Finitely additive strategies, zero-sum games, and decision making

SUMMARY

A noncooperative game is a model of strategic interaction between players. A player's payoff might depend on the decisions taken by other players. Decision theory can be viewed as a special case of game theory where there is only one player, called a decision maker. Game theory has many applications in economics, biology, computer science, logic, political science and psychology.

ZERO-SUM GAMES In zero-sum games there are two players. Given any pair of strategies of the players, the payoffs of the two players add up to zero. Hence, it is sufficient to consider the payoff of player 1 and also interpret it as the payment of player 2. These types of games are also called strictly competitive or conflict games.

The value is the central solution concept of zero-sum games. If a zero-sum game has a value, then this is the highest amount that player 1 can guarantee to receive, irrespective of what the other player plays. Similarly, the value is the lowest amount that player 2 can guarantee to pay, irrespective of what player 1 plays.

The theory of zero-sum games starts with von Neumann (1928), who showed that zero-sum games with finite action spaces admit a value. Games with infinite action spaces are much more complex, and Wald (1945) demonstrated that such games do not always have a value. In Wald's game there are two players who can each say any natural number. Player 1 wins, if he says a higher number or the same as player 2, otherwise he loses.

PROBABILITIES When a player makes a choice, he bases his decision on probabilities. In game theory the usual approach is to define mixed strategies as countably additive probabilities on the actions. A notable, but less frequent alternative is to define mixed strategies as finitely additive probabilities, so-called charges. Since finite additivity is a weaker requirement than countable additivity, the latter approach allows for a richer class of mixed strategies. Charges have regularly been argued for from a conceptual point of view, but they also provide technical advantages.

THESIS This thesis investigates how different models of probabilities have an effect on games. More specifically, Chapters 3 and 4 contain zero-sum games with infinite action spaces, and Chapter 5 contains a game with infinitely many time periods. In Chapter 3 we consider any game with a bounded payoff function, and mainly focus on finitely additive strategies. In Chapter 4 we consider only a certain type of zero-sum games, and focus on countably additive strategies.

In Chapter 3 we consider two-player zero-sum games with infinite action spaces and bounded payoff functions. The players' strategies are finitely additive probability measures, called charges. Since a strategy profile does not always induce a unique expected payoff, we distinguish two extreme attitudes of players. A player is viewed as pessimistic if he always evaluates the range of possible expected payoffs by the worst one, and a player is viewed as optimistic if he always evaluates it by the best one. This approach results in a definition of a pessimistic and an optimistic guarantee level for each player. We provide an extensive analysis of the relation between these guarantee levels, and connect them to the guarantee levels defined through countably additive strategies, and to other known techniques to define expected payoffs, based on computation of double integrals. In addition, we also examine existence of optimal strategies with respect to these guarantee levels.

Chapter 3 is centered around the problem that a finitely additive strategy profile does not always induce a unique expected payoff. One might reach the conclusion that finitely additive strategies are problematic, and so their use should be avoided. In Chapter 3 we only compute the guarantee levels through countably additive strategies when the action space is countable. When a game has action spaces with larger cardinalities, countably additive strategies can cause similar problems to finitely additive strategies. This is investigated in Chapter 4.

In Chapter 4 we examine the guarantee levels defined through countably additive strategies of the players in a type of zero-sum games, catch games. We show how these levels depend on the sigma-algebras that are being employed on the player's action spaces. We further argue that guarantee levels may therefore also depend on set theoretic considerations. Additionally, we calculate the guarantee levels for finitely additive strategies. The solutions of catch games essentially differ among these setups. We find optimal strategies for almost all cases.

In Chapters 3 and 4 we consider two-player games, however Chapter 5 includes a game with one player, so a decision theoretic model. Chapters 3 and 4 consider games where players move once and simultaneously. In Chapter 5 the decision maker is taking decisions over infinitely many time periods.

In Chapter 5 we consider the following class of decision problems. A decision maker chooses an action a_t from a given action space A at every period $t = 1, 2, \ldots$, and receives a payoff that is a function of the resulting sequence $(a_1, a_2, \ldots)$. A mixed strategy of the decision maker is a finitely additive probability measure on the space of pure strategies. A behavior strategy of the decision maker is a mapping that assigns to every history a finitely additive probability measure on the action space. In this setup, we address several questions, both from a conceptual and from a technical point of view. For a behavior strategy, it is generally not clear which finitely additive measure such a strategy induces on the set of plays, i.e., on the set of all infinite sequences of elements of A. Consequently, it is not clear which mixed strategies are equivalent to the given behavior strategy, and therefore what the expected payoff should be. We present and compare various approaches to this problem. Moreover, we investigate the equivalence between behavior and mixed strategies.