1 and 2. For $q=0.695$ convergence is to a value of around 2, indicating a lognormal distribution rather than a Pareto.

We also calculated LD plots, which are obtained by grouping the original observations into blocks of size b :

$$X_{t,b} = \sum_{i=(t-1)b+1}^{tb} X_i.$$

The right cumulative histograms are then plotted on double-log scales for different block sizes. If these curves are straight and parallel in the tails, then the data are in the domain of attraction of a Pareto-Levy stable distribution with $\alpha < 2$. If the curves seem to converge in the tails, this is an indication of a finite-variance distribution such as lognormal. Figures 20, 21 and 22 show the LD plots for the same runs as in Figures 17, 18 and 19. While they do not incontrovertibly demonstrate that the data are Pareto distributed, for $q=0.695$ the convergence of the curves is apparent, again indicating a finite-variance distribution.

Are such large jumps or sudden gains in performance consistent with the record of technological history? We think they are, even if they are relatively rare. Consider the speed of communication over long land distances or across oceans, for example. Until the advent of the telegraph, this speed was stagnant for millennia, limited by the speed of overland horse-

drawn transport or sailing ships to less than 10 km/h.⁴ Then within a very short period, albeit with somewhat limited bandwidth, it jumped to nearly the speed of light with the advent of the telegraph. This came about not through advances in materials transport but rather due to progress in a seemingly unrelated field, electricity. This kind of leapfrogging corresponds to the jumps induced in the BPF by overhanging cliffs in our percolation space, the height of which is in principle unbounded.

5. Refinements of the Model and Conclusions

Until now we have assumed that R&D effort is uniform all along the BPF. This is not exactly a very rational hypothesis for agents who are aware of the historical rates of advance of different parts of the frontier. For example, a section of the frontier may be blocked by an impregnable barrier of impossible sites. It would gradually become obvious that continuing to invest in R&D along this section after many cycles of stymied advance would be pointless. Progress would only be possible by making an ‘end run’ around the barrier and taking a detour through a more fertile technological region, such as electricity proved for communications and computation. Thus a mechanism to shift R&D effort from stagnant areas to progressive ones would probably make sense, and might even be discoverable by agents with learning capabilities. One scenario for realizing this reallocation of R&D effort would be to scan for discontinuities in the BPF. If they exceed a threshold value, such as the search radius m , R&D effort is shifted from the neighboring backward sites to the breakthrough sites.

The effect of learning in terms of the shortening of paths remains to be studied. We also intend to study the time pattern of major innovation arrivals (major meaning larger than some threshold value) to see if it resembles aspects of the empirical record, in particular overdispersion. The implications of the highly skewed, fat-tailed nature of the innovation size distribution for economic growth and R&D policy is another intriguing question (see Scherer and Harhoff 2000 and Sornette and Zajdenweber, 1999, for discussions of this issue).

The combination of a percolated technology search space with neighborhood-based probabilistic search enables us to endogenize the creation of technological trajectories and recover some of the characteristic statistical properties of the innovation process, in particular, the skewed and heavy-tailed distribution of innovation sizes. Innovation is revealed to be a highly path-dependent phenomenon in which excessive myopia can be a dangerous thing, since it can trap the system in dead ends. Whether the model parameters can be tuned to match specific features of real-world observations remains to be seen. Another possibility would be to eliminate the exogenous nature of some parameters by allowing the system to self-tune itself in the sense of self-organized criticality (SOC).⁵

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⁴ We neglect the historically rather limited role of visual semaphore systems over long distances such as were employed in France and England at the end of the 18th century. Their diffusion was undoubtedly limited by logistic factors and the necessity and unreliability of frequent relaying, and is impracticable over large bodies of water.

⁵ Cf. Bak (1966). On the relationship between SOC and percolation see Grassberger and Zhang (1996) and Sornette and Dornic (1996). Extremum dynamics along the lines of the Sneppen (1992) model might be an appropriate avenue of attack.

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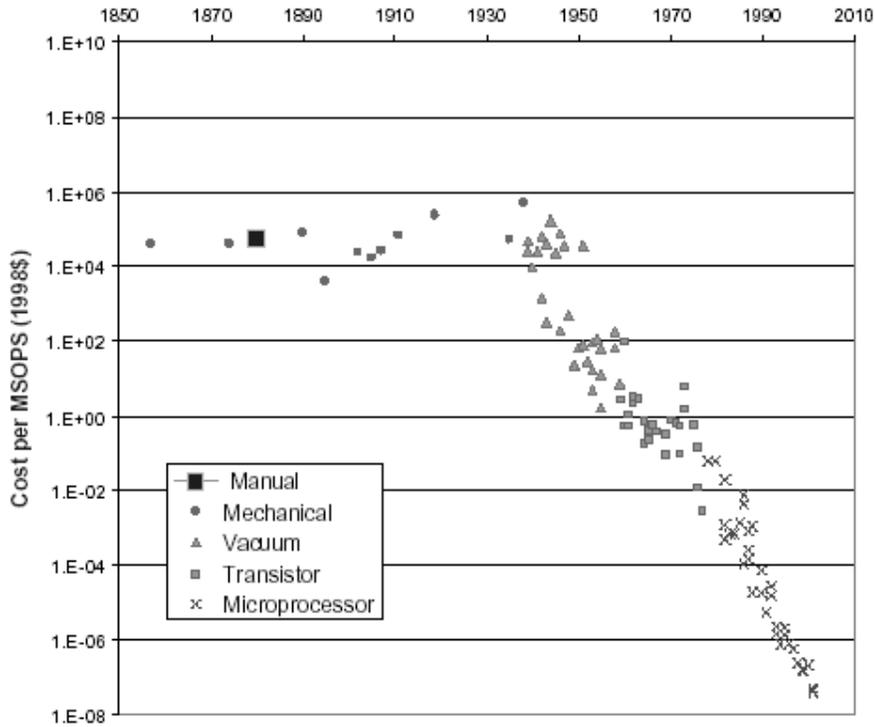


Figure 1. The real cost of computation for different technologies. The paradigm shift in the 1940s is apparent. Source: Nordhaus (2001).

(a) Innovation supersample

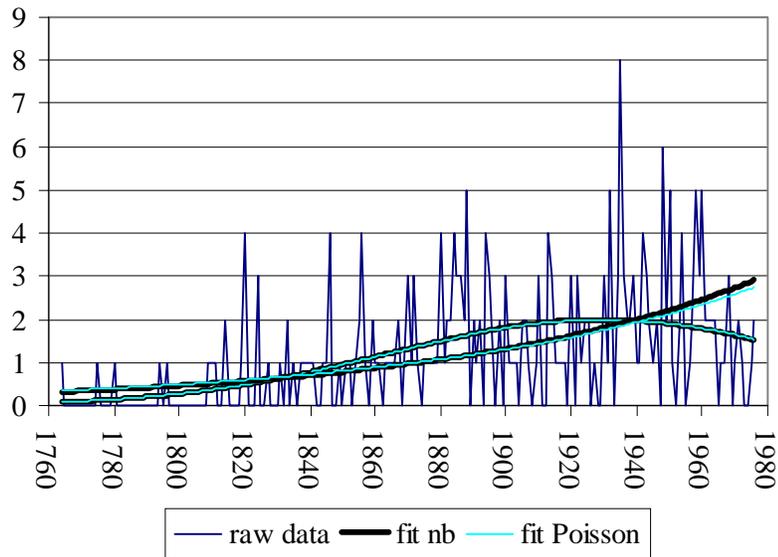


Figure 2. Poisson-regression analysis of radical innovation time series for first and third degree polynomial trends. While the point estimates of the Poisson and negative-binomial models are virtually identical, the latter is statistically preferred. The long-term trend is also apparent. Source: Silverberg and Verspagen (2000).

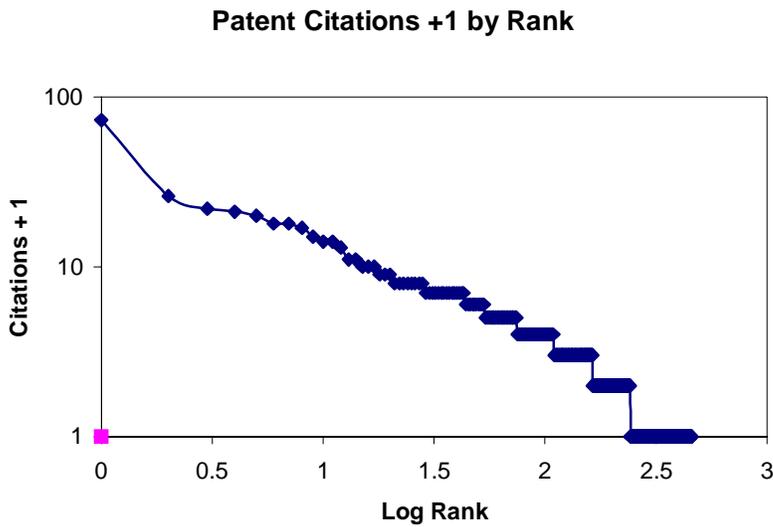
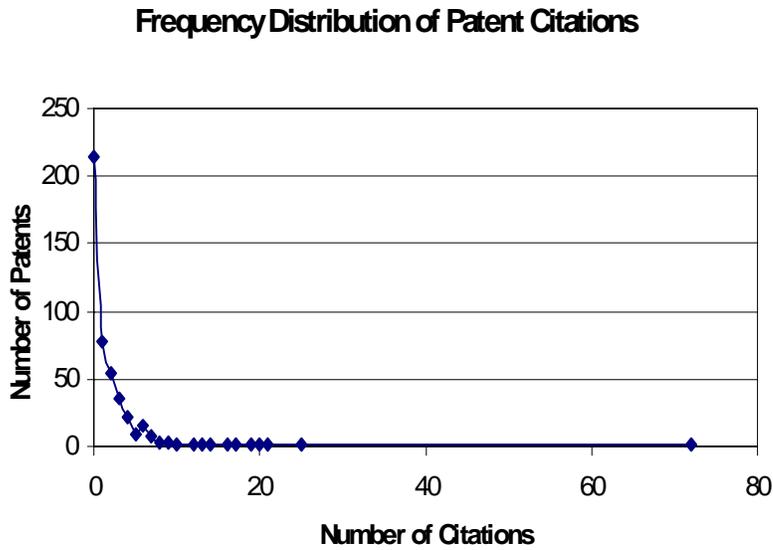


Figure 3. Innovation 'size' distributions based on CT scanner patent citations. Top: By number of citations. Bottom: Rank-order distribution counting self-citation, double-log scale.

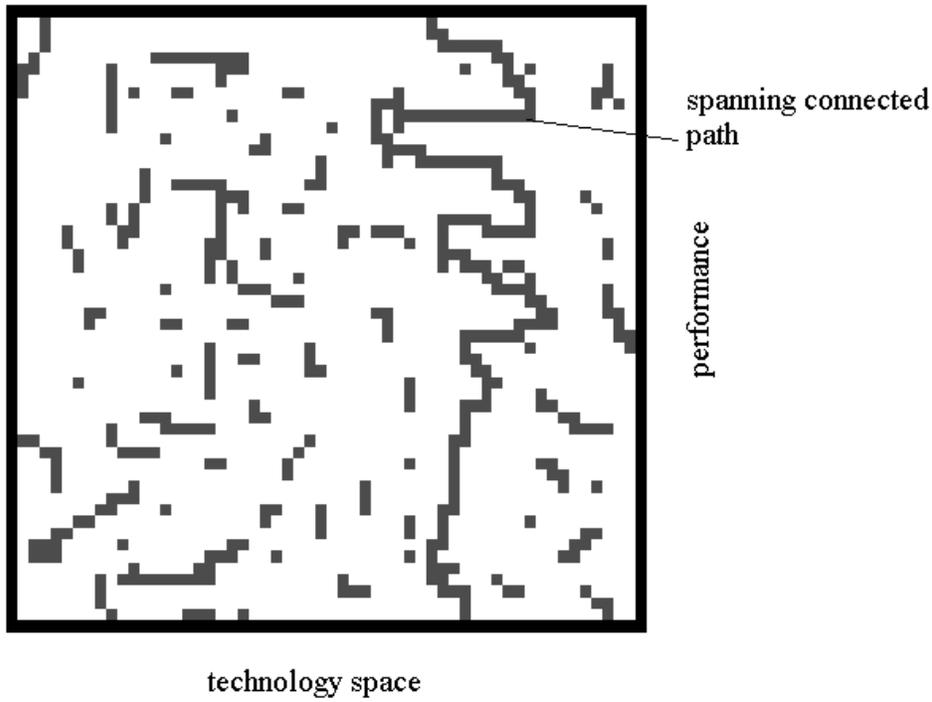


Figure 4. Technology-performance lattice. Discovered sites are marked in red, viable sites lie on the path connected to the baseline.

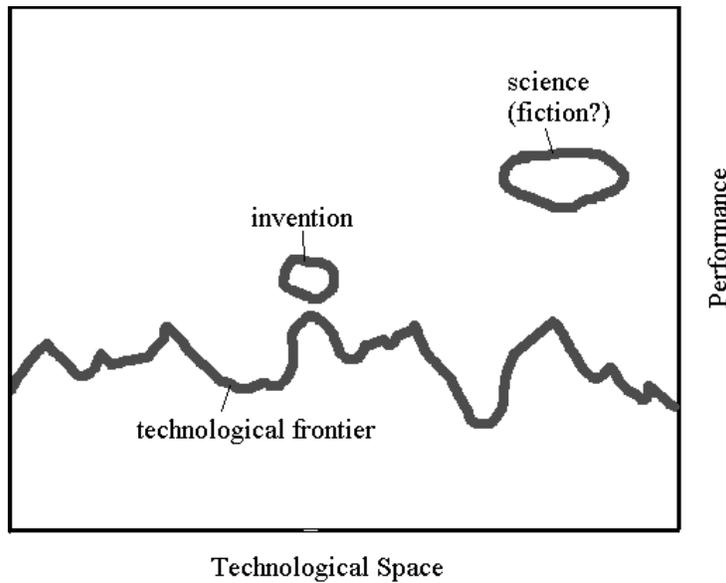


Figure 5. Best-practice frontier marks highest viable sites above each column of the baseline. Proximate discovered sites above the BPF are inventions, distant ones are scientific discoveries or fictions (not connectable).

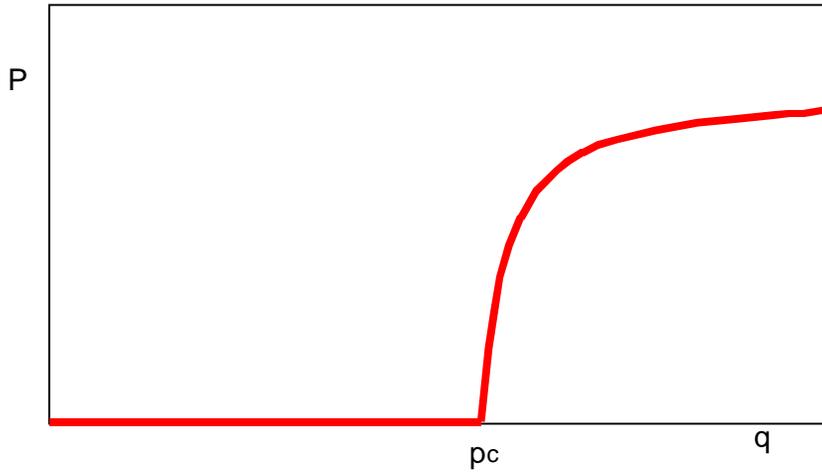


Figure 6. The probability that any site will be on the infinite cluster P as a function of the percolation probability q .

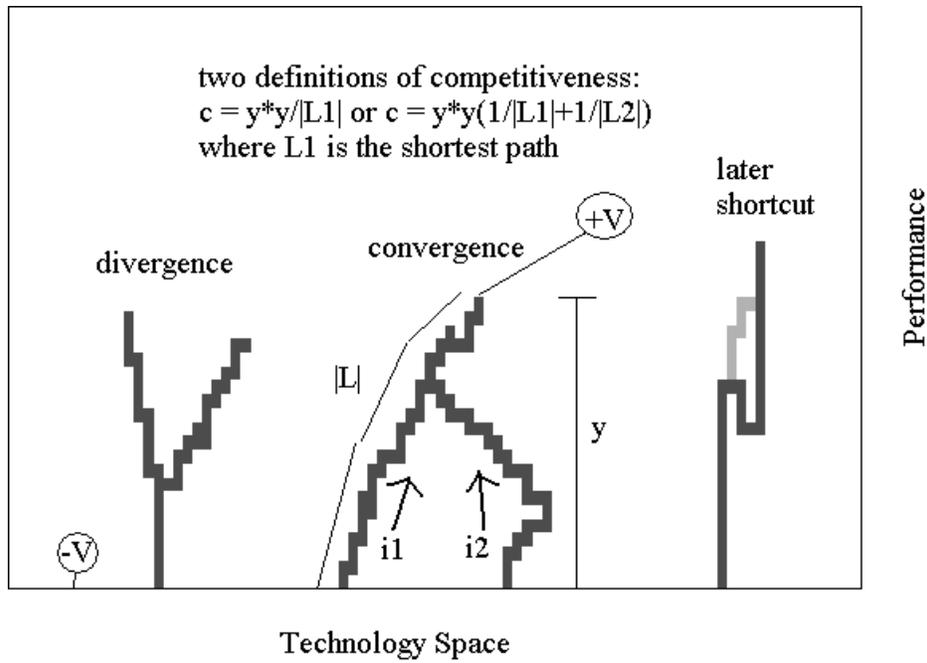


Figure 7. Diverging technological trajectories on the left, converging ones in the middle. 'Pruning' of paths leads to incremental innovation (right). The economic competitiveness is the square of technological performance divided by path length.

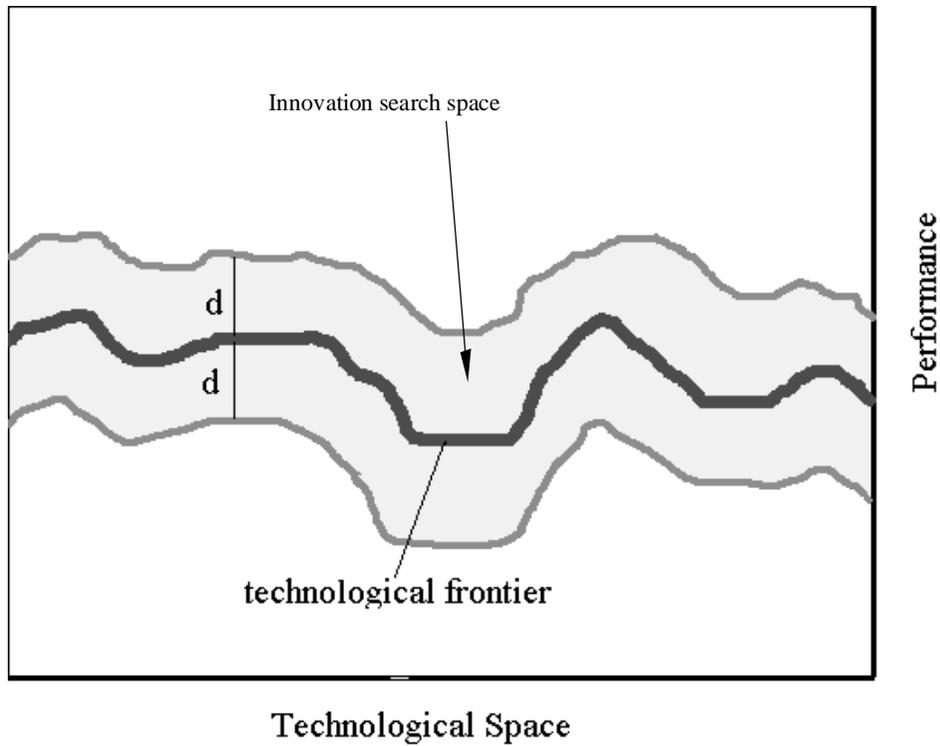


Figure 8. Search neighborhoods of radius d define an innovation search space enveloping the BPF.

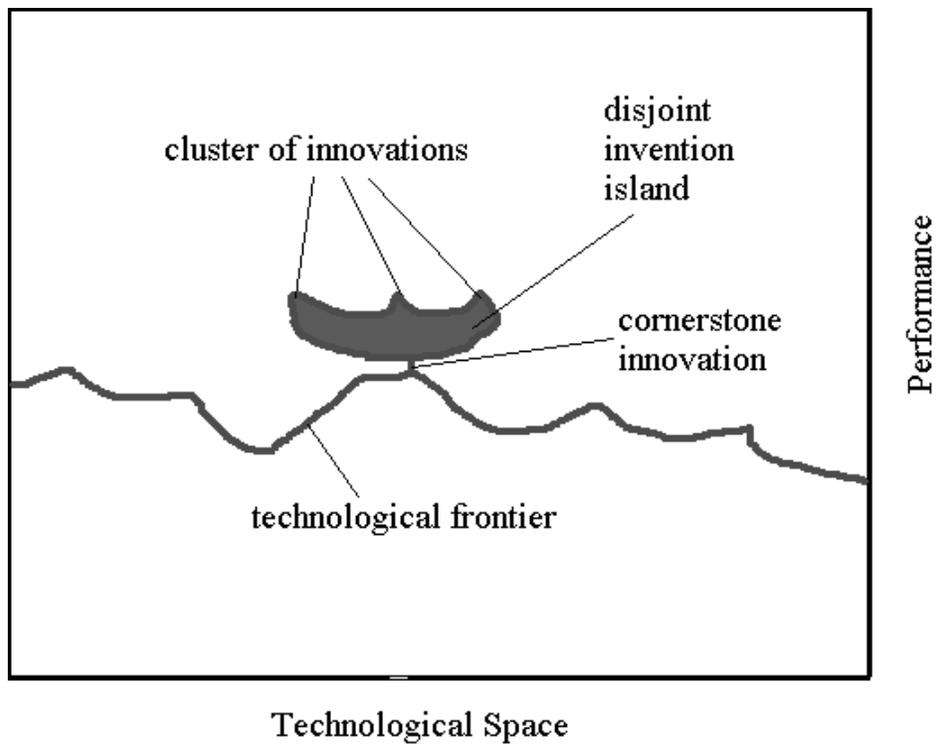


Figure 9. Clusters of innovations occur when disconnected islands of inventions are joined to the BPF by cornerstone innovations.

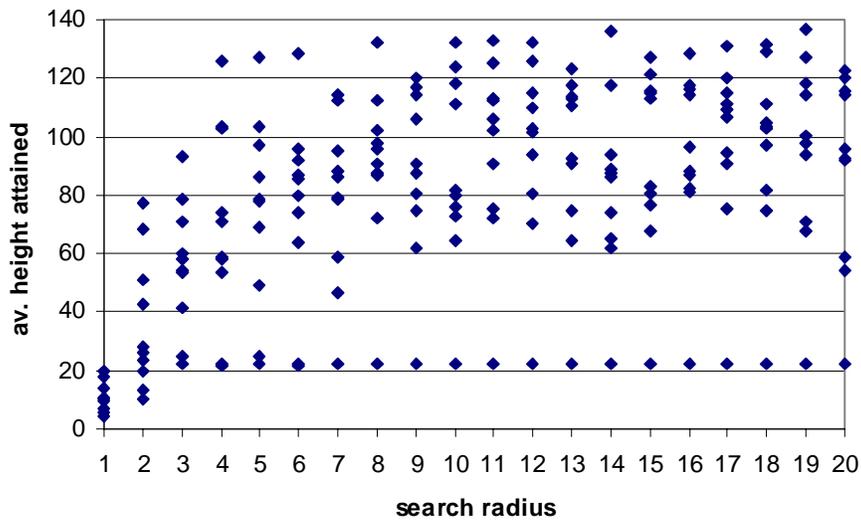


Figure 10. Mean height of BPF attained after 5000 periods for different values of the search radius with ten runs per value. $q=0.593$.

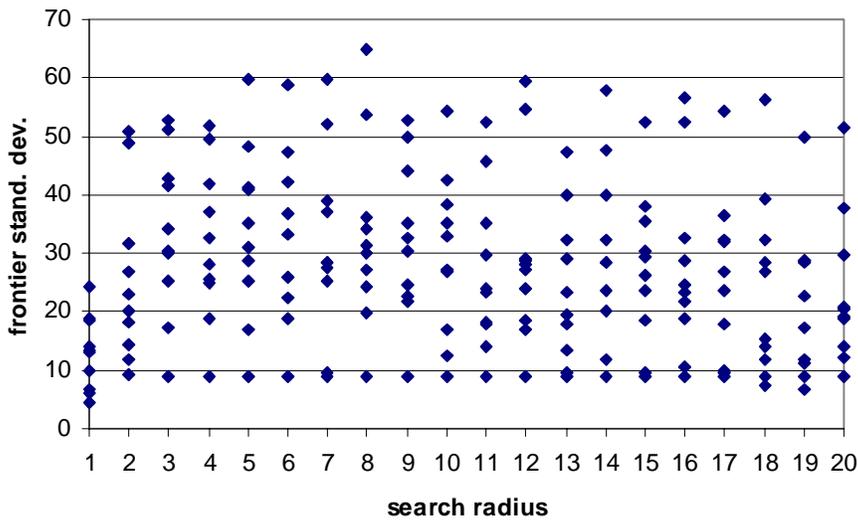


Figure 11. Standard deviation of BPF, a measure of technological unevenness, for the runs described in Fig. 10.

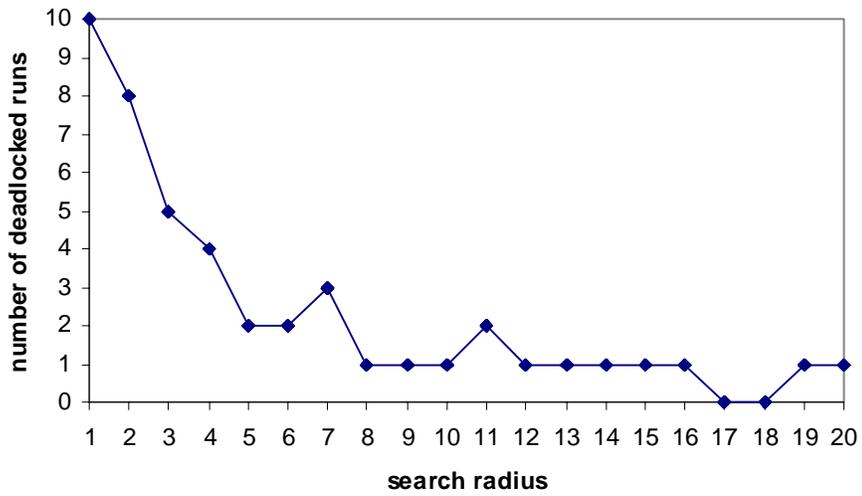


Figure 12. Number of runs, which deadlock out of batches of ten for different values of the search radius. $q=0.593$.

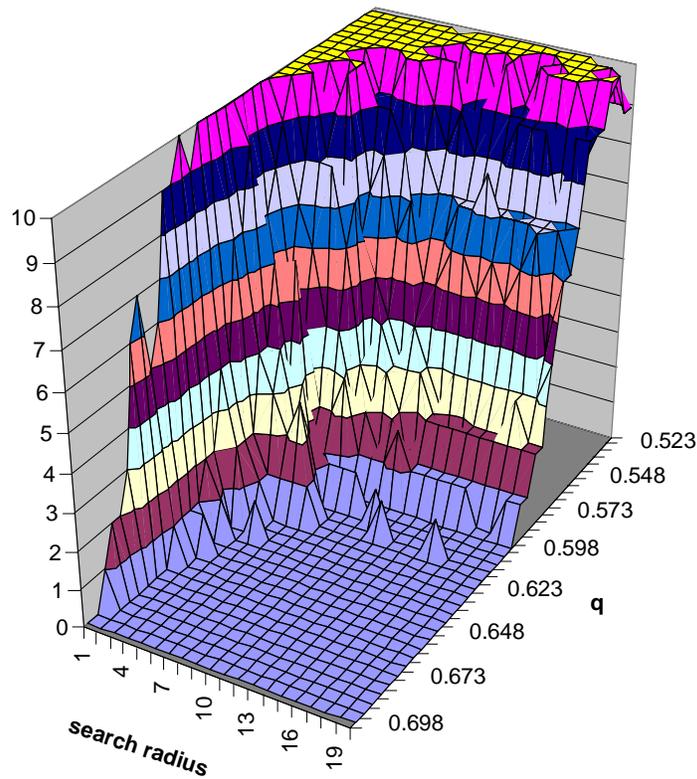


Figure 13. Number of deadlocked runs out of ten as a joint function of the search radius and the percolation probability q .

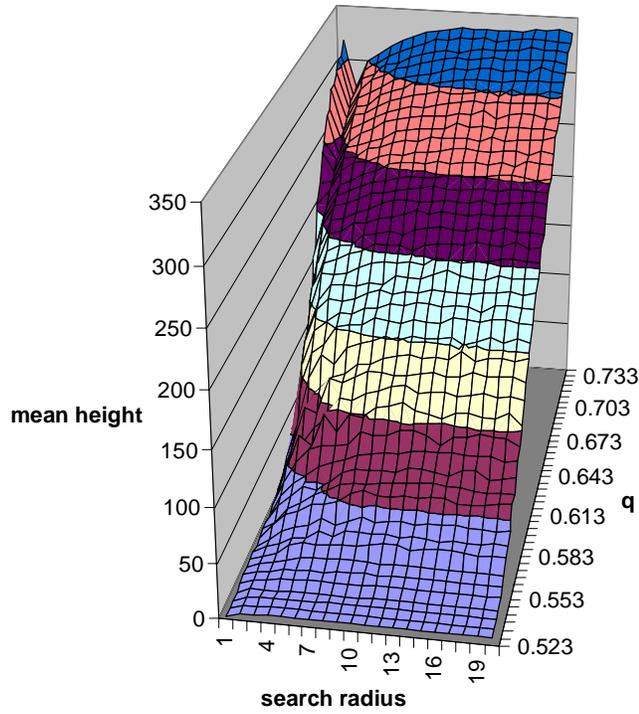


Figure 14. The mean height of the BPF attained after 5000 periods as a function of the search radius and percolation probability q .

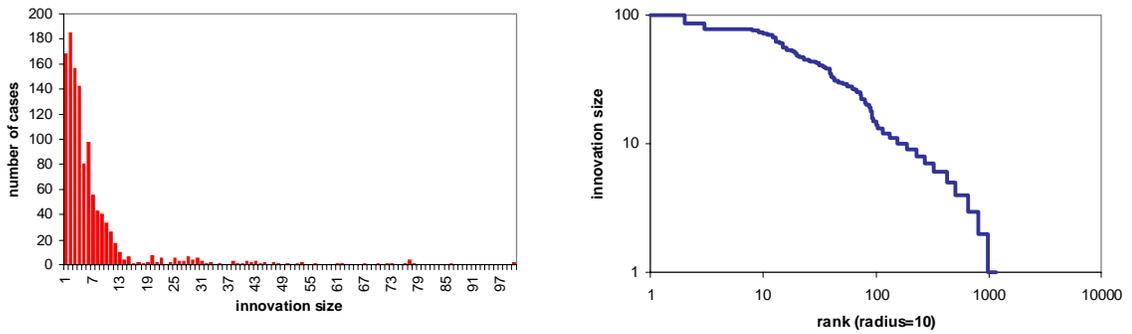


Figure 15. Size distribution of innovations (left) and rank-order distribution (right), $q=0.603$, $m=10$.

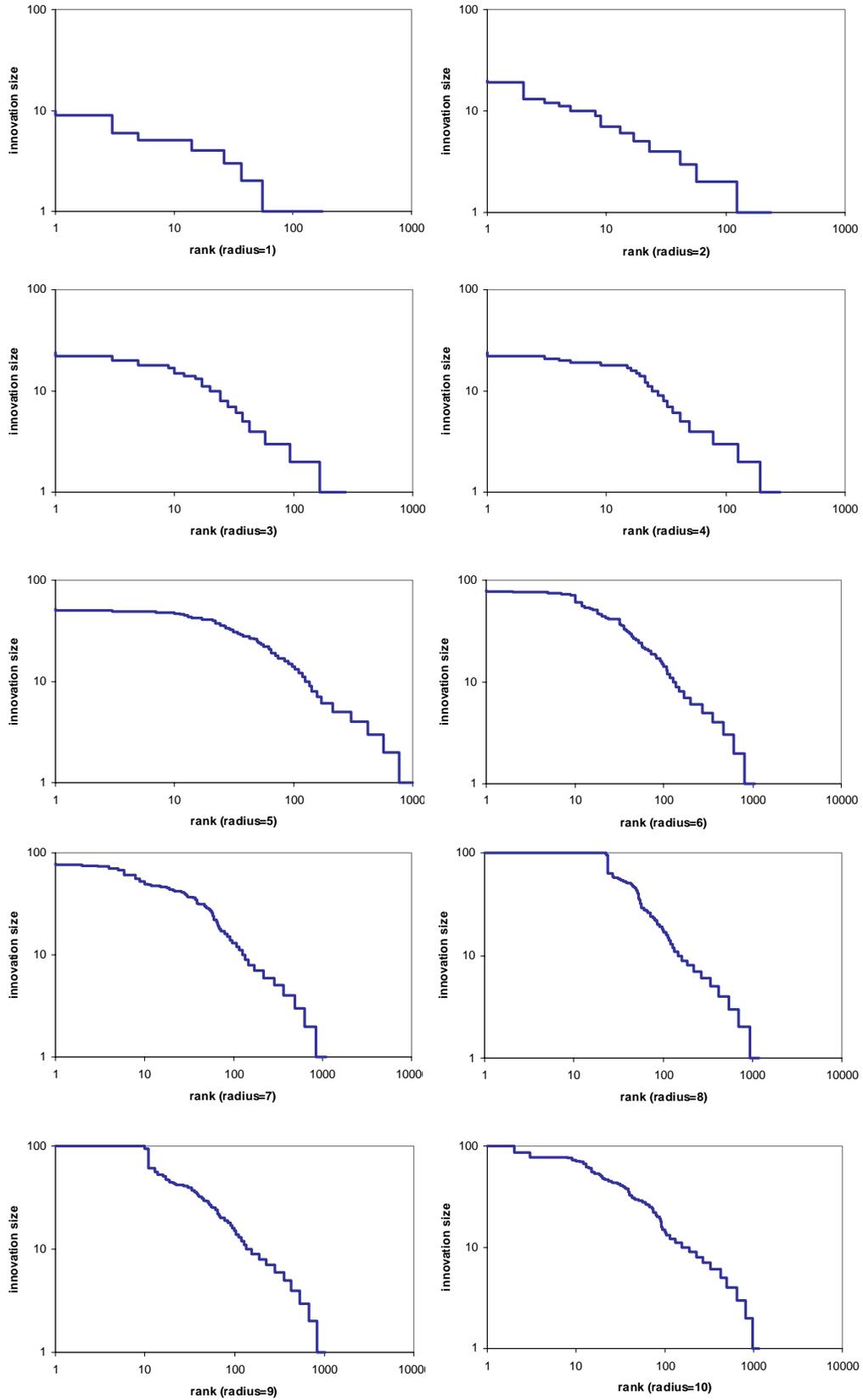


Figure 16. Rank-order distributions of innovation size for increasing search radius ($q=0.603$).

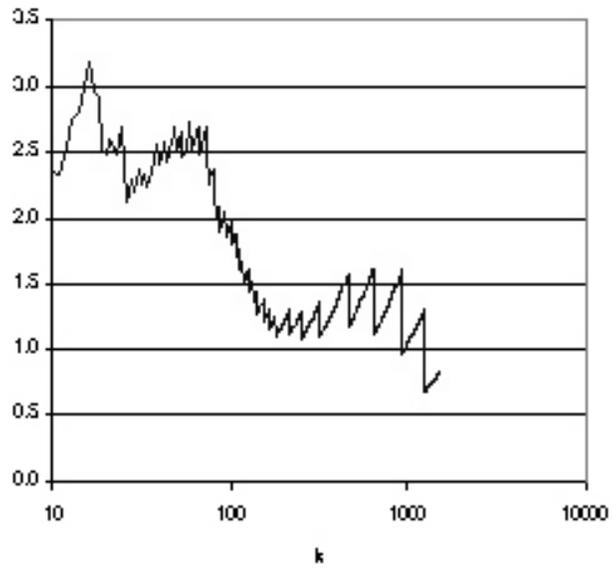


Figure 17. Hill estimator of Pareto α for innovation distribution generated with $q=0.6$ and $m=5$ plotted on a double-log scale for values of k up to 90% of number of observations.

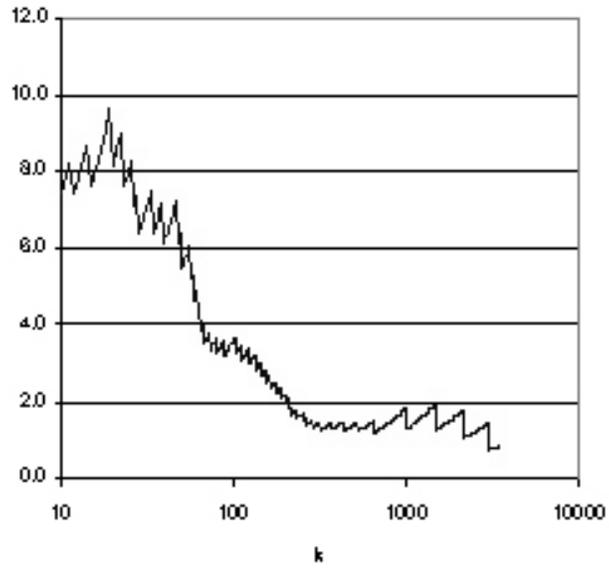


Figure 18. Hill estimator of Pareto α for innovation distribution generated with $q=0.645$ and $m=5$ plotted on a double-log scale for values of k up to 90% of number of observations.

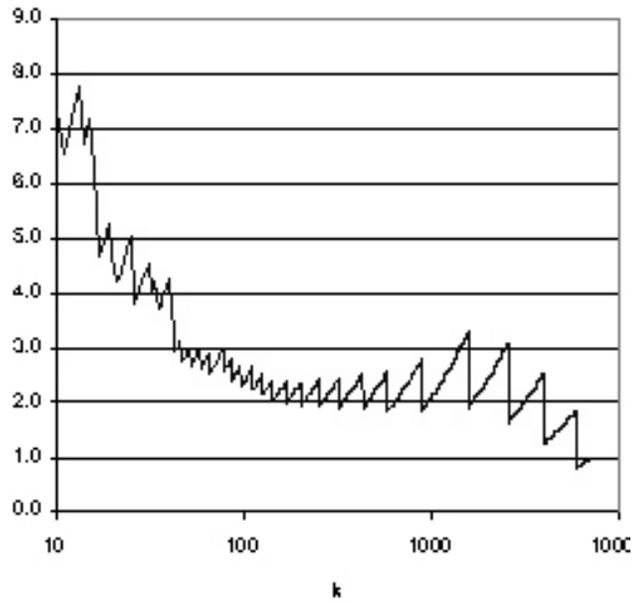


Figure 19. Hill estimator of Pareto α for innovation distribution generated with $q=0.695$ and $m=5$ plotted on a double-log scale for values of k up to 90% of number of observations.

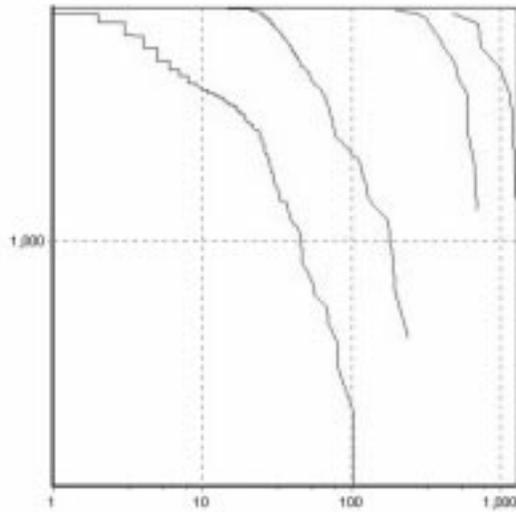


Figure 20. LD plot for innovation distribution generated with $q=0.60$ and $m=5$ (original data and aggregated data in blocks of 10, 100 and 200 observations).

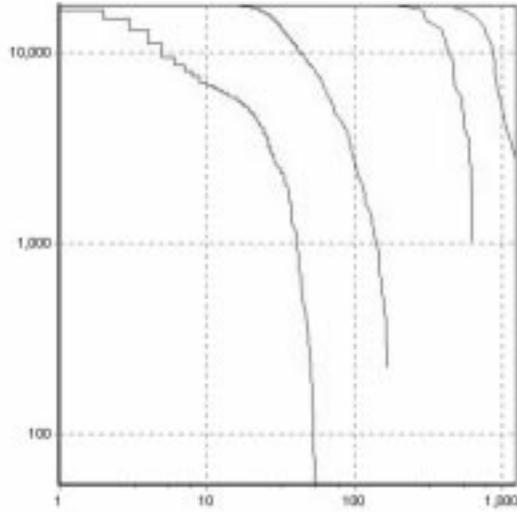


Figure 21. LD plot for innovation distribution generated with $q=0.645$ and $m=5$ (original data and aggregated data in blocks of 10, 100 and 200 observations).

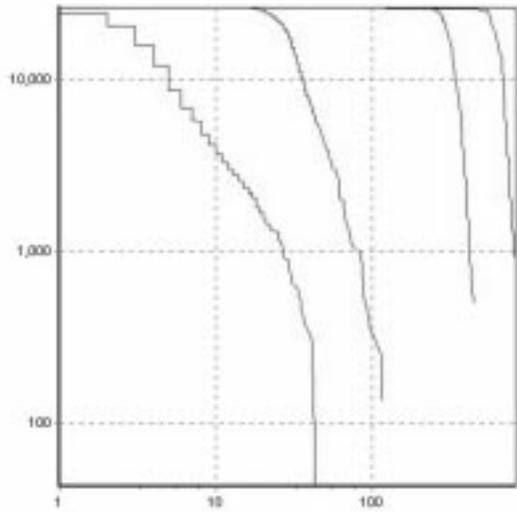


Figure 22. LD plot for innovation distribution generated with $q=0.695$ and $m=5$ (original data and aggregated data in blocks of 10, 100 and 200 observations).