Essays in games and decisions

Abstract

In this dissertation, I depict certain unexplored relations between uncertainties in games and decision problems. However, I do not take a side in the discussion of whether strategic games should be treated as multi-person decision problems, or decision problems as one-person games. Moreover, I propose an alternative solution that arguably provides novel insight into some non-cooperative games.

Chapter 2

In this chapter, I define a simple equivalence relation between the class of two-person symmetric games and the class of decision problems with a complete but not necessarily transitive preference relation. To illustrate this equivalence and show why transitivity is not assumed, consider the Rock-Paper-Scissors game, which is known by its cyclic structure (recall that rock beats scissors, scissors beats paper, and paper beats rock). The equivalent decision problem of this game is defined as the pair, $(X, \succeq)$, in which $X = \{R, P, S\}$ and $R \succ S \succ P \succ R$. In other words, the decision maker's preference relation over the choices is cyclic.

Under this equivalence, I highlight the links between some game and decision theoretical concepts. For example, a Nash equilibrium in a two-person symmetric zero-sum game and a pair of maximal elements in its equivalent decision problem coincide. Further, I show that a two-person symmetric zero-sum game can be extended to its von Neumann-Morgenstern (vN-M) mixed extension if and only if the extended decision problem satisfies SSB utility (Fishburn, 1982), which is a weaker notion than vN-M utility.

I also demonstrate that a decision problem satisfies vN-M utility if and only if its equivalent symmetric game is a potential game (Monderer and Shapley, 1996). Accordingly, I then provide a formula as a function of number of strategies for the number of linearly independent equations that must be satisfied for a potential function to exist in these games. The value of the formula grows

quadratically as the number of strategies increase.

Chapter 3

In this chapter, I introduce three sufficient conditions for the existence of a pure Nash equilibrium in two-person symmetric zero-sum games: (1) a quasiconcavity notion based on the preferences of a decision maker in the equivalent decision problem of a game; (2) a functional representation of an axiom in Fishburn and Rosenthal (1986); (3) the so-called sign-quasiconcavity notion, which is a generalization of quasiconcavity in Duersch et al. (2012a).

I show how these conditions are related to each other and other established sufficient conditions in the literature. The most general condition is sign-quasiconcavity which generalizes (1), (2), generalized ordinal potentials, and quasiconcave games. I also prove that if a two-person symmetric zero-sum game has a generalized ordinal potential, then the preference relation in its equivalent decision problem must be transitive.

Chapter 4

Because economic interactions are not typically zero-sum, maximin strategies cannot capture mutual gains and losses in these interactive situations. Thus, I introduce a new concept, *optimin equilibrium*, that extends the maximin strategy solution to nonstrictly competitive games by incorporating an individual rationality constraint whereby players do not harm themselves for the sake of harming others.

I show that optimin equilibrium consists of a pair of maximin strategies in zero-sum games, but this is not necessarily the case in nonzero-sum games. This result implies that the optimin equilibrium, the minimax solution, and the Nash equilibrium coincide at the mixed strategy profile $[(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})]$ in the Rock-Paper-Scissors game.

I also prove that every finite game in mixed extension possesses an optimin equilibrium, which may or may not be Pareto optimal. As an example, optimin equilibrium selects the two Pareto optimal profiles, (Football, Football) and (Opera, Opera), in the battle of the sexes game. However, it may select a Pareto dominated solution such as (Defect, Defect) in the prisoner's dilemma. Nevertheless, optimin equilibrium and Nash equilibrium may differ in the finitely repeated version in which the Tit-for-Tat strategy profile—whereby a player begins by cooperating in the first round and then does what the opponent did in the previous round—turns out to be an optimin equilibrium. For another example, consider the following one-shot game with ordinal payoffs.

	Top	Middle	Bottom
Top	$100, 100$	$100, 105$	$0, 0$
Middle	$105, 100$	$95, 95$	$0, 210$
Bottom	$0, 0$	$210, 0$	$5, 5$

In this game, the unique Nash equilibrium is (Bottom, Bottom), whereas the unique optimin equilibrium is (Top, Top) because each player guarantees the highest payoff (100) at this profile under any profitable deviation of the other player. For example, Player 2 might profitably deviate from (Top, Top) to Middle, but Player 1 would still receive 100 in this case. However, the maximin strategy concept says that Player 2 would deviate to any feasible strategy including Bottom, leaving Player 1 with a payoff of 0; though, this deviation is not in the self-interest of the deviator because the deviator also receives 0. Thus, the individual rationality constraint of a player rules out such deviations.

Finally, I apply this concept to the Arrow-Debreu economy and provide some sufficient conditions for its existence. My analysis shows that optimin equilibrium generalizes the concept of competitive equilibrium.

Chapter 5

In this chapter, I focus on the concept of rationality (à la vN-M) to answer the following question: 'Is it possible for an economic agent to be simultaneously rational in every situation she faces?' I show that there are games and decision problems in which an agent cannot be simultaneously rational in both. Put differently, the same person can be simultaneously rational and irrational, hence the paradox.

Naturally, one would like to assume that a player is rational both in the game and the decision problem. However, it turns out that a player's rationality in a domain does not necessarily imply her rationality in the other domain, which may challenge the belief that rationality is a personal trait.

I describe a necessary and sufficient condition called doubly linearity for a decision maker that makes her simultaneously rational in both situations. I show that if a player is rational in a game, then she cannot be rational in the associated decision problem unless her social preference function satisfies doubly linearity. This condition seems to rule out social preferences including inequality aversion and altruism.