Time series analysis under model uncertainty

Valorization Addendum

This chapter discusses the societal value of the research presented in this book, what is innovative about the presented results, who can benefit from them, and why they are relevant.

Before treating such a special case, the societal value of one single book, it is an interesting exercise to reflect on how scientific and technological development in general has shaped our society. Are we smarter or better off than the people were 100 years ago? What are we better at now than we were back then? The answer may be obvious when we talk about inventions of physical apparatuses like the airplane, that was a whole new method of transportation, like information technology, one might imagine what a typical insurance company looked like and how many contracts per employee it was able to manage in the past and compare this to the high-tech offices of today, or like medical procedures, whose usefulness is also rather apparent. But as we consider shorter time periods, say the last 10 years, and scientific discoveries with less direct impact the question becomes more difficult to answer and raises follow up questions. For example: How can societal value and its progress even be measured?

One of many approaches, too many to name here, that is often taken to address this question is to measure economic growth. And it is a common understanding that the advancement of technology is a central component of economic growth. Developed countries usually see long periods of such growth, some of which should be due to scientific progress, even if indirectly. Now, economic growth is typically measured as the gross domestic product, GDP, of one or several nation(s). This is were this book is of value: All of the chapters, and especially Chapter 4 in a very illustrative manner, treat the type of linear models that are to date commonly used for analyzing the dynamics of economic variables like GDP and inflation. These are vector autoregressive models, VARs. VARs describe time series data for groups of variables which interact with each other. The term time series means that the values of the variables can be observed at consecutive time points within a certain time period. Chapter 3 shows that when estimating this kind of model the proposed model averaging procedures can deliver more exact estimates than the more commonly used, corresponding model selection procedures. Here the exactness, or precision of an estimator is measured by the mean squared error, MSE. So this dissertation provides methods that improve the measurement of some important macroeconomic variables This also means that the results can be used to measure the progress of one common indicator of societal value. Or said

differently: One of the societal values of this book lies in being able to discuss societal value.

One group of users of VARs are central banks, which monitor the economy closely in order to find the level of interest rates that would lead to the inflation rate remaining in or returning to the mandated range.[1] Even though the main instruments of a central bank are designed only to set the lending rates for banks, their decisions impact the whole economy as the effect of changes in these interest rates trickle down to the interest rates that companies and retail clients are offered. So, in the end, the spending behavior and savings rate of the population, and hence the inflation rate, are influenced by the central bank's decisions. This illustrates how important it is that central banks use good models. Naturally they employ a wide range of different models and well staffed research departments, but even if after publication of this thesis the discussed methods are not immediately picked up by the research staff of central banks, who often are well connected to academia and stay up to date with the latest research, the economic impact that econometric innovations have are potentially huge: Slight changes in estimation methods, if adopted in multiple central banks, could lead to immense financial effects in total.

But the fact that a central bank can directly set certain types of lending rates, but cannot directly influence inflation, means that the application of VAR models requires some further attention when specifying the concrete model: Econometricians use model selection criteria to decide whether to use a VAR model with shorter or longer memory (lower or higher lag order). Many of these model selection criteria aim at a model that explains the observed values best, but that is not necessarily the same model as the one that describes the reaction profiles of the variables in the system to a change in one of the variables best. These reaction profiles are called impulse response functions and they are used as the leading example of focus parameters[2] throughout this book. Chapter 2 treats a model selection criterion that can be used to find the model that estimates these impulse response functions best. So, ultimately, the methods described in this book help central banks to better tune their monetary policy.

The model selection criterion called FIC - focused information criterion, that is being treated in Chapter 2, is not a new criterion. What is new about that chapter is that it extends the family of model types in which the FIC can be applied: The FIC was originally defined for usage with independent data, the type of data where one observation of the regressand is assumed not to be influenced by the other observations of the regressand. This book treats the case of time series data, where the value of a variable at one point in time does depend on the previous values. It is shown by means of mathematical analyses how the estimators of the impulse response functions, which are random variables, are distributed for a stable (in a certain sense defined in that chapter) VAR time series process that can be observed forever. Even though this can only be a thought experiment it is the cornerstone of the econometric science, where it goes under the name of

[1] Note that this is an exemplified depiction of the complex mission and range of instruments of most central banks.

[2] We use the term focus parameters, in analogy to the focused information criterion, for those parameters that are the main interest of the model's user.

asymptotic theory. The reason is that intuition gained from asymptotic theory often allows to draw conclusions for real world statistical applications with (only) a finite number of observations. The concept of consistency is an example for an asymptotic statement that is very important to econometricians. An estimator is named consistent if it delivers the true underlying parameter asymptotically. Some consistency results are also given and proven in Chapter 3, for example. Additionally, it is common practice to amend scientific publications about asymptotic theory with simulation results, treating some prototypical cases of the type of data that would be observed in real life. This is also the case for Chapters 2 and 3 of this book. Via these two contributions, mathematical theory and finite sample intuition, statistical tools are considered ready for application. This is what this book offers, it makes the focused information criterion "ready to use" for time series data.

We now come back to the definition of the focused information criterion: It is defined such that the model, which has the highest estimated precision, is selected based on the asymptotic results. The precision in question is that of estimating a certain focus parameter chosen by the user of the model. The leading example are impulse response functions, as mentioned above, but many other definitions of focus parameters are also covered by the results of this book: Any focus parameter is allowed that can be written as a smooth (continuously differentiable) function of the model's basic coefficients. This is especially interesting in VAR models since the number of basic coefficients is determined by the product of the number of variables one wants to consider with the length of the memory assumed by the model, a number that becomes relatively large even for low dimensional systems. It is hence sensible to expect that not all of the basic coefficients are equally interesting to the user of the model and by using the FIC the model (specification and) selection process can be geared towards those coefficients that are considered important.

One might wonder why there are different model selection criteria and why one would want to direct the selection process towards only some of the model coefficients and not all of them. Why do econometricians not simply find the correct model that fits the observed data perfectly? That is because econometrics is not a science of certainties. Instead it is a field of research that fine tunes its methods and makes them available for wider ranges of model settings. But in the end any model will just be an approximation, an attempt to explain some random outcomes. We see this from the discussion above, that explains that only if one was able to observe the outcomes of one process forever would it be possible to draw definite conclusions about its properties. And this would even only be the case if all the relevant variables were included in the model. That is why different researchers might use different models and methods for the same data. Some methods will explain one characteristic of the data better while others will explain other characteristics better. The model class, model specification, and model selection criterion a researcher applies to a set of data represent different ideas about the possible data generation process. It is one purpose of econometric literature to serve as a library of these different ideas. As discussed above, new ideas for approaching different kinds of data are made

"ready to use" by deriving their mathematical properties or the properties of existing methods are described for new kinds of processes. So even after decades of statistical research the literature is still growing as new and refined methods are designed inspired by users of random data who wonder how one or another procedure would perform for their application. This is another contribution of this book: It adds a piece to the library of mathematical derivations, and guidelines for the application of the presented methods. In Chapter 2, for example, the role of the locally misspecified parameters is discussed and it is shown that the problem of not being able to estimate them consistently leads to a worse performance of the FIC estimator than would be the case if the parameters were known. Other researchers can build on these derivations and do not need to formulate them themselves anymore in cases where they want to apply the FIC to time series data, for example.

Since all models can only be approximations, users of statistical methods do not have to confine themselves to the process of model selection, which is the process of finding the best suited specification for the data at hand from a predefined range of models. Model averaging is an alternative to model selection. In model averaging, instead of calculating scores for all the models under consideration, and subsequently choosing the one with the best (lowest) score for further use, weights are calculated for all the models. These are then used for calculating an averaged estimate as the weighted average of the estimates from all considered models. It turns out that such averaged estimators can be more precise than estimators based on single models. This is shown in Chapter 3. Like Chapter 2, Chapter 3 provides the asymptotic theory of methods that had already been defined for independent data settings, for the time series setting. The methods are smoothed AIC and smoothed BIC averaging. For those economists who want to use the idea of model averaging on a focus parameter that they are interested in in a time series setting, Chapter 2 shows how such a FIC averaging estimate can be meaningfully defined.

The theoretical results alluded to above lead to point estimates. As an example, an estimation procedure with a VAR model describing an economy could lead to the estimated result that if the central bank's monetary policy variable increased by one standard deviation from one quarter to the next unexpectedly, then the effect of this change on the gross domestic product two quarters later will be a reduction by 0.3%. But this number alone, this so called point estimate, does not give any information on the degree of certainty that the model attaches to this estimate. Such is usually represented by confidence intervals. In this case a 95% confidence interval might for example be described by the starting and end points -0.35% and -0.25%, meaning that with 95% probability the reduction in GDP will be in this range. Wider confidence intervals represent greater uncertainty. Confidence bands are the analog to confidence intervals for functions, and can be obtained by joining neighboring confidence intervals together. One common presentation of confidence bands is for temperature forecasts, where it can typically be observed that the confidence bands widen for longer prediction horizons as the precision of the estimates decreases. Chapter 2 of this book also presents a method of calculating confidence bands around the estimates from FIC model selection and FIC model averaging estimates. This method relies on the asymptotic distribution

of the estimators, which can be more or less relevant for finite sample results. But the estimators of Chapter 3 are also presented together with methods for constructing confidence bands. These, however, are based on the finite sample distribution of the estimators as they rely on so called bootstrapping methods. This means that the methods are relevant for all sample sizes, and hence very valuable for practitioners. Since the AIC is a heavily used model selection criterion, the smoothed AIC estimator together with the possibility of calculating confidence bands as presented in Chapter 3 should find many users among users of time series data.

While Chapters 2 and 3 provide the mathematical framework for some new econometric tools and simulation studies to illustrate their properties, Chapter 4 applies the aforementioned model selection and model averaging criteria and the bootstrap algorithm for constructing confidence bands to empirical data. The data is the same as has been used in previous econometric publications that analyse the effects of monetary policy on US macroeconomic variables by modelling VAR processes. These publications are well known among practitioners. Nowadays there are plenty of model selection criteria available whose theoretical properties are described in the literature, but it may not always be obvious how much the choice of model selection criterion affects estimation results. That is why the application of different model selection and model averaging criteria to these well known data sets helps practitioners to gain intuition on how sensible the results one might typically expect from monetary VAR models are to the choice of model selection criterion. Arguing from a slightly different perspective: There are many different methods and mathematical ideas that a researcher could spend a lot of time and paper on to write down the corresponding asymptotic theory only with the goal of producing material to impress publishers or employers. After all, the econometric literature mostly consists of methods that work, that have some use. So there would potentially still be a lot to be written down on methods that are of limited use only. But by comparing our methods with previous studies in empirical research and showing that our methods deliver estimates in similar ranges as existing, heavily used criteria, we show that our methods are not completely without application in empirically relevant cases.

To end this valorization section note that the vector autoregressive - VAR - models for which the methods in this book are discussed have autoregressive - AR - models as a special case. These have applications in many different fields, for example climate research, finance, physics, medicin, cf. e.g. Gani et al. (2009). And lastly, all the discussed methods can easily be implemented with standard statistical software packages like R and Matlab. All the necessary formulas and definitions are written down in this text.