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Time-inconsistent preferences in general equilibrium

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Time-inconsistent Preferences in General Equilibrium[1]

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Abstract

There is a growing body of research in economics that studies the consequences of time-inconsistent preferences. This paper introduces time-inconsistent preferences in a general equilibrium setting. We discuss how the standard notion of competitive equilibrium should be extended in order to allow for changes in intertemporal preferences. Depending on whether or not agents recognize that their intertemporal preferences change, agents are called sophisticated or naïve. We present competitive equilibrium notions for economies with naïve agents and economies with sophisticated agents and provide assumptions under which both types of equilibria exist. Time-inconsistency also raises conceptual issues on the appropriate notion of efficiency. Of particular importance is the way future intertemporal preferences are taken into account. An example shows that sophisticated equilibria may be less efficient than naïve equilibria. We specify suitable conditions for which both types of equilibria satisfy appropriate efficiency notions.

1 Introduction

The vast majority of the economic literature assumes that preferences are time-consistent. With time-consistent preferences a decision concerning a future date can be made at any period before that date and will not have to be reconsidered. Psychological research, however, suggests that observed behavior is often time-inconsistent. Individuals frequently have intertemporal preferences that change over time. An example is a phenomenon known as hyperbolic discounting. Experiments suggest that people tend to be more patient in the long run than in the short run. While a person may prefer at any point in time one apple today to two apples tomorrow, he might prefer two apples eleven days from the current period to one apple ten days from that period. Hyperbolic discounting can explain this phenomenon while maintaining the assumption of constant instantaneous preferences, but exponential discounting cannot[1].

A consequence of hyperbolic discounting is that a preference for commitment will arise.[2] O'Donoghue and Rabin (1999) show the implications of hyperbolic discounting for the timing of actions. As Laibson (1997, 1998) and Angeletos et al. (2001) show, a model which takes time-inconsistency into account can have more explanatory power than a traditional model. They study a representative agent economy, where the agent has a (quasi-)hyperbolic discount utility function and thus time-inconsistent preferences. They show that this time-inconsistency can explain empirical anomalies in the data, that cannot be explained by the exponential model. The hyperbolic model can explain the willingness of consumers to hold illiquid assets, the drop in consumption around retirement, extensive credit-card borrowing and the co-movement of income and consumption.

While the effects of time-inconsistent preferences are primarily studied in partial equilibrium settings, a sound assessment of the total effects of time-inconsistent preferences requires a general equilibrium approach. Partial equilibrium models might either overestimate or underestimate the effects of changing preferences. The study of asset pricing models like the CAPM with time-inconsistent preferences requires a general equilibrium set-up. A general equilibrium approach is needed to address the validity of the first and second welfare theorems in a world with time-inconsistent preferences. The need for a general equilibrium approach is also recognized in Luttmer and Mariotti (2002a, 2002b, 2002c), where competitive equilibrium is analyzed for specific classes of exchange economies, for instance when individuals have additively separable utility functions and subjective discount functions.

In this paper, time-inconsistent preferences are introduced into a full-fledged multi-

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[1]See for instance Thaler (1991), Ainslie and Haslam (1992), Loewenstein and Prelec (1992), Rachlin and Raineri (1992), and Frederick, Loewenstein and O'Donoghue (2002).

[2]See for instance Hoch and Loewenstein (1991).

period general equilibrium model. We take a general perspective on time-inconsistent preferences, that incorporates the models of hyperbolic and quasi-hyperbolic discounting as special cases. We model individuals as consisting of a different self in every period. Thus, no intrapersonal conflicts can arise when only one period is studied. This perspective differs from the one in Benhabib and Bisin (2002), who assume that at every period an individual has two conflicting preferences.

We follow Pollak (1968), in making a distinction between naïve and sophisticated individuals. Naïve individuals are not aware of their time-inconsistent behavior. They do not realize that in the future they might be willing to reconsider choices made today. Sophisticated individuals, on the other hand, are aware of their changing preferences and will take that into account when making decisions. They will only consider future plans that they expect to stick to.

In economies with naïve agents, planned future prices turn out not to be consistent with equilibrium in future periods, and have to be revised. The concept of competitive equilibrium in economies with, possibly time-inconsistent, naïve agents is extended to allow for discrepancies in planned and realized future prices. When agents are sophisticated, they optimize in the planning period subject to the constraints imposed upon them by their own, rational, future behavior. Moreover, they are assumed to realize that other agents are sophisticated as well. We define the appropriate equilibrium concept of sophisticated equilibrium. Both equilibrium notions are compared to the standard notion of competitive equilibrium. We provide equilibrium existence results for both economies with naïve agents, and economies with sophisticated agents.

One of the most important results of general equilibrium theory concerns the efficiency of competitive equilibrium. Under standard assumptions, competitive equilibria are Pareto optimal. When intertemporal preferences change over time, the very definition of Pareto optimality has to be reconsidered. We make a distinction between myopic and forward-looking social planners. Also, we consider planners that can change both actual and planned consumption and planners that can only change the former. The relationship between these concepts and the traditional Pareto efficiency concept is formalized. We show that if social planners can change both actual and expected consumption, then naïve and sophisticated equilibria are typically inefficient. However, when social planners can only change actual consumption, then naïve and sophisticated equilibria are efficient under suitable conditions.

The outline of this paper is as follows. Section 2 introduces the model. The definition and existence of competitive equilibria in naïve societies is the subject of Section 3. The definition of equilibrium and the proof of its existence for sophisticated economies is analyzed in Section 4. Sections 5 and 6 introduce the appropriate notions of constrained

optimality, and discuss them in relation to naïve and sophisticated economies. Section 5 considers myopic social planners, while section 6 considers forward-looking social planners. Finally, Section 7 concludes. For the proofs of the lemmas and theorems we refer to the Appendix.

2   The Model

We study sequences of deterministic exchange economies with non-storable commodities. There are T periods that are indexed by $t \in T$. In each period, the economy consists of H households, indexed by $h \in H$, and L commodities, indexed by $l \in L$.[3] We will abstract from the existence of securities that enable households to make commitments with respect to future deliveries of commodities. The dynamic links between periods are therefore solely dependent on preferences. The analysis of more complicated asset structures is a highly relevant issue. However, many of the new conceptual issues that are implicated by time-inconsistent preferences do already occur in this more simple structure.

With respect to periods or time, a distinction should be made between a planning period and a consumption period. At planning period t, plans are made for consumption in periods τ, where $\tau \geq t$. Planning periods are denoted by subscripts. Superscripts denote the period for which the plan is made.

At planning period 1, households expect to have a consumption set $X_1^h \subset \mathbb{R}^{LT}$ for the remaining T periods. It is assumed that households have correct expectations about their future consumption sets. This implies that the consumption set at a planning period t can be derived from the consumption plan realized so far and the consumption set X_1^h. Throughout the paper, we assume that the consumption sets are independent of past consumption.[4] Moreover, we will assume that $X_1^h = \mathbb{R}_+^{LT}$.

At every planning period t, households foresee an initial endowment $e^{h,\tau} \in \mathbb{R}^L$ for period τ. Here again, households are assumed to have correct expectations, so $e^{h,\tau}$ is independent of the planning period t. At planning period t, the vector of all expected future endowments for household h is represented by $e_t^h = (e^{h,t}, \ldots, e^{h,T})$.

At every planning period t, every household h makes a consumption plan, which indicates how much it expects to consume in the current and future periods. For household h the consumption in period τ, planned in period t, is denoted by $x_t^{h,\tau} \in \mathbb{R}^L$. The planned

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[3]Notice that T indicates both the number of time periods, and the set of time periods. Similarly, H (L) indicates both the number of households (commodities) and the set of households (commodities). The context in which the symbol is used will make sure that no confusion can arise.

[4]Making the consumption sets depend on past consumption complicates the proofs of existence of equilibria. For instance, even when the endowments are in the interior of X_1^h, for certain realized consumption plans they might be on the boundary of the consumption set at a future planning period.

consumption path for household h at period t is denoted by $x_t^h = (x_t^{h,t}, \ldots, x_t^{h,T})$. For practical purposes some other notations will be used: $x_t^{h,-} = (x_1^{h,1}, \ldots, x_{t-1}^{h,t-1})$ equals actual consumption up to period t, $x_t^{h,\tau,\tau'} = (x_t^{h,\tau}, \ldots, x_t^{h,\tau'})$ is consumption planned at period t for the periods τ up to τ', and $x^h = (x_1^h, \ldots, x_T^h)$ denotes a consumption bundle, i.e. T consumption paths, of household h. Moreover, when we drop the superscript h, the product over all households is taken, for instance $x_t = (x_t^1, \ldots, x_t^H)$. Similarly, if we drop the subscript t, the product over all time periods is considered, $x = (x_1, \ldots, x_T)$. For all the foregoing vectors, a subscript l is added if attention is restricted to a particular commodity l. We define the set $X_t^{h,-} = \mathbb{R}_+^{L(t-1)}$. Given $x_t^{h,-} \in X_t^{h,-}$, we define the sets $X_t^{h,\tau}$, X_t^h, $X_t^{h,\tau,\tau'}$, X^h, X_t^τ, X_t, and X correspondingly by taking the appropriate projections and Cartesian products. In particular, we define $X_t^{h,\tau} = \mathbb{R}_+^L$, $X_t^h = \mathbb{R}_+^{L(T-t+1)}$, $X_t^{h,\tau,\tau'} = \mathbb{R}_+^{L(\tau'-\tau+1)}$, $X^h = \mathbb{R}_+^{LT(T+1)/2}$, $X_t = \prod_{h \in H} X_t^h$, and $X = \prod_{h \in H} X^h$.

Notice that $x \in X$ consists of T consumption paths for the entire economy, each one starting at a different time period. Consumption paths starting at different time periods are not necessarily consistent. We explicitly allow for the possibility that $x_t^\tau \neq x_{t'}^\tau$. We call x an allocation. An allocation is called time-consistent if at all periods the same consumption is planned for a given future period. This is formally expressed by the following definition.

Definition 2.1 Time-consistent Allocation

An allocation x is *time-consistent* if, for every $h \in H$, for every $t \in T$ it holds that $x_t^h = x_1^{h,t,T}$.

At every planning period, every household has preferences over present and future consumption bundles. These preferences may depend on consumption in the past. Preferences of household h at planning period t, given past consumption $x_t^{h,-}$, are represented by the preference relation $\succeq_{x_t^{h,-}}^{h,t}$ defined on X_t^h. When past consumption is clear from the context, it is sometimes omitted from the notation, and the preferences of household h at planning period t are denoted by $\succeq^{h,t}$. The preference $\succeq^h$ of household h is the collection of preferences at all possible planning periods, contingent on all possible historical consumption paths, $\succeq^h = (\succeq_{x_t^{h,-}}^{h,t})_{t \in T, x_t^{h,-} \in X_t^{h,-}}$.

An economy is described by its primitives, consumption sets, preferences, and endowments, $\mathcal{E} = (X^h, \succeq^h, e^h)_{h \in H}$.

Consider two consumption paths that coincide up to period $t' > t$. Preferences of a household are said to be time-consistent if the household prefers the one consumption path over the other at period t' if and only if it also does so at period t.

Definition 2.2 Time-consistent Preferences

Preferences for household h are *time-consistent* if for all periods $t, t' \in T$ with $t < t'$, for

every $x_t^{h,-} \in X_t^{h,-}$, and $x_t^h, \overline{x}_t^h \in X_t^h$ with $x_t^{h,t,t'-1} = \overline{x}_t^{h,t,t'-1}$ it holds that

$$x_t^h \quad \succeq_{x_t^{h,-}}^{h,t} \quad \overline{x}_t^h,$$

if and only if

$$x_t^{h,t',T} \quad \succeq_{(x_t^{h,-}, x_t^{h,t,t'-1})}^{h,t'} \quad \overline{x}_t^{h,t',T}.$$

Preferences are said to be time-inconsistent if they are not time-consistent.[5]

The following lemma shows that for the verification of time-consistency of preferences it suffices to make only comparisons involving period 1 and period t. The proofs of all lemmas and theorems are in the appendix.

Lemma 2.3
If the preferences of household h are such that for every $x_1^h, \overline{x}_1^h \in X_1^h$ with $x_1^{h\tau} = \overline{x}_1^{h\tau}$, $\tau = 1, \ldots, t-1$, it holds that

$$x_1^h \succeq^{h,1} \overline{x}_1^h$$

if and only if

$$x_1^{h,t,T} \succeq_{x_1^{h,1,t-1}}^{h,t} \overline{x}_1^{h,t,T},$$

then the preferences of household h are time-consistent.

One of the implications of the lemma is that knowledge of the preference relation $\succeq^{h,1}$, together with the requirement of time-consistency, is sufficient for the derivation of all preference relations $\succeq^{h,t}$. Time consistency makes it possible to derive all future preferences from the one at time period 1.

The consumption paths chosen by the households depend on current and expected future prices. In period t, the expected prices for period τ are denoted by $p_t^\tau \in P_t^\tau = \mathbb{R}^L$. As before, the vector of expected prices, at planning period t, for present and future periods is denoted by $p_t = (p_t^t, \ldots, p_t^T)$. The set of admissible price systems P_t is defined appropriately. The expected prices, at planning period t, for periods τ up till period τ' are denoted by $p_t^{\tau,\tau'} = (p_t^\tau, \ldots, p_t^{\tau'})$, and the complete price system over all periods is represented by $p = (p_1, \ldots, p_T)$, where P is defined appropriately. Finally, realized prices up till period t are represented by $p_t^- = (p_1^1, \ldots, p_{t-1}^{t-1})$.

A distinction will be made between naïve and sophisticated households (Pollak (1968)). Naïve households are not aware of their changing preferences. Thus, when making a

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consumption decision in planning period t, a naïve household h only takes into account the prevailing preferences at that particular period, $\succeq^{h,t}_{x^h_t,-}$. A sophisticated household, however, is aware of its changing preferences and takes these changes into account when planning consumption. That is, when planning future consumption in period t, it incorporates $\succeq^{h,\tau}_{x^h_\tau,-}$ for all $\tau \geq t$.

First, the behavior of naïve households will be addressed. Demand and supply of commodities is identified and the existence of an equilibrium established. An example should enhance the intuition on the model. The following assumptions will be made:

Ass. 1 For every $h \in H$, for every $t \in T$, the consumption set $X^h_t = \mathbb{R}^{L(T-t+1)}_+$.

Ass. 2 For every $h \in H$, $t \in T$, and $x^{h,-}_t \in X^{h,-}_t$, the preference relation $\succeq^{h,t}_{x^h_t,-}$ is complete, transitive, and continuous.

Ass. 3 For every $h \in H$, $t \in T$, and $x^{h,-}_t \in X^{h,-}_t$, the preference relation $\succeq^{h,t}_{x^h_t,-}$ is monotone, i.e. for $x^h_t, \overline{x}^h_t \in X^h_t$ with $\overline{x}^h_t \geq x^h_t$ and $\overline{x}^{h,\tau}_t \gg x^{h,\tau}_t$ for some $\tau \geq t$, it holds that $\overline{x}^h_t \succ^{h,t}_{x^h_t,-} x^h_t$.

Ass. 4 For every $h \in H$, $t \in T$, and $x^{h,-}_t \in X^{h,-}_t$, the preference relation $\succeq^{h,t}_{x^h_t,-}$ is convex in present and future consumption, i.e. for $x^h_t, \overline{x}^h_t \in X^h_t$ with $\overline{x}^h_t \succ^{h,t}_{x^h_t,-} x^h_t$ it holds that $\alpha\overline{x}^h_t + (1-\alpha)x^h_t \succ^{h,t}_{x^h_t,-} x^h_t$ for any $\alpha \in (0,1)$.

Ass. 5 For every $h \in H$, $e^h_1 \gg 0$.

The completeness, transitivity and continuity assumptions on preferences make sure that there are continuous utility functions $u^{h,t}_{x^h_t,-}$ representing the preferences. Furthermore, the utility functions are weakly increasing and quasi-concave in present and future consumption by Assumptions 3 and 4.

3 Naïve Societies

Throughout this paper, endowments are assumed to be fixed. The dependence of behavior of individuals on endowments will therefore not be made explicit. In planning period t, given a price vector p_t, the naïve household will have to make sure that in each future period the value of its consumption bundle in that period does not exceed the value of its endowment. That is, the opportunity set of the naïve household h at period t is defined by

$$\gamma^h_t(p_t) = \{x^h_t \in X^h_t \mid p^\tau_t x^{h,\tau}_t \leq p^\tau_t e^{h,\tau} \text{ for all } \tau \geq t\}.$$

The demand set of household h at period t is then given by

$$\delta_t^h(p_t, x_t^{h,-}) = \{\tilde{x}_t^h \in \gamma_t^h(p_t) \mid \tilde{x}_t^h \succeq_{x_t^h,-}^{h,t} x_t^h \text{ for all } x_t^h \in \gamma_t^h(p_t)\}.$$

In a standard competitive analysis, preferences are implicitly assumed to be time-consistent. In our more general setting, one could define a competitive equilibrium as follows.

Definition 3.1 Competitive Equilibrium
A pair $(p_1^*, x_1^*) \in P_1 \times X_1$ is a *competitive equilibrium* of the economy $\mathcal{E}$ if

(a) for all households $h \in H$, $x_1^{*h} \in \delta_1^h(p_1^*)$,

(b) $\sum_{h \in H} x_1^{*h} = \sum_{h \in H} e_1^h$.

Note that this definition only concerns the behavior in the first period. Obviously, this makes sense only when preferences are time-consistent. Another implicit assumption in the definition of competitive equilibrium in the standard setting is that allocations are time-consistent, as well as expectations on future prices. This observation leads to the following notion of extended competitive equilibrium.

Definition 3.2 Extended Competitive Equilibrium
A pair $(p^*, x^*) \in P \times X$ is an *extended competitive equilibrium* of the economy $\mathcal{E}$ if

(a) (p_1^*, x_1^*) is a competitive equilibrium,

(b) $p_t^* = p_1^{*t,T}$ for every $t \in T$, and

(c) $x_t^{*h} = x_1^{*h,t,T}$ for every $h \in H$ and every $t \in T$.

To define a competitive equilibrium notion that is appropriate for the study of economies with time-inconsistent preferences, we first suppose that all households are naïve and maximize their utilities given past consumption. Thus, at any given price system, every household demands a future consumption path that is in its demand set. The price system and demanded consumption bundles will constitute an equilibrium if at any planning period, for every commodity, the total demand for that commodity does not exceed the total endowment of that commodity. Since preferences can be time-inconsistent, it may well be that the expected consumption bundles and prices will not be equal to the actual consumption bundles and prices. However, naïve households are not able to foresee their changing preferences and the resulting changing consumption bundles and prices. Thus, at an equilibrium price system there is no household that wants to deviate from the consumption plan at any period, given the prices and price expectations at that period. This leads to the following definition of an equilibrium for naïve households.

Definition 3.3 Naïve Equilibrium
A pair $(p^*, x^*) \in P \times X$ is a *naïve equilibrium* of the economy $\mathcal{E}$ if

(a) $x_t^{*h} \in \delta_t^h(p_t^*, x_t^{*h,-})$ for all $h \in H$ and all $t \in T$,

(b) $\sum_{h \in H} x_t^{*h} = \sum_{h \in H} e_t^h$ for all $t \in T$.

The following theorem claims that the set of extended competitive equilibria is a subset of the set of naïve equilibria if preferences are time-consistent.

Theorem 3.4

If preferences of all households are time-consistent, then an extended competitive equilibrium of the economy $\mathcal{E}$ is a naïve equilibrium.

The following example shows that the converse is not necessarily true. A naïve equilibrium is constructed such that the expectations in the first period of the second period prices are not correct. Moreover, would the expected prices have been replaced by the realized prices in the second period, such that expectations would have been correct, then this would not have yielded an equilibrium in the first period. We even show that the first-period naïve equilibrium allocation is not compatible with any extended competitive equilibrium, even though all preferences are time-consistent.

Example 3.5

Consider an economy with two naïve households, two goods and two periods. The endowments of the households are $e_1^1 = (e_1^{1,1}, e_1^{1,2}) = (1, 2, 0, 4)$ and $e_1^2 = (e_1^{2,1}, e_1^{2,2}) = (2, 1, 4, 0)$. The time-consistent preferences are given by

$$
u^{1,1}(x_1^{1,1}, x_1^{1,2}) = \begin{cases} \min(x_{1,1}^{1,1}, x_{1,2}^{1,1}, x_{1,1}^{1,2}, x_{1,2}^{1,2}) \\ \qquad\qquad \text{if } \min(x_{1,1}^{1,1}, x_{1,2}^{1,1}, x_{1,1}^{1,2}, x_{1,2}^{1,2}) \le 1 \\ \\ \left[(x_{1,1}^{1,1} - 1)(x_{1,2}^{1,1} - 1)(x_{1,1}^{1,2} - 1)(x_{1,2}^{1,2} - 1)\right]^{1/4} + 1 \\ \qquad\qquad \text{if } \min(x_{1,1}^{1,1}, x_{1,2}^{1,1}, x_{1,1}^{1,2}, x_{1,2}^{1,2}) \ge 1 \end{cases}
$$

for household 1 and

$$
u^{2,1}(x_1^{2,1}, x_1^{2,2}) = \min(x_{1,1}^{2,1}, x_{1,2}^{2,1}, x_{1,1}^{2,2}, x_{1,2}^{2,2})
$$

for household 2.

Consider prices p^* such that $p_1^* = (1, 2, 4, 1)$ and $p_2^* = (3, 4)$. Then for household 1 it holds that $\min(x_{1,1}^{1,2}, x_{1,2}^{1,2}) \le 4/5 < 1$. Thus, $x_1^{*1,1} = (1\frac{2}{3}, 1\frac{2}{3}, \frac{4}{5}, \frac{4}{5})$ is an optimal consumption bundle for household 1. Moreover, $x_1^{*2,1} = (1\frac{1}{3}, 1\frac{1}{3}, \frac{16}{5}, \frac{16}{5})$ is an optimal consumption bundle for household 2. By time-consistency of preferences, when arriving in the second period,

the households maximize the following utility functions

$$
u^{1,2}(x_1^{*1,1}, x_2^{1,2}) = \begin{cases} \min(1\tfrac{2}{3}, 1\tfrac{2}{3}, x_{2,1}^{1,2}, x_{2,2}^{1,2}) \\ \qquad\qquad\qquad\qquad \text{if } \min(x_{2,1}^{1,2}, x_{2,2}^{1,2}) \leq 1 \\[2em] \left[\tfrac{2}{3} \cdot \tfrac{2}{3} \cdot (x_{2,1}^{1,2} - 1)(x_{2,2}^{1,2} - 1)\right]^{1/4} + 1 \\ \qquad\qquad\qquad\qquad \text{if } \min(x_{2,1}^{1,2}, x_{2,2}^{1,2}) \geq 1 \end{cases}
$$

and

$$
u^{2,2}(x_2^{*2,1}, x_2^{2,2}) = \min(1\tfrac{1}{3}, 1\tfrac{1}{3}, x_{2,1}^{2,2}, x_{2,2}^{2,2})
$$

Then, with prices p_2^*, the second-period budget constraint for household 1 implies that $x_{2,2}^{1,2} = 4 - 3x_{2,1}^{1,2}/4$. The first household then maximizes $(x_{2,1}^{1,2} - 1)(x_{2,1}^{1,2} - 1)$ subject to that budget constraint, which yields $x_2^{*1,2} = (2\tfrac{1}{2}, 2\tfrac{1}{8})$. For household 2, $x_2^{*2,2} = (1\tfrac{1}{2}, 1\tfrac{7}{8})$ is an optimal consumption bundle. Thus, (p^*, x^*) is a naïve equilibrium.

Note that $((x_1^{*1,1}, x_2^{*1,2}), (x_1^{*2,1}, x_2^{*2,2})) = ((1\tfrac{2}{3}, 1\tfrac{2}{3}, 2\tfrac{1}{2}, 2\tfrac{1}{8}), (1\tfrac{1}{3}, 1\tfrac{1}{3}, 1\tfrac{1}{2}, 1\tfrac{7}{8}))$ can not be a competitive equilibrium allocation. Suppose to the contrary that this allocation is a competitive equilibrium allocation. Then, since household 1 demands more than one unit of each good for the second period, it will maximize $(x_{1,1}^{1,1} - 1)(x_{1,2}^{1,1} - 1)$ in the first period subject to the budget constraint. By deriving the first-order conditions of that problem, it can easily be seen that household 1 will demand an equal amount of both goods in the first period only if $p_{1,1}^1 = p_{1,2}^1$. But then again, it would demand $1\tfrac{1}{2}$ units of each good in the first period, instead of $1\tfrac{2}{3}$ units. Thus, we arrive at a contradiction. This shows that $((x_1^{*1,1}, x_2^{*1,2}), (x_1^{*2,1}, x_2^{*2,2}))$ can not be a competitive equilibrium allocation. Moreover, by similar arguments, $p = (1, 2, 3, 4)$ cannot be a competitive equilibrium price system.

$\square$

Though a naïve equilibrium allocation might be incompatible with any extended competitive equilibrium a weaker result can be obtained. If preferences are time-consistent and a naïve equilibrium exists, then at least one of the naïve equilibria is an extended competitive equilibrium as well. This is shown in the next theorem.

Theorem 3.6
Suppose preferences are time-consistent. If a naïve equilibrium exists in the economy $\mathcal{E}$, then also an extended competitive equilibrium exists.

Now the result will be established that a naïve equilibrium exists. We will compactify the consumption sets. Following the approach of Debreu (1959) compounded with an induction argument, we show that the resulting compactified economy has an equilibrium. This then implies that the unrestricted economy has an equilibrium.

9

First of all, let $\varepsilon > 0$ and define

$$\widehat{X}_t^{h,\tau} = \left\{ x_t^{h,\tau} \in X_t^{h,\tau} \; \middle| \; x_{t,l}^{h,\tau} \leq \sum_{h \in H} e_l^{h,\tau} + \varepsilon \text{ for all } l \in L \right\}$$

for some $\varepsilon > 0$. Let $\widehat{\gamma}_t^h$ and $\widehat{\delta}_t^h$ denote the corresponding budget and demand correspondences. The economy $\widehat{\mathcal{E}}$ is the compactified economy. We show some properties of the demand correspondence $\widehat{\delta}^h$.

We denote the $(L-1)$-dimensional unit simplex by Δ, so $\Delta = \{ p \in \mathbb{R}_+^L \mid \sum_{l=1}^L p_l = 1 \}$, and we denote the k-fold Cartesian product of Δ by Δ^k. The price vectors are now restricted to the sets $\widehat{P}_t = \Delta^{T-t+1}$.

In the next lemma the box product in $p_t \square \delta_t^h(p_t, x_t^{h,-})$ is defined by taking for all $\tau \geq t$ the product of p_t^τ and any demand $x_t^{h,\tau}$ planned at period t for period τ, i.e.

$$p_t \square \delta_t^h(p_t, x_t^{h,-}) = \{ (w_t, \ldots, w_T) \in \mathbb{R}^{T-t+1} \mid \text{there is an } x_t^h \in \delta_t^h(p_t, x_t^{h,-})$$
$$\text{such that } w_\tau = p_t^\tau x_t^{h,\tau}, \tau \in t, \ldots, T \}.$$

Lemma 3.7

Suppose the economy $\mathcal{E}$ satisfies Assumptions 1-5. Consider a naïve household $h \in H$, a planning period $t \in T$, and a realized consumption plan $x_t^{h,-} \in \widehat{X}_t^{h,-}$. Then, at prices $p_t \in \widehat{P}_t$, $\widehat{\delta}_t^h(\cdot, x_t^{h,-})$ is a non-empty, compact and convex-valued, upper-hemi continuous correspondence that satisfies:

- *Walras' law, $p_t \square \widehat{\delta}_t^h(p_t, x_t^{h,-}) = \{ (p_t^t e^{h,t}, \ldots, p_t^T e^{h,T}) \}$,*

- *Homogeneity property, $\widehat{\delta}_t^h(p_t, x_t^{h,-}) = \widehat{\delta}_t^h(\overline{p}_t, x_t^{h,-})$, where for $\tau' \geq t$, for $\lambda > 0$, $\overline{p}_t^{\tau'} = \lambda p_t^{\tau'}$ and $\overline{p}_t^\tau = p_t^\tau$ for $\tau \neq \tau'$.*

Theorem 3.8 (Existence of naïve equilibrium)

If the economy $\mathcal{E}$ satisfies Assumptions 1-5, then there exists a naïve equilibrium (p^*, x^*).

The proof of the theorem requires an induction argument. That is, we first establish the existence of equilibrium prices and allocations in the first period. Then given the first period equilibrium allocations, we show that there exist equilibrium prices and allocations in the second period, and so on.

4 Sophisticated Societies

This section will consider sophisticated households. The introduction of sophisticated households generates additional complications.

The difference between a naïve and a sophisticated household is that the former is not aware of its changing preferences, whereas the latter is. A sophisticated household will only make consumption plans for the future that it expects to actually stick to. A sophisticated household can be seen as consisting of different selves, where the first self acts first and the next selves act subsequently. The behavior of the household can then be modeled as a game where the players are the different selves. A sophisticated household will then only play a subgame-perfect Nash equilibrium of that game.[6]

In the last period, no plans for the future are made. Thus, in the last period, the opportunity and demand sets of the sophisticated households resemble those for the naïve households. More specifically, the opportunity set in the last period is defined by

$$\phi_T^h(p_T, x_T^{h,-}) = \{x_T^h \in X_T^h \mid p_T^T x_T^{h,T} \leq p_T^T e^{h,T}\}.$$

The set of optimal consumption bundles in the last period is then given by

$$\xi_T^h(p_T, x_T^{h,-}) = \{\tilde{x}_T^h \in \phi_T^h(p_T, x_T^{h,-}) \mid \tilde{x}_T^h \succeq_{x_T^{h,-}}^{h,T} x_T^h \text{ for all } x_T^h \in \phi_T^h(p_T, x_T^{h,-})\}.$$

The opportunity sets in earlier periods are similar to those for the naïve households, except for the fact that the sophisticated household restricts itself to future consumption plans that are in its future demand sets at the expected future prices. That is, the opportunity set for the sophisticated household h in period t, $t < T$, is defined by

$$\phi_t^h(p_t, x_t^{h,-}) = \{x_t^h \in X_t^h \mid p_t^\tau x_t^{h,\tau} \leq p_t^\tau e^{h,\tau} \text{ for all } \tau \geq t, \text{ and } \\ x_t^{h,t+1,T} \in \xi_{t+1}^h(p_t^{t+1,T}, x_t^{h,-}, x_t^{h,t})\}.$$

Since preferences depend on past consumption, the opportunity sets also depend on past consumption. The demand set for household h in period t, $t < T$, is then given by:

$$\xi_t^h(p_t, x_t^{h,-}) = \{\tilde{x}_t^h \in \phi_t^h(p_t, x_t^{h,-}) \mid \tilde{x}_t^h \succeq_{x_t^{h,-}}^{h,t} x_t^h \text{ for all } x_t^h \in \phi_t^h(p_t, x_t^{h,-})\}.$$

The equilibrium concept for sophisticated households will be different from the one for naïve households. Since sophisticated households make plans that they will stick to in the future, following Radner (1972), we define an equilibrium price system in such a way that expected prices are equal to actual prices, i.e that households have correct point expectations concerning future prices.[7] Furthermore, it is then also reasonable to assume that the consumption choices will not have to be reconsidered. This all leads to the following equilibrium notion.

---

[6]Intrapersonal conflicts only arise when two or more periods are studied. This differs from the perspective of Benhabib and Bisin (2002), who assume that at every period an individual has two conflicting preferences.

[7]See Dutta and Morris (1997) for alternatives to the concept of rational expectations as used by Radner (1972).

Definition 4.1 Sophisticated Equilibrium

A pair $(p^*, x^*) \in P \times X$ is a *sophisticated equilibrium* if

(a) $x_t^{*h} \in \xi_t^h(p_t^*, x_t^{*h,-})$ for all $h \in H$ and all $t \in T$,

(b) $\sum_{h \in H} x_t^{*h} = \sum_{h \in H} e_t^h$ for all $t \in T$,

(c) $p_t^{*t',T} = p_{t'}^*$ for all $t, t' \in T$ with $t \leq t'$,

(d) $x_t^{*h,t',T} = x_{t'}^{*h}$ for all $t, t' \in T$ with $t \leq t'$.

Note that this sophisticated equilibrium notion requires a stronger form of rationality than the naïve equilibrium notion. In a sophisticated equilibrium, it has to be common knowledge that expectations with respect to prices are correct. This would be the case if all sophisticated households would not only be able to calculate their own optimal consumption bundles, but also the optimal consumption bundles of other households. Sophisticated households would then be able to calculate equilibria, and thus correctly anticipate equilibrium prices.

The next theorem defines an equivalent equilibrium notion.

Theorem 4.2

A pair $(p^*, x^*) \in P \times X$ is a sophisticated equilibrium if and only if it satisfies the following conditions:

(i) $x_1^{*h} \in \xi_1^h(p_1^*)$ for all $h \in H$,

(ii) $\sum_{h \in H} x_1^{*h} = \sum_{h \in H} e_1^h$ for all $h \in H$,

(iii) $p_t^* = p_1^{*t,T}$ for all $t \in T$,

(iv) $x_t^{*h} = x_1^{*h,t,T}$ for all $h \in H$ and all $t \in T$.

The next result shows that if preferences are time-consistent, then the set of sophisticated equilibria coincides with the set of extended competitive equilibria.

Theorem 4.3

Suppose that the preferences of all households are time-consistent and that Assumptions 1-2 hold. A pair $(p^*, x^*) \in P \times X$ with $p^* \gg 0$ is a sophisticated equilibrium of the economy $\mathcal{E}$ if and only if it is an extended competitive equilibrium.

In order to derive positive results we will need preferences to be independent of past consumption. Independence of past consumption is defined next.

Definition 4.4 Independence of past consumption

Preferences are independent of past consumption when $\succeq_{x_t^{h,-}}^{h,t} = \succeq_{\overline{x}_t^{h,-}}^{h,t}$ for every $x_t^{h,-}, \overline{x}_t^{h,-} \in X_t^{h,-}$.

In this section the following additional assumptions will be made:

Ass. 4' For every $h \in H$, $t \in T$, and $x_t^{h,-} \in X_t^{h,-}$, the preference relation $\succeq_{x_t^{h,-}}^{h,t}$ is strictly convex in present and future consumption, i.e. for $x_t^h, \overline{x}_t^h \in X_t^h$ with $\overline{x}_t^h \succeq_{x_t^{h,-}}^{h,t} x_t^h$ and $\overline{x}_t^h \neq x_t^h$ it holds that $\alpha \overline{x}_t^h + (1-\alpha)x_t^h \succ_{x_t^{h,-}}^{h,t} x_t^h$ for any $\alpha \in (0,1)$.

Ass. 6 Preferences are independent of past consumption.

Assumption 6 requires preferences to be independent of past consumption. This restriction for instance does not allow for habit formation, where consumption depends on consumption in the past. However, it does allow for intertemporal utility functions that discount hyperbolically or quasi-hyperbolically (Frederick, Loewenstein and O'Donoghue (2002)). Thus, Assumption 6 does not rule out typical examples of time-inconsistent preferences as considered in the literature.

If the assumptions, and in particular Assumption 6, are not satisfied, it may well happen that an equilibrium does not exist. In that case, it cannot be guaranteed that demand correspondences are convex-valued. This is illustrated in the following example.

Example 4.5

Consider an economy with two sophisticated households $h \in \{1,2\}$, two goods $l \in \{1,2\}$ and two periods $t \in \{1,2\}$. The endowments of the first and the second household are respectively given by $e_1^1 = (1,0,1,1)$ and $e_1^2 = (0,1,1,0)$. Let the preferences of household 1 in respectively the first and the second period be

$$
\begin{aligned}
u_1^1(x_1^{1,1}, x_1^{1,2}) &= x_{1,1}^{1,1} + x_{1,1}^{1,2} + \tfrac{1}{2}(x_{1,2}^{1,1} + x_{1,2}^{1,2}), \\
u_2^1(x_1^{1,1}, x_2^{1,2}) &= \begin{cases} \dfrac{\ln x_{2,1}^{1,2}}{x_{1,1}^{1,1}} + \ln x_{2,2}^{1,2} & \text{if } x_{1,1}^{1,1} > 1/3 \\ 3\ln x_{2,1}^{1,2} + \ln x_{2,2}^{1,2} & \text{otherwise.} \end{cases}
\end{aligned}
$$

Note that $u_2^1(x_1^{1,1}, x_2^{1,2})$ is continuous in $x_{2,1}^{1,2}$ and $x_{2,2}^{1,2}$. Given second-period prices $p_2 = p_1^2 \gg 0$, the demand of household 1 for the second period satisfies

$$
\begin{aligned}
x_{2,1}^{1,2} &= \frac{p_{2,1} + p_{2,2}}{p_{2,1}} \frac{1}{x_{1,1}^{1,1}} \frac{1}{\frac{1}{x_{1,1}^{1,1}} + 1} = \frac{p_{2,1} + p_{2,2}}{p_{2,1}} \frac{1}{1 + x_{1,1}^{1,1}}, \\
x_{2,2}^{1,2} &= \frac{p_{2,1} + p_{2,2}}{p_{2,2}} \frac{x_{1,1}^{1,1}}{1 + x_{1,1}^{1,1}}
\end{aligned}
$$

if $x_{1,1}^{1,1} > 1/3$, and

$$
\begin{aligned}
x_{2,1}^{1,2} &= \frac{p_{2,1} + p_{2,2}}{p_{2,1}} \frac{1}{1 + 1/3}, \\
x_{2,2}^{1,2} &= \frac{p_{2,1} + p_{2,2}}{p_{2,2}} \frac{1/3}{1 + 1/3}
\end{aligned}
$$

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if $x_{1,1}^{1,1} \leq 1/3$.

By substituting these second period demands and the first period budget constraint in the first period utility function, household 1 maximizes the indirect utility function v in period 1, where

$$
\begin{aligned}
v(x_{1,1}^{1,1}) &= x_{1,1}^{1,1} + \frac{p_{2,1} + p_{2,2}}{p_{2,1}} \frac{1}{1 + x_{1,1}^{1,1}} + \frac{1}{2} \left(\frac{p_{1,1}^1 - p_{1,1}^1 x_{1,1}^{1,1}}{p_{1,2}^1} + \frac{p_{2,1} + p_{2,2}}{p_{2,2}} \frac{x_{1,1}^{1,1}}{1 + x_{1,1}^{1,1}} \right) \\
&= \frac{p_{1,1}^1}{2p_{1,2}^1} + \left(1 - \frac{p_{1,1}^1}{2p_{1,2}^1} \right) x_{1,1}^{1,1} + \frac{p_{2,1} + p_{2,2}}{1 + x_{1,1}^{1,1}} \left(\frac{1}{p_{2,1}} + \frac{x_{1,1}^{1,1}}{2p_{2,2}} \right)
\end{aligned}
$$

if $x_{1,1}^{1,1} > 1/3$, and

$$
v(x_{1,1}^{1,1}) = x_{1,1}^{1,1} + \frac{p_{2,1} + p_{2,2}}{p_{2,1}} \frac{1}{1 + 1/3} + \frac{1}{2} \left(\frac{p_{1,1}^1 - p_{1,1}^1 x_{1,1}^{1,1}}{p_{1,2}^1} + \frac{p_{2,1} + p_{2,2}}{p_{2,2}} \frac{1/3}{1 + 1/3} \right)
$$

if $x_{1,1}^{1,1} \leq 1/3$. Note that in the former case v is a nonlinear function, whereas in the latter case it is linear. The demand correspondence resulting from v is not everywhere convex-valued.

The time-consistent preferences of household 2 in respectively the first and the second period are given by

$$
\begin{aligned}
u_1^2(x_1^{2,1}, x_1^{2,2}) &= \min(x_{1,1}^{2,1}, x_{1,2}^{2,1}) + x_{2,1}^{2,2} + 2x_{2,2}^{2,2}, \\
u_2^2(x_1^{2,1}, x_2^{2,2}) &= x_{2,1}^{2,2} + 2x_{2,2}^{2,2}.
\end{aligned}
$$

In the appendix we show that there is no sophisticated equilibrium in this economy.

$\square$

The next lemma states that Assumptions 1-6 and 4' suffice to obtain convex-valuedness of demand of sophisticated individuals. In particular, it is shown that demand is either empty or single-valued. Another approach would be to study a continuum economy as in Luttmer and Mariotti (2002a, 2002b).

Lemma 4.6

Suppose the economy $\mathcal{E}$ satisfies Assumptions 1-6 and 4'. Consider a sophisticated household $h \in H$, a planning period $t \in T$, and a realized consumption plan $x_t^{h,-} \in X_t^{h,-}$. Then, at prices $p_t \in P_t$, $\xi_t^h(\cdot, x_t^{h,-})$ is either empty or single-valued.

We will now establish an existence theorem. The first step is again to compactify the consumption sets and examine the compactified economy $\widehat{\mathcal{E}}$. The next lemma shows that demand in the compactified economy satisfies standard properties needed to show existence.

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Lemma 4.7

Suppose the economy $\mathcal{E}$ satisfies Assumptions 1-6, and 4'. Then, at prices $p_t \in \widehat{P}_t$, $\widehat{\xi}_t^h$ is a non-empty, compact-valued and continuous function that satisfies the homogeneity property and Walras' law for every $h \in H$, $t \in T$.

The existence proof is fairly standard. It can be found in the appendix. The major complication to be taken care of is the part of the proof that shows a sophisticated equilibrium of the compactified economy to remain an equilibrium after the bounds on consumption sets have been removed.

Theorem 4.8 Existence of sophisticated equilibrium

If the economy $\mathcal{E}$ satisfies Assumptions 1-6 and 4', then there exists a sophisticated equilibrium (p^*, x^*).

5   Myopic Social Planners

The next two sections address the issue to what degree the equilibria in both naïve and sophisticated societies are efficient. An allocation is called efficient if there is no feasible way for a social planner to improve on that allocation. A specification of what is feasible for a social planner and a definition of "improve on", leads to a particular efficiency notion.

We distinguish between social planners that care only about the current self of every household and social planners that care about all selves of every household. Another distinction is made between social planners that can only reallocate the consumption that will actually be consumed in the period they are operating in and social planners that can also affect expectations concerning future consumption. This section will consider social planners that care about only the current self of each household. The next section will consider social planners that care about all selves.

When a social planner takes into account only the intertemporal preferences of the households in one particular period, this can either mean that the social planner cares only about the selves corresponding to the period in which the planner is active and is myopic in that it forgets to realize that the preferences of future selves might differ from the ones of current selves. Another interpretation is that the social planner has reasons to believe that the preferences of the current selves of the households are the true underlying preferences of the households and that the preferences of the future selves of the households are distorted preferences. Nevertheless, we will refer to the social planners in this section as being myopic.

We first assume that social planners can alter both actual consumption and expectations concerning future consumption. We show that in that case naïve and sophisticated

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equilibria are typically not efficient. We also introduce a weaker efficiency concept that is satisfied by equilibrium allocations under certain assumptions. For this weaker concept we assume that social planners can only alter actual consumption.

5.1 Myopic Overall Pareto Efficiency

In this subsection we assume that myopic social planners can alter both actual consumption and expectations concerning future consumption. An allocation will be called myopic overall Pareto efficient if there is no planning period t where actual consumption in that particular period could be reallocated in such a way that every household would be at least as well off in that period as at the original allocation, whereas one household would be strictly better off than at the original allocation. This is given by the following definition.

Definition 5.1 Myopic Overall Pareto (MOP) Efficiency

The allocation x^* is *myopic overall Pareto (MOP) efficient* if there is no other allocation $\tilde{x}$ and no period t' such that

(i) $\sum_{h \in H} \tilde{x}_{t'}^h = \sum_{h \in H} e_{t'}^h$,

(ii) $\tilde{x}_{t'}^h \succeq_{x_{t'}^{*h},-}^{h,t'} x_{t'}^{*h}$ for all $h \in H$, and

(iii) $\tilde{x}_{t'}^{h'} \succ_{x_{t'}^{*h'},-}^{h',t'} x_{t'}^{*h'}$ for some $h' \in H$.

The intuition behind this definition is as follows. In every planning period t there is a social planner who tries to maximize only the preferences of the selves of the households at period t. The social planner reallocates both current consumption and expected future consumption plans. Now an equilibrium is called MOP efficient if there is no sequence of social planners that behave as described and that can make at least one household better off than in equilibrium, while not making any household worse off.[8]

MOP efficiency is closely related to Pareto efficiency. In our multi-period context, by restricting attention to the preferences of households at period 1, Pareto efficiency could be defined as follows.

Definition 5.2 Pareto Efficiency

The allocation x^* is *Pareto efficient* if there is no other allocation $\tilde{x}$ such that

(i) $\sum_{h \in H} \tilde{x}_1^h = \sum_{h \in H} e_1^h$,

(ii) $\tilde{x}_1^h \succeq^{h,1} x_1^{*h}$ for all $h \in H$, and

---

[8]Note that a MOP efficient allocation is renegotiation-proof in the sense of Luttmer and Mariotti (2002b).

(iii) $\tilde{x}_1^{h'} \succ^{h',1} x_1^{*h'}$ for some $h' \in H$.

The following theorem shows that the two concepts are equivalent when preferences are time-consistent and when attention is restricted to time-consistent allocations.

Theorem 5.3

Suppose preferences are time-consistent. Then a time-consistent allocation is MOP efficient if and only if it is Pareto efficient.

In settings with incomplete markets, it has been shown that equilibria are typically not Pareto efficient. They typically are not even efficient when weaker efficiency concepts are used.[9] Therefore, since we have a sequence of markets that do not allow for intertemporal income transfers, examples showing that naïve and sophisticated equilibria may be MOP inefficient should not come as a surprise.

Example 5.4

Consider an economy with two households, two periods and two goods in each period. Let the endowments be given by $e^{h,\tau} = (1,1)$ for all h, τ. The first period preferences of respectively the first and the second household are given by:

$$
\begin{aligned}
u_1^1(x_1^1) &= 2(x_{1,1}^{1,1} + x_{1,2}^{1,1}) + x_{1,1}^{1,2} + x_{1,2}^{1,2} \\
u_1^2(x_1^2) &= x_{1,1}^{2,1} + x_{1,2}^{2,1} + 2(x_{1,1}^{2,2} + x_{1,2}^{2,2}),
\end{aligned}
$$

and in the second period by

$$
\begin{aligned}
u_2^1(x_2^1) &= x_{2,1}^{1,2} + x_{2,2}^{1,2} \\
u_2^2(x_2^2) &= 2(x_{2,1}^{2,2} + x_{2,2}^{2,2}).
\end{aligned}
$$

Consider prices $p_t^\tau = (1,1)$ for every τ, t with $\tau \geq t$. Then the allocation

$$
\begin{aligned}
(x_{1,1}^{*h,1}, x_{1,2}^{*h,1}, x_{1,1}^{*h,2}, x_{1,2}^{*h,2}) &= (1,1,1,1), \\
(x_{2,1}^{*h,2}, x_{2,2}^{*h,2}) &= (1,1)
\end{aligned}
$$

is a naïve and sophisticated equilibrium. However, with the allocation $\tilde{x}$ as defined next, the households are better off in the first period.

$$
\begin{aligned}
(\tilde{x}_{1,1}^{1,1}, \tilde{x}_{1,2}^{1,1}, \tilde{x}_{1,1}^{1,2}, \tilde{x}_{1,2}^{1,2}) &= (2,2,0,0), \\
(\tilde{x}_{1,1}^{2,1}, \tilde{x}_{1,2}^{2,1}, \tilde{x}_{1,1}^{2,2}, \tilde{x}_{1,2}^{2,2}) &= (0,0,2,2), \\
(\tilde{x}_{2,1}^{h,2}, \tilde{x}_{2,2}^{h,2}) &= (1,1).
\end{aligned}
$$

Thus, this naïve and sophisticated equilibrium is not MOP efficient.

$\square$

---

[9]See for instance Geanakoplos and Polemarchakis (1986), Citanna, Kajii and Villanacci (1998) and Herings and Polemarchakis (2003).

The next example shows that a sophisticated equilibrium may even be dominated by a naïve equilibrium.

Example 5.5

Consider a two-period economy with two households and two goods. Let $e^{h,\tau} = (1,1)$ for all h, τ. The first-period utilities for the two households are as follows:

$$
\begin{aligned}
u_1^1(x_1^1) &= 2(x_{1,1}^{1,1} + x_{1,1}^{1,2}) + x_{1,2}^{1,1} + x_{1,2}^{1,2}, \\
u_1^2(x_1^2) &= x_{1,1}^{2,1} + x_{1,1}^{2,2} + 2(x_{1,2}^{2,1} + x_{1,2}^{2,2}).
\end{aligned}
$$

The utilities in the second period are given by:

$$
\begin{aligned}
u_2^1(x_2^{1,2}) &= x_{2,1}^{1,2} + 2x_{2,2}^{1,2}, \\
u_2^2(x_2^{2,2}) &= 2x_{2,1}^{2,2} + x_{2,2}^{2,2}.
\end{aligned}
$$

Note that the utility functions are continuous and strictly increasing in every argument.

Suppose that the prices of the two goods are equal in the second period, i.e. $p_{2,1}^2 = p_{2,2}^2$. Since household 1 (2) cares more about consumption of good 2 (1) in period 2, it will spend all of its wealth on good 2 (1). More specifically, the demanded consumption bundles are as follows:

$$
\begin{aligned}
(x_{2,1}^{*1,2}, x_{2,2}^{*1,2}) &= (0,2), \\
(x_{2,1}^{*2,2}, x_{2,2}^{*2,2}) &= (2,0).
\end{aligned}
$$

Since markets clear this allocation can be sustained in equilibrium.

Note that the foregoing analysis does not depend on whether households are naïve or sophisticated. Both types of households would face the same maximization problem. For the analysis of period 1, however, a distinction has to be made between naïves and sophisticates. To avoid confusion, a superscript "n" is added for naïve households and a superscript "s" for sophisticated households.

Suppose that for each period the expected prices of both goods are equal, i.e. $p_{1,1}^1 = p_{1,2}^1$ and $p_{1,1}^2 = p_{1,2}^2$. Since the naïve household 1 (2) now cares most about consumption of good 1 (2), the demanded consumption bundles are as follows:

$$
\begin{aligned}
(x_{1,1}^{*n1,1}, x_{1,2}^{*n1,1}, x_{1,1}^{*n1,2}, x_{1,2}^{*n1,2}) &= (2,0,2,0), \\
(x_{1,1}^{*n2,1}, x_{1,2}^{*n2,1}, x_{1,1}^{*n2,2}, x_{1,2}^{*n2,2}) &= (0,2,0,2).
\end{aligned}
$$

Thus, again, the assumed relative prices can be sustained in equilibrium.

Now the assumption will be made that all households are sophisticated. Suppose again that the prices of the two goods are equal in both periods. Demand for sophisticated

households is as follows:

$$(x^{*s1,1}_{1,1}, x^{*s1,1}_{1,2}, x^{*s1,2}_{1,1}, x^{*s1,2}_{1,2}) = (2, 0, 0, 2),$$
$$(x^{*s2,1}_{1,1}, x^{*s2,1}_{1,2}, x^{*s2,2}_{1,1}, x^{*s2,2}_{1,2}) = (0, 2, 2, 0),$$

and again markets clear.

Note that both households are better off if they are both naïve than if they are both sophisticated. This can be seen by calculating their utility levels in both cases.

$$(u^{n1}_1, u^1_2) = (8, 4),$$
$$(u^{s1}_1, u^1_2) = (6, 4),$$

$$(u^{n2}_1, u^2_2) = (8, 4),$$
$$(u^{s2}_1, u^2_2) = (6, 4).$$

By reallocating the goods in the sophisticated equilibrium, all households can be made strictly better off as at the equilibrium allocations in period 1. This means that the sophisticated equilibrium is not MOP efficient. Moreover, all households could benefit from being naïve instead of sophisticated. Here the sophisticated households are too forward-looking.

□

5.2   Constrained Myopic Periodical Efficiency

In this subsection we assume that a social planner can only reallocate commodities in the current period. This leads to the following definition.

Definition 5.6 Constrained Myopic Periodical (CMP) Efficiency[10]
The feasible allocation x^* is *constrained myopic periodically (CMP) efficient* if there is no other allocation $\tilde{x}$ and no period t' such that

(i) $\tilde{x}^{h,t'+1,T}_{t'} = x^{*h,t'+1,T}_{t'}$ for every $h \in H$,

(ii) $\sum_{h \in H} \tilde{x}^{h,t'}_{t'} = \sum_{h \in H} e^{h,t'}$,

(iii) $\tilde{x}^h_{t'} \succeq^{h,t'}_{x^{*h,-}_{t'}} x^{*h}_{t'}$ for all $h \in H$, and

(iv) $\tilde{x}^{h'}_{t'} \succ^{h',t'}_{x^{*h',-}_{t'}} x^{*h'}_{t'}$ for some $h' \in H$.

---

[10]Note that a CMP efficient allocation is renegotiation-proof in the sense of Luttmer and Mariotti (2002b).

The interpretation of this concept is similar to the interpretation of the concept of MOP efficiency. The following theorem says that this concept is weaker than the concept of myopic overall Pareto efficiency. Its proof is obvious and therefore omitted.

Theorem 5.7

If an allocation is myopic overall Pareto efficient, then it is CMP efficient.

The following theorem shows that a naïve equilibrium allocation must be CMP efficient under the assumptions of the foregoing sections.

Theorem 5.8

In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3 and 4 a naïve equilibrium allocation is CMP efficient.

As the next example shows, a sophisticated equilibrium allocation is not necessarily CMP efficient under Assumptions 2, 3 and 4.

Example 5.9

Consider an economy with two sophisticated households, two commodities and two periods. Let the preferences of household 1 be given by

$$u_1^1(x_1^{1,1}, x_1^{1,2}) \;=\; x_{1,1}^{1,1} + \frac{1}{4}x_{1,2}^{1,1} + x_{1,1}^{1,2} + \frac{1}{4}x_{1,2}^{1,2}$$

$$u_2^1(x_1^{1,1}, x_2^{1,2}) \;=\; \begin{cases} x_{2,1}^{1,2} + \frac{1}{4}x_{2,2}^{1,2} & \text{if } \quad x_{1,1}^{1,1} \le 1 \\[2mm] \frac{1}{4}x_{2,1}^{1,2} + x_{2,2}^{1,2} & \text{if } \quad x_{1,1}^{1,1} > 1. \end{cases}$$

Let the preferences of household 2 be given by

$$u_1^2(x_1^{2,1}, x_1^{2,2}) \;=\; \frac{1}{4}x_{1,1}^{2,1} + x_{1,2}^{2,1} + \frac{1}{4}x_{1,1}^{2,2} + x_{1,2}^{2,2}$$

$$u_2^2(x_1^{2,1}, x_2^{2,2}) \;=\; \begin{cases} \frac{1}{4}x_{2,1}^{2,2} + x_{2,2}^{2,2} & \text{if } \quad x_{1,2}^{2,1} \le 1 \\[2mm] x_{2,1}^{2,2} + \frac{1}{4}x_{2,2}^{2,2} & \text{if } \quad x_{1,2}^{2,1} > 1. \end{cases}$$

Let the endowments be given by $e_l^{h,t} = 1$ for every good l, for every period t, for every household h.

Now consider prices and allocation (p^*, x^*), where $p_{t,l}^{*\tau} = 1$ for every $l \in L$ and every $t, \tau \in T$ with $t \le \tau$, $x_1^{*1} = (1, 1, 2, 0)$, $x_1^{*2} = (1, 1, 0, 2)$, $x_2^{*1} = (2, 0)$, and $x_2^{*2} = (0, 2)$. It can easily be seen that the pair (p^*, x^*) constitutes a sophisticated equilibrium.

However, consider the allocation $\tilde{x}$ where $\tilde{x}_1^1 = (2,0,2,0)$, $\tilde{x}_1^2 = (0,2,0,2)$, $\tilde{x}_2^1 = (2,0)$, and $\tilde{x}_2^2 = (0,2)$. Note that in the first period both households are better off. Therefore, the sophisticated equilibrium allocation x^* is not CMP efficient. $\qquad\qquad\square$

By restricting the degree of time-inconsistency in such a way that consumption decisions do not depend on past consumption, a sophisticated equilibrium allocation can be shown to be CMP efficient. This is done in the following theorem.

Theorem 5.10
In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3, 4 and 6, a sophisticated equilibrium allocation is CMP efficient.

Thus, if the degree of time-inconsistency is restricted, then a sophisticated equilibrium is CMP efficient. Note that the same restriction was needed in order to derive existence of equilibria. As soon as consumption decisions are allowed to depend on past consumption, a sophisticated equilibrium does no longer have to exist. Moreover, as this section shows, even if a sophisticated equilibrium does exist in that case, it may be inefficient. This illustrates once again the importance of the degree of sophistication in the model.

6 Non-Myopic Social Planners

In this section we will consider social planners that care about all selves. A social planner will reallocate consumption only if by doing so he can make one self of one household better off, while not making any self of any household worse off. Note that on the one hand one might expect social planners to have more opportunities to improve welfare now, in the sense that there are more selves to be made better off. On the other hand, social planners have less opportunities to improve welfare since preferences depend on consumption in the past. In the foregoing section a social planner was allowed to make future selves worse off, but here that is no longer allowed. Thus, there is no clear direct relationship between the concepts in this section and the concepts in the foregoing section.

In order to define the concepts in this section, we need to extend the preferences of households. We now want to enable households to compare consumption bundles with different realized past consumption. Although we assumed up to now that households have preferences over future consumption *given* past consumption, people also often seem to have preferences over past consumption. People cannot change the past, but one may assume that they know what past they would have liked if they could have chosen it. In this subsection we will extend the preferences of a household h in period t to preferences with domain X_1^h. We denote these preferences by $\succeq^{*h,t}$ and impose the following restriction

on them

$$(x_t^{h,-}, x_t^h) \quad \succeq^{*h,t} \quad (x_t^{h,-}, \overline{x}_t^h)$$

$$\text{iff}$$

$$x_t^h \quad \succeq_{x_t^{h,-}}^{h,t} \quad \overline{x}_t^h$$

for $x_t^h, \overline{x}_t^h \in X_t^h$ and $x_t^{h,-} \in X_t^{h,-}$. Notice that on top of comparing consumption bundles with identical past consumption, $\succeq^{*h,t}$ can also be used to compare consumption bundles with different past consumption.

We need to redefine independence of preferences on past consumption in order to obtain positive results in this section.

Definition 6.1 Strong independence of past consumption
Preferences $\succeq^{*h,t}$ are strongly independent of past consumption if the following holds: $(\widehat{x}_t^{h,-}, \widehat{x}_t^h) \succeq^{*h,t} (\overline{x}_t^{h,-}, \overline{x}_t^h)$ if and only if $(x_t^{h,-}, \widehat{x}_t^h) \succeq^{*h,t} (\breve{x}_t^{h,-}, \overline{x}_t^h)$ for every $x_t^{h,-}, \breve{x}_t^{h,-} \in X_t^{h,-}$.

We replace Assumption 6 by Assumption 6', which is as follows.

Ass. 6' Preferences $\succeq^{*h,t}$ are strongly independent of past consumption.

Note that this assumption is stronger than Assumption 6. Consider a household with preferences $\succeq^{*h,t}$ represented by $U(x_t^{h,-}, x_t^h) = \sum_{t'<t} \sum_{l \in L} x_{t',l}^{h,t'} + \sum_{t' \geq t} \sum_{l \in L} x_{t,l}^{h,t'}$. These preferences do satisfy Assumption 6, but not Assumption 6'. With these specific preferences past consumption does not influence current behavior, but past consumption does influence current utility.

We will again first consider social planners that can alter both actual consumption and expectations about future consumption.

6.1 Overall Pareto Efficiency

In this subsection we consider social planners that can alter expectations concerning future consumption. Combined with the assumption that social planners care about all selves, the natural extension of the myopic overall Pareto efficiency concept yields the following definition.

Definition 6.2 Overall Pareto (OP) Efficiency
The feasible allocation x^* is *overall Pareto (OP) efficient* if there is no other allocation $\tilde{x}$ and no period t' such that

(i) $\sum_{h \in H} \tilde{x}_t^h = \sum_{h \in H} e_t^h$ for all $t \geq t'$,

(ii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^{h,t'}, \ldots, \tilde{x}_{t-1}^{h,t-1}, \tilde{x}_t^h) \succeq^{*h,t} (x_t^{*h,-}, x_t^{*h})$ for all $h \in H$ and all $t \geq t'$, and

(iii) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h',t'}, \ldots, \tilde{x}_{t''-1}^{h',t''-1}, \tilde{x}_{t''}^{h'}) \succ^{*h',t''} (x_{t''}^{*h',-}, x_{t''}^{*h'})$ for some $h' \in H$ and some $t'' \geq t'$.

The intuition underlying this definition is as follows. The same social planner is now assumed to be active in every period, or social planners can specify binding contracts for their successors. A social planner will reallocate commodities only if by doing so one self of one household is made better off and no self of any household is made worse off.

Theorem 6.3
Suppose preferences are time-consistent and satisfy Assumption 6'. If a time-consistent allocation is Pareto efficient, then it is OP efficient.

Note that an OP efficient allocation might not be Pareto efficient, even if preferences are time-consistent and satisfy Assumption 6'. Consider for instance an economy with two households, where one household has a much lower discount factor than the other. Consider an initial allocation where consumers have strictly positive endowments both in period 1 and in period 2. Then a social planner that cares only about the selves in the first period would let one household consume only in the first period and the other only in the second period. However, when the social planner would also have to take care of future selves of the households, this would not be possible. The household that would not consume in the second period would be better off in the first period, but worse off in the second period.

Consider Example 5.4. Note that, while households are better off in the first period at the new allocation $\tilde{x}$ compared to the naïve and sophisticated equilibrium allocation x^*, they are not worse off in the second period. Thus, the naïve and sophisticated equilibrium allocation x^* is not OP efficient. Similarly, the sophisticated allocation in Example 5.5 is not OP efficient.

The overall Pareto efficiency concept is not very appropriate for application to sophisticated societies. We allowed a social planner to change actual and expected consumption in such a way that the resulting reallocation might be time-inconsistent. Consider what might happen in a sophisticated society. In every period a social planner announces actual and expected consumption bundles for that period. It might happen that the expected consumption that a previous planner announced is not the same as the consumption the current planner announces. A sophisticated household will then realize that the current announcement might also not come true the next period. Therefore, there is no reason why the sophisticated household would believe the announcement of the social planner, which contradicts the fact that a social planner would be able to alter expectations of households.

In sophisticated societies, it makes sense to allow a social planner to change allocations only in such a way that the reallocation is time-consistent. This is formalized in the following definition.

Definition 6.4 TC Overall Pareto (TCOP) Efficiency

The feasible time-consistent allocation x^* is *time-consistent overall Pareto (TCOP) efficient* if there is no other time-consistent allocation $\tilde{x}$ and no period t' such that

(i) $\sum_{h \in H} \tilde{x}_t^h = \sum_{h \in H} e_t^h$ for all $t \geq t'$,

(ii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^{h,t'}, \ldots, \tilde{x}_{t-1}^{h,t-1}, \tilde{x}_t^h) \succeq^{*h,t} (x_t^{*h,-}, x_t^{*h})$ for all $h \in H$ and all $t \geq t'$, and

(iii) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h',t'}, \ldots, \tilde{x}_{t''-1}^{h',t''-1}, \tilde{x}_{t''}^h) \succ^{*h',t''} (x_{t''}^{*h',-}, x_{t''}^{*h'})$ for some $h' \in H$ and some $t'' \geq t'$.

The following theorem simplifies this definition, which we will need in later results.

Theorem 6.5

A feasible time-consistent allocation x^* is TCOP efficient if and only if there is no other time-consistent allocation $\tilde{x}$ and no period t' such that

(i) $\sum_{h \in H} \tilde{x}_1^h = \sum_{h \in H} e_1^h$,

(ii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{*h,t} x_1^{*h}$ for all $h \in H$ and all $t \geq t'$, and

(iii) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{*h',t''} x_1^{*h'}$ for some $h' \in H$ and some $t'' \geq t'$.

Theorem 6.6

Suppose preferences are time-consistent and satisfy Assumption 6'. If a time-consistent allocation is Pareto efficient then it is TCOP efficient.

A similar argument as before shows that a TCOP efficient allocation might be Pareto inefficient.

The following theorem claims that if preferences are independent of past consumption, then a sophisticated equilibrium allocation is TCOP efficient. We refer to Example 6.11 for an illustration that we indeed need independence of preferences on past consumption.

Theorem 6.7

In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3, 4 and 6', a sophisticated equilibrium allocation is TCOP efficient.

6.2 Constrained Periodical Efficiency

In this section we assume that social planners care about all selves, but that once equilibrium expected consumption is given, the planners cannot alter those expectations anymore. That is, a social planner can only alter actual consumption. This yields the following definition.

Definition 6.8 Constrained Periodical (CP) Efficiency
The feasible allocation x^* is *constrained periodically (CP) efficient* if there is no other allocation $\tilde{x}$ and no period t' such that

(i) $\tilde{x}_{t'}^{h,t'+1,T} = x_{t'}^{*h,t'+1,T}$ for every $h \in H$,

(ii) $\sum_{h \in H} \tilde{x}_{t'}^{h,t'} = \sum_{h \in H} e^{h,t'}$,

(iii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{*h,t} (x_t^{*h,-}, x_t^{*h})$ for all $h \in H$ and all $t \geq t'$, and

(iv) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{*h',t''} (x_{t''}^{*h',-}, x_{t''}^{*h'})$ for some $h' \in H$ and some $t'' \geq t'$.

Note that since expectations concerning consumption are kept fixed, that is, only present consumption can be changed, it is not necessary to introduce a time-consistent variant of this definition as we did in the foregoing section. Contrary to the concept of TCOP efficiency, the CP efficiency concept is also applicable to naïve societies.

The following theorem says that the CP efficiency concept is weaker than the OP efficiency concept. Since its proof is obvious, it will be omitted.

Theorem 6.9
If an allocation is OP efficient, then it is CP efficient.

A similar theorem holds for time-consistent allocations and TCOP efficiency.

Theorem 6.10
If a time-consistent allocation is TCOP efficient, then it is CP efficient.

The next example shows that under Assumptions 2, 3, 4 and 6, naïve and sophisticated equilibria need not be constrained periodical efficient.

Example 6.11
Consider an economy with two periods, two households and two goods. Let the endowments of all households be equal to 1 for each good in each period, i.e. $e^{h,\tau} = (1,1)$ for all h, τ. For both households the preferences in the first period can be represented by the following utility function:

$$u_1^h(x_1^h) \;\; = \;\; x_{1,1}^{h,1} + x_{1,2}^{h,1} + x_{1,1}^{h,2} + x_{1,2}^{h,2}.$$

In the second period, the preferences of the household are represented by the following utility functions:

$$u_2^1(x_1^{1,1}, x_2^1) = 2x_{1,1}^{1,1} + x_{1,2}^{1,1} + x_{2,1}^{1,2} + x_{2,2}^{1,2},$$
$$u_2^2(x_1^{2,1}, x_2^2) = x_{1,1}^{2,1} + 2x_{1,2}^{2,1} + x_{2,1}^{2,2} + x_{2,2}^{2,2}.$$

Consider a price system where the prices of all goods are equal in both periods, i.e. $p_{1,1}^{*1} = p_{1,2}^{*1} = p_{1,1}^{*2} = p_{1,2}^{*2}$ and $p_{2,1}^{*2} = p_{2,2}^{*2}$. The following allocations are consistent with optimizing behavior of both naïve and sophisticated households at the price system p^*:

$$(x_{1,1}^{*h,1}, x_{1,2}^{*h,1}, x_{1,1}^{*h,2}, x_{1,2}^{*h,2}) = (1, 1, 1, 1),$$
$$(x_{2,1}^{*h,2}, x_{2,2}^{*h,2}) = (1, 1),$$

for every household h. Thus, at the price system p^*, markets clear, and (p^*, x^*) is both a naïve and a sophisticated equilibrium. Note, however, that this equilibrium is not CP efficient. In this equilibrium the first-period utility equals 4 for both households, while the utilities in the second period equal 5. Now consider the allocation

$$(\tilde{x}_{1,1}^{1,1}, \tilde{x}_{1,2}^{1,1}, \tilde{x}_{1,1}^{1,2}, \tilde{x}_{1,2}^{1,2}) = (2, 0, 1, 1),$$
$$(\tilde{x}_{1,1}^{2,1}, \tilde{x}_{1,2}^{2,1}, \tilde{x}_{1,1}^{2,2}, \tilde{x}_{1,2}^{2,2}) = (0, 2, 1, 1),$$

where the utility levels in the first period remain equal to 4 for both households, whereas the utilities in the second period both equal 6 under correct expectations. Thus, at the allocation $\tilde{x}$, households are not worse off in the first period, and better off in the second period. Moreover, the allocation $\tilde{x}$ is feasible. Therefore, the equilibrium allocation x^* is not CP efficient.

$\square$

If preferences are strongly independent of the past, however, then naïve and sophisticated equilibria are constrained periodically efficient.

Theorem 6.12
In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3, 4 and 6', naïve and sophisticated equilibrium allocations are CP efficient.

7 Conclusion

Although the economic literature typically assumes that people have time-consistent preferences, psychological research indicates that time-inconsistent behavior of humans and

animals cannot be ignored. Due to time-inconsistency a preference for commitment can arise, which does not arise if preferences are time-consistent.

In this paper, changing preferences are introduced in a multi-period general equilibrium model. Changing preferences require new concepts of for instance behavior, equilibrium, and welfare. Equilibrium concepts are defined that satisfy a particular form of efficiency and some surprising results are obtained about economies with time-consistent preferences.

A distinction is made between naïve and sophisticated societies. Appropriate equilibrium notions of naïve and sophisticated equilibrium are defined. We extend the standard competitive equilibrium notion and call it an extended competitive equilibrium. It is shown that, in the case of time-consistent preferences, an extended competitive equilibrium is a naïve equilibrium and a sophisticated equilibrium coincides with an extended competitive equilibrium. An intriguing result is that with time-consistent preferences there can be naïve equilibrium allocations that are not compatible with any competitive equilibrium. For naïve societies an equilibrium is shown to exist under quite general conditions. For sophisticated societies, however, the existence of an equilibrium can be established only under certain assumptions on time-inconsistency.

Several efficiency criteria are introduced. A distinction has been made between social planners that take into account only the preferences of the current selves and planners that take into account the preferences of both the current and future selves. Moreover, we distinguish the cases where social planners can change only current consumption or both current consumption and expectations concerning future consumption. When social planners can change expected consumption, naïve and sophisticated equilibria are typically not efficient. When social planners cannot change expected consumption, however, we provide sufficient conditions for both naïve and sophisticated equilibria to be efficient.

Main interesting issues for future work remain. These involve existence and efficiency issues, as well as extensions of the model to settings with a general specification of incomplete markets. Such a specification would be interesting, since it gives rise to new options for commitment by agents. There are also many conceptual issues left over. Up to now we considered the extreme cases where it is either common knowledge that behavior of individuals is naïve, or common knowledge that individuals behave sophisticated. Alternative assumptions on expectations concerning individuals' own future behavior or the behavior of others are possible, leading to distinct equilibrium concepts and related efficiency notions.

8   Appendix

Lemma 2.3
If the preferences of household h are such that for every $x_1^h, \overline{x}_1^h \in X_1^h$ with $x_1^{h\tau} = \overline{x}_1^{h\tau}$,

$\tau = 1, \ldots, t-1$, *it holds that*

$$x_1^h \succeq^{h,1} \overline{x}_1^h$$

if and only if

$$x_1^{h,t,T} \succeq_{x_1^{h,1,t-1}}^{h,t} \overline{x}_1^{h,t,T},$$

then the preferences of household h are time-consistent.

Proof

Let $x_t^{h,-} \in X_t^{h,-}$. We first show that when $x_t^h, \overline{x}_t^h \in X_t^h$ are such that $x_t^{h,t,t'-1} = \overline{x}_t^{h,t,t'-1}$, then

$$x_t^h \quad \succeq_{x_t^{h,-}}^{h,t} \quad \overline{x}_t^h$$

implies

$$x_t^{h,t',T} \quad \succeq_{(x_t^{h,-}, x_t^{h,t,t'-1})}^{h,t'} \quad \overline{x}_t^{h,t',T}.$$

If $x_t^h \succeq_{x_t^{h,-}}^{h,t} \overline{x}_t^h$, then it follows from the "if" part of the hypothesis of the lemma that

$$\left(x_t^{h,-}, x_t^h\right) \succeq^{h,1} \left(x_t^{h,-}, \overline{x}_t^h\right).$$

The "only if" part of the hypothesis yields

$$x_t^{h,t',T} \quad \succeq_{(x_t^{h,-}, x_t^{h,t,t'-1})}^{h,t'} \quad \overline{x}_t^{h,t',T}.$$

The proof that

$$x_t^{h,t',T} \quad \succeq_{(x_t^{h,-}, x_t^{h,t,t'-1})}^{h,t'} \quad \overline{x}_t^{h,t',T}$$

implies

$$x_t^h \succeq_{x_t^{h,-}}^{h,t} \overline{x}_t^h$$

is similar. Q.E.D.

Theorem 3.4

If preferences of all households are time-consistent, then an extended competitive equilibrium is a naïve equilibrium.

Proof

Suppose (p^*, x^*) is an extended competitive equilibrium. Since $x_t^{*h\tau} = x_1^{*h\tau}$ for every h and every $\tau \geq t$ and $\sum_{h \in H} x_1^{*h} = \sum_{h \in H} e_1^h$, it can easily be seen that (b) of Definition 3.3 is satisfied. It remains to be shown that Condition (a) of that definition is satisfied.

Notice that if $x_1^{*h} \in \gamma_1^h(p_1^*)$, then $x_1^{*h,t,T} \in \gamma_t^h(p_1^{*t,T})$. Moreover, for every $x_t^h \in \gamma_t^h(p_1^{*t,T})$ there is $x_1^h \in \gamma_1^h(p_1^*)$ with $x_1^{h,t,T} = x_t^h$ and $x_1^{h,1,t-1} = x_1^{*h,1,t-1}$.

We know that $x_1^{*h} \in \delta_1^h(p_1^*)$. Thus, $x_1^{*h} \in \gamma_1^h(p_1^*)$ and $x_1^{*h} \succeq^{h,1} x_1^h$ for all $x_1^h \in \gamma_1^h(p_1^*)$. Then, by time-consistency of preferences, $x_1^{*h,t,T} \succeq_{x_1^{*h,1,t-1}}^{h,t} x_t^h$ for all $x_t^h \in \gamma_t^h(p_1^{*t,T})$, so $x_1^{*h,t,T} \in \delta_t^h(p_1^{*t,T}, x_1^{*h,1,t-1}) = \delta_t^h(p_t^*, x_t^{*h,-})$. Thus, the extended competitive equilibrium is a naïve equilibrium. $\hspace{2cm}$ Q.E.D.

Theorem 3.6

Suppose preferences are time-consistent. If a naïve equilibrium exists, then also an extended competitive equilibrium exists.

Proof

Let (p^*, x^*) be a naïve equilibrium. First of all, note that $\gamma_1^{h,t,T}(p_1^*) = \gamma_t^h(p_1^{*t,T})$. Then we know that $x_1^{*h,t,T} \in \gamma_t^h(p_1^{*t,T})$. Furthermore, $(x_1^{*h,1,t-1}, x_1^{*h,t,T}) \succeq^{h,1} (x_1^{*h,1,t-1}, x_1^{h,t,T})$ for all $x_1^{h,t,T} \in \gamma_t^h(p_1^{*t,T})$.

By time-consistency of preferences it holds that $x_1^{*h,t,T} \succeq_{x_1^{*h,1,t-1}}^{h,t} x_1^{h,t,T}$ for all $x_1^{h,t,T} \in \gamma_t^h(p_1^{*t,T})$. Thus, $x_1^{*h,t,T} \in \delta_t^h(p_1^{*t,T}, x_1^{*h,1,t-1})$. Let $\widehat{p}$ and $\widehat{x}$ be defined by $\widehat{p}_t = p_1^{*t,T}$ and $\widehat{x}_t^h = x_1^{*h,t,T}$. It then follows immediately that $(\widehat{p}, \widehat{x})$ is a naïve equilibrium. $\hspace{1cm}$ Q.E.D.

Lemma 3.7

Suppose the economy $\mathcal{E}$ satisfies Assumptions 1-5. Consider a naïve household $h \in H$, a planning period $t \in T$, and a realized consumption plan $x_t^{h,-} \in \widehat{X}_t^{h,-}$. Then, at prices $p_t \in \widehat{P}_t$, $\widehat{\delta}_t^h(\cdot, x_t^{h,-})$ is a non-empty, compact and convex-valued, upper-hemi continuous correspondence that satisfies:

- *Walras' law, $p_t \square \widehat{\delta}_t^h(p_t, x_t^{h,-}) = \{p_t^t e^{h,t}, \dots, p_t^T e^{h,T}\}$,*

- *Homogeneity property, $\widehat{\delta}_t^h(p_t, x_t^{h,-}) = \widehat{\delta}_t^h(\overline{p}_t, x_t^{h,-})$, where for $\tau' \geq t$, for $\lambda > 0$, $\overline{p}_t^{\tau'} = \lambda p_t^{\tau'}$ and $\overline{p}_t^{\tau} = p_t^{\tau}$ for $\tau \neq \tau'$.*

Proof

(i) Since $e_t^h \in \widehat{\gamma}_t^h(p_t)$, $\widehat{\gamma}_t^h(p_t)$ is non-empty.

(ii) Consider a sequence $\{p_t^m\}_{m=1}^{\infty}$ with $p_t^m \to p_t$. Let the sequence $\{x_t^{hm}\}_{m=1}^{\infty}$ be such that $x_t^{hm} \in \widehat{\gamma}_t^h(p_t^m)$ for every m and $x_t^{hm} \to x_t^h$. By closedness of $\widehat{X}_t^h$ and since $p_t^{\tau m} x_t^{h,\tau m} \leq p_t^{\tau m} e_t^{h,\tau}$, it follows that $x_t^h \in \widehat{\gamma}_t^h(p_t)$. Since $\widehat{\gamma}_t^h$ is bounded, $\widehat{\gamma}_t^h$ is upper-hemi continuous.

(iii) Let $\{p_t^m\}_{m=1}^{\infty}$ be a sequence of prices with $p_t^m \to p_t$. Let $x_t^h \in \widehat{\gamma}_t^h(p_t)$. Then $p_t^{\tau} x_t^{h,\tau} \leq p_t^{\tau} e_t^{h,\tau}$. Define $a^{\tau m}$ such that $p_t^{\tau m} a^{\tau m} x_t^{h,\tau} = p_t^{\tau m} e_t^{h,\tau}$.

If $p_t^{\tau} x_t^{h,\tau} < p_t^{\tau} e_t^{h,\tau}$, then $p_t^{\tau m} x_t^{h,\tau} \leq p_t^{\tau m} e_t^{h,\tau}$ for m larger than a certain value M^1. In that case define $x_t^{h,\tau m} = x_t^{h,\tau}$ for $m > M^1$.

Otherwise, if $p_t^{\tau} x_t^{h,\tau} = p_t^{\tau} e_t^{h,\tau} > 0$, it holds that $p_t^{\tau m} e_t^{h,\tau} > 0$ and $p_t^{\tau m} x_t^{h,\tau} > 0$ for m larger than a certain M^2. Now, if $a^{\tau m} > 1$, then define $x_t^{h,\tau m} = x_t^{h,\tau}$ and if $a^{\tau m} \leq 1$, then

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define $x_t^{h,\tau\,m} = a^{\tau m} x_t^{h,\tau}$ for m larger than M^2. Note that in this case $a^{\tau m}$ is unique and tends to one, since $a^{\tau m} = p_t^{\tau m} e_t^{h,\tau} / p_t^{\tau m} x_t^{h,\tau} > 0$.

For all m smaller than or equal to M^1 or M^2 define $x_t^{h\,m}$ arbitrarily such that $x_t^{h\,m} \in \widehat{\gamma}_t^h(p_t^{\,m})$.

Then $x_t^{h\,m} \in \widehat{\gamma}_t^h(p_t^{\,m})$ for every m and $x_t^{h\,m} \to x_t^h$. Thus, $\widehat{\gamma}_t^h$ is lower-hemi continuous. It follows that $\widehat{\gamma}_t^h$ is continuous.

We can then apply the Theorem of the Maximum to establish that $\widehat{\delta}_t^h(\cdot, x_t^{h,-})$ is non-empty, compact-valued and upper-hemi continuous.

Convex-valuedness of $\widehat{\delta}_t^h$ is straightforward. Walras' law follows from monotonicity. The homogeneity property follows immediately from the definition of the budget constraints $\widehat{\gamma}_t^h$.

<div align="right">Q.E.D.</div>

Theorem 3.8 (Existence of naïve equilibrium)

If the economy $\mathcal{E}$ satisfies Assumptions 1-5, then there exists a naïve equilibrium (p^*, x^*).

Proof

Define $\widehat{Z}_t = \sum_{h \in H} \widehat{X}_t^h - \sum_{h \in H} \{e_t^h\}$ and for any $x_t^- \in \widehat{X}_t^-$, $\widehat{\zeta}_t(p_t, x_t^-) = \sum_{h \in H} \widehat{\delta}_t^h(p_t, x_t^{h,-}) - \sum_{h \in H} \{e_t^h\}$. Using Lemma 3.7, the correspondence $\widehat{\zeta}_t(\cdot, x_t^-)$ is non-empty, compact-valued, convex-valued and upper-hemi continuous on $\widehat{P}_t$.

Define $\mu_t(z_t) = \{\tilde{p}_t \in \widehat{P}_t \mid \tilde{p}_t^\tau z_t^\tau \geq p_t^\tau z_t^\tau \text{ for all } p_t \in \widehat{P}_t \text{ for all } \tau \geq t\}$. By the theorem of the maximum, μ_t is non-empty and upper-hemi continuous. Moreover, μ_t is convex-valued. For $x_t^- \in \widehat{X}_t^-$, define $\phi_t(\cdot, x_t^-) : \widehat{P}_t \times \widehat{Z}_t \to \widehat{P}_t \times \widehat{Z}_t$ as $\phi_t(p_t, z_t, x_t^-) = \mu_t(z_t) \times \widehat{\zeta}_t(p_t, x_t^-)$.

First, consider period 1. By Kakutani's fixed point theorem $\phi_1(\cdot)$ has a fixed point $(p_1^*, \overline{z}_1) \in \mu_1(\overline{z}_1) \times \widehat{\zeta}_1(p_t^*)$.

Since then $p_1^{*\tau} \overline{z}_1^\tau \leq 0$ for every τ, we know, by the definition of μ_1, that $\overline{z}_1 \leq 0$. The corresponding consumption bundles are denoted by $\overline{x}_1^h \in \widehat{\delta}_1^h(p_1^*)$.

By Walras' law (Lemma 3.7), we know that $p_{1,l}^{*\tau} = 0$ if $\overline{z}_{1,l}^\tau < 0$. By monotonicity, the excess supply of good l for period τ can be given to any household without making that household worse off and without violating the budget constraints. Thus, given prices p_1^*, $z_1^* = 0 \in \widehat{\zeta}_1(p_1^*)$. Denote the corresponding demands by x_1^{*h}.

It remains to be shown that $x_1^{*h} \in \delta_1^h(p_1^*)$ for every h. Suppose that this is not the case, i.e. suppose that there is a household h with $x_1^{*h} \notin \delta_1^h(p_1^*)$. That would mean that there is an $\tilde{x}_1^h \in \delta_1^h(p_1^*)$ with $\tilde{x}_1^h \succ^{h,1} x_1^{*h}$. Since $x_{1,l}^{*h,\tau} < \sum_{h \in H} e_l^{h,\tau} + \varepsilon$ for every τ, and every l, there would be a small positive number $\lambda \in (0,1)$ such that $\lambda \tilde{x}_1^h + (1-\lambda) x_1^{*h} \in \widehat{\gamma}_1^h(p_1^*)$, and $\lambda \tilde{x}_1^h + (1-\lambda) x_1^{*h} \succ^{h,1} x_1^{*h}$, which would contradict $x_1^{*h} \in \widehat{\delta}_1^h(p_1^*)$. Thus, $x_1^{*h} \in \delta_1^h(p_1^*)$ for every h.

Now suppose that for every $\tau \leq t$ there exist p_τ^* such that $0 \in \zeta_\tau(p_\tau^*, x_\tau^{*-})$. Then, by a similar argument as before it can be shown that there exists a p_{t+1}^* such that

$0 \in \zeta_{t+1}(p_{t+1}^*, x_{t+1}^{*-})$. This argument of induction then establishes the existence of a naïve equilibrium.

<div align="right">Q.E.D.</div>

Theorem 4.2

A pair $(p^*, x^*) \in P \times X$ is a sophisticated equilibrium if and only if it satisfies the following conditions:

(i) $x_1^{*h} \in \xi_1^h(p_1^*)$ for all $h \in H$,

(ii) $\sum_{h \in H} x_1^{*h} = \sum_{h \in H} e_1^h$ for all $h \in H$,

(iii) $p_t^* = p_1^{*t,T}$ for all $t \in T$,

(iv) $x_t^{*h} = x_1^{*h,t,T}$ for all $h \in H$ and all $t \in T$.

Proof

It can immediately be seen that a sophisticated equilibrium pair (p^*, x^*) satisfies (i)-(iv). It remains to be shown that a pair that satisfies (i)-(iv) is a sophisticated equilibrium. Let (p^*, x^*) satisfy (i)-(iv) and let $t < t'$. Then, by (iii) $p_t^* = p_1^{*t,T}$ and $p_{t'}^* = p_1^{*t',T}$. So $p_t^{*t',T} = p_{t'}^*$ and (c) is satisfied. Furthermore, $x_t^* = x_1^{*t,T}$ and $x_{t'}^* = x_1^{*t',T}$. So $x_t^{*t',T} = x_{t'}^*$ and (d) is satisfied. It also holds that $\sum_{h \in H} x_t^{*h} = \sum_{h \in H} x_1^{*h,t,T} = \sum_{h \in H} e_1^{h,t,T} = \sum_{h \in H} e_t^h$, so (b) is satisfied. Finally, $x_1^{*h} \in \xi_1^h(p_1^*)$, so $x_1^{*h} \in \phi_1^h(p_1^*)$, which implies that $(x_1^{*h2}, \ldots x_1^{*hT}) \in \xi_2^h(p_1^{*2,T}, x_1^{*h1})$. But then, $x_2^{*h} \in \xi_2^h(p_2^*, x_1^{*h1})$. Now, by an argument of induction it can be shown that $x_t^{*h} \in \xi_t^h(p_t^*, x^{*h,-})$ for all t. So (a) is satisfied too. Thus, a pair (p^*, x^*) that satisfies (i)-(iv) is a sophisticated equilibrium.

<div align="right">Q.E.D.</div>

Theorem 4.3

Suppose that the preferences of all households are time-consistent and that Assumptions 1-2 hold. A pair $(p^*, x^*) \in P \times X$ with $p^* \gg 0$ is a sophisticated equilibrium of the economy $\mathcal{E}$ if and only if it is an extended competitive equilibrium.

Proof

First of all, let (p^*, x^*) be an extended competitive equilibrium. Since $x_1^{*h} \in \delta_1^h(p_1^*)$ we know that $x_1^{*h} \succeq^{h,1} x_1^h$ for every $x_1^h \in X_1^h$ with $p_1^{*\tau} x_1^{h,\tau} \leq p_1^{*\tau} e_1^{h,\tau}$ for every τ. By time-consistency we then know that $x_1^{*h,T} \succeq_{x_1^{*h,1,T-1}}^{h,T} x_1^{h,T}$ for every $x_1^{h,T} \in X_T^h$ with $p_1^{*T} x_1^{h,T} \leq p_1^{*T} e_1^{h,T}$. It follows that $x_1^{*h,T} \in \xi_T^h(p_1^{*T}, x_1^{*h,1,T-1})$. We show next that $x_1^{*h,t,T} \in \xi_t^h(p_1^{*t,T}, x_1^{*h,1,t-1})$ for every t. Assume that $x_1^{*h,\tau,T} \in \xi_\tau^h(p_1^{*\tau,T}, x_1^{*h,1,\tau-1})$ for every $\tau > t$. Suppose that $x_1^{*h,t,T} \notin \xi_t^h(p_1^{*t,T}, x_1^{*h,1,t-1})$. Then there must be a consumption bundle that is strictly preferred to $x_1^{*h,t,T}$, but is in the opportunity set at time t, which, by time-consistency, leads to a contradiction of x_1^{*h} being an optimal consumption bundle for household h in period 1. Thus, (p^*, x^*) is a sophisticated equilibrium.

<div align="center">31</div>

Now let (p^*, x^*) be a sophisticated equilibrium. Suppose that (p^*, x^*) is not an extended competitive equilibrium. Then there must be a household h and an $\widehat{x}_1^h \in X_1^h$ such that $\widehat{x}_1^h \succ^{h,1} x_1^{*h}$ with $p_1^{*\tau} \widehat{x}_1^{h,\tau} \leq p_1^{*\tau} e_1^{h,\tau}$ for every τ. Consider the maximum of those $\widehat{x}_1^h$ with respect to $\succ^{h,1}$. Such an $\widehat{x}_1^h$ exists because preferences are continuous. Since $\widehat{x}_1^h$ is not chosen by h, there must be a $t_1 > 1$ such that $\widehat{x}_1^{h,t_1,T} \notin \xi_{t_1}^h(p_1^{*t_1,T}, \widehat{x}_1^{h,1,t_1-1})$. So there must be a $t_1' \geq t_1$ and an $\tilde{x}_{t_1'}^h \in X_{t_1'}^h$ such that $\tilde{x}_{t_1'}^h \succ^{h,t_1'}_{\widehat{x}_1^{h,1,t_1'-1}} \widehat{x}_1^{h,t_1',T}$ and $p_1^{*\tau} \tilde{x}_{t_1'}^{h,\tau} \leq p_1^{*\tau} e_1^{h,\tau}$ for every $\tau \geq t_1'$, and by time-consistency $(\widehat{x}_1^{h,1,t_1'-1}, \tilde{x}_{t_1'}^h) \succ^{h,1} \widehat{x}_1^h \succ^{h,1} x_1^{*h}$, which contradicts our assumption on $\widehat{x}_1^h$.

<div align="right">Q.E.D.</div>

Example 4.5

An important role is played by the second derivative of v with respect to $x_{1,1}^{1,1}$. It is given by

$$\frac{d^2 v}{d{x_{1,1}^{1,1}}^2} = \frac{p_{2,1} + p_{2,2}}{(1 + x_{1,1}^{1,1})^3}\left(\frac{2}{p_{2,1}} - \frac{1}{p_{2,2}}\right)$$

for $x_{1,1}^{1,1} > 1/3$ and is equal to 0 otherwise. Thus, when $x_{1,1}^{1,1} > 1/3$, it depends on the prices whether v is convex ($p_{2,1} < 2p_{2,2}$), linear ($p_{2,1} = 2p_{2,2}$) or concave ($p_{2,1} > 2p_{2,2}$). We distinguish three cases accordingly.

Case 1

First consider the case where $p_{2,1} < 2p_{2,2}$. Note first that in this case the second derivative of v with respect to $x_{1,1}^{1,1}$ is larger than 0 for $x_{1,1}^{1,1} > 1/3$ and equal to 0 if $x_{1,1}^{1,1} \leq 1/3$.

First, assume that $2p_{1,2}^1 < p_{1,1}^1$, so that the first derivative of v is negative when $x_{1,1}^{1,1} > 1/3$, i.e.

$$\frac{dv}{dx_{1,1}^{1,1}} = 1 - \frac{p_{1,1}^1}{2p_{1,2}^1} + \frac{p_{2,1} + p_{2,2}}{(1 + x_{1,1}^{1,1})^2}\left(\frac{1}{2p_{2,2}} - \frac{1}{p_{2,1}}\right) < 0$$

for $x_{1,1}^{1,1} > 1/3$. It is easily verified that the first derivative of v is negative as well for $x_{1,1}^{1,1} \leq 1/3$. Then $x_{1,1}^{*1,1} = 0$. Note that if $x_{1,1}^{*1,1} = 0$, then in equilibrium it must hold that $x_{1,1}^{*2,1} = 1$, which is only possible if $p_{1,1}^1 = 0$. However, then demand for goods would be infinite in the first period, which cannot yield an equilibrium.

Now assume that $2p_{1,2}^1 \geq p_{1,1}^1$. Since the second derivative of v is larger than 0 if $x_{1,1}^{1,1} > 1/3$, it holds that v is convex for $x_{1,1}^{1,1} > 1/3$. Thus, the optimal $x_{1,1}^{1,1}$ will not be an interior solution if $x_{1,1}^{1,1} > 1/3$. Thus, either $x_{1,1}^{*1,1} \in [0, 1/3]$ or $x_{1,1}^{*1,1} = 1$. If $x_{1,1}^{*1,1} = 1$ then $p_{1,2}^1 = 0$, since otherwise household 2 would never be willing to consume $x_{1,1}^{*2,1} = 0$. But if $p_{1,2}^1 = 0$, then household 1 will demand an infinite amount of good 2 in period 1. Thus, this can also not yield an equilibrium. So consider $x_{1,1}^{*1,1} \in [0, 1/3]$. Then, for the second

period demand it holds that $x_{2,1}^{*1,2} > 9/8$. However, it also holds that $x_{2,1}^{*2,2} = 1$. Thus, there would be excess demand of good 1 in period 2.

Case 2

Now consider the case where $p_{2,1} > 2p_{2,2}$. Then $x_{2,2}^{*2,2} = \frac{p_{2,1}}{p_{2,2}}$ and $x_{2,1}^{*2,2} = 0$. So in equilibrium it would have to hold that $x_{2,1}^{*1,2} = 2$ and $x_{2,2}^{*1,2} = \frac{p_{2,2}-p_{2,1}}{p_{2,2}}$. First of all, assume that $x_{1,1}^{*1,1} > 1/3$. This would imply the following two conditions:

$$\left(1 + \frac{p_{2,2}}{p_{2,1}}\right) \frac{1}{1 + x_{1,1}^{*1,1}} = 2,$$

$$\left(\frac{p_{2,1}}{p_{2,2}} + 1\right) \frac{x_{1,1}^{*1,1}}{1 + x_{1,1}^{*1,1}} = 1 - \frac{p_{2,1}}{p_{2,2}}.$$

By manipulating these two conditions, we arrive at

$$x_{1,1}^{*1,1} = \frac{1}{2}\left(\frac{p_{2,2}}{p_{2,1}} - 1\right) < -\frac{1}{4},$$

which is not possible.

Now assume $x_{1,1}^{*1,1} \leq 1/3$. Then the following two conditions should hold:

$$\left(1 + \frac{p_{2,2}}{p_{2,1}}\right) \frac{1}{1 + 1/3} = 2,$$

$$\left(\frac{p_{2,1}}{p_{2,2}} + 1\right) \frac{1/3}{1 + 1/3} = 1 - \frac{p_{2,1}}{p_{2,2}},$$

which would imply that $p_{2,2}/p_{2,1} = 5/3$, which contradicts the fact that $p_{2,1} > 2p_{2,2}$.

Case 3

Finally, consider the case where $p_{2,1} = 2p_{2,2}$. Then the second derivative of v is equal to zero and

$$\frac{dv}{dx_{1,1}^{1,1}} = 1 - \frac{p_{1,1}^1}{2p_{1,2}^1}.$$

Thus, if $p_{1,1}^1 \neq 2p_{1,2}^1$, then either $x_{1,1}^{*1,1} = 0$ or $x_{1,1}^{*1,1} = 1$, which is incompatible with equilibrium for the same reasons as before. Now assume that $p_{1,1}^1 = 2p_{1,2}^1$. For household 2 it then holds that $x_{1,1}^{*2,1} = 1/3$. Then in equilibrium it follows that $x_{1,1}^{*1,1} = 2/3$. This leads to $x_{2,1}^{*1,2} = 9/10$ and $x_{2,2}^{*1,2} = 6/5$. This yields a contradiction, since the total endowment of good 2 in period 2 equals 1. Thus, in this economy no equilibrium exists.

Lemma 4.6

Suppose the economy $\mathcal{E}$ satisfies Assumptions 1-6 and 4'. Consider a sophisticated household $h \in H$, a planning period $t \in T$, and a realized consumption plan $x_t^{h,-} \in X_t^{h,-}$. Then, at prices $p_t \in P_t$, $\xi_t^h(\cdot, x_t^{h,-})$ is convex-valued and either empty or single-valued.

Proof

Since preferences are independent on past consumption, the demand correspondences will also be independent on past consumption. Therefore, the opportunity sets

$$\phi_t^h(p_t, x_t^{h,-}) = \{x_t^h \in X_t^h \mid \begin{aligned} p_t^\tau x_t^{h,\tau} &\leq p_t^\tau e^{h,\tau} \quad \text{for all } \tau \geq t, \text{ and} \\ x_t^{h,t+1,T} &\in \xi_{t+1}^h(p_t^{t+1,T}, x_t^{h,-}, x_t^{h,t}) \} \end{aligned}$$

will be convex-valued. Then it is straightforward that the demand correspondences are convex-valued.

Suppose that a demand correspondence contains two elements. By convex-valuedness of the demand correspondence and by strict convexity of preferences this yields a contradiction. Thus, the demand correspondence is either empty or single-valued.

Q.E.D.

Lemma 4.7

Suppose the economy $\mathcal{E}$ satisfies Assumptions 1-6, and 4'. Then, at prices $p_t \in \widehat{P}_t$, $\widehat{\xi}_t^h$ is a non-empty, compact-valued and continuous function that satisfies the homogeneity property and Walras' law for every $h \in H$, $t \in T$.

Proof

The homogeneity property is straightforward.

Since in the last period the maximization problem for the sophisticated household is identical to that of the naïve household and since $\widehat{\phi}_T^h(p_T, x_T^{h,-})$ is independent of $x_T^{h,-}$, the characteristics of $\widehat{\xi}_T^h$ follow immediately from Lemma 3.7. By single-valuedness and upper-hemi continuity, continuity of $\widehat{\xi}_T^h$ follows immediately. We will establish the properties of the other demand correspondences by an argument of backwards induction.

Let $t \in T$. Assume that $\widehat{\xi}_\tau^h$ is non-empty, compact-valued and continuous for $\tau \in T$, $\tau \geq t + 1$. We need to show that $\widehat{\xi}_t^h$ is non-empty, compact-valued and upper-hemi continuous. Thus, it is necessary to show that $\widehat{\phi}_t^h$ satisfies the conditions that are needed in order to be able to apply the theorem of the maximum.

(i) Since $\widehat{\xi}_{t+1}^h(p_{t+1}, x_t^{h,-}, e^{h,t})$ is non-empty, $p_t^t e^{h,t} \leq p_t^t e^{h,t}$, and $p_t^\tau x_t^{h,\tau} \leq p_t^\tau e^{h,\tau}$, $\tau \geq t + 1$ for $x_t^{h,t+1,T} = \widehat{\xi}_{t+1}^h(p_{t+1}, x_t^{h,-}, e^{h,t})$, it can be seen that $(e^{h,t}, \widehat{\xi}_{t+1}^h(p_{t+1}, x_t^{h,-}, e^{h,t})) \in \widehat{\phi}_t^h(p_t, x_t^{h,-})$. Thus, $\widehat{\phi}_t^h(p_t, x_t^{h,-})$ is nonempty.

(ii) Consider the sequence $\{p_t^m\}_{m=1}^\infty$ with $p_t^m \to p_t$. Let $\{x_t^{h,-m}, x_t^{hm}\}_{m=1}^\infty$ be a sequence of consumption plans converging to $(x_t^{h,-}, x_t^h)$, where $x_t^{h,-m} \in X_t^{h,-}$ and $x_t^{hm} \in \widehat{\phi}_t^h(p_t^m, x_t^{h,-m})$ for all m. Then $p_t^{\tau m} x_t^{h,\tau^m} \leq p_t^{\tau m} e^{h,\tau}$ for every $\tau \geq t$ and $x_t^{h,t+1,T^m} = \widehat{\xi}_{t+1}^h(p_t^{t+1,T^m}, x_t^{h,-m}, x_t^{h,t^m})$. By continuity it follows that $p_t^\tau x_t^{h,\tau} \leq p_t^\tau e^{h,\tau}$ for every $\tau \geq t$. Moreover, by continuity of $\widehat{\xi}_{t+1}^h$, $x_t^{h,t+1,T} = \widehat{\xi}_{t+1}^h(p_t^{t+1,T}, x_t^{h,-}, x_t^{h,t})$. Therefore, $x_t^h \in \widehat{\phi}_t^h(p_t, x_t^{h,-})$. Thus, the graph of $\widehat{\phi}_t^h$ is closed. By boundedness of $\widehat{X}_t^h$ it can easily be seen, for a compact set B, that $\widehat{\phi}_t^h(B)$ is bounded. Therefore, $\widehat{\phi}_t^h$ is upper-hemi continuous.

(iii) Consider a sequence $\{p_t{}^m, x_t^{h,-m}\}_{m=1}^\infty$ with $(p_t{}^m, x_t^{h,-m}) \to (p_t, x_t^{h,-})$. Let $x_t^h \in \widehat{\phi}_t^h(p_t, x_t^{h,-})$. Then, for m large enough, there are $x_t^{h,t^m} \in \widehat{X}^{h,t}$ such that $p_t^{t^m} x_t^{h,t^m} \leq p_t^{t^m} e^{h,t}$ and $x_t^{h,t^m} \to x_t^{h,t}$. Let $x_t^{h,t+1,T^m} = \widehat{\xi}_{t+1}^h(p_t^{t+1,T^m}, x_t^{h,-m}, x_t^{h,t^m})$. It follows immediately that $p_t^{\tau^m} x_t^{h,\tau^m} \leq p_t^{\tau^m} e^{h,\tau}$ for $\tau \geq t+1$. Continuity of $\widehat{\xi}_{t+1}^h$ then implies that $x_t^{h,t+1,T^m} \to \widehat{\xi}_{t+1}^h(p_t^{t+1,T}, x_t^{h,-}, x_t^{h,t}) = x_t^{h,t+1,T}$. Therefore, $x_t^{hm} \in \widehat{\phi}_t^h(p_t{}^m, x_t^{h,-m})$ and $x_t^{hm} \to x_t^h$. Thus, $\widehat{\phi}_t^h(\cdot)$ is lower-hemi continuous.

Since $\widehat{\phi}_t^h$ is both upper-hemi and lower-hemi continuous, it is continuous.

To conclude, $\widehat{\phi}_t^h$ satisfies the conditions that are needed in order to be able to apply the theorem of the maximum. Also, since Walras' law holds for period $t+1$, and since consumption in period t does not influence the optimal consumption in period $t+1$, Walras' law holds for period t. The characteristics of $\widehat{\xi}_t^h$ then follow immediately.

<div align="right">Q.E.D</div>

Theorem 4.8 Existence of sophisticated equilibrium

If the economy $\mathcal{E}$ satisfies Assumptions 1-6 and 4', then there exists a sophisticated equilibrium (p^*, x^*).

Proof

Note that in order to prove the existence of a sophisticated equilibrium, we can restrict ourselves to the first planning period. By similar arguments as in the foregoing section, there exists a restricted equilibrium pair (p_1^*, z_1^*) such that $z_1^* \in \widehat{\zeta}_1(p_1^*)$ and $z_1^* \leq 0$. By monotonicity and strict convexity of preferences, it must be the case that $p^* \gg 0$. Therefore, and by Walras' law, it must hold that $z_1^* = 0$. Denote the corresponding consumption bundles by x_1^{*h}. It remains to be shown that $x_1^{*h} \in \xi_1^h(p_1^*)$. Suppose that this is not the case. Then two cases can be distinguished. First assume that $x_1^{*h,2,T} \in \xi_2^h(p_1^{*2,T})$. Then, since consumption in period 1 does not influence optimal consumption in period 2, a similar argument as in the proof of Theorem 3.8 leads to a contradiction. Now assume that $x_1^{*h,2,T} \notin \xi_2^h(p_1^{*2,T})$. Then, either $x_1^{*h,3,T} \in \xi_3^h(p_1^{*3,T})$, which again leads to a contradiction, or $x_1^{*h,3,T} \notin \xi_3^h(p_1^{*3,T})$. Continuing in this way, we end up with $x_1^{*h,T} \notin \xi_T^h(p_1^{*T})$, which leads to a contraction by the same arguments as before. Thus, a sophisticated equilibrium exists.

<div align="right">Q.E.D.</div>

Theorem 5.3

Suppose preferences are time-consistent. Then a time-consistent allocation is MOP efficient if and only if it is Pareto efficient.

Proof

Let the time-consistent allocation x^* be Pareto efficient. Suppose that it is not MOP efficient. Then there must be a $\tilde{x}$ and a period t' such that

(i) $\sum_{h \in H} \tilde{x}_{t'}^h = \sum_{h \in H} e_{t'}^h$,

<div align="center">35</div>

(ii) $\tilde{x}_{t'}^h \succeq_{x_{t'}^{*h,-}}^{h,t'} x_{t'}^{*h}$ for all $h \in H$, and

(iii) $\tilde{x}_{t'}^{h'} \succ_{x_{t'}^{*h',-}}^{h',t'} x_{t'}^{*h'}$ for some $h' \in H$.

But then, by time-consistency of preferences it holds that $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{h,1} (x_{t'}^{*h,-}, x_{t'}^{*h})$ for all $h \in H$, and $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{h',1} (x_{t'}^{*h',-}, x_{t'}^{*h'})$ for some $h' \in H$. Then it follows from time-consistency of x^* that $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{h,1} x_1^{*h}$ for all $h \in H$, and $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succeq^{h',1} x_1^{*h'}$ for some $h' \in H$. This yields a contradiction to x^* being Pareto efficient.

That a MOP efficient allocation is Pareto efficient, follows immediately from the definitions.

<div align="right">Q.E.D.</div>

Theorem 5.8

In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3 and 4 a naïve equilibrium allocation is CMP efficient.

Proof

Let (p^*, x^*) be a naïve equilibrium. Suppose that x^* is not CMP efficient, i.e. that there is a reallocation $\tilde{x}$ and a period t' that satisfy

(i) $\tilde{x}_{t'}^{h,t'+1,T} = x_{t'}^{*h,t'+1,T}$ for every $h \in H$,

(ii) $\sum_{h \in H} \tilde{x}_{t'}^{h,t'} = \sum_{h \in H} e^{h,t'}$,

(iii) $\tilde{x}_{t'}^h \succeq_{x_{t'}^{*h,-}}^{h,t'} x_{t'}^{*h}$ for all $h \in H$, and

(iv) $\tilde{x}_{t'}^{h'} \succ_{x_{t'}^{*h',-}}^{h',t'} x_{t'}^{*h'}$ for some $h' \in H$.

Then, since $\tilde{x}_{t'}^h$ was not chosen in equilibrium, it must hold that

$$
\begin{aligned}
p_{t'}^{*t'} \tilde{x}_{t'}^{h',t'} &> p_{t'}^{*t'} x_{t'}^{*h',t'}, \text{and} \\
p_{t'}^{*t'} \tilde{x}_{t'}^{h,t'} &\geq p_{t'}^{*t'} x_{t'}^{*h,t'} \text{ for every household } h \in H.
\end{aligned}
$$

By summing over all households, this leads to

$$
\sum_{h \in H} p_{t'}^{*t'} \tilde{x}_{t'}^{h,t'} > \sum_{h \in H} p_{t'}^{*t'} x_{t'}^{*h,t'},
$$

which can be written as

$$
p_{t'}^{*t'} \sum_{h \in H} \tilde{x}_{t'}^{h,t'} > p_{t'}^{*t'} \sum_{h \in H} x_{t'}^{*h,t'}.
$$

This leads to a contradiction, since, by assumption, it must hold that

$$\sum_{h \in H} \tilde{x}_{t'}^{h,t'} = \sum_{h \in H} e^{h,t'} = \sum_{h \in H} x_{t'}^{*h,t'}.$$

Thus, it follows that the naïve equilibrium allocation x^* must be CMP efficient.

<div align="right">Q.E.D.</div>

Theorem 5.10

In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3, 4 and 6, a sophisticated equilibrium allocation is CMP efficient.

Proof

Let (p^*, x^*) be a sophisticated equilibrium. Suppose that x^* is not CMP efficient. Then there must be a reallocation $\tilde{x}$ and a period t' that satisfy

(i) $\tilde{x}_{t'}^{h,t'+1,T} = x_{t'}^{*h,t'+1,T}$ for every $h \in H$,

(ii) $\sum_{h \in H} \tilde{x}_{t'}^{h,t'} = \sum_{h \in H} e^{h,t'}$,

(iii) $\tilde{x}_{t'}^h \succeq_{x_{t'}^{*h},-}^{h,t'} x_{t'}^{*h}$ for all $h \in H$, and

(iv) $\tilde{x}_{t'}^{h'} \succ_{x_{t'}^{*h'},-}^{h',t'} x_{t'}^{*h'}$ for some $h' \in H$.

Since preferences are independent of consumption in the past, optimal consumption is also not dependent on consumption in the past. Similarly, optimal future consumption is independent of current and past consumption. Therefore, the only reason why household h' has not chosen $\tilde{x}_{t'}^{h'}$ is that its period-t' component must be too expensive. Similarly, for every household h the period-t' component of $\tilde{x}_{t'}^h$ must be at least as expensive as the period-t' component of $x_{t'}^{*h}$. This can be summarized as

$$\begin{aligned} p_{t'}^{*t'} \tilde{x}_{t'}^{h',t'} &> p_{t'}^{*t'} x_{t'}^{*h',t'}, \text{and} \\ p_{t'}^{*t'} \tilde{x}_{t'}^{h,t'} &\geq p_{t'}^{*t'} x_{t'}^{*h,t'} \text{ for every household } h \in H. \end{aligned}$$

As in the proof of Theorem 5.8 this leads to a contradiction. It follows that the sophisticated equilibrium allocation x^* must be CMP efficient.

<div align="right">Q.E.D.</div>

Theorem 6.3

Suppose preferences are time-consistent and satisfy Assumption 6'. If a time-consistent allocation is Pareto efficient, then it is OP efficient.

Proof

Let x^* be a time-consistent allocation that is Pareto efficient. Suppose that x^* is not OP efficient. Then there must be an allocation $\tilde{x}$ and a period t' such that

<div align="center">37</div>

(i) $\sum_{h \in H} \tilde{x}_t^h = \sum_{h \in H} e_t^h$ for all $t \geq t'$,

(ii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^{h,t'}, \ldots, \tilde{x}_{t-1}^{h,t-1}, \tilde{x}_t^h) \succeq^{*h,t} (x_t^{*h,-}, x_t^{*h}) = x_1^{*h}$ for all $h \in H$ and all $t \geq t'$, and

(iii) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h',t'}, \ldots, \tilde{x}_{t''-1}^{h',t''-1}, \tilde{x}_{t''}^{h'}) \succ^{*h',t''} (x_{t''}^{*h',-}, x_{t''}^{*h'}) = x_1^{*h'}$ for some $h' \in H$ and some $t'' \geq t'$.

Then, by Assumption 6', it holds that

(ii) $(x_t^{*h,-}, \tilde{x}_t^h) \succeq^{*h,t} x_1^{*h}$ for all $h \in H$ and all $t \geq t'$, and

(iii) $(x_{t''}^{*h',-}, \tilde{x}_{t''}^{h'}) \succ^{*h',t''} x_1^{*h'}$ for some $h' \in H$ and some $t'' \geq t'$.

By time-consistency of preferences it then follows that for some $t'' \geq t'$

(ii) $(x_{t''}^{*h,-}, \tilde{x}_{t''}^h) \succeq^{*h,1} x_1^{*h}$ for all $h \in H$, and

(iii) $(x_{t''}^{*h',-}, \tilde{x}_{t''}^{h'}) \succ^{*h',1} x_1^{*h'}$ for some $h' \in H$.

Since $\sum_{h \in H}(x_{t''}^{*h,-}, \tilde{x}_{t''}^h) = e_1^h$ by definition of x^*, this would imply that x^* is not Pareto efficient, which is a contradiction. Thus, x^* must be OP efficient.

<div align="right">Q.E.D.</div>

Theorem 6.5

A feasible time-consistent allocation x^* is TCOP efficient if and only if there is no other time-consistent allocation $\tilde{x}$ and no period t' such that

(i) $\sum_{h \in H} \tilde{x}_1^h = \sum_{h \in H} e_1^h$,

(ii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{*h,t} x_1^{*h}$ *for all $h \in H$ and all $t \geq t'$, and*

(iii) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{*h',t''} x_1^{*h'}$ *for some $h' \in H$ and some $t'' \geq t'$.*

Proof

This can easily be derived from the definition of TCOP efficiency.

<div align="right">Q.E.D.</div>

Theorem 6.6

Suppose preferences are time-consistent and satisfy Assumption 6'. If a time-consistent allocation is Pareto efficient then it is TCOP efficient.

Proof

Let the time-consistent allocation x^* be Pareto efficient. Then it follows by Theorem 6.3 that x^* is OP efficient. By the definitions it then follows immediately that x^* is TCOP efficient.

<div align="right">Q.E.D.</div>

Theorem 6.7

In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3, 4 and 6', a sophisticated equilibrium allocation is TCOP efficient.

Proof

Let (p^*, x^*) be a sophisticated equilibrium. Suppose that x^* is not TCOP efficient. Then, there must be a time-consistent reallocation $\tilde{x}$ and a period t' that satisfy

(i) $\sum_{h \in H} \tilde{x}_1^h = \sum_{h \in H} e_1^h$,

(ii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{*h,t} x_1^{*h}$ for all $h \in H$ and all $t \geq t'$, and

(iii) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{*h',t''} x_1^{*h'}$ for some $h' \in H$ and some $t'' \geq t'$.

If $t'' = T$ then, since preferences are independent of past consumption, it must hold that

$$
\begin{aligned}
p_1^{*T} \tilde{x}_{t'}^{h',T} &> p_1^{*T} x_1^{*h',T}, \text{and} \\
p_1^{*T} \tilde{x}_{t'}^{h,T} &\geq p_1^{*T} x_1^{*h,T} \text{ for every household } h \in H,
\end{aligned}
$$

which yields a contradiction as before. Now assume that for every household h and every $\tilde{t} > t$ it holds that

$$
\begin{aligned}
p_1^{*\tilde{t}} \tilde{x}_{t'}^{h,\tilde{t}} &\leq p_1^{*\tilde{t}} x_1^{*h,\tilde{t}} \text{ , and} \\
(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) &\succeq^{*h,\tilde{t}} x_1^{*h}.
\end{aligned}
$$

Since preferences are independent of past consumption it follows that $\tilde{x}_{\tilde{t}}^h$ must be an optimal consumption in period $\tilde{t}$ given prices $p_1^{*\tilde{t}}$. Now assume that $t'' = t$. Then it must hold that

$$
\begin{aligned}
p_1^{*t} \tilde{x}_{t'}^{h',t} &> p_1^{*t} x_1^{*h',t}, \text{and} \\
p_1^{*t} \tilde{x}_{t'}^{h,t} &\geq p_1^{*t} x_1^{*h,t} \text{ for every household } h \in H,
\end{aligned}
$$

which again leads to a contradiction. Continuing like this we end up with this contradiction for $t = t'$, so that case (iii) can never hold.

It follows that the sophisticated equilibrium allocation x^* must be TCOP efficient.

<div align="right">Q.E.D.</div>

Theorem 6.10

If a time-consistent allocation is TCOP efficient, then it is CP efficient.

Proof

Let the time-consistent allocation x^* be TCOP efficient. Suppose that x^* is not CP efficient. Then there is an allocation $\tilde{x}$ and a period t' such that

<div align="center">39</div>

(i) $\tilde{x}_{t'}^{h,t'+1,T} = x_{t'}^{*h,t'+1,T} = x_1^{*h,t'+1,T}$ for every $h \in H$,

(ii) $\sum_{h \in H} \tilde{x}_{t'}^{h,t'} = \sum_{h \in H} e^{h,t'}$,

(iii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{*h,t} (x_t^{*h,-}, x_t^{*h}) = x_1^{*h}$ for all $h \in H$ and all $t \geq t'$, and

(iv) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{*h',t''} (x_{t''}^{*h',-}, x_{t''}^{*h'}) = x_1^{*h'}$ for some $h' \in H$ and some $t'' \geq t'$,

which contradicts the fact that x^* is TCOP efficient.

<div align="right">Q.E.D.</div>

Theorem 6.12

In an economy $\mathcal{E}$ that satisfies Assumptions 2, 3, 4 and 6', naïve and sophisticated equilibrium allocations are CP efficient.

Proof

For sophisticated equilibria the result follows directly from Theorems 6.10 and 6.7, since sophisticated equilibrium allocations are time-consistent. Now let (p^*, x^*) be a naïve equilibrium. Suppose that x^* is not CP efficient. Then there must be a reallocation $\tilde{x}$ and a period t' that satisfy

(i) $\tilde{x}_{t'}^{h,t'+1,T} = x_{t'}^{*h,t'+1,T}$ for every $h \in H$,

(ii) $\sum_{h \in H} \tilde{x}_{t'}^{h,t'} = \sum_{h \in H} e^{h,t'}$,

(iii) $(x_{t'}^{*h,-}, \tilde{x}_{t'}^h) \succeq^{*h,t} (x_t^{*h,-}, x_t^{*h})$ for all $h \in H$ and all $t \geq t'$, and

(iv) $(x_{t'}^{*h',-}, \tilde{x}_{t'}^{h'}) \succ^{*h',t''} (x_{t''}^{*h',-}, x_{t''}^{*h'})$ for some $h' \in H$ and some $t'' \geq t'$.

Since $\tilde{x}_{t'}^{h',t'',T}$ was not demanded in equilibrium by household h' in period t'', it must hold that for some $\tilde{t} \geq t''$

$$
\begin{aligned}
p_{t''}^{*\tilde{t}} \tilde{x}_{t'}^{h',\tilde{t}} &> p_{t''}^{*\tilde{t}} x_{t'}^{*h',\tilde{t}}, \text{and} \\
p_{t''}^{*\tilde{t}} \tilde{x}_{t'}^{h,\tilde{t}} &\geq p_{t''}^{*\tilde{t}} x_{t'}^{*h,\tilde{t}} \text{ for every household } h \in H,
\end{aligned}
$$

which leads to a contradiction as before. Thus, x^* must be CP efficient.   Q.E.D.

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