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Abstract

A model in which agents on both sides of the market are subject to informational cascades is examined. In an uncertain environment with asymmetric information agents tend to be overoptimistic about the state of the world, a result that fits with empirical evidence on financing new technologies. This overoptimism based on mutual illusions makes the system vulnerable to two-sided bubbles, and may be one of the reasons behind "dot com" crash.

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1. Introduction

Many authors writing on human behaviour have noticed that individual decisions are often influenced by decisions made by others. They have documented a number of situations in which individuals prefer to follow the ‘crowd’, while their own feelings are against it. There is a range of different social mechanisms, which may cause conformist behaviour of individuals such as punishment of deviators (Akerlof, 1980), positive payoff externalities (Arthur, 1989) and so on. It is also possible that herding arises as consequence of the bounded rationality of individuals (Shiller, 1989).

In the last decade there has been a surge of interest in a particular kind of mechanism behind conformist behaviour, which can explain *voluntary rational* ‘herding’. After the seminal works of Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992) (BHW henceforth), Welch (1992) this social phenomenon is often referred to as an ‘informational cascade’.

The basic idea of informational cascades is that in certain environments where private information can be revealed only through individual actions because of information externalities, truly rational agents may find it optimal to follow the choice of others, rejecting their own information. Perhaps the most striking feature of informational cascades is that when there is noise in private information there is always a positive probability that the overall outcome will be suboptimal, i.e. agents will form a cascade in which they reject optimal actions in favour of inferior ones, regardless of their private information. Therefore, information structures that are vulnerable to information cascades in this way can have negative effects on social welfare.

The original BHW model has been extended in a number of ways and the robustness of the model with respect to changes in assumptions has been examined. The informational cascade framework has been used to explain a wide range of social phenomena such as fads, fashions, medical (mal)practice, collapse of political regimes among the others (see (Bikhchandani et al. 1998) for a review). There are also some important applications of this kind of model in economics and finance. Welch (1992) applied informational cascades to the IPO market and explains underpricing incentives of issuers. Avery and Zemsky (1998) argued that short-run mispricing on financial markets might be a consequence of investors’ herd behaviour.

Most of the informational cascade models assume that the agents forming a cascade are the same either with respect to the sort of information available to them, or with respect to roles they play, or both. For instance, in Avery and Zemsky’s model of financial market investors have different roles: some of them are sellers and others are buyers of assets, but the sort of information available to agents is essentially the same.

However, there is no *a priori* reason to suppose that the agents on the two sides of the market have identical information sets, and modelling some situations may require us to take into account that two sides of the market having access to different information. Consider financing new technology. It is widely recognized today that one of the main problems of external financing in new high-tech industries is information asymmetry between firms developing new technologies and their potential investors. On the one hand, financial institutions and individual investors often do not have enough expertise to judge 'state-of-the-art' technologies. On the other hand, firms working in new industries, especially new small ventures, which mainly contribute to development of 'at-the-edge' technologies, have problems with evaluating both market and financial potential for the products they are developing. It seems quite reasonable to assume that investors have better knowledge of the *market perspectives* for new technologies, while firms know the *technology* with which they are working. This is an example of the situation in which information sets available to the opposite sides of the market are different.

There is also no *a priori* reason to believe that only one side of the market is subject to information cascades. In our example of financing new technology the process of acquiring information about the technology and about the market for new products is costly, and the information has commercial value. As a consequence, agents have incentives not to share their private information and so spillovers of this kind of knowledge are limited, at least in the short term. Nevertheless, their actions, or 'outcomes' arising from the actions, in many cases are observable and it may create conditions necessary for information cascades. This argument is valid for both entrepreneurs and venture capitalists. Hence we may expect informational cascades on the two sides.

In this paper we examine a simple setting of two-sided informational cascades and show that taking into account both sides of the market with different information sets may generate interesting learning dynamics on both sides of the market and emphasise the conclusion about suboptimality of some information structures.

2. The model

The model captures the features of two-side interactions described above. Consider two populations of agents: the population of potential investors, whom we call venture capitalists (VCs) and the population of entrepreneurs (EPs). Success of a project is determined by two factors: by technology itself (e.g. quality of the product if the project involves product innovation, productivity gains if it is about process innovation), and by the market prospects for the new technology (e.g. its prospective demand). We assume that EPs observe the state of technology E, while VCs observe the market prospects for the new technology V; but neither do EPs know V, nor do VCs know E. That is, entrepreneurs and venture capitalists have different information sets.

There is a market place, where an EP meets with a VC to discuss the potential project. From the discussion they attempt to infer what their counterparts know. We can model agents' subjective 'guesses' about the content of their partners' knowledge as private signals of limited precision. Besides private signals, agents can observe the results of negotiations of all previous pairs. On the basis of this information each of the parties independently decides whether to make a deal or not. The project goes ahead only if both sides agree to participate. Information about whether the project goes ahead or not becomes public knowledge.

As in BHW we assume V and E be binary variables: $E \in \{h,l\}$, $V \in \{H,L\}$, with equal prior probabilities of ½ . Each period t=1,2,... a pair of agents EP_t, VC_t meets. The t-th entrepreneur, EP_t, observes a conditionally independent identically distributed signal about state of V, $v \in \{H,L\}$, and the t-th venture capitalist, VC_t, gets a signal about E, $e \in \{h,l\}$. The signal probabilities are the same as in the model of BHW (Table 1, $p,q > ½$).

Table1.

Signal probabilities for EPs

	$\Pr(v=H\|V)$	$\Pr(v=L\|V)$
$V=H$	p	$1-p$
$V=L$	$1-p$	p

Signal probabilities for VCs

	$\Pr(e=h\|E)$	$\Pr(e=l\|E)$
$E=h$	q	$1-q$
$E=l$	$1-q$	q

If any of the agents chooses *Reject*, then the deal fails and the payoff to both of them will be 0 (the 'status quo' value. If both *Agree* to launch the project, they will get payoffs according to the following payoff matrix (Table 2)

Table 2.

Payoff matrix (EP,VC)

	$V=H$	$V=L$
$E=h$	(1, 1)	(-1, 1)
$E=l$	(1,-1)	(-1,-1)

The rationales behind signs of payoffs are as follows: whenever one the parties has low state of market it may free ride on the counterpart if the opposite side of the market is high. If both sides are mediocre, both parties lose, while if both the technology and market perspectives are in the high state, then both parties gain from the project.

We distinguish between 'actions' and 'outcomes'. The set of possible actions for every agent is {*Agree, Reject*}. The set of outcomes for every pair (EP,VC) is {*Proceed, Fail*}. But notice that the outcome is determined by a pair of actions, so that even if, for example every entrepreneur *Agree*s, all projects would *Fail* if every venture capitalist *Reject*s. We denote outcome of the negotiations at time t as Z_t: if the project *Proceed* then we assign $Z_t = 1$, otherwise $Z_t = -1$. The outcome of the negotiations becomes public knowledge.

3. Analysis

Before we proceed to the analysis of the model, it worth clarifying our notions of informational cascades. By the term '**1-cascade**' we refer to one-sided informational cascade of the BHW-type: the situation where agents on one side of the market follow their 'herd' rejecting private information. Hence term '1-cascade' describes agents' actions. In BHW model there is one-to-one relationship between the agents' actions and the outcomes of those actions. As a result, in the 'standard' model, an informational cascade related to agents' behavioural pattern always leads to a series of uniform outcomes.

Things are different for our two-sided model. Here an outcome is a function of actions taken by *both* parties, and there is no one-to-one correspondence between actions and outcomes. Observing a failure (outcome) one can cannot infer who rejected the deal (agents' actions).

'Behavioural' 1-cascades do not necessarily lead to cascades in outcomes. Suppose that EPs are in an *UP* 1-cascade, i.e. they choose *Agree* on the deal whatever is their private signal. Does it result in a series of positive outcomes (successful deals)? Not necessarily. For a deal to be settled both sides must agree to launch the project. Therefore, if VCs are not in an *UP* 1-cascade, outcomes may be negative (*Fail*). Therefore for a two-sided *UP* cascade both sides must be in *UP* 1-cascades. This is a necessary condition for two-sided *UP* cascades. But it is not sufficient as we will see further when discuss *revelation*.

In contrast with an *UP* 1-cascade, a *DOWN* 1-cascade always leads to a series of negative outcomes. Therefore we can state that a *DOWN* 1-cascade is a sufficient condition for a two-sided *DOWN* cascade. However, it is not a necessary condition: there are some situations in which never-ending series of negative outcomes start when there is no *DOWN* 1-cascade. Again, we will examine these 'effective' two-sided *DOWN* cascades, in the discussion of *revelation*.

Taking this into account, we define an *UP* (*DOWN*) **2-cascade** as series of successful deals (failures), which cannot be broken (probably with exception of one revealing outcome) by any combination of private signals.

Benchmark model. To highlight the ways in which two-sided cascade model defined above differs from 'standard' models, we introduce a 'benchmark' model. In the benchmark model we change the information structure, so that cascades of EPs and VCs become independent.

Let agents observe not only *outcomes* of negotiations but also *actions* of their predecessors (on the both sides).

For our payoff matrix, the payoff to an agent depends only on the state of the other side of the market. An entrepreneur gets 1, if the market prospects are high ($V=H$), and -1 if the prospects are low ($V=L$), whatever is the state of the technology. Similarly, the venture capitalist's payoff is determined by the state of technology. Obviously agents' actions depend entirely on their believes about the state of the opposite side, and do not depend on the state of their side. Since VCs actions are independent from V and driven only by their believes about E, the information about actions of preceding VCs has no value for an entrepreneur. His belief is built only on the history of entrepreneurs' actions[1]. The same can be said about a VC: he counts only preceding actions of VCs. This is exactly what happens in the BHW model. Therefore, 1-cascades of the benchmark model are the same as informational cascades in the BHW model.

A two-sided *UP* cascade in benchmark model corresponds to simultaneous EPs' and VCs' *UP* 1-cascades. A two-sided *DOWN* cascade happens whenever any side is in a *DOWN* 1-cascade.

The benchmark is a convenient tool for examination of our model of observable outcomes. On the one hand, as our model the benchmark is two-sided. On the other hand, the benchmark's information structure is such that both sides make their decisions independently and 2-cascades are just superposition of one-sided informational cascade of the BHW model.

3.1. Minimum series

We start analysis of our model by examining '*minimum series*', the shortest series that result in informational cascades, for the two reasons. First, as in the BHW model (and hence benchmark) the minimum series give the prime contribution into the probabilities to end up in cascades. Second, our approach for examination of minimum series will be extended further to analyse the general case.

[1] Note, that since outcomes arise from the action of both sides, in the environment where only *outcomes* are public knowledge, entrepreneur's belief is affected by the actions of preceding entrepreneurs.

Consider the first pair. The information available to agents EP_1 and VC_1 consists of their private signals only. EP_1's expected payoff to is

$$\pi EP_1 = \Pr(e_1 = h \mid E) \cdot [\Pr(V = H \mid v_1) - (1 - \Pr(V = H \mid v_1))] + 0 =$$
$$= \Pr(e_1 = h \mid E) \cdot [2\Pr(V = H \mid v_1) - 1].$$

As in BHW, Bayes' formula and Table 1 give posterior

$$\Pr(V = H \mid v_1) = \begin{cases} \frac{p/2}{p/2 + (1-p)/2} = p, & \text{if } v_1 = H, \\ \frac{(1-p)/2}{(1-p)/2 + p/2} = 1 - p, & \text{if } v_1 = L. \end{cases}$$

Since by assumption $p>1/2$, if $v_1=H$, then expected payoff $\pi EP_1>0$, and *Agree* is optimal; if $v_1=L$, then $\pi EP_1<0$ and the entrepreneur is better-off refraining from the project. The same is true for VC_1. Therefore the decision rule of the first agent in the BHW model (we can call it "follow your signal"): if the signal is favourable, then an agent chooses *Agree*, else *Reject*, is applied here. This strategy profile is an equilibrium.

There is another equilibrium, in which both agents always *Reject* regardless of the available information. However, this equilibrium is not stable with respect to 'trembling hand'. Indeed, suppose that the private signal is favourable. If there is any small, but non-zero probability that the other party plays *Agree*, then *Agree* is a strictly better strategy than *Reject*, i.e. (*Reject*, *Reject*) is no longer an equilibrium. Taking this into account, we will not analyse this equilibrium further.

3.1.1. Emergence of UP 2-cascade

Suppose that the first pair sets a deal, $Z_1=1$. Then the second pair unambiguously infers that $v_1 e_1=Hh$ (else one of the agents would have chosen *Reject* and the outcome would have been negative, $Z_1= -1$). If the private signal to the second entrepreneur is favourable $v_2=H$, he chooses *Agree*. If $v_2=L$, then he is indifferent between *Agree* of *Reject*. We assume that an indifferent agent chooses actions according to his private signal (an agent believes more in his own intuition).[2] Under this assumption, after a successful deal at $t=2$ ($Z_2=1$), agents infer that $v_2 e_2=Hh$. Even if an agent from the third pair got an adverse signal he would choose to join to the 'herd', because the signal is offset by the 'positive news' and his expected payoff of *Agree* is positive. The same would do his counterpart. Therefore, as in benchmark model the sides may start *UP* 2-cascade from $t=3$ for any values of p, $q > ½$.

[2] This kind of tie-breaking rule was employed by Anderson and Holt in their experimental study of informational cascades (1997).

3.1.2. Emergence of *DOWN* 2-cascade

***DOWN* 1-cascade.** Consider the case of Z_1= -1. The deal may fail if one of the following pairs of signals happens: v_1e_1=*Hl*, *Lh*, or *Ll* (i.e. any combination of signals except *Hh*). Thus the conditional probability of failure is $\Pr(Z_1= -1|V,E) = 1 - p_vq_e$, where $p_v = \Pr(v=H|V)$ is the conditional probability of a favourable signal about the market given that the true state of the market is *V*, i.e. $p_v = p$, if $V=H$, and $p_v = 1 - p$ when $V=L$; and $q_e = \Pr(e=h|E)$ is the conditional probability of a VC's positive signal about technology given *E*. Note, that unlike the case of $Z_1 = 1$, one cannot unambiguously infer private signals, v_1 and e_1, or put differently, $Z_1 = -1$ is less informative than Z_1= 1. It is easy to see, that EP_2 and VC_2 follow their signals.

Let $Z_2 = -1$. Again, the deal may fail only if v_2e_2 =*Hl*, or *Lh*, or *Ll*. The conditional probability of double failure is $\Pr(Z_1= Z_2= -1|V,E) = (1 - p_vq_e)^2$.

We denote the 'history', the list of outcomes in all preceding periods 1,..,*t* -1, as I_t, i.e. $I_t =\{Z_1, Z_2,.., Z_{t-1}\}$. To make our analysis easier we will use the following remark.

Remark: *Given the state of the technology be E, EP_t follows the 'herd' (DOWN 1-cascade) if and only if*[3]

$$p\Pr(I_t \mid V=H,E) < (1-p)\Pr(I_t \mid V=L,E). \tag{1}$$

Suppose that $v_t = H$. We will show that if (1) holds, then in spite of his favourable signal EP_t *Reject*s the deal. If EP_t agreed, according to Table 2 his expected payoff given I_t, and v_t would be

$$\pi EP_t = \Pr(V=H,E \mid I_t,v_t) - \left(1-2\Pr(V=H,E \mid I_t,v_t)\right) = 2\Pr(V=H,E \mid I_t,v_t) - 1$$

It is negative *iff* $\Pr(V=H,E \mid I_t,v_t) < ½$. Applying Bayes' formula we rewrite this inequality as

$$\Pr(V=H,E \mid I_t,v_t) = \frac{\Pr(I_t,v_t \mid V=H,E)\frac{1}{2}}{\Pr(I_t,v_t \mid V=H,E)\frac{1}{2} + \Pr(I_t,v_t \mid V=L,E)\frac{1}{2}} < \frac{1}{2},$$

which is equivalent to

$$\Pr(I_t,v_t \mid V=H,E) < \Pr(I_t,v_t \mid V=L,E).$$

Since the signals are independent, $\Pr(I_t ,v_t \mid V,E) = \Pr(v_t \mid V) \Pr(I_t |V,E)$. Taking this into account, the latter inequality can be rewritten as (1). It means that when (1) holds, EP_t's expected payoff is negative. If VC_t chooses *Agree* the entrepreneur is (*ex-ante*) strictly better off choosing *Reject* in spite of his positive private signal, and he is indifferent if VC_t plays *Reject*. But the venture capitalist is in the same situation, and chooses *Reject* regardless of his signal, and (*Reject*,*Reject*) is an equilibrium. Therefore, both join the 'herd'.

[3] *revelation* is not considered here

Let $I_t = \{Z_1 = .. = Z_{t-1} = -1\}$. To find the period $t=n+1$ starting from which EPs may start *DOWN* 1-cascade we apply the Remark:

$$p(1-pq_e)^n < (1-p)(1-(1-p)q_e)^n,$$

which is equivalent to

$$q_e > \Phi_{1/n}(p) \equiv \frac{p^{1/n} - (1-p)^{1/n}}{p^{1/n+1} - (1-p)^{1/n+1}}. \tag{2}$$

$\Phi_{1/n}(p)$ is continuous, monotonously increasing and convex on (0.5,1). The values of $\Phi_{1/n}(p)$ at the ends of the interval are: $\Phi_{1/n}(0.5) = 2/(n+1)$ and $\Phi_{1/n}(1) = 1$.

Let examine the case of $E=h$. By definition, $q_e = \Pr(e=h| E=h) = q$, therefore the inequality (2) has the form $q > \Phi_{1/n}(p)$. Given the number of failures, n, the condition can be represented graphically (Fig 1a). The curve $q = \Phi_{1/n}(p)$ divides the range of all possible values of p and q ($0.5 < p,q < 1$) in the two parts: 'cascade' region D and 'non-cascade' region N. For (p,q) from D the entrepreneur (EP_{n+1}) joins the herd, even if he would get a positive signal, while for (p,q) from the non-cascade region, EP_{n+1} always follows his signal. For $n \geq 2$, the value $\Phi_{1/n}(0.5) = 2/(n+1) < 1$ and D is non-empty. Therefore, there are always p and q for which a cascade can start after two failures.

When the length of series of failures, n, increases, the curve $q = \Phi_{1/n}(p)$ moves toward the south-east corner of the diagram expanding 'cascade' region D, as it is shown on the Fig. 1a, and as $n\rightarrow\infty$ 'cascade' region D tends to expand over the whole range of possible p and q.

In the case of $E=l$, the conditional probability of the favourable signal is $q_e = \Pr(e=h| E=h) = 1- q$. We can rewrite (2) as $q < 1 - \Phi_{1/n}(p)$. Area D at Fig.1b represents the pairs of parameters (p,q) for which EPs, given the series of n failures, start a *DOWN* 1-cascade. In contrast with $E=h$ case, the 'cascade' region D is situated in the southwest corner of the diagram, where p and q are relatively small. The maximum value of $(1 - \Phi_{1/n}(p))$ on $p\in(0.5,1)$ is achieved at $p = 0.5$ and it is $(1 - \Phi_{1/n}(0.5)) = (n - 1)/(n + 1)$. Note, that for $n < 4$ the maximum value, is less than or equal to ½. Hence there are no $p\in(0.5,1)$ such that $½ < q < 1 - \Phi_{1/n}(p)$, and for any p and q a *DOWN* 1-cascade cannot start before $n=4$. Therefore, the minimum length of failure series that spurs *DOWN* 1-cascade, $n \geq 4$.

An increase in the number of failures, n, pushes the curve $q = 1 - \Phi_{1/n}(p)$ toward the bigger values of p and q (northeast corner of the diagram Fig.1b). As before, for $n\rightarrow\infty$ a *DOWN* 1-cascade may start for any p and q.

***DOWN* 2-cascade.** The outcome of the negotiations is the result of the actions taken by both parties. Therefore to see how two-sided cascades emerge we shall examine interactions between EPs and VCs. Due to the symmetry of payoffs and similar information structure for EPs and VCs, our analysis made for EPs must be valid for VCs as well. As we have already mentioned, for a *DOWN* 2-cascade to start, a *DOWN* 1-cascade on one of the sides is sufficient.

Given p and q we can find the length of the shortest series of failures, which trigger a *DOWN* 2-cascade. First, we draw curves $q_e = \Phi_{1/n}(p)$ and $q_v = \Phi_{1/n}(p)$ for $n = 2$ and check whether the chosen pair (p,q) is in the 'cascade' region of EPs, or of VCs or of both EPs and VCs. Fig. 2a illustrates the point for the case $E = h$, $V=H$. Area *I* is the 'non-cascade' region, where both sides refrain from herding. For (p,q) from *II* EPs start a *DOWN* 1-cascade after two failures, while VCs still follow their signals. If (p,q) is in *IV*, then VCs follow the herd rejecting private information, while EPs do not. Finally, for (p,q) from *III* both parties start their *DOWN* 1-cascades simultaneously. When any of the parties start a *DOWN* 1-cascade, a *DOWN* 2-cascade takes place. Therefore, if $(p,q) \in II \cup III \cup IV$ two failures are enough to spur a *DOWN* 2-cascade. If it is not the case we proceed to $n=3$. Since $\Phi_{1/3}(0.5)= ½$ and $\Phi_{1/n}(\cdot)$ is convex, at $n = 3$ the 'non-cascade' region disappears and at least one of the parties starts a *DOWN* 1-cascade.

We can apply similar arguments for $E=l$, $V=L$. There are no 2-cascade until $n = 4$. Fig. 2b shows 'cascade' regions for $n = 10$ (notations are the same as in Fig. 2a). As before, both sides may start *DOWN* 1-cascades simultaneously if the parties are symmetric in precision of their signals. If the cascade is spurred only by one of the sides, it prevents the other from herding. In contrast with $E=h$, $V=H$ case, the 'non-cascade' region *I* does not disappear at any n. However, for any given pair (p,q) increasing number of failures, n, soon or later must result in 'capturing' (p,q) by one of the 'cascade' regions.

3.1.3. Examination of the minimum series

Comparing our model of observable outcomes with the benchmark we note that an *UP* 2-cascade arising from the shortest series of successes of our model is absolutely the same as an *UP* 2-cascade in the benchmark. One may note, that information content of positive outcome in the former model is the same as in the benchmark, because from the fact that $Z_t = 1$ agents learn that both sides chose to *Agree* on the project, i.e. agents can infer their predecessors' *actions*. Obviously, all features of the minimum *UP* cascade series in our model remain to be the same as in benchmark.

On the contrary, negative outcome of negotiations in the model of observable outcomes does not allow unequivocally conclude the predecessors' actions. This 'lack of informativeness' generates some degree of interaction. While in benchmark model 2 failures are enough to start an informational cascade, for any *E, V, p* and *q* (as far as $p,q > ½$), in the model of observable outcomes a *DOWN* 2-cascade may require longer series of failures and the length now crucially depends on *p* and *q*.

To see why there appear limits on signals precision for a *DOWN* 1-cascade to start, consider an EP observing a series of failures. Unlike an EP in the benchmark model he cannot observe actions of his predecessors and does not know who reject the deals. Let *E=h*. Given *p*, the higher is *q*, the higher is the probability that in preceding periods VCs have got correct signals which are positive (*E=h*) in the case of *E=h*, and the higher is probability that VCs have chosen *Agree*. But the deals have failed. It means, that more probably that the deals have been rejected by EPs. Finally, he can conclude that EPs must have got adverse signals about market prospective of the new technology (*V=L*) and the prospective is rather gloomy. If accuracy of his signal is not high enough, this conclusion will offset positive private signal about *V*. Therefore if *q* is high and *p* is low (region *D* at Fig.1a, *I* and *IV* at Fig.2a), an EP will prefer to refrain from the deal and 'herding' will occur.

Interestingly, that increasing precision of VCs' signal *q* would have opposite effect on EP's decision were *E=l*. Indeed, following the similar line of reasoning this time he would think that the deals have been rejected by VCs. Since the deals failed due to VCs, he does not have enough information about actions of preceding EPs and therefore has to put more weight on his own signal, i.e. for *q* high enough an EP does not join the 'herd' and acts according to his private signal (region *N* at Fig.1b, *I* and *IV* at Fig.2b).

An important point here is that although the chosen payoff matrix implies independence of agent's payoff from the state of his side of market, the state does affect the behaviour of an agent. As we have seen, decision of an EP depends on the state of the technology, *E*, even if there is no direct relationship between *E* and EP's payoff.

Revelation. In some situations this kind of relationship can be used by 'sophisticated' VCs to learn about *E*. VCs observe outcomes and check whether they are compatible with those which must have take place if *E=h*. When an observed outcome deviates from the 'expected' outcome VCs reveal that of *E=l* and stop the deals.

Let there be series of $n > 2$ failures. A VC analysing the outcome at time $t = n+1$ may depict 'cascade' regions drawing a diagram similar to one at Fig.3 (for $n = 4$). Note, that if *E=h*, then the areas *I*, *II*, and *III* are related to the 'cascade' region of EP_{n+1}, but if *E=l*, then they belong to 'non-cascade' region. Suppose, that $(p,q) \in I \cup II$ and a deal took place at $t = n+1$. It would have been impossible, if the technology had been of high quality ($E=h$), because then EPs had rejected the deals and the outcome must have been negative. Thus the VC observing this deviation from 'expected' behaviour concludes that $E=l$.

The revelation takes place not in all circumstances. Some conditions are required. First, for a successful deal both agents must get positive signals. Therefore revelation is a random event. Second, VC_{n+1} must be not in *DOWN* 1-cascade, because if it is the case, then Z_{n+1} = -1 regardless of the EP_{n+1}'s action. For the high market prospect, $V=H$, VC's 'non-cascade' region is denoted as *I* at Fig.3, for $E=l$ it consists of $I \cup II \cup III \cup VI$. Therefore, when $V=H$, $E=l$ for (p,q) from *I*, there is a chance (with the probability of simultaneous arrival of VC's and EP's positive signals, $p_v q_e$) that the truth about *E* will be revealed. When $V=L$, $E=l$ revelation may happen also if (p,q) is from *II*. If revelation takes place then all consequent VCs will reject the deal.

Note, that since payoff to an agent is paid once and does not depend on actions chosen by followers, he has no incentives to change his behaviour in order to mislead the other side (in our example EP_{n+1} chooses *Agree*, even if it may reveal true *E* and prevent consequent VCs from making deals with EPs).

As we have seen if $E=h$ and $V=H$, after three failures at t = 4 at least one of the sides must be in a *DOWN* 1-cascade, let it be EPs' side, and therefore all deals in consequent periods must fail. If, nevertheless, in any of consequent periods an outcome was positive, then VCs infer that $E=l$, and will reject deals. The same argument is applied to EPs and *V*. Hence, whatever are *E* and *V*, after the series of three failures (or even two if (p,q) does not belong to 'non-cascade' region *I* of Fig.2a) all consequent deals (probably with except one revealing $E=l$ or $V=L$) will fail and we can say that for any *E* and *V* a two-sided *DOWN* cascade 'effectively' starts. It implies that the probability of locking into a *DOWN* 2-cascade after three rounds in the model of observable outcomes must exceed one in benchmark, because to trigger a *DOWN* 2-cascade in the benchmark model two negative signals must arrive *on the same side*, while in the model of observable outcomes two (or three) negative signals trigger cascade even when they arrive to *different sides*.

3.2. General case

So far we have analysed only the shortest series of success/failures for cascades to emerge. Unfortunately, our model does not have one nice feature of one-side cascades: in BHW model, if a cascade has not happened by time $t = 2n$, then $(2n+1)$-th agent is in the same situation as the first one. This makes analysis of the general case in our model more complicated than in 'standard' model. On the other hand, we can apply the same approach that we use analysing the minimum series.

Let I_t consist of positive and negative outcomes, n_+ is the number of successful deals and n_- the number of failures ($n_+ + n_- = t - 1$). Now inequality (1), which is condition of herding in a *DOWN* 1-cascade, has the form

$$p(pq_e)^{n_+}(1-pq_e)^{n_-} < (1-p)((1-p)q_e)^{n_+}(1-(1-p)q_e)^{n_-}, \qquad (3)$$

which is equivalent to

$$q_e > \frac{p^{(n_+ +1)/n_-} - (1-p)^{(n_+ +1)/n_-}}{p^{(n_+ +1)/n_- +1} - (1-p)^{(n_+ +1)/n_- +1}}.$$

Modifying the Remark one can also obtain condition for herding in an *UP* 1-cascades

$$(1-p)(pq_e)^{n_+}(1-pq_e)^{n_-} < p((1-p)q_e)^{n_+}(1-(1-p)q_e)^{n_-}, \qquad (4)$$

which is equivalent to

$$q_e < \frac{p^{(n_+ -1)/n_-} - (1-p)^{(n_+ -1)/n_-}}{p^{(n_+ -1)/n_- +1} - (1-p)^{(n_+ -1)/n_- +1}}.$$

Introducing a function

$$q_e > \Phi_a(p) \equiv \frac{p^a - (1-p)^a}{p^{a+1} - (1-p)^{a+1}},$$

we rewrite the condition for a *DOWN* 2-cascades, (3), as

$$q_e > \Phi_{a_{down}}(p), \qquad \text{where} \quad a_{down} \equiv \frac{n_+ + 1}{n_-}.$$

Similarly (4) can be rewritten as

$$q_e < \Phi_{a_{up}}(p), \qquad \text{where} \quad a_{up} \equiv \frac{n_+ - 1}{n_-}.$$

For $0 < a < 1$, the function $\Phi_a(p)$ is continuous, monotonously increasing and convex for $p \in (0.5,1)$; $\Phi_a(0.5) = 2a/(a+1)$, and $\Phi_a(1) = 1$. For $a > 1$, $\Phi_a(p) > 1$ for $p \in (0.5,1)$.

We are particularly interested in 'incorrect herding', the situation in which the overall outcome is socially suboptimal. Therefore we will focus on two cases. One of them is a two-sided *DOWN* cascade when $E=h$, $V=H$. That is, although technology is good and demanded by the market, a chain of unlucky events causes a *DOWN* 2-cascade to lock in ('two-sided crunch'). The other case is a two-sided *UP* cascade when $E=l$, $V=L$. This is, in fact, a 'two-sided bubble'.

Case 1 (*E=h* , *V=H*). Consider one of the sides, for example EPs. EPs begin *DOWN* 1-cascade when $q > \Phi_{adown}(p)$, and an *UP* 1-cascade $q < \Phi_{aup}(p)$. The inequalities divide (p,q) domain into 3 regions: where EPs are in a *DOWN* 1-cascade, in *UP* 1-cascade, and region where they follow their signals, the ‘non-cascade’ region (Fig.4a for $n_+ = 3$, $n_- = 6$). Regions of (p, q) domain related to two-sided cascades are shown at Fig. 4b.

Having this picture in mind one might expect that the probability of a two-sided *UP* cascade (which is a *correct* cascade) for low precision of signals is higher than in the benchmark model, while if the signals have high precision, especially when the precision of signals is asymmetric, locking into a two-sided *UP* cascade is less probable in comparison with the benchmark. The intuition is the same as in our analysis of the minimum series. If the precision of signals is relatively low, then agents are more optimistic: they treat the failures as deals being rejected by preceding agents because of wrong signals. But if p and q are high (the probability of receiving a false signal is low) they think that the other side more probably has got favourable signals (because $E=h$, $V=H$), hence observing failures they would suspect that the deals have been rejected by their own side and the other side of market is bad. As we will see the results of simulations support this intuition.

Note that in the case of $E=h$, $V=H$ ‘sophisticated learning’ makes no difference. If the agents were able to reveal the true state of the other side the result would be an *UP* cascade. Revelation has sense only when the sides are not in a *DOWN* 1-cascade because if, for instance, EPs are in a *DOWN* 1-cascade, VCs may reveal that $E=h$, but they cannot switch EPs from *DOWN* to *UP* 1-cascade. Revelation does not change the situation when both sides are already in an *UP* 1-cascade. The revelation cannot occur when the sides are outside the cascade regions: necessary condition for revelation is both sides being in a cascade in order to treat the outcome unambiguously. The case of $E=l$, $V=L$ is more rich in this respect.

Case 2 (*E=l* , *V=L*). Cascade regions for EP ($E=l$) are shown in Fig. 4c (for $n_+ = 2$, $n_- = 10$). As in the analysis of minimum series there are two situations, which we identify as two-sided *DOWN* cascades. First, if any of the parties starts a *DOWN* 1-cascade, the cascade will not be broken. Even if the true state of market is revealed, the opposite side will also start rejecting the deals.

The other situation in which a *DOWN* 2-cascade ‘effectively’ starts is related to revelation. It realizes if (p,q) is in the *DOWN* 2-cascade region corresponding to the case of $E=h$, $V=H$: $q > \Phi_{adown}(p)$, or $p > \Phi_{adown}(p)$. Although the parties, in fact, are not in a 1-cascade, any positive outcome will lead to revelation and will turn one of them (or both) to *DOWN* 1-cascade. If the revelation does not happen, i.e. there are no positive outcomes in the series, series of failures long enough will spur a *DOWN* 1-cascade. A *DOWN* 1-cascade is sufficient for a *DOWN* 2-cascade. Therefore, whatever are the private signals in this situation, series of failures cannot be broken (probably with exception of one revealing outcome), and the sides are 'effectively' in the *DOWN* 2-cascade.

There is also hypothetical situation in which a *DOWN* 2-cascade starts because of revelation: if both sides are *expected* to be in an *UP* 1-cascade, i.e. (p,q) belongs to the *UP* 2-cascade region for $E=h$, $V=H$: $q < \Phi_{aup}(p)$, or $p < \Phi_{aup}(p)$, and the outcome is negative. However, it never realizes: if (p,q) belongs to the *UP* 2-cascade region for $E=h$ and $V=H$ then it must belong to *UP* 2-cascade region for $E=l$ and $V=L$. For instance, for $E=h$ the condition for EPs' an *UP* 1-cascade is $q < \Phi_{aup}(p)$, which given $q > ½$ implies that the condition for an *UP* 1-cascade for $E=h$ must hold: $1 - q < \Phi_{aup}(p)$.

Random walk. When both sides are outside the cascade regions and follow their signals, an outcome is a random variable distributed according to Bernoulli law with the parameter $p_v q_e$. To see the emergence of two-sided cascades we can use representation of Bernoulli series as a random walk.

Let us introduce variable $S_t = Z_1 + .. + Z_{t-1}$. When the sides are not in a cascade and follow their signals, an increment ΔS_t is independent identically distributed random variable

$$\Delta S_t = \begin{cases} 1, & \text{with probability} \quad p_v q_e, \\ -1, & \text{with probability}\ (1 - p_v q_e). \end{cases}$$

Conditions (3) and (4) represent absorption barriers: *DOWN* (S_t^-) and *UP* (S_t^+) respectively.

One-sided cascades also can be analysed as a random walk. The absorption barriers for one-sided cascade are $S^+ = +2$ and $S^- = -2$. The nice property simplifying analysis having been mentioned above is the stationary nature of absorption barriers in this case. As can be seen from (3) and (4) in our model S_t^- and S_t^+ depends on t (on p and q as well). The position of absorption barriers in the two-sided model also depends on E and V, while in one-side model the state of market affects only a random walk path.

An example of a random walk path and the absorption barriers for EPs ($p = 0.75$, q = 0.65, $E=h$, $V=H$) are shown at Fig.5a. The series corresponds to the case where EPs starts their *UP* cascade at $t = 20$. However this does not necessary mean that a two-sided *UP* cascade will start. A two-sided *UP* cascade requires both sides being in *UP* 1-cascades.

The different states of the world (E and V) are related to the different absorption barriers. It is shown at Fig 5b: absorption barriers for $E=h$ and $V=H$ are denoted as H_u and H_d, those for $E=l$ and $V=L$ as L_u and L_d. 'Non-cascade corridor', the area between *UP* and *DOWN* barriers, in the case of $E=l$ and $V=L$ (between L_u and L_d) is going down steeper than for $E=h$ and $V=H$.

If *E=l* and *V=L*, then if S_t is between H_d and L_d revelation may take place. If in this region the outcome happens to be positive, the true state of *E*, *E=l*, will be revealed, and VCs will turn to a *DOWN* 1-cascade. This means that whenever S_t falls below H_d there is no chance to escape from VCs' *DOWN* 1-cascade, and hence a two-sided *DOWN* cascade becomes inevitable. Therefore we may assume that a two-sided cascade 'effectively' starts as soon as S_t hits H_d.

For further analysis we can use direct examination of all possible realizations of private signals, outputs and their probabilities. Although following this way we can obtain precise results, with increasing length of analysed series the time needed for calculations is drastically increasing. Because of this we applied direct examination of short series such as minimum series, but for longer time spans we use simulations.

3.3. Results of simulations

To analyse our model series of simulations have been done. Estimated probabilities of *DOWN* 2-cascades (p = 0.75, q = 0.65) for the model of observable outcomes are presented at Fig.6a (*E=h*, *V=H*). In the benchmark model (of observable actions) *DOWN* 2-cascades follow the pattern similar to informational cascades in the BHW model: absorption take place after each even period except t =1 (at t = 3,5,7 etc.). In the model of observable outcomes absorption at the *DOWN* barrier happens at t = 3,6,9,11 etc.

This result may be anticipated if look at the absorption barriers at Fig.5a and 5b. The shortest series of failures hit the *DOWN* barrier, H_d, at t = 3. If series include one success it have a chance to be absorbed at t = 6. Series with two successful deals may reach the *DOWN* barrier at t = 9 if it escape from absorption by the *UP* barrier at t = 3,4,6 and 7. Next absorption points on the *DOWN* barrier are at t = 14,19, …

For the given parameters most of series are absorbed in the beginning at t = 3 (and t = 4 on the *UP* barrier), while in the benchmark absorption spreads over time more evenly. Due to this the average time before locking in a cascade in the model of observable outcomes is lower than in the benchmark (3.69 for *DOWN* and 4.43 for *UP* 2-cascades vs. 4.50 and 5.56 respectively).

Does introduction of the restrictions on the publicly available information, which makes our model of observable outcomes different from the benchmark, affects social welfare? To answer this question we estimated the probability to end up in a 'wrong' cascade for both models. For the case of *E=h*, *V=H* (p = 0.75, q = 0.65) the estimated probability of incorrect (*DOWN*) 2-cascade in the former (p_1 = 0.34) exceeds one in benchmark (p_2 = 0.34). To check whether $p_1 > p_2$ has statistical significance, we run series of simulations to estimate the difference (p_1 - p_2) and apply t-test to check the hypothesis that the probabilities are equal ($\mathbf{H_0}$) vs. the hypothesis that the difference is positive ($\mathbf{H_1}$). The hypothesis $\mathbf{H_0}$ can be rejected in favour of $\mathbf{H_1}$ at 1% level. It allows us to state that for given values of parameters probability of 'wrong' cascade in the model of observable outcomes is higher than in benchmark model, i.e. the restrictions imposed on public information decrease the efficiency of the system.

We estimated cumulative probabilities to end up in a *DOWN* 2-cascade ($E=h$, $V=H$) in both models for different values of parameters p and q. The results of our estimations are shown at Fig.7a and Fig.7b. One can see that in the model of observable outcomes the probabilities to be locked in a *DOWN* 2-cascade in the region of low p and q ($p,q \leq 0.66$) is clearly distinct form the rest, while in the benchmark the probability is monotonously decreasing as in p and q increase on the whole domain. This pattern fits with the remark made earlier in the paper: for low precision of private signals agents are more 'optimistic': they put more weight on successful deals than on failures. The probability to start a *DOWN* 2-cascade is decreasing with t (e.g. see Fig.6a), and the prime contribution to the cumulative probability comes from the shortest series. As we already know, for small p and q the shortest series for a *DOWN* 2-cascade consist of 3 failures, while for large values of p and q two failures are enough. Therefore the border between 'small' and 'large' p and q must lie about $\Phi_{1/2}(0.5) = 2/3 \approx 0.66$, which matches with the results of simulations. Hence the region of the low parameters corresponds to relatively lower cumulative probability to start a *DOWN* 2-cascade because the minimum series in this region are longer.

With the help of Fig 7a and 7b we conclude that, except the area of small values, for most pairs of values of p and q cumulative probability of locking in the wrong (*DOWN*) 2-cascade in the model of observable outcomes is higher than in the benchmark. It means that the restrictions on the information structure are responsible for lowering the efficiency of the system. In other words, in the case of $E=h$, $V=H$ for most values of parameters p and q if agents observe outcomes of their predecessors' actions instead of actions by themselves, agents are 'too sceptic' about the other side of the market, which lead to socially inefficient rejections of developing and financing of the new technology.

The other situation to be examined is one in which both market prospect for the new product is and the technology are mediocre. Obviously, financing of the technology in this case ($E=l$, $V=L$) is undesirable for everyone. How probably is that the technology nevertheless will get finance?

As have been pointed out in our discussion of revelation, we can assume that H_d is 'effective' *DOWN* barrier in the case of $E=l$, $V=L$. Two-sided *UP* cascade requires both sides being in *UP* 1-cascade and both sides being above H_d so that revelation is excluded. Since the two-sided

DOWN absorption barrier in the case of *E=l*, *V=L* is H_d i.e. the same as in the case of *E=h*, *V=H* average time necessary to form two-sided *DOWN* cascade must be close to one for *E=h*, *V=H*. Indeed, our estimates gives 3.40 which is close to 3.96 for *E=h*, *V=H*. Average time before *UP* 2-cascade for *E=l*, *V=L* (4.68) is also close to one for *E=h*, *V=H* (4.43).

Estimated cumulative probabilities to end up in a wrong cascade for *E=l*, *V=L* (which is an *UP* 2-cascade) for the model of observable outcomes and for the benchmark are presented at Fig.7c and Fig.7d. As before, the region with small values of *p* and *q* differs the former model (Fig.7d) from benchmark (Fig.7d), although this time it is not the result of that one of the sides fall in a 'cascade' region but rather because of revelation. The probability of an *UP* 2-cascade, which is 'wrong' in this case, is higher in this region. Analysis of the difference between probabilities of an *UP* 2-cascade in the model of observable outcomes and one in the benchmark shows that in the case of *E=l*, *V=L* the cumulative probability in the former model exceeds one in the benchmark for all values of *p* and *q*. It implies that in the situation where both sides of the market know that their part of the market is 'weak', agents' inability to observe actions of preceding agents makes them 'overoptimistic' and may result in undesirable 'two-sided bubble'.

'Overoptimism' While the 'inferiority' of the information structure in model of observable outcomes with respect to the benchmark might be anticipated from the very beginning - the less information is available to agents, the higher is the probability of 'incorrect herding', the other result related to 'overoptimism' of agents independent of *E* and *V* when precision of private signals is relatively low, seems to be peculiar for interactions between the two side of the market based on learning.

The impact of 'overoptimism' is substantial. While for *E=l*, *V=L* in the area of large *p* and *q* (>0.66) the difference between cumulative probabilities of *UP* 2-cascade in the model of observable outcomes and one in the benchmark is in the range of 0.1÷5%. In the area of low *p* and *q* the difference is about 10÷15%. Hence we can estimate consequences of excess optimism as 5÷10% increase in probability of *UP* 2-cascade. For *E=h*, *V=H* the overoptimism is even higher. As was already mentioned it alters the sign of the difference: for large *p* and *q* cumulative probability of *UP* 2-cascade in the former model is lower than in the benchmark by 0.1÷5%, but for low *p* and *q* it is greater by 12÷15%. Therefore, overoptimism results in increase of 10÷20% of cumulative probability of *UP* 2-cascade.

The region of low values of *p* and *q* is of particular interest if financing new technology is concerned. The degree of 'asymmetry' of information in this case is high, because neither investors know the technology well, nor entrepreneurs have knowledge of the market. Overoptimistic bias of individuals in uncertain environment is well known in the field of cognitive science. There are studies documenting overconfidence of among entrepreneurs (Buenitz, 1997) and venture capitalists (Zachariakis, 2001). Not contesting explanations of this phenomenon from the point of view of cognitive psychology, we conjecture that two-sided interaction and information constraints discussed in this paper may contribute to overconfidence of the agents in real economy.

Recent ‘dot com’ crash rises questions why market overvalued many “new economy” companies with immature products, which have vague market perspectives. As we have seen, low quality of information (precision of private signals and incompleteness of information in public domain) results in overoptimistic bias in interpretation of the history: successful deals get more weight than failures, and agents tend to overvalue performance of their counterpart. Overoptimism based on mutual illusions makes the system more vulnerable to two-sided ‘high-tech bubbles’.

Summarizing, we conclude with the following. First, dynamics of two-sided cascade in information structure where only history of outcomes (rather than history of predecessor’s actions) are observable is non-trivial and can be characterized by interactions between the two sides of market arising from learning. In comparison with benchmark model, where agents’ actions are public information, in the model where only outcomes of the actions’ are publicly observable, although with some exceptions, the probability to end up in socially inferior cascade is higher. Second, in the situation where precision of the private signals is low for both of the sides of the market, agents tend to be ‘overoptimistic’ about the state of the world.

4. Conclusions

We examine a model in which agents on the both sides of a market have different information sets and are subject to information cascades. We assume some restrictions on available information: instead of observing actions of their predecessors as in one side information cascade models, agents observe only successes or failures of negotiations. The changes in the information structure lead to increasing probability of locking in socially inferior informational cascade. The results support general conclusion that can be drawn from literature about information cascades: information structure does matter, and the more restrictions on publicly available information are imposed, the higher is the probability that collective behaviour will be suboptimal. Another finding of the paper is that in uncertain environment agents tend to be overoptimistic about the state of the world, which fits with results of empirical studies of financing new technologies. Overoptimism based on mutual illusions makes the system vulnerable to two-sided ‘high-tech’ bubbles, and may be one of the reasons behind ‘dot com’ crash.

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Figures

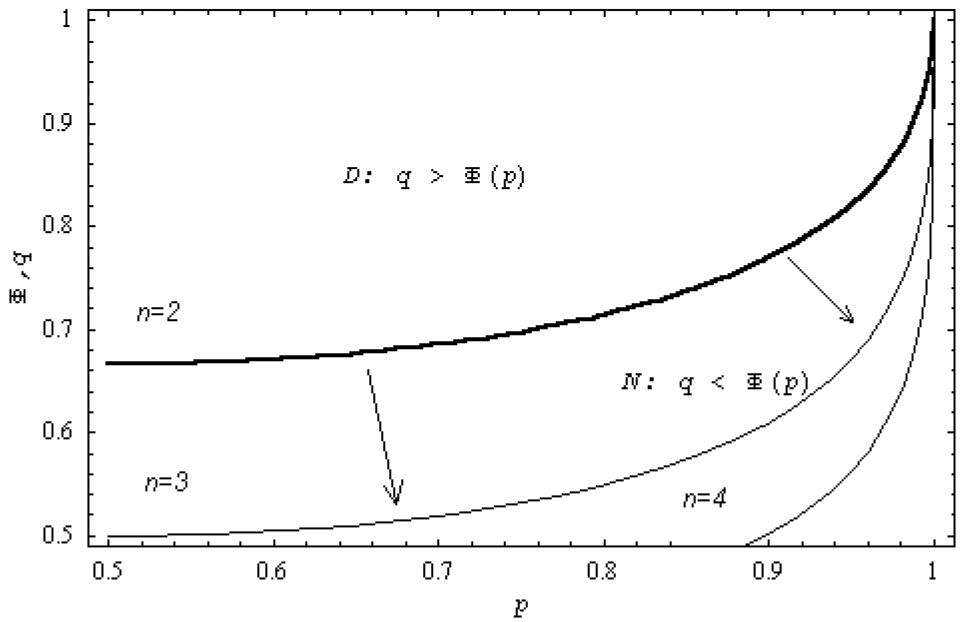

Fig. 1a

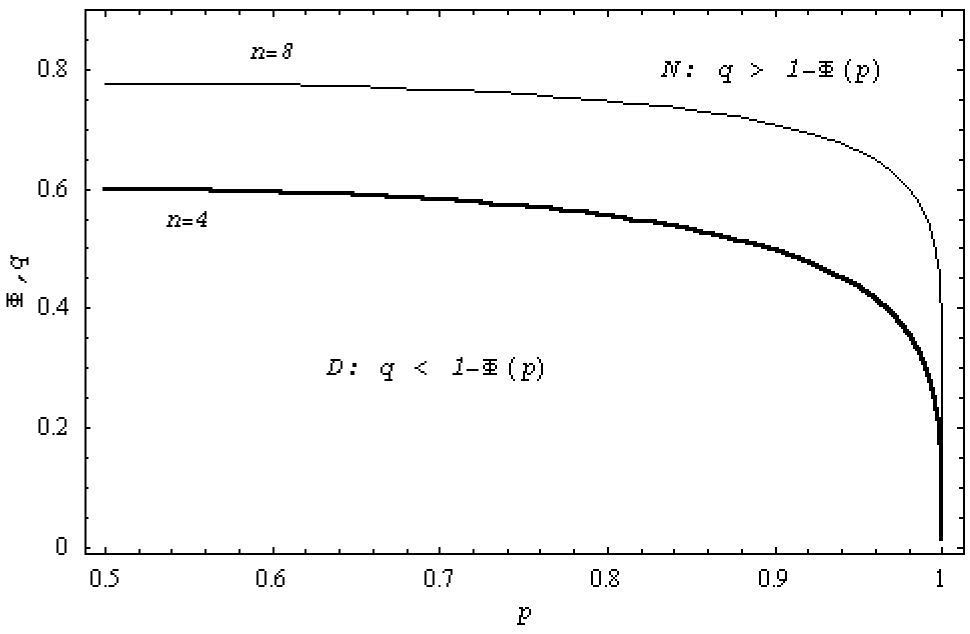

Fig. 1b

Fig.1. Range of values of p given q for which *DOWN* cascade at t may start: *a*. $E = h$, $q = 0.7$, $t = 3$; *b*. $E = l$, q = 0.6, $t = 7$.

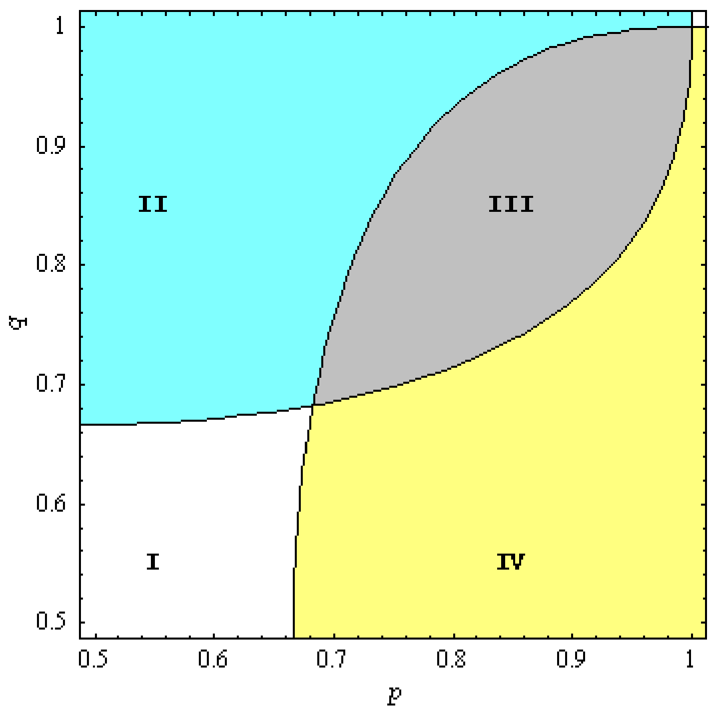

Fig. 2a

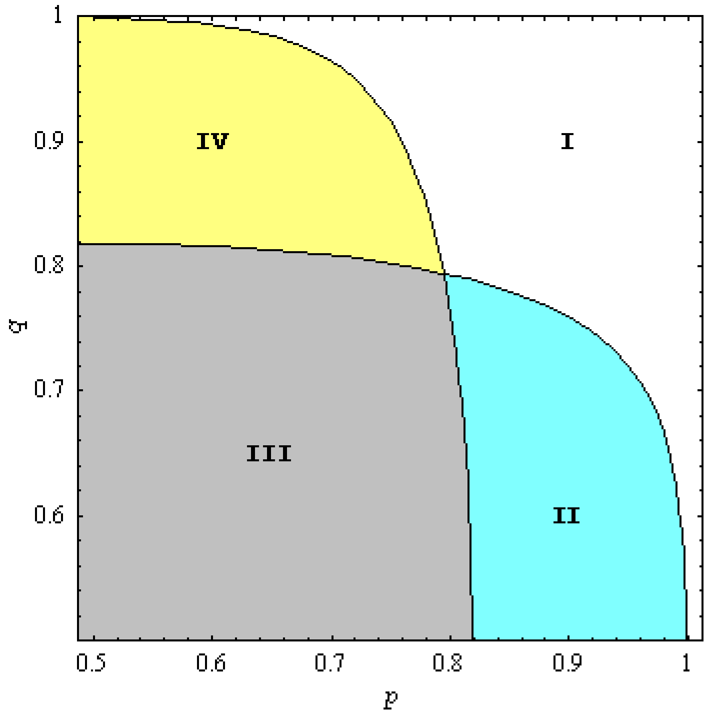

Fig. 2b

Fig.2. Range of (p, q) for which two-sided *DOWN* cascade at t may start: *a*. $E = h$, $V = H$, $t = 3$; *b*. $E = l$, $V = L$, $t = 11$.

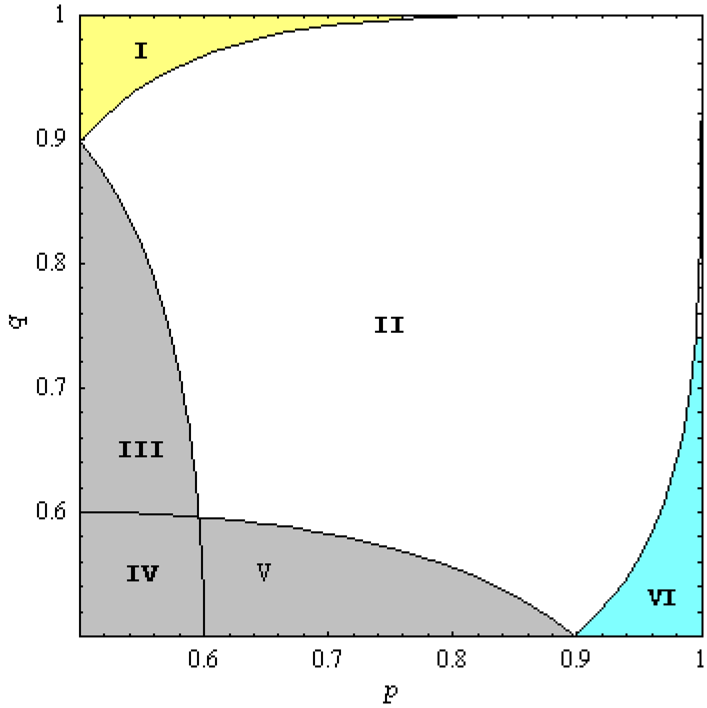

Fig. 3

Fig.3. Cascade regions in (*p*, *q*) domain for an EP (minimum series), $n = 4$, $E = h$ and $E = l$.

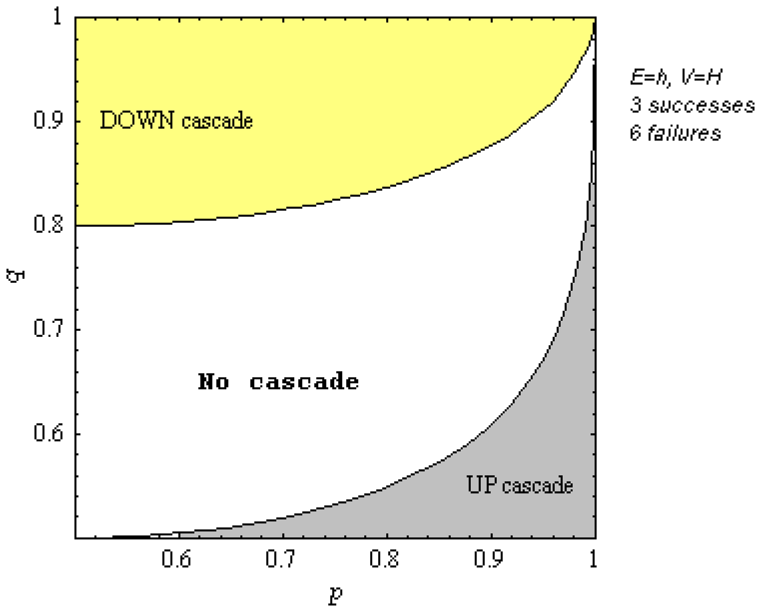

Fig. 4a

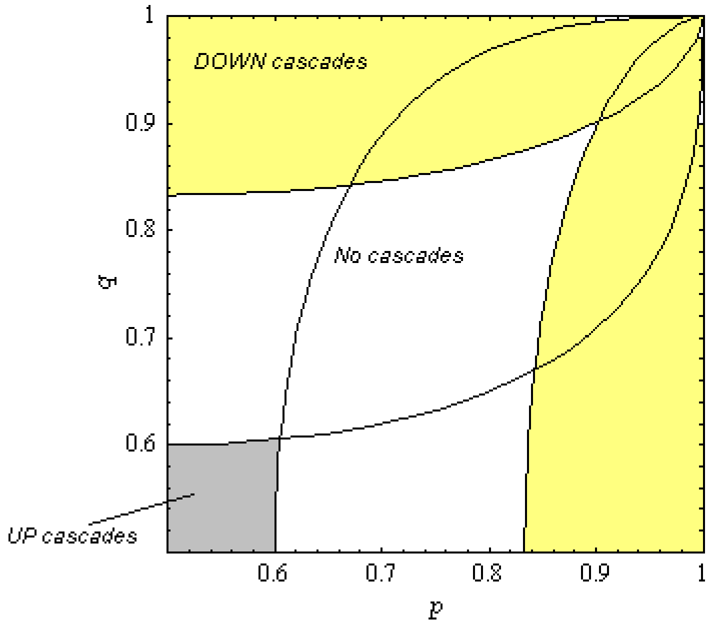

Fig. 4b

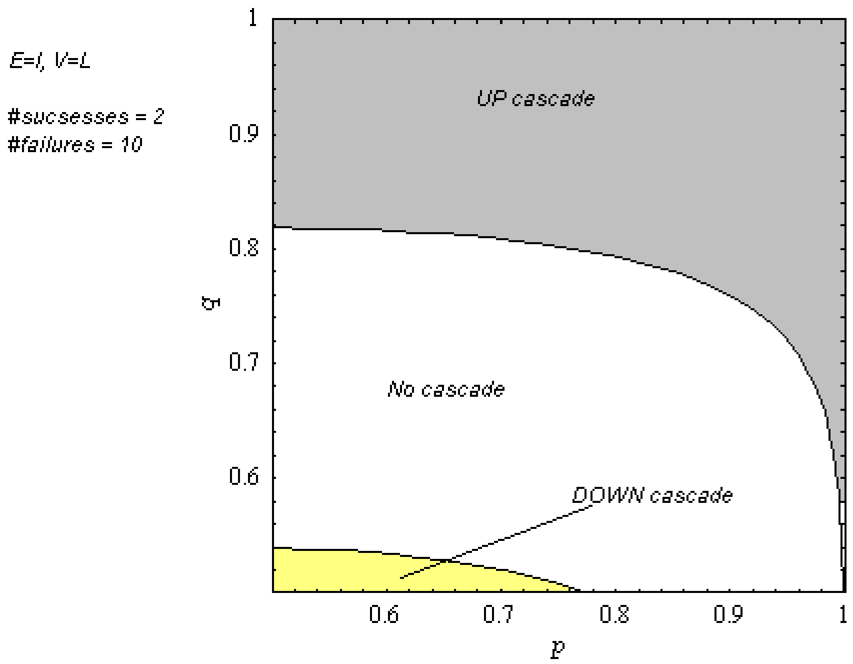

Fig. 4c

Fig.4. Two sided cascades (general case): *a.* 1-cascade regions for an EP *E=h*, *V=H*; *b.* 2-cascade regions *E=h*, *V=H*; *c.* 1-cascade regions for an EP *E=l*, *V=L*,

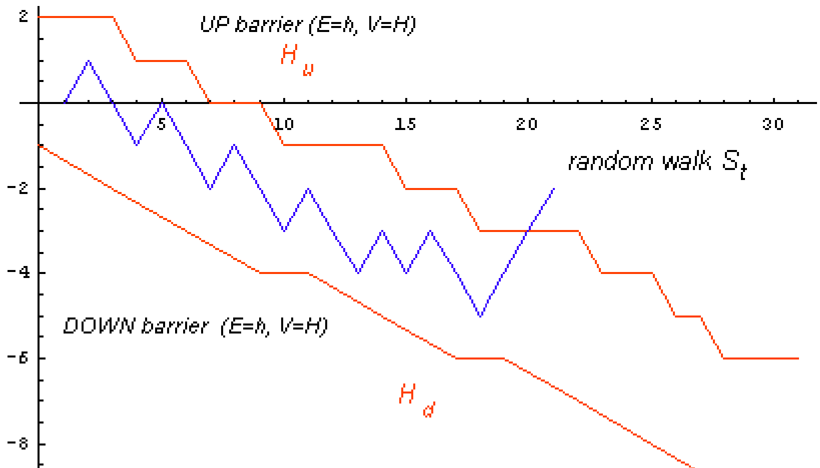

Fig. 5a

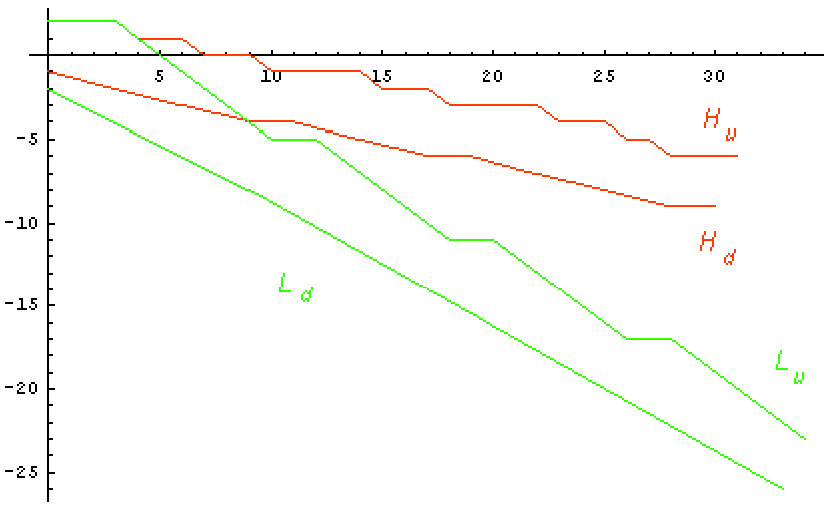

Fig. 5b

Fig.5. Random walk and absorption barriers (p = 0.65, q = 0.75): ***a***. random walk and absorption barriers for E=h, V=H; ***b.*** absorptionbarriers for E=h, V=H, and E=l, V=L.\medskip

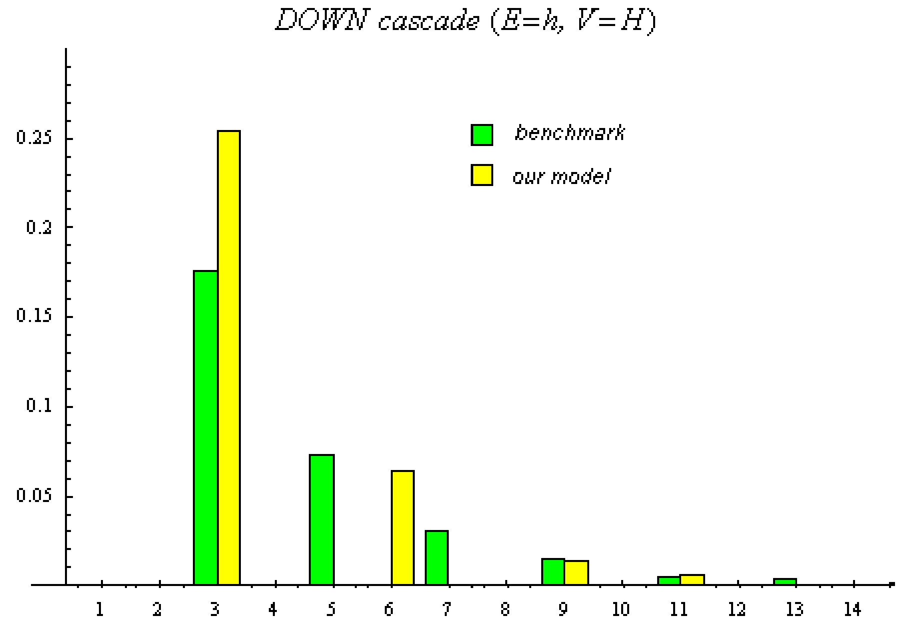

Fig. 6a

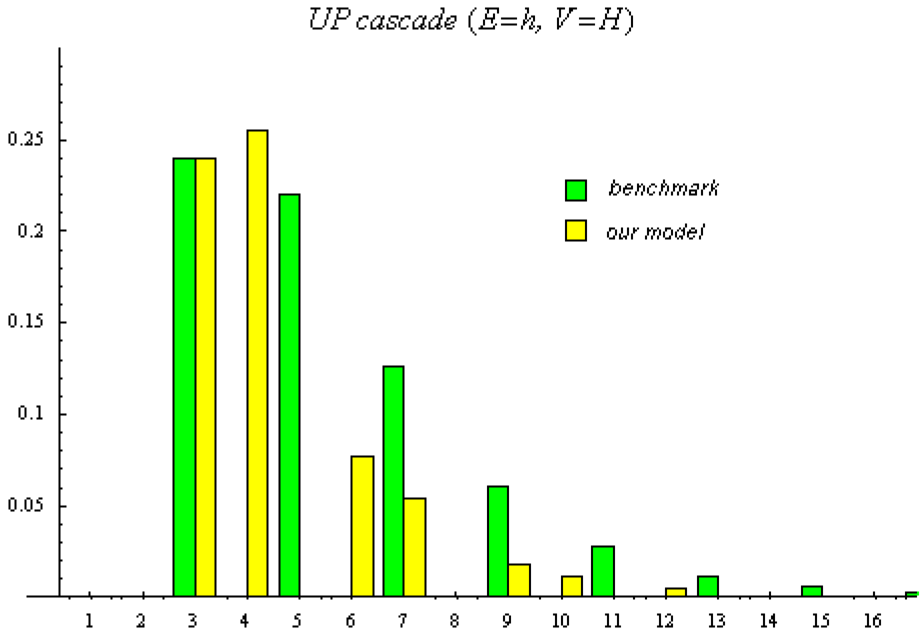

Fig. 6b

Fig.6. Probability of emergence of two-sided cascades (p = 0.65, q = 0.75): ***a***. *E=h*, *V=H*, *DOWN* cascade, benchmark; ***b***. *E=h*, *V=H*, *UP* cascade, our model.

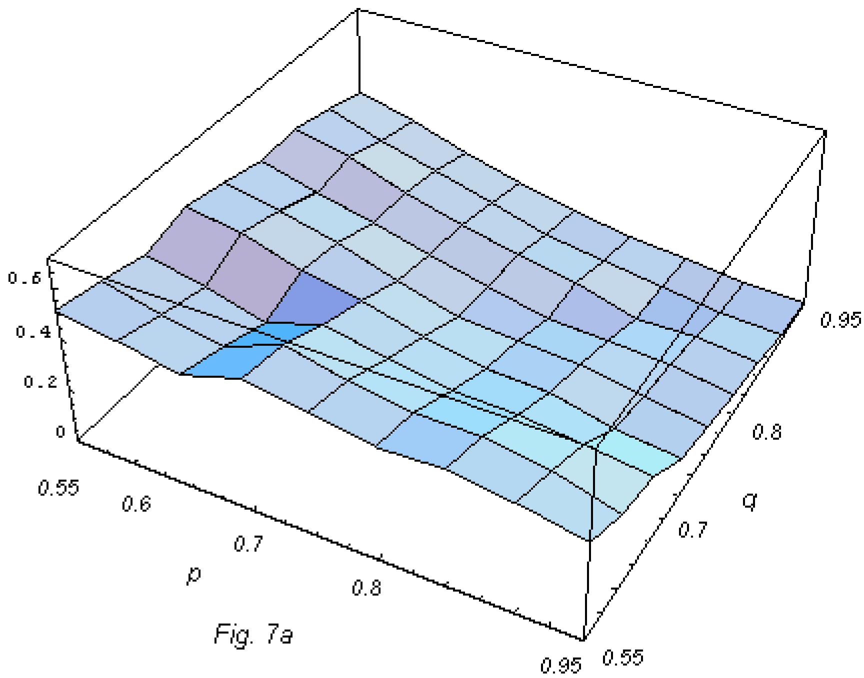
0.6
0.4
0.2
0
0.55
0.6
0.7
p
0.8
0.95
0.55
0.7
q
0.8
0.95

Fig. 7a

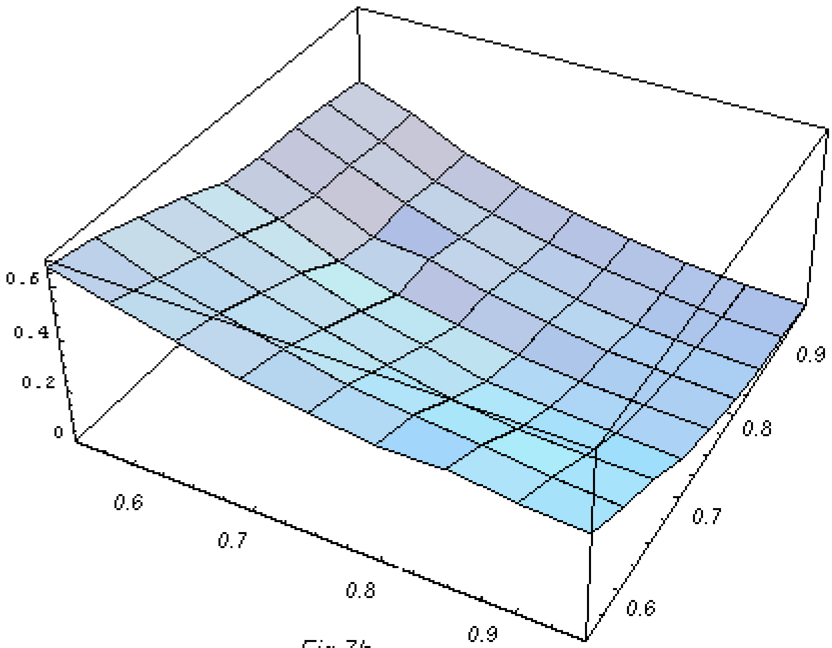
0.6
0.4
0.2
0
0.6
0.7
0.8
0.9
0.6
0.7
0.8
0.9

Fig.7b

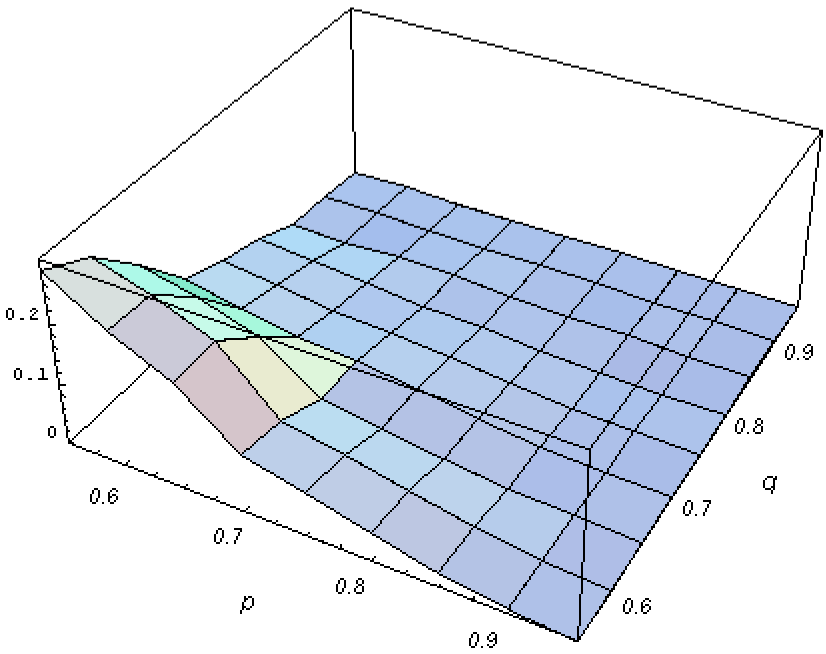

Fig. 7c

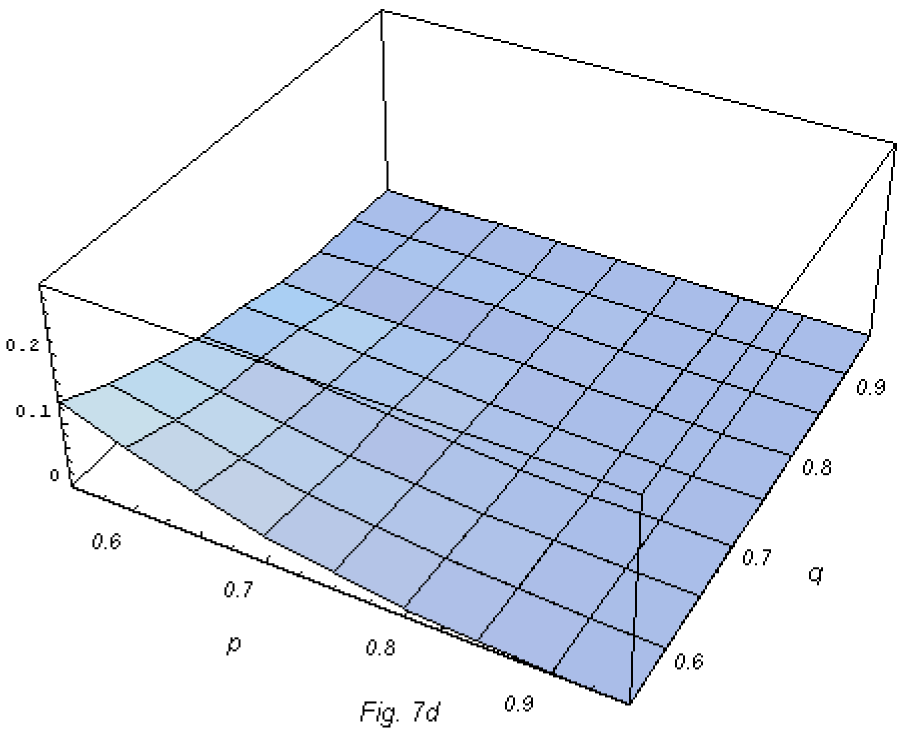

Fig. 7d

Fig.7. Probability of emergence of two-sided 'incorrect' cascades: ***a***. our model: *E=h*, *V=H*, *DOWN* cascade; ***b***. benchmark model: *E=h*, *V=H*, *DOWN* cascade; ***c***. our model: *E=l*, *V=L*, *UP* cascade; ***d.*** benchmark model: *E=l*, *V=L*, *UP* cascade.