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Decision Support

Safe dike heights at minimal costs: An integer programming approach

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ABSTRACT

Optimal dike heights are of crucial importance to the Netherlands as almost 60% of its surface is under threat of flooding from sea, lakes, or rivers. This area is protected by more than 3500 kilometres of dunes and dikes. These dunes and dikes require substantial annual investments of over 1 billion euro. In this paper we propose an integer programming model for a cost-benefit analysis to determine optimal dike heights. We improve upon the model proposed by Brekelmans, den Hertog, Roos and Eijgenraam (2012). Our model provides an alternative approach with almost complete flexibility towards input-parameters for flood probabilities, damage costs and investment costs for dike heightening. We present an easy-to-implement algorithm that provides an optimal solution to the problem. The method has been implemented and tested for the most recent data on flood probabilities, damage and investment costs, which were recently used by the government to determine the new safety standards in the Dutch Water Act.

1. Introduction

The 1953 flood in the South-western part of the Netherlands is, after more than 60 years, still in the Dutch collective memory. It resulted into the death of 1835 people. Almost 200,000 hectares of land were flooded, 67 dike breaches arose and immense economic damage resulted (10% of the Dutch GDP). The government rapidly appointed the so-called Delta Committee in order to design measures for preventing similar disasters in the future. The Delta Committee asked Van Dantzig (1956) to develop a mathematical approach to formulate and solve the economic cost-benefit decision model concerning the optimal dike height problem.

The work of the Delta Committee, including the work by Van Dantzig (1956), resulted in statutory minimal safety standards which were in place until the year 2016. These safety standards against flooding were defined on the basis of a dike ring area. A dike ring is an uninterrupted ring of water defences. In total, there are 53 dike ring areas, each having a certain minimum safety standard (i.e. maximum flood probability). The tightest (i.e. lowest) flood probability was 1/10,000 per year for the most populated part of the Netherlands. This number is derived from Van Dantzig (1956).

At present, protection against flooding is an important issue worldwide (Adikari & Yoshitania, 2009; Syvitski, Kettner, & Overeem, 2009). Devastating floods occur more and more frequently, e.g. in Bangladesh (2004, 2007, 2012), Pakistan (2010), and the well-known serious flooding in and around New Orleans in 2005, which killed about 1500 people and created enormous damage.

Renewed interest in determining optimal dike heights in the Netherlands arose – again – after a critical situation in 1995. The rising water levels of the major rivers Rhine and Meuse forced 200,000 people to evacuate. Fortunately, no serious flooding occurred. This event triggered the Dutch government to ask CPB Netherlands Bureau for Economic Policy Analysis to develop an economic cost-benefit analysis to determine optimal safety levels for dike rings along the river Rhine. The results of this analysis are presented in Eijgenraam (2005, 2006) and Eijgenraam, Brekelmans, den Hertog, and Roos (2017). The government initiated an investment project of several billion euros to bring the dikes adjacent to the major Dutch rivers up to standards (Ministry of Infrastructure & the Environment, 2009).

Gradually, awareness grew in the Netherlands that the safety levels against flooding were from the 1950s and were therefore in need of a thorough reconsideration. Since then, both the population size and the economic value of the protected land have increased significantly. Moreover, the knowledge about the causes of flooding has increased, as well as the technical measures to prevent flooding or reduce its consequences. Finally, the sea level and the discharge levels of the rivers during winter have risen in this period. Therefore, the Dutch Central Government initiated a safety programme as part of an overall new Delta Programme (Delta Pro-

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gramme Commissioner, 2012), with the aim of developing and setting down new water safety standards and implementing the EU Flood Risks Directive (EU, 2007).

Several research projects have been initiated to prepare these new standards. A new economic cost-benefit analysis (CBA) was carried out by the hydraulic research and consultancy company Deltares (Kind, 2011). This CBA is based upon a mathematical model developed by Brekelmans, den Hertog, Roos, and Eijgenraam (2012), an extension of the previous models by Van Dantzig (1956) and Eijgenraam (2006). The Minister of Infrastructure and the Environment (Schultz van Haegen-Maas Geesteranus, 2013) approved the result of the cost-benefit analysis to increase safety in specific regions (Eijgenraam et al., 2013, see also the video presentation by the Minister at the Franz Edelman Award 2013). In the past few years, a decision process of central and local governments (municipalities, counties and district waterboards) has resulted in the new legally binding safety standards for flood risks (Delta Programme Commissioner, 2016).

We present a new modelling approach to determine the optimal timing and extent of dike heightening or strengthening. We will use the terms heightening and strengthening interchangeably throughout this paper. Our new modelling approach entails three major advantages in comparison with Brekelmans et al. (2012).

The first advantage is that the model provides substantially more flexibility with respect to the input data. Some crucial assumptions of the underlying model by Brekelmans et al. (2012) are rather restrictive. Hence, the State Secretary on Water Infrastructure (Atsma, 2011) asked for a more made-to-measure approach, which is able to include more local-details for safety measures, especially low-cost solutions, to increase safety. The model is well equipped to include this type of measures in coming research projects. Situations in which this flexibility is crucial:

- For the major rivers a maximum exists to the discharge that can enter the Netherlands. Hence, the overflow probability of dikes will become zero in cases where these dikes are above a certain height.
- Damage that occurs when a dike (ring) fails differs up to a factor 20–100, depending on the exact location of the breach (CPB, 2011; VNK2, 2011).
- For some dikes (e.g. the Afsluitdijk, Grevers & Zwaneveld, 2011), it is possible to renovate certain constructions, like vessel locks and drainage sluices, up to a certain safety level at relatively low costs.
- After a ‘standard’ strengthening of a dike (by increasing its height and width), additional safety can be obtained by means of tailor-made, low cost measures (like ‘strengthened’ grass for a more robust dike covering), which yield a safety equivalent of 50 centimetres dike heightening.
- A time-varying discount rate may be appropriate and is already prescribed in France and the UK (UK DfT, 2011; Hepburn, 2007).

The second advantage is that our solution procedure guarantees optimality as it is based on solving an integer linear programming (ILP) formulation. This ILP formulation is solved very quickly by standard ILP solvers such as CPLEX. The approach by Brekelmans et al. (2012) can only guarantee optimality for quadratic cost functions since these allow a reformulation to a convex problem. For most instances, Brekelmans et al. (2012) use a heuristic and case-specific approach with no optimality guarantee.

The third and final advantage is that the procedure is easy to implement. Ease of implementation is not only very important for the use of our results in Dutch practice, but also to disseminate our results to less wealthy countries.

The ILP formulation is obtained by two ideas that are implemented consecutively. First, the data are discretized: the time horizon is divided in fixed length periods, and the heights are discretized in fixed length steps. Of course, the finer the discretization the more accurate the approximation of the continuous model. However, theoretically finer discretizations do not substantially influence the quality of the optimal solution. In practice, finer discretizations do not really make much sense: height increases more accurate than 20 centimeters are not really realizable. Moreover, with respect to time, intervals of less than three months (on a time horizon of 50 years or longer) never influenced the optimal costs with more than 1%. In fact, since every height increase takes five to ten years it is not realistic to consider periods smaller than one year. These two discretizations allow us to compute costs for all time-height combinations directly from the formulas as developed by Brekelmans et al. (2012), and any other cost functions that arise from practice (see above). The discretizations of time and height also allow us to introduce binary variables for every time-height combination (and change from one pair to another). The ILP formulation contains shortest path constraints for each dike segment, which are coupled by type 1 special ordered set constraints (SOS1), see Beale and Tomlin (1970). Thus, we can use discrete dynamic programming and branch-and-cut as methods to solve the problem. Unfortunately, dynamic programming still takes a lot of time for solving, specifically when finer discretizations are used. Therefore, we use branch-and-cut (B & C) to solve the ILP which uses very little time to solve the problem to optimality. In fact, all considered problem instances were solved with the standard package CPLEX.

This manuscript is structured as follows. We start in Section 2 with the introduction of the integer linear program for the nonhomogeneous case, the most general problem. In Section 3, we discuss some implementation issues, such as the discretizations used, and variable elimination rules (preprocessing). Then the computational results are presented. We conclude in Section 4 with some remarks and future research. In the appendices we present three more integer linear programs. In appendix A we present a model, model A, for the homogeneous case, where there is essentially exactly one segment. This model is considerably simpler than the non-homogeneous model, namely a min-cost flow problem. In appendix B we present model B, a simplification of our main model C. Model B introduces cost variables which replace the binary connecting variables. This makes the constraints simpler, but introduces non-integrality and therefore this model is not really performing better than model C. Finally, in appendix C, we present model D, which generalizes model C by allowing for height dependent damage costs.

2. Cost-benefit model as an integer programming model

In this section, we define the problem of determining optimal dike heights, and we present an integer programming (IP) formulation for the problem.

The optimal timing and heightening of a dike ring is based on a cost-benefit analysis, where we attempt to minimize the total (discounted) social costs, consisting of investment costs for heightening the dikes and the expected loss by flooding. In this section we will first describe the problem of optimally heightening the dikes in more detail, and then we will develop the IP-formulation.

2.1. The cost-benefit problem

The basic question is on when and to which safety level to heighten the dikes. The basic dilemma is the trade-off between paying up the investment costs of heightening a dike ring or accepting a (higher) probability of dike failure with all associated costs of flooding.

The costs of flooding include damage costs, cost of evacuation, rescue costs and immaterial costs (e.g. victims, sufferings). These costs of flooding for a specific dike ring are based upon extensive simulations by engineers of Deltares, using the information system HIS-SSM (Kind, 2011).

The expected loss of flooding is modelled as the product of the probability of flooding and the damage done (loss by flooding). In Brekelmans et al. (2012), the damage is defined as

$$V_t = V_0 e^{\gamma t} e^{\eta(h_t - h_0)}$$

Here V_t is the damage of the dike at time t that is heightened to h_t starting from h_0. The parameters γ and η include the economic growth rate, and the increase of loss per cm of dike heightening, respectively. The flood probability is defined as the original probability used by Van Dantzig:

$$P_t = P_0 e^{\beta\alpha t} e^{-\beta(h_t - h_0)}$$

Here P_t is the flood probability at time t of the dike that is heightened to h_t starting from h_0. The parameters α and β include the structural increase of the water level in cm per year, and the exponential distribution parameter for extreme water levels, respectively.

Finally, dike investment costs are dependent of the realised height h_t, the height in the previous year h_{t-1} and the starting height h_0. In Brekelmans et al. (2012), two functions are used to model this dependence: a quadratic function and an exponential function. The exponential function is defined as

$$I_t = \begin{cases} (a + b(h_t - h_{t-1}))e^{\lambda(h_t - h_0)} & \text{if } h_t > h_{t-1} \\ 0 & \text{if } h_t = h_{t-1} \end{cases} \tag{1}$$

for a, b and $\lambda > 0$. All costs are automatically discounted with a fixed interest.

Given the above functions, the problem is to find the best times and height increases for the dike. This problem is the homogeneous version, where the dike ring is assumed to consist of one piece or segment. It has been solved in Eijgenraam et al. (2017) by a closed form formula. This method is in actual practical use by The Netherlands.

We discuss the problem of non-homogeneous dike rings. This problem was first proposed by Brekelmans et al. (2012) and considers the case in which a dike ring consists of multiple segments. Each segment has its own characteristics with respect to flood probability as a function of height. Moreover, all segments can be heightened independently of each other, with segment-specific costs of realising certain heights or safety levels.

A dike ring fails first at its weakest point. In other words, a dike ring starts to fail at a segment with the highest flood probability. Hence, the flood probability of a dike ring as a whole is the maximum of the individual flood probabilities over all segments. Since damage in case of flooding is presumed to be identical for all segments, the overall expected damage of a dike ring is the maximum of the expected damage per segment. The latter formulation is used in the model of Brekelmans et al. (2012), where the cost functions and the probabilities of all segments have the same shape as given above for the homogeneous case (of course, with the difference that the costs are now segment dependent).

In the next subsection, we present a generalization of the model for the non-homogeneous case, which is able to include segment-specific damage costs, and a variety of investment costs and flood probabilities. Hence, we relax upon the above mentioned characteristics.

2.2. General IP-formulation: model C

The main difficulty of the problem is the non-linearity of the cost functions and probabilities in the time and height variables. The non-linearity of both will therefore be removed by discretizing them. For both there are sound practical arguments as we will show.

The set of all segments of a dike ring will be denoted by L ($|L| \geq 1$). To indicate the dependence of a model parameter on a particular segment, an index l ($l \in \{1, \ldots, |L|\}$) will be added to this parameter or decision variable.

The set T ($|T| \geq 1$) represents all considered time periods in the planning horizon. We denote the time periods by $T = \{0, 1, 2, \ldots, |T| - 1\}$, where '0' represents the starting period. A time period $t \in T$ may represent any period such as a month, a year, or a decade. In practice, the exact timing of a dike heightening cannot be planned with great precision due to legislation, communication with land owners, planning uncertainties etc. Moreover, the minimal time between consecutive strengthenings of a dike ring involves 10 years due to legal and civil engineering restrictions.

The set H ($|H| \geq 1$) represents all possible safety levels/heights of a dike ring. We denote the different safety levels by $H = \{0, 1, 2, \ldots, |H| - 1\}$, where '0' is the starting level. Since not all safety levels may be available to each segment, we define the set H^l ($H^l \subseteq H$) as the set of levels available for link l ($l \in L$). Though differences in consecutive levels can be measured in theory in very small sizes, in practice, steps smaller than 20 centimeters need not be considered. Dikes are heightened with the use of clay. Due to a-priori unknown thickness of the clay, it is almost impossible to assess a priori the new height of a dike after heightening with more precision than 20 centimeters.

The aim of our paper is to provide a model formulation with maximum flexibility with respect to the input-parameters that represent investment costs and expected damage costs. Hence, we set up our model (model C) with generally applicable exogenous input data.

The decision variables of model C are:

$CY(t, l, h_1, h_2) = 1$, if segment or link l of the dike ring is updated in time period t from height/safety level h_1 up to height/safety level h_2. If $h_1 = h_2$, then the dike ring segment is not strengthened in period t and remains at its previous height. This decision variable is used for bookkeeping investment (and maintenance) costs.
0, otherwise.

$DY(t, l, h) = 1$, if segment l with height/safety level h represents the 'weakest link' in period t, i.e. the segment with the highest flood probability such that a dike ring starts to fail at this segment. This decision variable is used for bookkeeping flood probabilities and related expected damage costs.
0, otherwise.

The input-parameters are:

cost (t, l, h_1, h_2) = costs for investment and maintenance, if segment l of the dike ring is strengthened in time period t from h_1 to h_2. If $h_1 = h_2$, the dike ring segment is not strengthened in period t and these costs only represent maintenance costs.

prob (t, l, h) = flood probability, if the resulting height of segment l in period t equals h.

damage (t, l, h) = damage (i.e. 'damage in case the dike ring starts to fail at segment l'), if the resulting height of segment l in period t equals h. In the next section, we use the fact that in all cases we have encountered, damage in case of flooding only depends on a specific year. Hence, we can replace the parameter *damage(t,l,h_2)* by the parameter *damage(t)*.

Now that the time and height are discretized it is easy to compute the above parameters for given cost functions such as those used in the homogeneous case. Moreover, the model is not restricted to these cost functions. Note that all input parameters are calculated in net present value of a certain year (i.e. 2015, which is the starting year for our calculations) and represent price levels in a certain year. In our calculations, we also presume that dike heightening takes place immediately at the start of the time period $t \in T$. The final time period $|T| - 1$ includes the expected damage from this time period on.

Model C reads as follows:

$$\text{Minimize} \quad \sum_{t \in T} \sum_{l \in L} \sum_{h_1, h_2 \in H^l : h_2 \geq h_1} cost(t, l, h_1, h_2) \cdot CY(t, l, h_1, h_2) + \sum_{t \in T} \sum_{l \in L} \sum_{h \in H^l} prob(t, l, h) \cdot damage(t, l, h) \cdot DY(t, l, h) \tag{2}$$

subject to

$$CY('0', l, '0', '0') = 1; \; CY('0', l, h_1, h_2) = 0 \quad \forall l \in L; h_1, h_2 \in H^l; h_2 \geq h_1 \wedge h_2 >' 0' \tag{3}$$

$$\sum_{h_1 \in H^l : h_1 \leq h_2} CY(t-1, l, h_1, h_2) = \sum_{h_3 \in H^l : h_3 \geq h_2} CY(t, l, h_2, h_3) \quad \forall t \in T / \{'0'\}, l \in L, h_2 \in H^l \tag{4}$$

$$\sum_{h_1 \in H^l} \sum_{\substack{h_2 \geq h_1 \in H^l : \\ prob(t, l_0, h_2) > prob(t, l^*, h^*)}} CY(t, l_0, h_1, h_2) + \sum_{l \in L} \sum_{\substack{h \in H^l : \\ prob(t, l, h) \leq prob(t, l^*, h^*)}} DY(t, l, h) \leq 1 \quad \forall t \in T, l_0, l^* \in L, h^* \in H^{l^*} \tag{5}$$

$$\sum_{l \in L} \sum_{h \in H^l} DY(t, l, h) = 1 \qquad \forall t \in T \tag{6}$$

$$CY(t, l, h_1, h_2) \in \{0, 1\} \qquad \forall t \in T, l \in L, h_1, h_2 \in H^l, h_2 \geq h_1 \tag{7}$$

$$DY(t, l, h) \in \{0, 1\} \qquad \forall t \in T, l \in L, h \in H^l \tag{8}$$

The objective function (2) minimizes the total cost for investments (first term) and expected damage (second term). Constraints (3) define the starting condition for each segment l of the dike ring (i.e. its present height/strength).

Constraints (4) ensure that the final height of each segment l of the dike ring in a period $t-1$ equals the starting height of the segment in the consecutive period t.

Constraints (5) determine the value of $DY(t,l,h)$ in each period. If the overall safety level associated with segment l^* and height h^* is selected, then no segment is allowed to have a strength (l, h) below this safety level. Note that this is equivalent to a higher failing probability: $prob\ (t, l, h) > prob\ (t, l^*, h^*)$. Hence, all segments need to be at least as safe as level (l^*, h^*) prescribes.

Constraints (6) state that in each time period only one link l and one safety level h must be selected. Together with the constraints (5) these constraints ensure that the selected combination of link l and height h is the one with the highest flood probability.

Constraints (7) and (8) declare the decision variables as binary.

Note that, for every $l \in L$, $h_l \in H^l$, the constraints (4) are flow balance constraints in a network with vertices defined by all valid (t, h_l) combinations. In fact, the constraints determine a path from the starting node ('0', '0') to the final height at time $|T| - 1$ for each link $l \in L$.

The remaining non-trivial constraints, i.e. (5) and (6) are type 1 SOS-constraints, see Beale and Tomlin (1970), or independent set constraints: from a given subset of variables at most one or exactly one of the variables should be set to 1. These constraints allow the software to generate new constraints and to do (pre)processing on the variables with conflict graphs, see Atamturk, Nemhauser, and Savelsbergh (2000), Zwaneveld, Kroon, and van Hoesel (2001) and the next section.

An additional constraint that must be added to the model comes from the already mentioned fact, that a dike ring segment cannot be updated twice within a certain time period. In fact, there is a minimum update period of each segment l of $up(l)$. Hence, we can add the following constraints:

$$\sum_{t^* = t+1, \ldots, t+up(l)} \sum_{h_1 \in H^l} \sum_{h_2 > h_1 \in H^l} CY(t^*, l, h_1, h_2) \leq 1 \quad \forall l \in L, t = 0, \ldots, |T| - up(l) \tag{9}$$

Another real-world example of a crucial side-constraint is the obligation that a segment must be updated before a certain year. This is due to the fact, that certain segments wear out and need to be thoroughly reconstructed at a certain point in time.

3. Implementation, solution procedure and numerical results

We have implemented model C in GAMS and used CPLEX to solve the models to optimality by using branch-and-cut. We intentionally use these standard and easily available software packages and we avoid complex programming solutions to allow for easy usage in other environments such as less wealthy countries.

3.1. Discretization schemes

As briefly mentioned earlier, the problem size of all models depends on the chosen number of time periods and possible heightenings. Several aspects play a role in the choice of the discretization scheme for time periods and heigthenings. First, the level of refinement determines the problem size and consequently the solution time. Secondly, using a more refined discretization scheme provides the possibility to determine the optimal timing and extent of dike heightening more precisely. Finally, practical considerations, as earlier mentioned, indicate, that very fine discretization schemes are only of theoretical importance.

In addition, only model results with respect to the near future, say 20–30 years from now, are being used in practice. The Dutch government only makes decisions on actions that need to be initiated during this time period. Postponing the decision on the required dike heigthenings 30 years from now, is also economically sensible. More information will then be available on climate change, flood risks, new technical solutions to prevent flooding, and the occurring economic damage in case of flooding.

It is still important, however, to take time periods in the distant future into consideration. Possible dike heightenings in the distant future may influence dike heightening decisions in the first few decades. Therefore, we consider a planning horizon of 300 years, which is in line with Brekelmans et al. (2012) and Kind (2011). Consequently, discretization of time and height in the (distant) future can be done with larger intervals. We use the following discretization scheme:

- Heightenings in steps of 10 centimeters up to 100 centimeters, in steps of 20 centimeters for heights between 100 and 200 centimeters, and steps of 30 centimeters for larger heights.
- Up to the year 2100 5-year periods are used, after that 10-year periods are used.

After solving the model with several other discretization schemes, we concluded that the optimal solution hardly depends on the used discretization scheme.

3.2. Preprocessing: reducing the problem size

By using preprocessing techniques based upon the structure of our problem, the problem size can be reduced substantially. This is useful, because it limits computing times of the branch-and-cut procedure. We present three preprocessing techniques. Each technique aims at reducing the number of decision variables.

Technique 1: identical heightening one period later always better?

This technique searches for decision variables that can be replaced by another decision variable in any feasible solution such that the objective function value of this adjusted solution improves and the solution remains feasible.

Consider any feasible solution in which a segment l is heightened in period $t < |T| - 1$ from height h_1 up to $h_2 > h_1$. Given this solution, another feasible solution can be constructed in which this heightening takes place one period later, i.e. in $t+1$. The minimal improvement of the objective function value if segment l is not further heightened in period $t+1$, equals:

$$\begin{aligned} MxChngT(t,l,h_1,h_2) = &+ cost(t,l,h_1,h_2) - cost(t+1,l,h_1,h_2) \\ &+ (prob(t,l,h_2) - prob(t,l,h_1)) \cdot damage(t) \\ &+ cost(t+1,l,h_2,h_2) - cost(t,l,h_1,h_1) \end{aligned}$$

The first line represents the improvement in investment costs due to the incurred postponing of the dike heightening. The second line represents the *maximum* change (i.e. worsening) in expected damage. The last line represents the difference in maintenance costs in period t at height h_1 and in period $t+1$ at height h_2.

The change in expected damage is the maximum change since we implicitly assume that segment l determines the overall flood probability of the dike ring area in period t. If another segment determines the overall flood probability, then the change in expected damage equals zero. Note that the following holds due to the fact that h_2 is safer than h_1:

$$(prob(t,l,h_2) - prob(t,l,h_1)) \cdot damage(t) < 0 \quad \forall t \in T, l \in L, h_2 > h_1 \in H^l$$

It is possible that segment l is also heightened in period $t+1$ in the considered feasible solution. Suppose that this segment obtains height h_3 in period *t+1*, then the improvement will be higher, due the high fixed costs of heightening a dike ring segment twice, i.e. in $t+1$ first from h_1 to h_2 and then from h_2 to h_3. This is more expensive than heightening at once from h_1 to h_3 in period $t+1$ due to strict positive parameters of cost function (1). Formally, the following condition must hold for this technique:

$$cost(t,l,h_1,h_2) + cost(t,l,h_2,h_3) \geq cost(t,l,h_1,h_3) \quad \forall t \in T, l \in L, h_1 < h_2 < h_3 \in H^l$$

Hence, if $\exists t < |T| - 1, l \in L, h_2 > h_1 \in H^l : MxChngT(t,l,h_1,h_2) \geq 0$ and the condition above holds, then variable $CY(t, l, h_1, h_2)$ is dominated by variable $CY(t+1,l,h_1,h_2)$ and the former can be set equal to zero (or removed from the problem formulation).

Technique 2: late heightening not optimal?

This technique searches for heightenings of a segment in one of the latest time periods that are not efficient: the total costs of the heightening (including maintenance and flooding costs) are larger than the total costs of not heightening. For a specific time period t^* and segment l this happens for a heightening from h_1 to $h_2 > h_1$ if

$$cost(t^*,l,h_1,h_2) + \sum_{t>t^*} cost(t,l,h_2,h_2) + \sum_{t \geq t^*} prob(t,l,h_2) \cdot damage(t) > \sum_{t \geq t^*} cost(t,l,h_1,h_1) + \sum_{t \geq t^*} prob(t,l,h_1) \cdot damage(t)$$

The first line gives the costs of heightening segment l in t^* from h_1 to h_2 and no heightening in all following time periods $t > t^*$. The second line states the costs of doing nothing in t^* and all following time periods. If the condition above also applies to later periods $t > t^*$ and all possible heightenings $\forall h_1, h_2 \in h^l, h_2 > h_1$, then we can set $CY(t^*, l, h_1, h_2) := 0$.

Technique 3: heightening in two steps better than in one step?

This third technique searches for large heightenings at once that are so expensive that it is always better to heighten the segment in two consecutive steps. The procedure is as follows. We use mxh_l to denote the height of segment l with the lowest flood probability (i.e. the biggest height).

Let $t_1, t_2 \in T;\ t_1 < t_2;\ l \in L;\ h_1, h_2, h_3 \in H^l;\ h_3 > h_2 > h_1$

If

$$\begin{aligned} &cost(t_1,l,h_1,h_3) + \sum_{t_2 \geq t > t_1} cost(t,l,h_3,h_3) \\ &- cost(t_1,l,h_1,h_2) - \\ &\sum_{t_2 > t > t_1} cost(t,l,h_2,h_2) - cost(t_2,l,h_2,h_3) \\ &+ \sum_{t_2 > t \geq t_1} \{prob(t,l,mxh_l) - prob(t,l,h_2)\} \cdot damage(t) > 0 \end{aligned} \quad (10)$$

then $CY(t_1, l, h_1, h_3) := 0$

If condition (10) is met, then we can improve any feasible problem instance in which segment l is heightened in time period t_1 from h_1 up to h_3. This improved solution implies heightening segment l in time period t_1 from h_1 up to h_2 and heightening segment l in time period t_2 from h_2 up to h_3. The first three terms of (10) calculate the cost reduction if this segment is heightened in two consecutive steps. Note that this may result in a negative number, i.e. a cost increase. Due to higher flood probability, the second term of (10) represents the maximum difference in expected damage. Maximum due to the assumption that segment l determines the overall flood probability of the dike ring area. We use minimum flood probability (due to mxh_l) to be sure, that we do not underestimate the increase in expected damage. The reason is because segment l may be heightened from height h_3 again in the given feasible solution between time periods t_1 and t_2.

3.3. Results

The data used in our computational experiments were kindly provided by Ruud Brekelmans (Tilburg University), and thus are the same data from the experiments in Brekelmans et al. (2012).

Our computing times are measured on a Windows Server 2003 based computer with Intel Xeon E5-2670 processors and refer to the CPLEX-based branch-and-cut procedure only. We specify that CPLEX should first branch on variable *DY*. Apart from this, we use default CPLEX settings.

Table 1 shows, besides the objective function values from Eq. (2), also the True Objective values. The difference between these two objective values results from using time periods of 5–10 years. This is a consequence from our (and Brekelmans et al. (2012)) assumption that in each time period one segment only determines the expected damage for all dike rings. However, which segment is the weakest link may change within each time period due to the annual increase of flood probability resulting from climate change and subsidence. For example, at the start of a certain time period segment one is the weakest link. However at of the end of this time period segment two could become the weakest link, since this link suffers more from annual degradation. The True Objective recalculates the objective value of the optimal solution with the expected damage determined by the weakest segment per year. Therefore, the True Objective will always be larger than the objective function value of model C. The procedure of Brekelmans et al. (2012) also uses similar discretization schemes of time periods. Hence, their results show similar differences be-

Table 1
Comparison of MINLP-approach of Brekelmans et al. (2012) and model C.

Dike ring	Number of segments	MINLP approach of Brekelmans et al. (2012)		Model C		Difference in true objective (%)
		MINLP objective (M)	True objective (M)	Model C objective (M)	True objective (M)	
10	4	107.51	107.51	108.26	108.26	0.69
13	4	10.38	10.38	10.33	10.33	−0.48
14	2	94.04	94.04	94.57	94.57	0.56
16	8	1044.45	1046.08	1064.65	1064.80	1.79
17	6	377.05	377.05	380.65	380.66	0.96
21	10	217.40	217.71	221.62	221.62	1.80
22	5	373.98	374.08	378.67	378.68	1.23
36	6	395.65	395.65	395.34	395.34	−0.08
38	3	136.26	136.29	136.75	136.76	0.34
43	8	486.72	488.10	492.66	492.66	0.93
47	2	16.57	16.57	16.63	16.63	0.36
48	3	42.92	42.92	43.37	43.37	1.05

Table 2
Effect of the preprocessing techniques on the solution of branch-and-cut algorithm of model C.

Dike ring	$\|L\|$	Total number of variables	Reduction in the number of variables				Solution time (minutes)	
		before pre-	Technique			Total (%)	With pre-	Without pre-
		processing	1 (%)	2 (%)	3 (%)		processing	processing
10	4	42,988	44	0	11	56	0.06	0.12
13	4	42,988	64	0	3	67	0.03	0.11
14	2	21,494	35	0	10	45	0.03	0.04
16	8	85,976	33	0	18	51	0.36	0.55
17	6	64,482	49	0	14	63	0.10	0.27
21	10	107,470	34	0	16	50	0.89	1.34
22	5	53,735	36	0	15	51	0.16	0.50
36	6	64,482	25	0	15	40	0.10	0.12
38	3	32,241	29	0	12	41	0.05	0.08
43	8	85,976	53	1	15	68	0.48	0.70
47	2	21,494	63	1	10	74	0.03	0.03
48	3	32,241	45	0	18	64	0.04	0.07

$|L|$: number of segments.

tween the objective function value of their MINLP-model and the True Objective.

As Table 1 shows, the difference in values for the True Objective function is small for both methods, i.e. less than 2%. A detailed investigation of the optimal solutions of both models shows that they are almost identical in all instances. Due to the fact that the solution procedure of Brekelmans et al. (2012) allows for very fine heightenings (e.g. 7.292 centimeters), their True Objective is in most cases slightly better than the corresponding True Objective of model C. In a few cases (dike ring 13 and 36), model C provided a slightly better solution. This may be a consequence of the fact that we solve model C to proven optimality, while the MINLP method is a heuristic procedure. It may also be a consequence of the fact that both methods apply different discretization schemes. Clearly, we can refine our discretization scheme to find mathematically better solutions. However, this has no relevance in practice whatsoever. From the results we conclude, that both methods find (almost) identical solutions in all tested situations. Furthermore, Table 2 shows that model C could be solved to optimality within one minute for all 12 dike rings. The average solution time of the branch-and-cut procedure is 0.19 minutes. The procedure of Brekelmans et al. (2012) needed on average a solution time of 12.75 minutes.

Table 2 provides information on the effect of using preprocessing techniques for the solution time of model C. The reduction in solution time for the branch-and-cut procedure is substantial, i.e. roughly 40%. However, in terms of absolute solution time, preprocessing techniques reduce the solution times of the branch-and-cut procedure in most cases by a few seconds. The table below indicates furthermore, that the use of preprocessing technique 1 only, already reduces the problem size substantially. Although the use of preprocessing techniques does not appear to be very relevant for the solution time of the investigated problem instances, they could be relevant for more detailed or complex problem instances.

3.4. Effect of considering failure mechanisms other than overflow

The main advantage of model C is its flexibility with respect to the functional form for flood probabilities, damage and investment costs. In principle, every type of functional form could be specified.

Here, we present an example in which this flexibility is crucial for the acceptance of the results. Our model not only allows for height-based failure mechanisms, like overflow, but also for strength-based failure mechanisms, like piping or lack of structural quality. As Brekelmans et al. (2012) state on page 1343, their modelling approach presumes that actual problems with piping and the quality of some of the structures are solved before further improvements in the safety level are considered.

In practice however, there is a need to asses the optimal timing of these anti-piping measures or renovations of constructions and whether or not a simultaneous heightening should be included. In their MKBA WV21 study (p. 28, second reference situation, Kind, 2011), Deltares states that currently the overall flood probability of

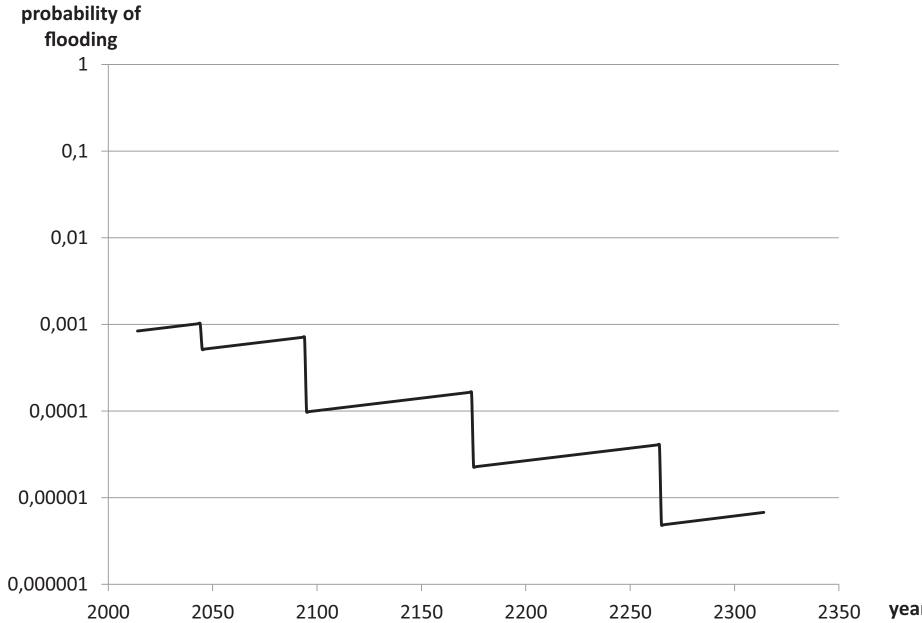

Fig. 1. Flood probabilities for dike ring 10 according to the optimal investment strategy, with the option of constructing anti piping measures only.

about half of all dike rings is to a large extent (>50%) determined by other failure mechanisms than overflow. Hence, the ability to correctly model these other failure mechanisms is of utmost importance to obtain optimal dike strengthenings. Due to the flexibility of our modelling approach, these mechanisms can be easily included.

To illustrate this flexibility, we adjust the input parameters for dike ring 10 to correctly represent the actual flood probability. The failure mechanisms piping and slope instability contribute substantially to the flood probability of dike ring 10. According to the latest insights (Kind, 2011), the total flood probability of this dike ring area is twice as high as the overflow probability. Hence, we have to double the previously used initial flood probability.

Possible measures to solve the problems related to piping and slope instability are the construction of a sheetpile wall or a partial broadening of the dike. The construction costs of these anti-piping measures are in general low in comparison with the (fixed) cost of a dike heightening. For dike ring 10, the cheapest solution to solve the piping and slope instability problems costs 78 million euro. The minimum costs of heightening (including anti-piping measures) dike ring 10 amount to 122 million euro. Given that on average a dike is heightened by about 60 centimeters, the ‘regular’ costs of a dike heightening are 273 million euro for dike ring 10. After the implementation of these anti-piping measures the flood probability is reduced by 50%.

This ‘jump’ in the flood probability and the specific costs of these anti-piping measures can be easily modelled in model C. The construction of anti-piping measures only is represented in the model by introducing safety level ‘$h = 1$’. Higher safety levels involve both the actual heightening of the dike and the construction of anti-piping measures. This represents the fact that in the case of dike heightening, the additional costs of anti-piping measures are relatively low.

Fig. 1 depicts a typical pattern for economic optimal dike heightening. Due to the existence of fixed investment costs, the dikes are periodically heightened/strengthened. After heightening, the probability of flooding will gradually increase as a result of higher water levels and the subsidence of the dike. In the long run, the economic optimal level of flooding probabilities will decline, because the economic value of goods and people behind the dikes will increase with continued economic growth.

The figure shows the relevance of allowing the construction of anti-piping measures only. It turns out to be optimal to take these anti-piping measures in the year 2045, without any further heightening at that time. This can be seen in the figure since the flood probability is halved in 2045. The dike ring should be heightened in 2095. Clearly, it is non-optimal to take these anti-piping measures directly in 2015 as assumed by Brekelmans et al. (2012).

4. Concluding remarks

This paper considers the dike height optimization problem: what is the economical optimal dike investment strategy to protect against floods? This has been a very important problem in the Netherlands for decades and recent flooding in other deltas shows that it is becoming an important issue all over the world.

We propose an integer programming model for a cost-benefit analysis to determine optimal dike heights and strengths. Our approach, as discussed in this paper, has three important advantages:

1. Virtually complete flexibility towards input-parameters and functional specifications for flood probabilities, damage costs and investments costs for dike heightening. This flexibility facilitates the inclusion of more location specific safety measures and is crucial for the acceptance of the model results by policy makers.
2. Proven optimal solutions are found for all problem instances.
3. The model is easy to implement with the use of standard software. Ease of implementation is not only important for the use of our results in Dutch practice, but also for the dissimilation of our results to less wealthy countries.

The only possible drawback of our formulation of the dike safety problem, is the required a priori discretization of the time periods and the amount of dike strengthenings. However, our results show a difference in values for the objective function smaller than 2%. This is small given the amount of uncertainty in the input data of the model.

Use of preprocessing techniques reduces the number of decision variables by roughly 50%. This improves the solution time of the branch-and-cut procedure with around 40%. This might become relevant in more elaborate problem instances, for instance in cases where a finer discretization is necessary.

In the years to come, more and more detailed local information on flood risk and different prevention measures will become available. Due to high costs and significant landscape consequences of dike heightenings, policy makers will increasingly ask for tailor-made low costs measures. The models presented in this paper and its e-companion are well suited to meet this demand.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2018.03.012.

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