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Constrained Suboptimality when Prices Are Non-competitive

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Abstract

The paper addresses the following question: how efficient is the market system in allocating resources if trade takes place at some prices p that are not necessarily competitive? Even though there are many partial answers to this question, an answer that stands comparison to the rigor by which the first and second welfare theorems are derived is lacking. We first prove a "Folk Theorem" on the generic suboptimality of equilibria at non-competitive prices. The more interesting problem is whether equilibria are constrained optimal, i.e. efficient relative to all allocations that are consistent with prices at which trade takes place. We give a necessary condition, called the separating property, for constrained optimality: each constrained household should be constrained on each constrained market. If the number of commodities is less than or equal to two, then this necessary condition is also sufficient. In that case equilibria are constrained optimal. In all other cases, this necessary condition is typically not sufficient and equilibria are generically constrained suboptimal.

Key words: Non-competitive prices, welfare, Pareto improvement.

JEL classification numbers: D45, D51, D61.

1 Introduction

More than two centuries ago, Adam Smith described how the pursuit of self-interest can promote the interest of society. Since then, economists have devoted much of their time in providing rigorous foundations to this claim, which finally resulted in the first and second fundamental welfare theorems. These theorems are valid only in idealized circumstances, among which the requirement that all trade takes place at competitive prices, including trade in contracts contingent on all imaginable future events.

The case where the assumption of complete financial markets is relaxed has received much attention in the recent literature. When markets are incomplete, then a competitive equilibrium is typically suboptimal. The appropriate question to ask, however, is not whether competitive equilibria are optimal, but whether competitive allocations are optimal relative to the restrictions imposed by market incompleteness. When a fully informed central planner, who takes into account the implications of market incompleteness, is able to improve upon a competitive allocation, then competitive equilibria are said to be constrained suboptimal. Geanakoplos and Polemarchakis (1986) show that competitive equilibria are typically constrained suboptimal, by showing that Pareto improvements can be obtained by making the appropriate redistributions in households' initial asset portfolios and next restricting all trade in asset markets. More recently, similar results have been obtained that show the possibility of generating Pareto improvements by introducing new financial assets, see Cass and Citanna (1998), or Citanna, Kajii and Villanacci (1998) for a more general perspective, and the possibility of generating Pareto improvements by price regulation, see Drèze and Gollier (1993) and Herings and Polemarchakis (1999).

The assumption that all trade takes place at competitive prices has also been relaxed. During the last quarter of the 20th century, traditional Walrasian theory has accommodated in a general equilibrium setting the possibility of sluggish price adjustment, short-run price rigidities, and, as a consequence, non-clearing markets. For seminal contributions, see Bénassy (1975), Drèze (1975) and Younès (1975). Attention has been focused on issues of equilibrium existence, and on explanations why prices and wages may not adjust freely to equate supply and demand. Instances of the latter are cases with information imperfectness, menu costs, renegotiation costs and so on. Many empirical studies show that quantity constraints, like involuntary unemployment in the labor market, and infrequent price adjustments, like nominal wage rigidities, are common in the real world, see Romer (1996). Other examples where the analysis of non-clearing markets is relevant are situations involving market power, planned economies and markets for agricultural products, see Bénassy (1993). More generally, application of standard tools from public choice theory shows that governments have incentives to intervene in the price formation process to gain votes, see Herings (1997) and Tuinstra (1998).

Most economists share the strong conviction that imperfections in the price formation process, and in particular trade at non-competitive prices, has strongly negative welfare consequences. Given the strength of this conviction, it is surprising that most of its foundations come from partial settings. No rigorous general results that stand comparison to the first and second welfare theorems, or the constrained suboptimality results in the case of market incompleteness, are available. It is therefore that we label the claim on the detrimental effects of trade at non-competitive prices as a Folk Theorem.

The paper addresses the following question: how efficient is the market system in allocating resources when prices are not competitive. To get an answer, we analyse the equilibria of the cleanest fixed price model available, the one of Drèze (1975). In his seminal paper, Drèze introduced the concept of quantity rationing in a general equilibrium model with price rigidities. In this approach a household chooses that commodity bundle which is most preferred by it, subject to both the budget constraint and the quantity constraints on net trades. The quantity rationing may affect either supply or demand of a commodity, but it never affects both simultaneously to reflect the transparency of markets. The first main result we show is the Folk Theorem on the generically suboptimal allocation of resources when prices are non-competitive.

Inspired by the incomplete markets literature, we continue our investigation by analyzing a concept of constrained optimality, that takes into account the restrictions imposed by trading at false prices. Suppose that trade takes place at prices p and that an allocation is efficient relative to the set of physically feasible allocations for which the net trades of all households have value zero at the price vector p . Such an allocation is said to be p -optimal.

Böhm and Müller (1977) give an example of an economy, whose equilibria are not p -optimal. Maskin and Tirole (1984) observe that if all traders have strictly positive weights in a welfare program, then non-competitive p -optimal allocations involving trade in all markets are never voluntary, that is imply forced trade, and, therefore, are not equilibria. However, satiation or non-constrained maximization is typical for a fixed price model, see Aumann and Drèze (1986), which implies that p -optimal allocations need not be solutions to welfare programs with strictly positive weights. The question rises whether trade at non-competitive prices leads typically to constrained suboptimal allocations.

A household is said to be constrained if it is subject to quantity rationing in at least one market. A market is said to be constrained if at least one household faces constraints in that market. We give an easy to verify necessary condition for equilibria to be p -optimal: each constrained household should be constrained on each constrained market. If the number of commodities is less than or equal to two, then this necessary condition is also sufficient. In that case equilibria are constrained optimal. This case is not entirely without interest as it is the general equilibrium equivalent of the partial equilibrium textbook picture that

analyzes the effects of a minimum or a maximum price on a single good. In cases with more than two goods, this necessary condition is not sufficient and generic constrained suboptimality of equilibria is obtained.

The paper has been organized as follows. Section 2 exposes a model of an exchange economy where trade takes place at non-competitive prices. Section 3 shows that in such an economy, equilibria are typically suboptimal. Section 4 shows that in the two commodity case constrained optimality holds. Section 5 derives the necessary condition that all constrained households be constrained on each constrained market for constrained optimality to hold. It is shown that this criterion is typically not met when the number of commodities is greater than or equal to three. Finally, Section 6 concludes.

2 The model

We consider an exchange economy denoted by $\mathcal{E} = \langle \mathcal{N}, \mathcal{L}, \{X^i, u^i, w^i\}_{i \in \mathcal{N}} \rangle$. Here $\mathcal{N} = \{1, \dots, N\}$ is the set of households, indexed by i , and $\mathcal{L} = \{0, 1, \dots, L\}$ is the set of commodities, indexed by l . Each household is characterized by a consumption set X^i , a subset of \mathbb{R}^{L+1} , a utility function u^i defined on X^i , and a vector of initial endowments w^i in X^i . Price systems belong to the set $P = \{p \in \mathbb{R}^{L+1} \mid p_0 = 1, p \gg 0\}$. Commodity 0 serves as a numeraire commodity.

To evaluate the welfare consequence of trade at non-competitive prices, we fix a price system p in P at which trade is supposed to take place. In general, since prices p might be not compatible with a competitive equilibrium, traders will face quantity constraints on supply and demand. The description of the market mechanism is now extended in the sense that the information transmitted by it is no longer only the price system, but also the maximal amount a household is able to supply of every commodity, called the rationing scheme on supply, and the maximal amount a household is able to demand of every commodity, called the rationing scheme on demand. In this we follow the approach and formulation of Drèze (1975). All exchange takes place against the numeraire commodity, which is not rationed. A household faces rationing $\underline{z}_l \in \mathbb{R}_-$ and $\bar{z}_l \in \mathbb{R}_+$ in the market for each commodity $l \neq 0$, which represents the minimal and the maximal amount of good l household i is able to trade. For the sake of simplicity we only consider uniform rationing, which means that \underline{z}_l and \bar{z}_l are the same for every household i .

Given a price system p and a rationing scheme $(\underline{z}, \bar{z}) \in \mathbb{R}_-^L \times \mathbb{R}_+^L$, a household maximizes its utility function u^i over its constrained budget set defined by

$$B^i(\underline{z}, \bar{z}, p) = \{x^i \in X^i \mid px^i \leq pw^i \text{ and } \underline{z}_l \leq x_l^i - w_l^i \leq \bar{z}_l, l = 1, \dots, L\}.$$

Throughout the paper we will use the following assumptions with respect to the economy \mathcal{E} :

A1 For every household $i \in \mathcal{N}$, $X^i = \mathbb{R}_{++}^{L+1}$.

A2 For every household $i \in \mathcal{N}$, the utility function u^i is C^2 on X^i , u^i is differentially strictly increasing, i.e., $Du^i(x^i) \gg 0$ for all $x^i \in X^i$, and u^i is differentially strictly quasiconcave, i.e., the Gaussian curvature of $I_{x^i} = \{y^i \in X^i \mid u^i(y^i) = u^i(x^i)\}$ is different from zero for any $x^i \in X^i$. Moreover, u^i satisfies the boundary condition, that is, for every $x^i \in X^i$ the set $\{y^i \in X^i \mid u^i(y^i) \geq u^i(x^i)\}$ is closed relative to \mathbb{R}^{L+1} .

A3 For every household $i \in \mathcal{N}$, $w^i \in \mathbb{R}_{++}^{L+1}$.

The demand function of individual i is defined by

$$d^i(\underline{z}, \bar{z}, p) = \operatorname{argmax}_{x^i \in B^i(\underline{z}, \bar{z}, p)} u^i(x^i).$$

Assumptions A1–A3 guarantee that d^i is a function indeed. Given $(\underline{z}, \bar{z}, p) \in \mathbb{R}_-^L \times \mathbb{R}_+^L \times P$, household i is said to be constrained on its supply in market l if for any $\tilde{z} \in \mathbb{R}_-^L$, such that $\tilde{z}_k = \underline{z}_k$ for $k \neq l$ and $\tilde{z}_l = \underline{z}_l - \varepsilon$ for some positive ε , $u^i(d^i(\tilde{z}, \bar{z}, p)) > u^i(d^i(\underline{z}, \bar{z}, p))$. If household i is constrained on its supply in the market for commodity l , then it follows from the strict quasiconcavity of the utility function that $d_l^i(\underline{z}, \bar{z}, p) - w_l^i = \underline{z}_l$. A household that is constrained on its supply in market l can improve its utility if the rationing on supply in market l is relaxed. A similar definition is made with respect to rationing on demand. The market for commodity l is said to be constrained if there is at least one household constrained on it, either on supply or on demand.

Following Drèze (1975), we introduce a Drèze equilibrium of the economy \mathcal{E} at prices $p \in P$.

Definition 2.1 A *Drèze equilibrium* at prices $p \in P$ of an economy \mathcal{E} is an allocation $(\bar{x}^1, \dots, \bar{x}^N) \in \prod_{i \in \mathcal{N}} X^i$ such that there exists $(\underline{z}, \bar{z}) \in \mathbb{R}_-^L \times \mathbb{R}_+^L$ satisfying the following conditions:

- (i) for all $i \in \mathcal{N}$, \bar{x}^i maximizes u^i on $B^i(\underline{z}, \bar{z}, p)$;
- (ii) $\sum_{i \in \mathcal{N}} \bar{x}^i = \sum_{i \in \mathcal{N}} w^i$;
- (iii) for every $l \in \mathcal{L} \setminus \{0\}$,

$$\begin{aligned} \bar{x}_l^{i'} - w_l^{i'} = \bar{z}_l \text{ for some } i' \in \mathcal{N} &\text{ implies } \bar{x}_l^i - w_l^i > \underline{z}_l \text{ for all } i \in \mathcal{N}, \\ \bar{x}_l^{i'} - w_l^{i'} = \underline{z}_l \text{ for some } i' \in \mathcal{N} &\text{ implies } \bar{x}_l^i - w_l^i < \bar{z}_l \text{ for all } i \in \mathcal{N}. \end{aligned}$$

The first two conditions of the definition are standard, they state that every household behaves optimally given the price system and the rationing scheme, and that all markets

clear. Condition (iii) guarantees that markets are transparent. Constraints are on one side of the market at most. The requirement that (\underline{z}, \bar{z}) belong to $\mathbb{R}_-^L \times \mathbb{R}_+^L$ implies that there is no forced trading. Nothing precludes that the prices p are competitive. A competitive equilibrium is indeed a special case of a Drèze equilibrium, it is a Drèze equilibrium without binding rationing.

Notice that the case with two commodities, $L = 1$, is the exact general equilibrium analogue of the standard textbook analysis of partial equilibrium, where, for instance, a minimum price is imposed in the market for commodity 1, which is exchanged against commodity 0. If at the minimum price supply exceeds demand, which is the case with the standard upward sloping supply curves and downward sloping demand curves, then the quantity actually traded is determined by the short side of the market, the total demand for commodity 1. Some of the suppliers will be constrained. They are only able supply part of their preferred supply.

Now consider the general equilibrium set-up with $L = 1$ and suppose that at prices p total net supply exceeds total net demand. A Drèze equilibrium will necessarily involve only rationing on the supply side, so \bar{z}_1 equals a number sufficiently large not to affect the households' decision problems. Total net demand is not affected by rationing on the supply side, so when \underline{z}_1 is such that constrained net supply equals total net demand for commodity 1, the unique Drèze equilibrium is obtained.

3 Suboptimality of equilibrium

Our first aim is to prove the Folk Theorem that, given a tuple of utility functions u^1, \dots, u^N , and prices $p \in P$, for almost all initial endowments $(w^1, \dots, w^N) \in \mathbb{R}_{++}^{N(L+1)}$ every Drèze equilibrium is suboptimal. An allocation (x^1, \dots, x^N) is said to be feasible if $x^i \in X^i$, $i \in \mathcal{N}$, and $\sum_{i \in \mathcal{N}} x^i = \sum_{i \in \mathcal{N}} w^i$.

Definition 3.1

A feasible allocation (x^1, \dots, x^N) is *optimal* if there is no feasible allocation y such that $u^i(y^i) \geq u^i(x^i)$ for all $i \in \mathcal{N}$ with at least one inequality strict.

As a consequence of the boundary condition on the utility function, an allocation (x^1, \dots, x^N) is optimal if and only if

$$\frac{\partial_{x^i} u^i(x^i)}{\partial_{x_0^i} u^i(x^i)} = \frac{\partial_{x^{i'}} u^{i'}(x^{i'})}{\partial_{x_0^{i'}} u^{i'}(x^{i'})}$$

for every $i, i' \in \mathcal{N}$.

Lemma 3.2

Suppose that $(\bar{x}^1, \dots, \bar{x}^N)$ is a Drèze equilibrium at prices $p \in P$ of an economy \mathcal{E} for a rationing scheme $(\underline{z}, \bar{z}) \in \mathbb{R}_-^L \times \mathbb{R}_+^L$. Then there exist $\lambda^i \in \mathbb{R}_{++}$, $\underline{\mu}_l^i, \bar{\mu}_l^i \in \mathbb{R}_+$, $i \in \mathcal{N}$, $l \in \mathcal{L} \setminus \{0\}$, such that

$$\frac{\partial_{x_l^i} u^i(\bar{x}^i)}{\partial_{x_0^i} u^i(\bar{x}^i)} = p_l - \frac{\underline{\mu}_l^i - \bar{\mu}_l^i}{\lambda^i}, \quad i \in \mathcal{N}, \quad l \in \mathcal{L} \setminus \{0\},$$

$\lambda^i = \partial_{x_0^i} u^i(\bar{x}^i)$, $\underline{\mu}_l^i > 0$ implies $x_l^i - w_l^i = \underline{z}_l$, and $\bar{\mu}_l^i > 0$ implies $x_l^i - w_l^i = \bar{z}_l$.

Proof. The conclusion of the lemma follows from the first-order conditions for the maximization problem of household i :

$$\begin{aligned} \max_{x^i \in X^i} \quad & u^i(x^i) \\ \text{s.t.} \quad & px^i - pw^i \leq 0, \\ & \underline{z}_l \leq x_l^i - w_l^i \leq \bar{z}_l, \quad l \in \mathcal{L} \setminus \{0\}. \end{aligned}$$

The Lagrangian of the problem is

$$\begin{aligned} \mathcal{L}^i(x^i, \lambda^i, \underline{\mu}^i, \bar{\mu}^i) = & u^i(x^i) - \lambda^i(px^i - pw^i) - \\ & \sum_{l=1}^L \left(\underline{\mu}_l^i(\underline{z}_l - x_l^i + w_l^i) + \bar{\mu}_l^i(-\bar{z}_l + x_l^i - w_l^i) \right). \end{aligned}$$

The derivatives of \mathcal{L}^i with respect to x^i are equal to zero at \bar{x}^i :

$$\begin{aligned} \partial_{x_0^i} \mathcal{L}^i &= \partial_{x_0^i} u^i(\bar{x}^i) - \lambda^i = 0, \\ \partial_{x_l^i} \mathcal{L}^i &= \partial_{x_l^i} u^i(\bar{x}^i) - \lambda^i p_l + \underline{\mu}_l^i - \bar{\mu}_l^i = 0, \quad l \neq 0. \end{aligned}$$

Notice that λ^i is never equal to 0, since the numeraire commodity is always desirable. The first part of Lemma 3.2 is now straightforward. Moreover, the Kuhn-Tucker conditions imply that

$$\begin{aligned} \underline{\mu}_l^i(\underline{z}_l - x_l^i + w_l^i) &= 0, \\ \bar{\mu}_l^i(\bar{z}_l - x_l^i + w_l^i) &= 0, \end{aligned}$$

which gives the second part of the lemma. □

The interpretation of the lemma is very natural. The marginal rate of substitution between good l and the numeraire equals the price of good l , if the household is unconstrained in market l . It is less than p_l , if the household is constrained on its supply in market l , and it is greater than p_l , if the household is constrained on its demand in market l .

Drèze equilibria may be optimal, for instance, if the initial allocation of resources is optimal. This is necessarily the case if there is just one commodity or just one household. But when the number of commodities and households is greater than one, we show this situation to be rather exceptional.

When $L \geq 2$ and $N \geq 2$, we can construct an example of an economy with an optimal Drèze equilibrium at non-competitive prices \bar{p} but an inefficient initial distribution of resources. Suppose that there are two households and three commodities, and competitive equilibrium prices p^* are such that the excess demand for commodity 2 is zero for both households, so only two goods are traded in non-zero amounts. Consider prices \bar{p} such that $\bar{p}_2 < p_2^*$, and $\bar{p}_l = p_l^*$, $l = 0, 1$. It is possible to choose utility functions such that the demand for good 2 becomes positive for both households under the price system \bar{p} . In fact, this is the natural case. Choose a rationing scheme (\underline{z}, \bar{z}) with $\bar{z}_2 = 0$ and other components of (\underline{z}, \bar{z}) non-binding. Strict quasi-concavity of preferences implies that $\bar{z}_2 = 0$ is binding in an optimal solution to the household's decision problem. But then the competitive allocation is a Drèze equilibrium of the economy \mathcal{E} at prices \bar{p} , which means that the Drèze equilibrium is optimal.

For $L = 1$ such an example cannot be constructed. If $L = 1$ and the initial distribution of resources is inefficient, then a Drèze equilibrium at non-competitive prices p is necessarily inefficient. Suppose $L = 1$ and let \bar{x} be an optimal Drèze equilibrium at non-competitive prices p of an economy \mathcal{E} , whereas the initial resources of \mathcal{E} are distributed inefficiently. There is at least one household i who is rationed, and at least one household i' , who is not. The former holds because p is non-competitive. The latter because trade is needed to go from an inefficient initial distribution of resources to an optimal allocation. Condition (iii) of Definition 2.1 implies that one side of the market of commodity 1 is unconstrained. By Lemma 3.2 it holds that

$$\frac{\partial_{x^i} u^i(\bar{x}^i)}{\partial_{x_0^i} u^i(\bar{x}^i)} \neq \frac{\partial_{x^{i'}} u^{i'}(\bar{x}^{i'})}{\partial_{x_0^{i'}} u^{i'}(\bar{x}^{i'})},$$

which contradicts the optimality of \bar{x} .

The following proposition gives a useful characterization of optimal Drèze equilibria. It claims that each optimal Drèze equilibrium coincides with a competitive equilibrium allocation.

Proposition 3.3

A Drèze equilibrium \bar{x} at prices p of an economy \mathcal{E} for a rationing scheme (\underline{z}, \bar{z}) is optimal if and only if \bar{x} corresponds to a competitive equilibrium allocation.

Proof. Let \bar{x} be an optimal Drèze equilibrium at prices p and a rationing scheme (\underline{z}, \bar{z}) . Optimality together with Assumption A1 implies that

$$\frac{\partial_{x_i^i} u^i(x^i)}{\partial_{x_0^i} u^i(x^i)} = \frac{\partial_{x_{i'}^i} u^i(x^i)}{\partial_{x_0^i} u^i(x^i)}, \quad \text{for any } i, i' \in \mathcal{N}.$$

Then it follows from Lemma 3.2 that $(\underline{\mu}_l^i - \bar{\mu}_l^i)/\lambda^i$ does not depend on i . Together with Condition (iii) in the definition of a Drèze equilibrium, it implies that for each market $l \in \mathcal{L} \setminus \{0\}$, either every $\underline{\mu}_l^i$ is positive, or every $\bar{\mu}_l^i$ is positive, or both these multipliers are equal to zero. The first two cases mean that everyone in market l is constrained, so from market clearing it follows that $\bar{x}_l^i = w_l^i$, for all $i \in \mathcal{N}$. The last possibility is equivalent to the situation of a free market without rationing. Consider a vector of prices $p' \in P$ such that

$$\begin{aligned} p'_0 &= p_0 = 1, \\ p'_l &= p_l - \frac{\underline{\mu}_l^i - \bar{\mu}_l^i}{\lambda^i}, \quad l \neq 0. \end{aligned}$$

Since prices p' are different from p only for markets without trade, \bar{x} satisfies the budget condition under the price system p' , $p' \bar{x}^i = p' w^i$, $i \in \mathcal{N}$. It follows immediately that \bar{x} is a competitive equilibrium allocation at competitive prices p' , which proves the “only if” part of the proposition.

It is immediate that a Drèze equilibrium \bar{x} , which corresponds to a competitive equilibrium allocation, is optimal. □

The next step is to show the generic suboptimality of Drèze equilibria.

Theorem 3.4

Fix any price system $\bar{p} \in P$ and utility functions u^1, \dots, u^N satisfying Assumption A2. Then there is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}_{++}^{N(L+1)}$ such that every Drèze equilibrium at prices \bar{p} of the economy \mathcal{E} is suboptimal.

Proof. By Proposition 3.3, an optimal Drèze equilibrium corresponds to a competitive equilibrium allocation. It follows from the results in Laroque (1978) that for an open set of full Lebesgue measure of initial endowments, for every competitive equilibrium allocation x^* ,

$$x_l^{*i} - w_l^i \neq 0,$$

for every household i and every commodity l . Therefore, using Lemma 3.2, generically in initial endowments, cases where all $\underline{\mu}_l^i > 0$ and all $\bar{\mu}_l^i > 0$ are excluded. Generically in

endowments, an optimal Drèze equilibrium consists of competitive equilibrium prices and a competitive equilibrium allocation.

To complete the proof we need to show that for generic w the price system \bar{p} is not competitive. Let $z(p, w)$ denote aggregate excess demand at prices p and endowments $w = (w^1, \dots, w^N)$. Let $F_l(p, w)$ be equal to $z_l(p, w)$ for $l = 1, \dots, L$, and define $F_{L+1}(p, w) = p_L - \bar{p}_L$. If (p, w) is such that $z(p, w) = 0$, then the rank of the matrix $\partial_w z(p, w)$ is L , see Mas-Colell (1985), p. 227. Therefore, the rank of the system $\partial_{w, p_L} F(p, w)$,

$\partial_w z(p, w)$	$\partial_{p_L} z(p, w)$
0	1

is equal to $L + 1$ if $F(p, w) = 0$. By the Transversality Theorem, $\partial_{\tilde{p}} F^w(p)$ has full rank for almost all $w \in \mathbb{R}_{++}^{N(L+1)}$ if $F^w(p) = 0$, where $F^w(p) = F(p, w)$, $\tilde{p} = (p_1, \dots, p_L)$. Since the rank of $\partial_{\tilde{p}} F^w(p)$ can at most be L , generically in initial endowments, $z(p, w) = 0$ and $p_L = \bar{p}_L$ has no solution. Consequently, generically in initial endowments, the price system \bar{p} is not competitive. We conclude that for a set of endowments of full Lebesgue measure, all Drèze equilibria are suboptimal.

We now show that we can choose the set of initial endowments of full Lebesgue measure for which all Drèze equilibria are suboptimal to be open. Notice that the set of $(p, w) \in P \times \mathbb{R}_{++}^{N(L+1)}$ such that $F(p, w) = 0$ is closed due to the continuity of F . It follows from Balasko (1988), page 89, that the natural projection function, which maps (p, w) into w , is proper. This implies that the set of initial endowments, for which some competitive equilibrium price has its last component equal to \bar{p}_L , is closed, since the image of a closed set under a proper function is closed. The complement of this set is open and of full Lebesgue measure. The intersection of this set with the open set of full Lebesgue measure for which there is trade for every household for every commodity at a competitive equilibrium, is open and of full measure and contains only endowments with suboptimal Drèze equilibria.

□

The theorem gives a rigorous statement of the Folk Theorem on the generic suboptimality of equilibria at non-competitive prices. The next step is whether we can even strengthen this conclusion to generic constrained suboptimality.

4 Constrained optimality when $L = 1$

It is apparent that as long as prices are not competitive, full optimality is too much to be expected. The appropriate criterion in this case is the one of constrained optimality or p -optimality, that is optimality relative to all allocations for which trades of all households have zero value at an admissible price system p . The notion of p -optimality was introduced for the first time in Younès (1975).

Definition 4.1

Fix a price system $p \in P$. A feasible allocation $(x^1, \dots, x^N) \in \prod_{i \in \mathcal{N}} X^i$ is p -optimal if there is no allocation $(y^1, \dots, y^N) \in \prod_{i \in \mathcal{N}} X^i$ such that

- (i) $\forall i \in \mathcal{N} \quad py^i = pw^i$,
- (ii) $\sum_{i \in \mathcal{N}} y^i = \sum_{i \in \mathcal{N}} w^i$,
- (iii) $\forall i \in \mathcal{N} \quad u^i(y^i) \geq u^i(x^i)$ with strict inequality for at least one $i' \in \mathcal{N}$.

We start by analyzing the case with two commodities, so $L = 1$. Strict quasi-concavity of utility functions implies that the preferences of households over the set of all attainable amounts of good 1 given fixed prices and the budget constraint are single-peaked. In this case it is possible to show uniqueness and constrained optimality of a Drèze equilibrium.

Proposition 4.2

If $L = 1$, then a Drèze equilibrium $(\bar{x}^1, \dots, \bar{x}^N)$ at prices $p \in P$ of an economy \mathcal{E} is unique and p -optimal.

Proof. If $L = 1$, then quantity constraints are present only on the market of commodity 1. As far as an analysis of equilibrium is concerned, there is no loss of generality by indexing all relevant rationing schemes as a function of $\underline{q}, \bar{q} \in [0, 1]$:

$$\underline{z}_1(\underline{q}) = -\underline{q}(\sum_{i \in \mathcal{N}} w_1^i + \varepsilon),$$

$$\bar{z}_1(\bar{q}) = \bar{q}(\sum_{i \in \mathcal{N}} w_1^i + \varepsilon),$$

where ε is an arbitrarily chosen positive real number. The aggregate excess demand for commodity 1 at prices p and rationing parameters \underline{q}, \bar{q} is given by

$$z_1(\underline{q}, \bar{q}) = \sum_{i \in \mathcal{N}} d_1^i(\underline{z}_1(\underline{q}), \bar{z}_1(\bar{q}), p) - \sum_{i \in \mathcal{N}} w_1^i.$$

Since households face constraints either on demand, or on supply, but not on both of them, it is possible to represent all relevant rationing schemes by a single parameter $q \in [0, 1]$ as follows:

$$\tilde{z}_1(q) = z_1(\min\{2q, 1\}, \min\{2 - 2q, 1\}).$$

Here $q = 0$ corresponds to the full rationing on supply, $q = 1$ corresponds to the full rationing on demand, and when $q = 1/2$ there is no rationing at all. It is immediate that $\tilde{z}_1(0) \geq 0$ and $\tilde{z}_1(1) \leq 0$. The function \tilde{z}_1 is continuous and is easily shown to be weakly decreasing in q .

Proposition 4.2 is clearly true if p is competitive. If p is non-competitive, assume that $\tilde{z}_1(1/2) < 0$. Let q_s be the minimal value of q such that $\tilde{z}_1(q_s) = \tilde{z}_1(1/2)$. Since the rationing scheme $\tilde{z}_1(2q)$ is binding for at least one household for all $q \in [0, q_s)$, it is easy to see that the function \tilde{z}_1 is strictly decreasing on the interval $[0, q_s]$. Moreover, $\tilde{z}_1(0) \geq 0$ and $\tilde{z}_1(q_s) = \tilde{z}_1(1/2) < 0$. This implies that a Drèze equilibrium exists and is unique. Households on the short side of the market, the demand side in this case, are not rationed and get the most preferred consumption bundle they can reach under the given fixed prices, households on the long side cannot improve without making some other household worse off. Therefore, a Drèze equilibrium is p -optimal.

A similar argument applies when $\tilde{z}_1(1/2) > 0$.

□

Without any doubt the case $L = 1$ is special. We think it has some importance, as it is the case that is typically analyzed in textbooks. In Section 3 we have argued that optimality of equilibrium typically fails when $L = 1$. More precisely, we have argued above Proposition 3.3 that an equilibrium at non-competitive prices p is efficient for $L = 1$ if and only if the initial distribution of resources is efficient. Proposition 4.2 makes clear that a weaker notion of optimality, p -optimality holds for all equilibria.

5 Constrained suboptimality when $L \geq 2$

If L is greater than or equal to 2, the situation becomes different. Then a Drèze equilibrium is not necessarily p -optimal. Two counter-examples can be found in Böhm and Müller (1977). Using a modified Edgeworth box diagram, they showed that equilibria and constrained optima constitute two disjoint sets. At the same time, robust examples of constrained optimal Drèze equilibria can be easily found as well. Figure 1 shows such an example. There are three goods in an economy and two households endowed with the same amount of initial resources. The big triangle CDE corresponds to the set

$\{x^i \in X^i \mid px^i = pw^i\}$, where the price system p is fixed. The second household consumes its most preferred consumption bundle on CDE . The triangle $\bar{x}^1 AB$ corresponds to the constrained budget set of the first household. This household faces lower bounds on the net trade in the market for both commodities 1 and 2. An indifference curve through \bar{x}^1 is depicted. It is easily verified that (\bar{x}^1, \bar{x}^2) is a p -optimal Drèze equilibrium. Our next aim is to provide mild conditions that rule out examples like this one.

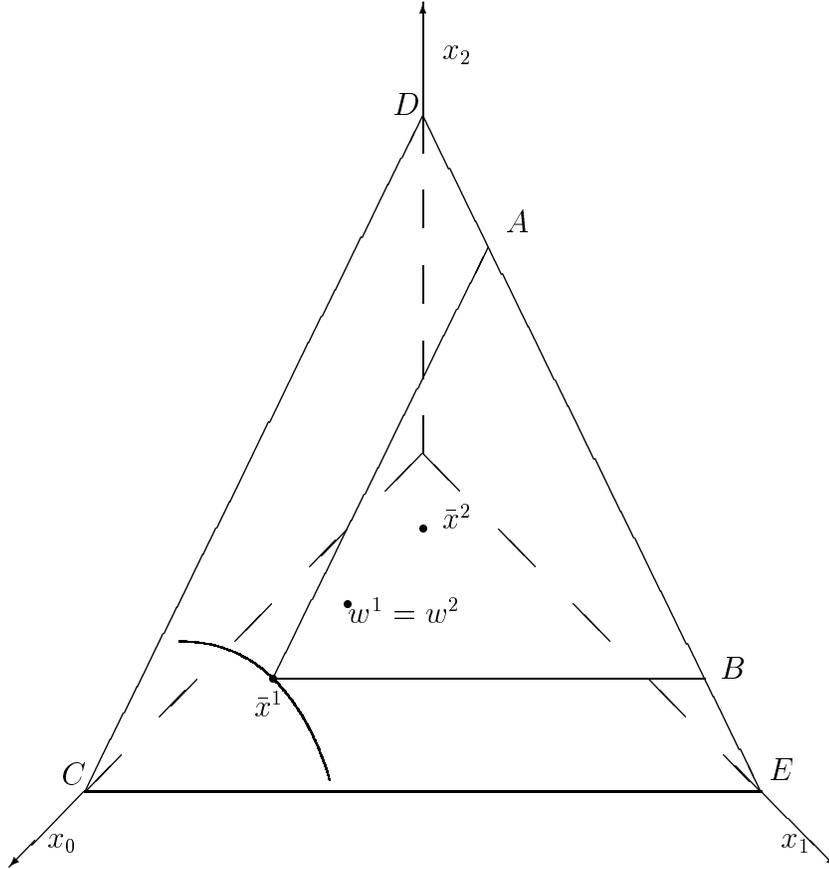


Figure 1: Example of a p -optimal equilibrium.

Let $p \in P$ be a fixed price system. To study the matter of constrained optimality, consider a “transformed” economy $\tilde{\mathcal{E}}$ with the same set of traders \mathcal{N} as in the original economy \mathcal{E} , and the set of goods $\tilde{\mathcal{L}} = \mathcal{L} \setminus \{0\}$. The economy $\tilde{\mathcal{E}}$ is derived from \mathcal{E} by using the budget equality to eliminate commodity 0. Initial endowments, consumption sets and utility functions of household $i \in \mathcal{N}$ are specified as follows:

$$\begin{aligned} \tilde{w}^i &= w_{-0}^i, \\ \tilde{X}^i &= \{(\tilde{x}_1^i, \dots, \tilde{x}_L^i) \in \mathbb{R}_{++}^L \mid \tilde{p}(\tilde{x}^i - \tilde{w}^i) \leq w_0^i\}, \\ \tilde{u}^i(\tilde{x}^i) &= u^i(w_0^i - \tilde{p}(\tilde{x}^i - \tilde{w}^i), \tilde{x}_1^i, \dots, \tilde{x}_L^i), \end{aligned}$$

where $\tilde{p} = (p_1, \dots, p_L)$. The first order conditions for an optimum of the economy $\tilde{\mathcal{E}}$ give the following characterization of a p -optimal allocation for \mathcal{E} . It holds that $(\bar{x}^1, \dots, \bar{x}^N) \in \prod_{i \in \mathcal{N}} X^i$ is a p -optimum if and only if there exist some $q \in \mathbb{R}^L \setminus \{0\}$ and $\alpha \in \mathbb{R}_+^N$ such that

$$(\partial_{x_i^l} u^i(\bar{x}^i) - p_l \partial_{x_0^i} u^i(\bar{x}^i))_{l=1, \dots, L} = \alpha^i q.$$

As we have seen before, at any Drèze equilibrium allocation \bar{x} ,

$$\partial_{x_i^l} u^i(\bar{x}^i) - p_l \partial_{x_0^i} u^i(\bar{x}^i) = -\underline{\mu}_l^i + \bar{\mu}_l^i,$$

for some non-negative real $\underline{\mu}_l^i, \bar{\mu}_l^i$ such that $\underline{\mu}_l^i \bar{\mu}_l^i = 0$. Let $\mathcal{N}^C \subseteq \mathcal{N}$ be the set of all constrained households, given a Drèze equilibrium $(\bar{x}^1, \dots, \bar{x}^N)$. Then $\alpha^i = 0$ implies that household i consumes its most preferred element of the set $\{x^i \in X^i \mid px^i = pw^i\}$. For each $i \in \mathcal{N}^C$, $\alpha^i \neq 0$. Therefore, if a Drèze equilibrium is p -optimal, then

$$\begin{aligned} q_l = 0 &\Rightarrow \underline{\mu}_l^i = \bar{\mu}_l^i = 0, \quad \forall i \in \mathcal{N}^C, \\ q_l > 0 &\Rightarrow \bar{\mu}_l^i > 0, \quad \forall i \in \mathcal{N}^C, \\ q_l < 0 &\Rightarrow \underline{\mu}_l^i > 0, \quad \forall i \in \mathcal{N}^C. \end{aligned}$$

The above allows us to formulate a necessary condition for a Drèze equilibrium to be p -optimal. We call this condition the separating property.

Proposition 5.1 (Separating property)

If a Drèze equilibrium is p -optimal, then every constrained household is constrained in every constrained market.

The separating property is quite powerful since it is stated in observable data only. Whenever there are two households that face constraints, but in different markets, constrained suboptimality is the case. The separating property is a very strong requirement, so very stringent conditions are needed to achieve constrained optimality. Notice that in the example in Figure 1, the separating property is satisfied.

The separating property is trivially satisfied if $L = 1$, or if there is only one constrained household. The first case has been analyzed in Section 4, where it has been concluded that constrained optimality results in the case with two commodities.

The vector q is also called a vector of coupons prices in the literature, see Drèze and Müller (1980). Note the one to one correspondence between the side of rationing and the sign of a component of a coupons price vector for p -optimal Drèze equilibria.

We already argued that the separating condition is strong, and if satisfied, only a necessary condition. The next result gives conditions for the separating condition to be sufficient for p -optimality.

Theorem 5.2

Any Drèze equilibrium at prices p of an economy \mathcal{E} with the number of constrained households or the number of constrained markets less than or equal to one, is p -optimal.

Proof. In the case where the number of constrained households or the number of constrained markets equals zero, the Drèze equilibrium corresponds to a competitive equilibrium allocation, so optimality and therefore p -optimality follows.

Suppose the number of constrained households equals one. Then define $\alpha^i = 1$ and $q_l = -\underline{\mu}_l^i + \bar{\mu}_l^i$, $l = 1, \dots, L$, with $\underline{\mu}_l^i$, $\bar{\mu}_l^i$, the Lagrange multipliers corresponding to the rationing constraints of the constrained household i . It follows that the condition for p -optimality is satisfied.

Suppose the number of constrained markets equals one, say market l is constrained. Then define $\alpha^i = -\underline{\mu}_l^i + \bar{\mu}_l^i$, $i \in \mathcal{N}$, and define q to be the l -th unit vector in \mathbb{R}^L . Again, it follows that the condition for p -optimality is satisfied. □

Our final claim is that under weak conditions, the separating property is typically not a sufficient condition for constrained optimality. More precisely, Drèze equilibria for which the set of constrained households \mathcal{N}^C and the set of constrained markets \mathcal{L}^C consist of more than one element each, are generically not p -optimal.

Theorem 5.3

Fix any price system $p \in P$ and utility functions u^1, \dots, u^N satisfying Assumption A2. There is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}_{++}^{N(L+1)}$ such that every Drèze equilibrium at prices p of the economy \mathcal{E} with the number of constrained households and the number of constrained markets greater than or equal to two, is not p -optimal.

Proof. It is helpful to introduce a vector r that describes the state of the markets. The vector r is an element of

$$R = \{r \in \mathbb{R}^L | r_l = -1, 0, \text{ or } 1\},$$

where $r_l = -1$ if there is supply rationing in market l , $r_l = 0$ if there is no rationing in market l , and $r_l = 1$ if there is demand rationing in market l . We also introduce a vector s that describes whether a household is rationed or not. The vector s is an element of

$$S = \{s \in \mathbb{R}^N | s^i = 0 \text{ or } 1\},$$

where $s^i = 1$ if and only if i belongs to the set of constrained households \mathcal{N}^C .

Denote by $c_l^i(x^i)$ the derivative with respect to x_l^i of the “transformed” utility function \tilde{u}^i we used before:

$$c_l^i(x^i) = \partial_{x_l^i} u^i(x^i) - p_l \partial_{x_0^i} u^i(x^i).$$

We know from Lemma 3.2 that for any Drèze equilibrium $(\bar{x}^1, \dots, \bar{x}^N) \in \prod_{i \in \mathcal{N}} X^i$,

$$c_l^i(\bar{x}^i) = -\underline{\mu}_l^i + \bar{\mu}_l^i,$$

for some non-negative $\underline{\mu}_l^i, \bar{\mu}_l^i$ — the Lagrange multipliers corresponding to the rationing constraints. If a Drèze equilibrium \bar{x} is p -optimal, then for any $l, l' \in \mathcal{L}$, $i, i' \in \mathcal{N}$,

$$\begin{vmatrix} c_l^i(\bar{x}^i) & c_l^i(\bar{x}^{i'}) \\ c_{l'}^i(\bar{x}^i) & c_{l'}^i(\bar{x}^{i'}) \end{vmatrix} = \begin{vmatrix} -\underline{\mu}_l^i + \bar{\mu}_l^i & -\underline{\mu}_{l'}^{i'} + \bar{\mu}_{l'}^{i'} \\ -\underline{\mu}_{l'}^i + \bar{\mu}_{l'}^i & -\underline{\mu}_l^{i'} + \bar{\mu}_l^{i'} \end{vmatrix} = \begin{vmatrix} \alpha^i q_l & \alpha^{i'} q_l \\ \alpha^i q_{l'} & \alpha^{i'} q_{l'} \end{vmatrix} = 0.$$

For given $r \in R$ and $s \in S$, consider the sets

$$M_{rs} = \{(\mu^i)_{i \in \mathcal{N}^C} \in \mathbb{R}^{|\mathcal{N}^C| |\mathcal{L}^C|} \mid \mu_l^i < 0 \text{ if } r_l = -1, \mu_l^i > 0 \text{ if } r_l = 1\},$$

$$Z_r = \{z \in \mathbb{R}^{|\mathcal{L}^C|} \mid z_l < \varepsilon \text{ if } r_l = -1, z_l > -\varepsilon \text{ if } r_l = 1\},$$

where ε is some given positive number.

By the Kuhn-Tucker theorem, if (x^1, \dots, x^N) is a Drèze equilibrium that satisfies the separating property, then there exists $r \in R$, $s \in S$, $\lambda \in \mathbb{R}_{++}^N$, $\mu \in M_{rs}$, and $z \in Z_r$ such that¹

$$px^i - pw^i = 0, \quad i \in \mathcal{N}, \quad (1)$$

$$\partial_{x_0^i} u^i(x^i) - \lambda^i = 0, \quad i \in \mathcal{N}, \quad (2)$$

$$\partial_{x_l^i} u^i(x^i) - \lambda^i p_l - \mu_l^i = 0, \quad i \in \mathcal{N}, \quad l \in \mathcal{L} \setminus \{0\}, \quad (3)$$

$$\sum_{i \in \mathcal{N}} (x_l^i - w_l^i) = 0, \quad l \in \mathcal{L} \setminus \{0\}, \quad (4)$$

$$x_l^i - w_l^i - z_l = 0, \quad i \in \mathcal{N}^C, \quad l \in \mathcal{L}^C. \quad (5)$$

The number of unknowns in this system equals $N(L+2) + |\mathcal{N}^C| |\mathcal{L}^C| + |\mathcal{L}^C|$, which is less than the number of equations $N(L+2) + |\mathcal{N}^C| |\mathcal{L}^C| + L$, or is equal to it if $|\mathcal{L}^C| = L$. Since there is a finite number of constrained markets and households, there is a finite number of such systems. Since a finite intersection of open sets of full Lebesgue measure is open and of full Lebesgue measure, it is enough to restrict attention to an arbitrary fixed r and s .

¹Notice that a Drèze equilibrium always leads to a solution to the system of equations. The other way around is not necessarily true, since non-binding inequality constraints have been omitted, and the definition of Z_r implies that a limited amount (ε) of forced trading is not excluded in a solution to the system of equations.

Suppose that (x^1, \dots, x^N) is a p -optimal Drèze equilibrium. Without loss of generality, $\{1, 2\} \subseteq \mathcal{N}^C$ and $\{1, 2\} \subseteq \mathcal{L}^C$. By the separating property and the equation in determinants derived before,

$$\mu_1^1 \mu_2^2 - \mu_2^1 \mu_1^2 = 0. \quad (6)$$

Thus, p -optimal Drèze equilibria satisfy a system of n equations, where

$$n = (N + 1)(L + 2) + |\mathcal{N}^C| |\mathcal{L}^C| - 1,$$

which is at least one more than there are unknowns.

Let $\psi(w, x, z, \lambda, \mu)$ be the function defined as the left-hand side of the equations (1) – (6). It is defined on $\prod_{i \in \mathcal{N}} X^i \times \prod_{i \in \mathcal{N}} X^i \times Z_r \times \mathbb{R}_{++}^N \times M_{rs}$ and takes its values in $\mathbb{R}^{(N+1)(L+2)+|\mathcal{N}^C||\mathcal{L}^C|-1}$. The key element of the proof is the fact that ψ is transversal to the origin, i.e. whenever $\psi(w, x, z, \lambda, \mu) = 0$, its Jacobian is of full rank.

Suppose that there exists $y \in \mathbb{R}^n$ such that

$$y^T \partial \psi(\cdot) = 0.$$

We write $y = (y_1, \dots, y_6)$, where $y_1 \in \mathbb{R}^N$, $y_2 \in \mathbb{R}^N$, $y_3 \in \mathbb{R}^{NL}$, $y_4 \in \mathbb{R}^L$, $y_5 \in \mathbb{R}^{|\mathcal{N}^C||\mathcal{L}^C|}$, and $y_6 \in \mathbb{R}$. Then, in particular,

$$y^T \partial_{w_0^i} \psi(\cdot) = -p_0 y_{1,i} = 0,$$

so $y_1 = 0$. If $l \in \mathcal{L} \setminus \mathcal{L}^C$, then taking into account the previous expression one gets

$$y^T \partial_{w_1^l} \psi(\cdot) = -y_{4,l} = 0.$$

For $l \in \mathcal{L}^C$ we have

$$\begin{aligned} y^T \partial_{w_i^i} \psi(\cdot) &= -y_{4,l} - y_{5,i,l} = 0, & \text{for } i \in \mathcal{N}^C \\ y^T \partial_{w_i^i} \psi(\cdot) &= -y_{4,l} = 0, & \text{for } i \in \mathcal{N} \setminus \mathcal{N}^C, \end{aligned}$$

and

$$y^T \partial_{z_l} \psi(\cdot) = - \sum_{i \in \mathcal{N}^C} y_{5,i,l} = 0.$$

Thus, $\sum_{i \in \mathcal{N}^C} y_{5,i,l} = |\mathcal{N}^C| y_{4,l}$, so $y_{4,l} = 0$, $l \in \mathcal{L}^C$. Therefore, $y_{5,i,l} = 0$, $i \in \mathcal{N}^C$, $l \in \mathcal{N}^C$. Moreover,

$$y^T \partial_{\mu_1^1} \psi(\cdot) = y_6 \mu_2^2 = 0,$$

so $y_6 = 0$. To complete the proof of regularity it is sufficient to show that the matrix

$$(\partial^2 u^i(\bar{x}^i), -p^\top)$$

has a full row rank for every $i \in \mathcal{N}$. This follows from the differentiable strict quasiconcavity of the utility function, Proposition 2.6.4 of Mas-Colell (1985), and the possibility to cover a consumption set by a countable number of compacts.

By the Transversality Theorem, $\partial_{(x,z,\lambda,\mu)}\psi^w(x,z,\lambda,\mu)$ has full rank for almost all $w \in \mathbb{R}_{++}^{N(L+1)}$ if $\psi^w(x,z,\lambda,\mu) = 0$, where $\psi^w(x,z,\lambda,\mu) = \psi(w,x,z,\lambda,\mu)$. Therefore, generically in w , the inverse image of $\{0\}$ has the same co-dimension as zero, which implies that $\psi^w(x,z,\lambda,\mu) = 0$ has no solution. We conclude that for a set of endowments of full Lebesgue measure, any Drèze equilibrium is constrained suboptimal.

Denote by S the set $\prod_{i \in \mathcal{N}} X^i \times Z_r \times \mathbb{R}_{++}^N \times M_{rs}$. To show that we can choose the set of initial endowments of full Lebesgue measure for which Drèze equilibria are constrained suboptimal to be open, consider the set Σ of all $(w,x,z,\lambda,\mu) \in \mathbb{R}_{++}^{N(L+1)} \times S$ such that $\psi(w,x,z,\lambda,\mu) = 0$. This set is closed by continuity of ψ . Moreover, it is not difficult to see that the set $\{(w,x,z,\lambda,\mu) \in \Sigma \mid w \in K\}$ is compact for any compact subset K of $\mathbb{R}_{++}^{N(L+1)}$. The latter means that the natural projection function $\pi : \Sigma \rightarrow \mathbb{R}_{++}^{N(L+1)}$ that maps (w,x,z,λ,μ) to w is proper. Therefore, the set of all w for which the conclusion of the theorem does not hold, is closed as the image of a closed set by a proper function. Its complement is open and, as has been shown before, contains a set of full Lebesgue measure. \square

The condition that the number of constrained markets is greater than or equal to two may be omitted from the statement of Theorem 5.3, since it is a generic property when $L > 1$. The proof of this fact goes along the same lines as the proof of the theorem above. Thus, we have the following corollary.

Corollary 5.4

Fix any price system $p \in P$ and utility functions u^1, \dots, u^N satisfying Assumption A2. There is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}_{++}^{N(L+1)}$ such that every Drèze equilibrium at prices p of the economy \mathcal{E} with the number of constrained households greater than or equal to two, is not p -optimal.

It is not possible to claim that the number of constrained households is generically greater than one. For any tuple of utility functions u^1, \dots, u^N satisfying Assumption A2, there is an open set of initial endowments $\Omega \subset \mathbb{R}_{++}^{N(L+1)}$ such that for every $w \in \Omega$ there is a Drèze equilibrium with only one constrained household. The example is constructed in such a way that the constrained household is constrained on its supply in all markets, whereas all other households have small net demands for all non-numeraire commodities.

Consider any tuple of utility functions u^1, \dots, u^N satisfying Assumption A2 and fix an arbitrary price system $p \in P$. Let \bar{x}^i be the unconstrained demand of household i at prices p when it has initial endowments e , where e is the vector of all ones in \mathbb{R}^{L+1} . Pick

initial endowments for household 1 and a rationing scheme (\underline{z}, \bar{z}) such that household 1, at prices p and rationing scheme (\underline{z}, \bar{z}) , is constrained on its supply in each market for non-numeraire commodities and $-\underline{z}$ is smaller than \bar{x}^i for $i = 2, \dots, N$. To achieve this, one may take $w^1 \in \{w \in X^1 \mid pw = pe\}$ such that $w_{-0}^1 \gg \bar{x}_{-0}^1$, so the unconstrained demand of household 1 at prices p equals \bar{x}^1 , and household 1 prefers to supply all non-numeraire commodities. Take $\underline{z}_1^1 = \bar{x}_1^1 - w_1^1 + \varepsilon^1$, with ε^1 a small positive number. By continuity, the demand \tilde{x}^1 of household 1 when taking \underline{z}_1 into account is close to \bar{x}^1 , in particular all non-numeraire commodities are still supplied by household 1, and rationing in the market for commodity 1 is binding. Take $\underline{z}_2 = \tilde{x}_2^1 - w_2^1 + \varepsilon^2$, with ε^2 a positive number that is small enough for rationing in the market for commodity 1 to remain binding. This construction is repeated until the rationing scheme \underline{z} is obtained that induces rationing on the supply of household 1 in the markets for all non-numeraire commodities. By taking w^1 sufficiently close to \bar{x}^1 , the requirement that $-\underline{z}$ be smaller than \bar{x}^i , $i = 2, \dots, N$, can be fulfilled as well. The rationing scheme \bar{z} is chosen as never to affect the choice of any household.

For $i = 2, \dots, N$, initial endowments are taken such that household i is on the short side of each market,

$$w^i = \bar{x}^i - \begin{pmatrix} \sum_{l=1}^L p_l \underline{z}_l / (N-1) \\ -\underline{z}_1 / (N-1) \\ \vdots \\ -\underline{z}_L / (N-1) \end{pmatrix}.$$

Since all households, but household 1, get their most preferred commodity bundle at prices p , it follows that $(d^1(\underline{z}, \bar{z}, p), \bar{x}^2, \dots, \bar{x}^N)$ is a p -optimal Drèze equilibrium. If we slightly perturb the initial endowments, total net demand of households excluding household 1 changes slightly, and a Drèze equilibrium is obtained by rationing the supply of household 1 by this amount. All other households remain unconstrained. The property of p -optimality is obviously kept. It follows that there is an open set of initial endowments with p -optimal Drèze equilibria.

6 Conclusions

Notwithstanding the strong conviction of most economists that trade at non-competitive prices has detrimental welfare consequences, it is not based on foundations derived with equal rigor as the first and second welfare theorems. This paper provides these foundations. We show that the Folk Theorem holds that equilibria are typically suboptimal when trade at non-competitive prices occurs.

The more appropriate question to answer is whether equilibria are perhaps not even constrained optimal. In this paper we formalized the notion of constrained optimality as optimality among allocations where budget constraints of all households at the non-competitive prices are met. A necessary condition for constrained optimality to prevail is the separability condition that all constrained households be constrained on all constrained markets. This condition is of interest in itself as it is formulated in terms that are observable. If the number of commodities is less than or equal to two, then this necessary condition is always satisfied. We show that in this case it is also a sufficient condition, so constrained optimality holds in the two commodity case. When there are three or more commodities, then this sufficient condition does typically not hold at equilibrium. As a consequence, even the stronger Folk Theorem, that equilibria are typically constrained suboptimal when trade at non-competitive prices occurs, is true as well under reasonable assumptions.

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