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Shifts and twists in the relative productivity of skilled labor

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Abstract

Skill-biased technical change is usually interpreted in terms of the efficiency parameters of skilled and unskilled labor. This implies that the relative productivity of skilled workers changes proportionally in all tasks. In contrast, we argue that technical changes also affect the curvature of the distribution of relative productivity. Building on Rosen (1978) [Rosen, S., 1978. Substitution and the division of labor. Economica 45, 235–250] tasks assignment model, this implies that not only the efficiency parameters of skilled and unskilled workers change, but also the elasticity of substitution between skill types of labor. Using data for the United States between 1963 and 2002, we find significant empirical support for a decrease in the elasticity of substitution at the end of the 1970s followed by an increase at the beginning of the 1990s. This pattern of the elasticity of substitution has contributed to the labor productivity slowdown in the mid-1970s through the 1980s and to a *speedup* in the 1990s.

JEL classification: J20; J31; O30; O40

Keywords: Assignment; SBTC; Output growth; Cointegration and change in regime

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1. Introduction

The skill premium of college graduates has increased in most developed countries in the last decades and especially in the US. Since the relative supply of college graduates increased at the same time, this means that the relative demand for college graduates increased even faster than the relative supply. In the literature on wage inequality, these demand shifts in favor of skilled labor are interpreted as the ensuing effects of technical changes. Recent new technologies have increased the marginal productivity of skilled relative to unskilled labor. These productivity shifts are usually associated with changes in the relative efficiency parameters of skilled and unskilled workers (Katz and Murphy, 1992) such that it is implicitly assumed that the relative productivity of skilled workers increased proportionally in every task.

The main contribution of this paper is two-fold. In the theoretical part we take a closer look at the possible effects of skill-biased technical change in the labor market, by analyzing how skill-biased technical change may affect the productivity of skilled workers relative to unskilled workers in a continuum of tasks. To this aim we use (Rosen's, 1978) tasks assignment model that not only offers a microfoundation for the CES production function, the workhorse model in the SBTC and growth literature, but also reveals a relationship between the elasticity of substitution across workers types and the slope of their productivity schedule across tasks. In this model, skill-biased technical change may lead to shifts and twists in the productivity schedule of skilled versus unskilled workers. Shifts correspond to increases in the relative efficiency parameter that are commonly associated with skill-biased technical change. Twists reflect changes in the elasticity of substitution between skilled and unskilled workers that have been absent in the skill-biased technical change literature.

The second contribution is that investigating for the stability of the parameters of a generalized (Katz and Murphy, 1992) framework, we show empirical evidence that the elasticity of substitution between skilled and unskilled labor has changed over time. This variability of the elasticity of substitution over time is of importance as it (*twist*) explains (i) a significant part of the rise in the skill premium after 1977 but also (ii) part of the productivity slowdown observed in the 1970s and 1980s and acceleration in the 1990s as the magnitude of the elasticity of substitution between inputs is directly linked to the growth rate of income per capita as already recognized in the literature on economic growth.[1]

This paper relates to the standard literature on skill-biased technical change (e.g. Katz and Murphy, 1992) by releasing the implicit assumption that the relative productivity of skilled workers increased proportionally in every task. In practice, indeed, new technologies will not necessarily increase the productivity of skilled relative to unskilled workers equally in all tasks and recent empirical evidence points into that direction. For instance, Autor et al. (2003) investigated the impact of recent technical change on the demand for skilled labor and found that although computers substitute for workers performing routine tasks, computers complement workers performing non-routine tasks: “the

[1] Solow (1956) first showed that for an elasticity of substitution between labor and capital of 2, income per head could grow forever if the saving rate s were to be larger than the threshold $s = n/a^2$ where n is population growth and a is the relative efficiencies of capital. De La Grandville (1989) generalized this finding and showed that the value of the threshold was of the form: $s = n\beta(\sigma)^{\sigma/(1-\sigma)}$. More recently, Klump and de La Grandville (2000) have proved that the higher σ the higher income per head.

substitution away from routine to non-routine tasks was not primarily accounted for by educational upgrading; rather, task shifts are pervasive at all educational levels'' (see Autor et al., 2003, p. 2). Although the net effect of new technologies is an increase in the demand for skilled labor, empirical evidence indicates that the demand for skilled relative to unskilled labor decreased in (non-routine) manual tasks and increased in (non-routine) cognitive tasks.[2]

In the debate between supporters of the steady demand hypothesis (see Katz and Murphy, 1992 and Card and Lemieux, 2001) and the acceleration hypothesis (see among others Bound and Johnson, 1992; Krueger, 1993; Berman et al., 1994; Autor et al., 1998 and Berman et al., 1998),[3] an important argument in support of the former has been that accelerating (skill-biased) technical change is difficult to reconcile with the slowdown in labor productivity growth[4] that we have witnessed since the 1970s (Acemoglu, 2002). The analysis of this paper also contributes to this discussion. We show that the decrease in the elasticity of substitution that we find at the end of the 1970s has contributed to the slowdown in labor productivity that started in the 1970s and the increase in the 1990s has contributed to the speedup in labor productivity in the 1990s. Therefore, by acknowledging that skill-biased technical change has affected the elasticity of substitution, we are able to reconcile acceleration of skill-biased technical change with the productivity slowdown and subsequent acceleration.

The assignment model presented in this paper explains the productivity slowdown from the mid-1970s to the late 1980s by a decrease in the elasticity of substitution between skilled and unskilled workers that is, a increase in the comparative advantage of skilled workers in certain tasks. In that sense, the assignment model offers a point of view similar to the hypothesis first formulated by Nelson and Phelps (1966) and more recently by Greenwood and Yorukoglu (1997) that skilled workers have a comparative advantage in implementing and adopting new technologies so that technological changes are followed by a transition period during which a growing proportion of skilled workers are assigned to ''new'' tasks that consist of experimenting, developing and implementing routines in order to use these new technologies. This transition period is characterized by an acceleration of the demand for skilled workers (a shift when more skills are required to perform the various tasks with the new technology and a twist since the comparative advantage of skilled workers has changed) and a fast growing skill premium but a slowdown in labor productivity. In a recent paper, using quarterly data from 1979:1, Castro and Coen-Pirani (2005) have shown empirical evidence for a decline in the degree of capital-skill complementarity in the late 1980s indicating the decline in the comparative advantage of skilled workers.

The remainder of this paper is organized as follows. In Section 2 we show how technical change may affect the relative productivity of skilled versus unskilled workers in an assignment model that is consistent with a CES production function for the economy. In Section 3, after a brief discussion of the data, we investigate the stability of the parameters of the generalized (Katz and Murphy, 1992) equation to investigate whether shifts and twists in

[2] See also Autor et al. (2006), Goos and Manning (2007) and Spitz-Oener (2006).

[3] Krusell et al. (2000) also argue in favor of an acceleration in SBTC brought about by the more rapid decline in the relative price of capital equipment in the early 1970s.

[4] See for instance Fischer (1988), Griliches (1994) and Kozicki (1997).

the relative productivity of skilled labor have occurred in the US. We draw our conclusions in Section 4.

2. Theoretical framework

Though technical changes may affect the productivity of skilled relative to unskilled workers, it is, *a priori*, not necessarily true that the relative productivity of skilled workers shifts in the same direction and with the same magnitude in all productive tasks. In some tasks, skilled workers may have an even larger productivity compared to unskilled workers while the new technologies may decrease the relative productivity of skilled workers in some other tasks. This suggests that the distribution of relative productivity could be affected by technical changes in a non-trivial way.

In this paper, we use (Rosen's, 1978) tasks assignment model to study how technical change affects the distribution of relative productivity between skilled and unskilled workers. In the model, there are two types of workers; skilled denoted s and unskilled denoted u. The supply of skilled and unskilled labor, denoted $\overline{S}$ and $\overline{U}$, respectively, is assumed exogenous and perfectly inelastic to wages. [5] Jobs refer to certain tasks and there is a continuum of tasks to be performed in order to produce output. Aggregate output Y is produced with fixed proportions of the output of each tasks, i.e. tasks are perfect complements. The problem is to find an assignment of the various tasks to skilled and unskilled workers in order to maximize output, denoted Y. In that sense, the model focuses essentially on the demand for labor.

The analytic setting is as follows. Let $1/\pi_s(v)$ and $1/\pi_u(v)$ measure the productivity, in units of output per worker, of skilled and unskilled workers at task v.[6] The continuum of tasks v is defined so that the relative productivity of skilled to unskilled workers, defined by the function $q(v) = \pi_u(v)/\pi_s(v)$, is increasing in v (i.e. $q' > 0$). The function $q(v)$ offers a convenient ordering of tasks by comparative advantage. Skilled workers have a comparative advantage in cognitive tasks v, v close to *1*, while unskilled workers have a comparative advantage in manual tasks v, v close to 0.

Consider the following functional form:

$$\frac{1}{\pi_s(v)} = \frac{\sigma - 1}{\sigma} a_s (1 - v)^{\frac{1}{1-\sigma}} \tag{1}$$

$$\frac{1}{\pi_u(v)} = \frac{\sigma - 1}{\sigma} a_u v^{\frac{1}{1-\sigma}} \tag{2}$$

so that:

$$\ln q(v) = \ln \frac{a_s}{a_u} + \frac{1}{1-\sigma} \ln\left(\frac{1-v}{v}\right) \tag{3}$$

[5] In practice, the skill premium and the relative number of skilled workers are determined simultaneously by demand and supply. This might lead to an identification problem when estimating structural parameters using the inverse (relative) demand curve. However, controling for endogenous human capital formation both Heckman et al. (1998) and Ciccone and Peri (2005) have found estimates of the elasticity of substitution between skilled and unskilled workers ''surprisingly'' similar (Ciccone and Peri (2005) preferred estimate is 1.5) to that of Katz and Murphy (1992) on the same period.

[6] Note that, reciprocally, the demand for workers per unit of output at task v is $\pi_s(v)$ and $\pi_u(v)$ for skilled and unskilled workers, respectively.

Suppose that between time periods $t-1$ and t new technologies are implemented. It is clear from Eq. (3) that new technologies can only affect the relative productivity of skilled workers through a_{su} and σ. If σ remains constant and a_{su} changes, the relative productivity *shifts* proportionally in every tasks, i.e. $d\ln q = \mathrm{d}\ln a_{su}$ independent of v. However, at constant a_{su}, if σ decreases (increases) then the relative productivity of skilled workers increases (decreases) in tasks $v > \frac{1}{2}$ and decreases (increases) in tasks $v < \frac{1}{2}$ since $\frac{\partial \ln q}{\partial \sigma} = (1-\sigma)^{-2}\ln\frac{1-v}{v}$. In other words, changes in σ lead to *twists* in the relative productivity schedule.[7] The fixed point of the twist, given by $v = \frac{1}{2}$, is the "technical marginal task" or the "anybody-can-do-it-as-efficiently" task.

As Rosen (1978) acknowledged, the efficient assignment is such that the marginal task ε with $\varepsilon \in (0,\ 1)$ divides the spectrum of v so that it is optimal to assign tasks $(0,\ \varepsilon)$ to unskilled workers and tasks $(\varepsilon,\ 1)$ to skilled workers. The unit isoquant is defined parametrically by integrating the demand for workers of each type per unit of output, the inverse of the workers' productivity, over the spectrum of v, which, using the functional form of workers' productivity as defined in Eqs. (1) and (2), read as:

$$\begin{aligned} \frac{U}{Y} &= \int_0^{\varepsilon} \pi_u(v)\mathrm{d}v = \frac{1}{a_u}\varepsilon^{\frac{\sigma}{\sigma-1}} \\ \frac{S}{Y} &= \int_{\varepsilon}^{1} \pi_s(v)\mathrm{d}v = \frac{1}{a_s}(1-\varepsilon)^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{4}$$

Using both equations to derive expressions of the marginal task as a function of U/Y and S/Y, respectively and equating both expressions, after some algebra, we derive the maximum output level Y as (equating exogenous supply to the demand for skilled and unskilled workers, i.e. $S = \overline{S}$ and $U = \overline{U}$):

$$Y = \left[(a_s\overline{S})^{\frac{\sigma-1}{\sigma}} + (a_u\overline{U})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

Eq. (5) reads as a CES production function.[8] In the literature on labor demand, the parameter σ, indicating the curvature of the distribution of relative productivity, is usually referred to as the elasticity of substitution between skilled and unskilled workers. The larger σ the larger the ease to substitute between skill types of workers or equivalently, the flatter the distribution of relative productivity. The indirect production function indicates that the existence of comparative advantages among workers imply imperfect substitution between the various types of workers.

2.1. Assignment and the skill premium

We use Eq. (5) to derive the marginal product of skilled and unskilled labor. Assuming perfect competition in the output and labor market, that is equating the marginal product

[7] An increase in the curvature of the distribution of relative productivity (a decrease in σ) increases the relative productivity of skilled workers in the tasks ranging from 0 to $\frac{1}{2}$ and decreases their relative productivity in the other tasks. Hence, the larger σ the flatter the shape of $q(v)$. As σ tends to infinity, the curvature of the relative productivity schedule disappears: There is equity of comparative advantage (see Willis, 1986 for instance).

[8] In general, solving the system yields Rosen's indirect production function. Imposing workers' productivity as in Eqs. (1) and (2) yields the CES form.

of skilled and unskilled workers, as derived from Eq. (5), to their respective wages, denoted w_s and w_u the expression of the (log) relative skill premium, $\ln \frac{w_s}{w_u} = \omega_{su}$ reads as:

$$\omega_{su} = \frac{\sigma - 1}{\sigma} \ln \frac{a_s}{a_u} - \frac{1}{\sigma} \ln \frac{\overline{S}}{\overline{U}}. \tag{6}$$

Eq. (6) is used by Katz and Murphy (1992) to link developments in the skill premium with developments in the relative supply of skilled and unskilled workers. Katz and Murphy argue that changes in the skill premium in the US are consistently explained by steady demand shifts. The shifts in the relative demand for skilled workers are further assumed to come about because of skill-biased technical change. In their model, skill-biased technical change only enters the equation via upward shifts in the relative productivity of skilled workers $\ln a_{su}$.

$$\omega_{su,t} = \frac{\sigma - 1}{\sigma} \ln a_{su,t} - \frac{1}{\sigma} \ln \frac{\overline{S}_t}{\overline{U}_t} \quad \text{with} \ln a_{su,t} = \ln a_{su} + \delta t. \tag{7}$$

Hence, the type of technical change Katz and Murphy consider is restricted to proportional shifts in the distribution of relative productivity of skilled (college graduates) and unskilled (high-school graduates) workers. We argue that, in addition, technical changes may twist the distribution of relative productivity of skilled workers and hence impact the skill premium via a change in σ.

2.2. Three sources of income per capita growth

To single out the various sources of income per capita growth, we normalize the CES production function at time 0, where time 0 corresponds to the timing of the structural break (see de La Grandville, 1989 and Klump and Preissler, 2000).

$$Y_t = Y_0 \left((1 - b_t) \left(\frac{U_t}{U_0} \right)^{\frac{\sigma-1}{\sigma}} + b_t \left(\frac{S_t}{S_0} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

with Y_0 the output at time 0, S_0 and U_0 the supply of skilled and unskilled workers at time 0 and b_t a parameter indicating the relative efficiency of skilled to unskilled workers at time t.

The long run labor productivity, denoted $y_t = Y_t/(S_t + U_t)$, can be written as a function of the proportion of skilled workers in the firm, denoted p_t ($p_t = \frac{S_t}{U_t + S_t}$) .

$$y_t = g_\sigma(p_t) = Y_0 \left((1 - b_t) U_0^{\frac{1-\sigma}{\sigma}} (1 - p_t)^{\frac{\sigma-1}{\sigma}} + b_t S_0^{\frac{1-\sigma}{\sigma}} p_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{9}$$

Writing $a_{u,t} = Y_0 (1 - b_t)^{\frac{\sigma}{\sigma-1}} U_0^{-1}$ and $a_{s,t} = Y_0 b_t^{\frac{\sigma}{\sigma-1}} S_0^{-1}$ or $b_t = \frac{\left(\frac{a_{s,t} S_0}{a_{u,t} U_0}\right)^{\frac{\sigma-1}{\sigma}}}{1 + \left(\frac{a_{s,t} S_0}{a_{u,t} U_0}\right)^{\frac{\sigma-1}{\sigma}}}$, the indirect production function derived from the assignment of tasks to workers reads as a normalized CES production function and the derived labor productivity function reads as the g function:

$$Y_t = \left((a_{u,t} U_t)^{\frac{\sigma-1}{\sigma}} + (a_{s,t} S_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{10}$$

$$y_t = g_\sigma(p_t) = \left((a_{u,t}(1 - p_t))^{\frac{\sigma-1}{\sigma}} + (a_{s,t} p_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{11}$$

Labor productivity growth is:

$$\frac{\dot{y}}{y}=\frac{1}{g_\sigma}\left(\frac{\partial g_\sigma}{\partial\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}t}+\frac{\partial g_\sigma}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t}+\frac{\partial g_\sigma}{\partial b}\frac{\mathrm{d}b}{\mathrm{d}t}\right). \tag{12}$$

Eq. (12) indicates the three sources of labor productivity growth:

1. *twists*, initiated by changes in the elasticity of substitution $\frac{\partial g_\sigma}{\partial\sigma}$,
2. *supply*, initiated by changes in the skill employment share $\frac{\partial g_\sigma}{\partial p}$ and,
3. *shifts*, initiated by changes in the relative efficiency units of skilled labor $\frac{\partial g_\sigma}{\partial b}$.

The following proposition, the proof of which is provided in Appendix, shows the relationship between *twists* and labor productivity $\frac{\dot{y}}{y}$.

Proposition 1. *If the economy is described by a CES production function and the elasticity of substitution between skilled and unskilled workers σ decreases through time, then the growth of labor productivity slows down.*

The following proposition indicates that labor productivity growth is partially driven by the growth of the skill employment share (*supply*).

Proposition 2. *If the economy is described by a CES production function, an increase in the employment share of skilled labor at time t will increase labor productivity growth if and only if the skill premium is strictly positive,* $\omega_{su,t}>0$.

Proof. Deriving Eq. (11) with respect to p_t yields:

$$\frac{\partial g_\sigma}{\partial p_t}=g_\sigma^{\frac{1}{\sigma}}\left[(a_{\mathrm{s},t})^{\frac{\sigma-1}{\sigma}}p_t^{-1/\sigma}-(a_{\mathrm{u},t})^{\frac{\sigma-1}{\sigma}}(1-p_t)^{-1/\sigma}\right].$$

Hence, $\frac{\partial g_\sigma}{\partial p_t}>0\iff\frac{p_t}{1-p_t}=\frac{S_t}{U_t}<\left(\frac{a_{\mathrm{s},t}}{a_{u,t}}\right)^{\sigma-1}$.

Note that since $\sigma>0$, this condition can be written as: $\left(\frac{S_t}{U_t}\right)^{1/\sigma}<\left(\frac{a_{\mathrm{s},t}}{a_{u,t}}\right)^{\frac{\sigma-1}{\sigma}}\iff\frac{1}{\sigma}\ln\frac{S_t}{U_t}<\frac{\sigma-1}{\sigma}\ln\frac{a_{\mathrm{s},t}}{a_{\mathrm{u},t}}$. Using Eq. (6) we conclude that $\frac{\partial g_\sigma}{\partial p_t}>0\iff\omega_{\mathrm{su},t}>0$. □

Finally, the following proposition indicates that the growth of labor productivity is driven by changes in the relative efficiency parameter of skilled labor (*shifts*).

Proposition 3. *If the economy is described by a CES production function, an increase in the relative efficiency of skilled labor, i.e.* b_t, *at time t will increase labor productivity growth if and only if the relative employment of skilled workers at time t is strictly greater than initial relative employment at time* 0, *i.e.* $\frac{S_t}{U_t}>\frac{S_0}{U_0}$.

Proof. Deriving Eq. (8) with respect to b_t yields:

$$\frac{\partial g_\sigma}{\partial b_t}=\frac{\sigma}{\sigma-1}Y_0^{\frac{\sigma-1}{\sigma}}g_\sigma^{\frac{1}{\sigma}}\left[S_0^{\frac{1-\sigma}{\sigma}}p_t^{\frac{\sigma-1}{\sigma}}-U_0^{\frac{1-\sigma}{\sigma}}(1-p_t)^{\frac{\sigma-1}{\sigma}}\right]$$

Hence, $\frac{\partial g_\sigma}{\partial b_t}>0\iff\frac{p_t}{1-p_t}=\frac{S_t}{U_t}>\frac{S_0}{U_0}$. □

3. Empirical analysis

3.1. Data

Our data consist of annual US time-series for the employment share of skilled workers and the skill premium between 1963 and 2002. The data for the period 1963–1992 are made available by Krusell et al. (2000).[9] We use the CPS March supplements files for the years 1993–2003 and derive changes in the relative supply of skills and the skill premium between 1992 and 2002. We use the procedure proposed by Katz and Autor (2000) and described in Acemoglu (2002). The relative supply of skills is calculated from a sample that includes all workers between the ages of 18 and 65 and defined by the ratio of college equivalents to non-college equivalents using weeks worked as weights. College equivalents equals the number of college graduates (at least 16 years of schooling) to which we add half of the workers with some college (strictly more than 12 years of schooling and less than 15 years of schooling). The non-college equivalents equal high-school dropouts plus high-school graduates to which we add the other half of workers with some college.

The college premium is the coefficient for workers with at least a college degree in a log weekly wage regression. The regression includes dummies for other educational categories, experience and its square, a nonwhite dummy, a female dummy and interactions between the female dummy and the nonwhite dummy and the experience controls. The sample includes full-time full-year workers between the ages of 18 and 65.

The series from 1963 to 1992 are then extended to 2002 by applying the calculated changes on the last year observation of the Krusell et al. series.

The relative supply of skilled workers to unskilled workers increased more than two-fold over the period considered. The skill premium increases through the 1960s, then declines through the 1970s to rise sharply after 1980.

3.2. Testing for shifts and twists in relative productivity of skilled labor

Consider the class of skill-biased technical change models described by Katz and Murphy (1992).

[KM Model]

$$\omega_{su,t} = \gamma_0 + \gamma_1 \ln\frac{S_t}{U_t} + \gamma_2 t + \xi_t \tag{13}$$

$$= -\frac{1}{\sigma}\ln\frac{S_t}{U_t} + \frac{\sigma-1}{\sigma}\delta t + \frac{\sigma-1}{\sigma}\ln a_{su} + \xi_t \tag{14}$$

where $\ln a_{su}$, the relative efficiency parameter in 1963, is a constant and ξ an error term satisfying the usual properties, *IID*.

In these models, new technologies increase the relative productivity of skilled workers proportionally in all tasks. The elasticity of substitution between labor types, given by $\sigma = -1/\gamma_1$, is assumed constant over time. The demand shifts are captured by a linear time

[9] The data can be obtained from Violante's website, http://www.ucl.ac.uk. For more details on the sources and construction of the data see (Krusell et al., 2000).

Table 1
OLS regression of wage inequality

KM Model			Augmented KM Model		
	Coefficient	Standard error[a]		Coefficient	Standard error[a]
$\ln \frac{S_t}{U_t}$	−0.639[b]	0.0517	$\ln \frac{S_t}{U_t}$	−0.596[b]	0.1112
t	0.022[b]	0.0014	t	0.028[b]	0.0080
Intercept	0.014[c]	0.0068	Intercept	0.002	0.0028
			$t^2 \times 10$	−0.011	0.0080
			$t^3 \times 100$	0.005	0.0035
			$t^4 \times 10{,}000$	−0.007	0.0045
R^2_{adj}	0.934		R^2_{adj}	0.951	

[a] Bootstrap estimated standard error (10,000 sampling with replacement) to account for the nonnormal distribution of the OLS estimator in the context of cointegration.
[b] sig 1%.
[c] sig 5%.

trend, i.e. $\gamma_2 = \frac{\sigma-1}{\sigma}\delta$ indicating the yearly growth rate in the relative demand for skills. The estimation results of the KM Model, reported in the first column of Table 1, are consistent with findings in the literature.[10] The elasticity of substitution between types of labor is 1.56 and the demand for skills shifts steadily at a yearly rate of 2.2%. The result indicates that technical change has increased the relative productivity of skilled workers in all tasks. However, in the KM Model, demand for skills shifts steadily over time whereas some authors (see Acemoglu, 2002) argued in favor of an acceleration of SBTC during the 1980s. We extent the model to capture a possible acceleration in SBTC during the 1980s (e.g. Acemoglu, 2002) by adding time squared, cubed and fourth time order in the regression. As reported in Table 1, the second, third and fourth time order are not significant, we find no evidence in favor of an acceleration in SBTC during the period 1963–2002.

We therefore investigate empirically the stability of not only the efficiency parameters but also the elasticity of substitution over time. Before testing for structural breaks in the KM model, we illustrate the (un-)stability of its parameters by estimating Eq. (13) using rolling regressions of 15-year window through the span 1963–2002. The results are presented in Fig. 1. The last year of each window[11] is reported on the horizontal axis whereas the magnitude of the respective estimates of $\gamma_1 = -1/\sigma$ and time trend and their respective 95% interval are reported on the vertical axis of panel a and b, respectively. Clearly γ_1 is not constant nor is the trend parameter. The first panel indicates that γ_1, and hence σ, is high before 1967–1981, low between 1968–1982 and 1976–1990 and return

[10] Note that the relative supply and skill premium series are nonstationary, i.e. $I(1)$. The Augmented Dickey-Fuller statistics, with drift, for the respective series are −1.09 and −1.1 and not significant. However, the series are cointegrated, $ADF - statistic = -3.89$ significant at 1%, such that the OLS estimates presented in Table 1 are consistent. Moreover, the t-statistics of the coefficients estimated by Error Correction Regression are all significant which confirms that the coefficients of the KM model reflect a structural (and not spurious) long run relationship. Moreover, we use Bootstrap estimated standard error (10,000 sampling with replacement) to account for the nonnormal distribution of the OLS estimator in the context of cointegration.
[11] For instance, for 1980, parameters of interest have been estimated using the span 1966–1980.

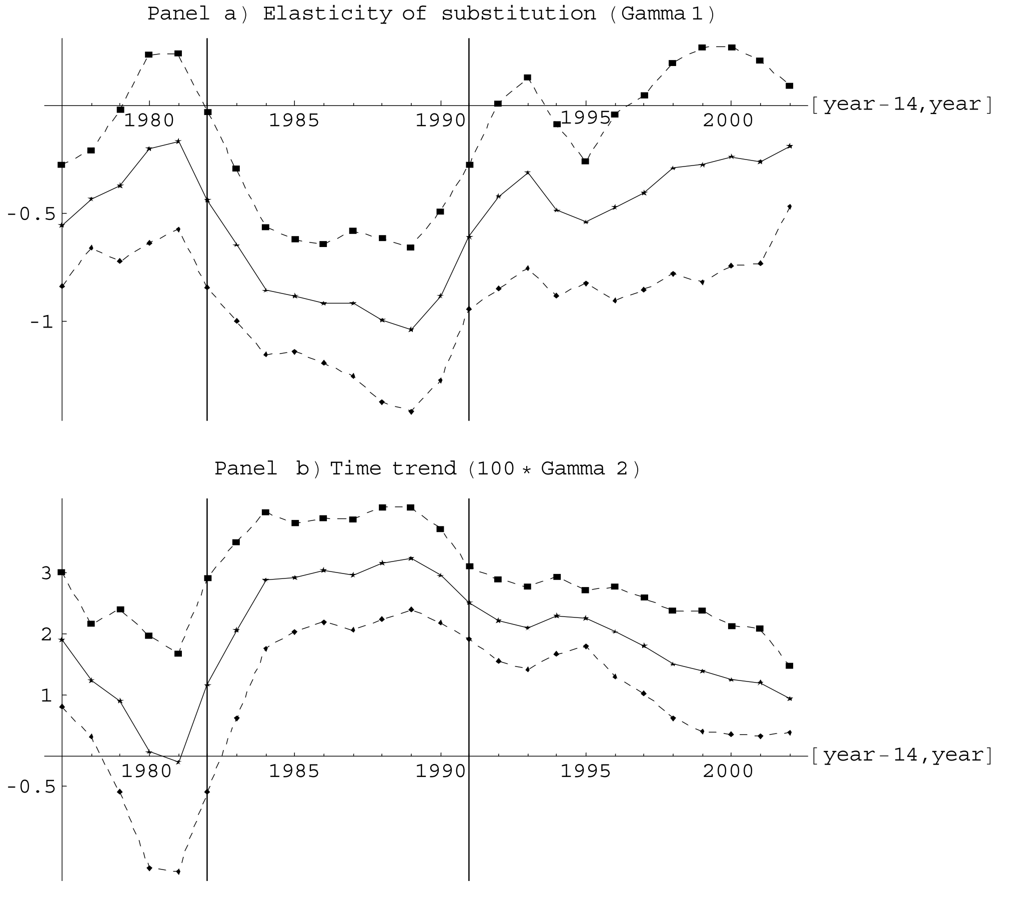

Fig. 1. Stability of the parameters of the Katz and Murphy (1992) Model. Estimates derived using rolling regression techniques of sample size 15.

to its initial level after 1977–1991. The second panel indicates that the time trend parameter is decreasing before 1968–1982 and after 1977–1991 at a roughly similar rate and increasing in the between. This suggests augmenting Eq. (13) with time squared, the parameter of which is negative before 1967–1981, positive between 1968–1982 and 1976–1990 and returns to its initial level after 1977–1991.

To formally determine the number of breaks in the augmented KM model, we follow (Bai, 1997) and compute the residual variance of the model with a single break for all possible break dates, looking for the presence of local minima. The plot of the residual variance indicates three minima: the global minimum in 1989, and two local minima in 1977 and 1984. However, following the literature on structural breaks (see among others Andrews, 1993), to avoid small sample biases in the Chow test at the sample-ends and between breaks, a trimming parameter of 15% is imposed. This restriction does not allow us to statistically distinguish between the break dates in 1984 and 1989. The inspection of the residual variance of the augmented KM model confirms the number of breaks suggested by the rolling regressions, i.e. 2.

Table 2
OLS regression of wage inequality with three regimes and breaks in 1977 and 1990

ST Model			
Three regimes	Estimates	Coefficient	Elasticity, $\widehat{\sigma}$
1963:1976	$\ln \frac{S_t}{U_t}$ $(\widehat{\gamma}_1)$	−0.450	2.22
	t $(\widehat{\gamma}_2)$	0.017	
	t^2 (×1000) $(\widehat{\gamma}_3)$	−0.099	
	Intercept	0.020	
1977:1989	$\ln \frac{S_t}{U_t}$ $(\widehat{\gamma}_1 + \widehat{\gamma}_{1T_{77\text{–}90}})$	−0.666	1.50
	t $(\widehat{\gamma}_2)$	0.017	
	t^2 (×1000) $(\widehat{\gamma}_3 + \widehat{\gamma}_{3T_{77\text{–}90}})$	0.227	
1990:2002	$\ln \frac{S_t}{U_t}$ $(\widehat{\gamma}_1)$	−0.450	2.22
	t $(\widehat{\gamma}_2 + \widehat{\gamma}_{2T_{90\text{–}02}})$	0.020	
	t^2 (×1000) $(\widehat{\gamma}_3)$	−0.099	
Parameters		Coefficient	Standard error[a]
	$\widehat{\gamma}_1$	−0.450[c]	0.1060
	$\widehat{\gamma}_2$	0.017[c]	0.0055
	$\widehat{\gamma}_3$	−0.099[d]	0.0449
	Intercept	0.020	0.0226
	$\widehat{\gamma}_{1T_{77\text{–}90}}$	−0.216[c]	0.0420
	$\widehat{\gamma}_{2T_{90\text{–}02}}$	0.003[c]	0.0130
	$\widehat{\gamma}_{3T_{77\text{–}90}}$	0.327[c]	0.0718
Statistic tests	R^2_{adj}	0.969	
	ADF-statistic	− 7.024[c]	
	Quandt-statistic[b] with two states	12.330[d]	
	Three regimes, and breaks in 1977 and 1990		

[a] Bootstrap estimated standard error (10,000 sampling with replacement) to account for the nonnormal distribution of the OLS estimator in the context of cointegration.

[b] No critical values exist for models with more than 1 break in cointegrated series. The critical values we used are obtained by bootstrapping. We first simulate the true data generating process as a augmented KM model fed with a random shock, sample size 40 and then, for each of the 10,000 sampling with replacement, calculate the Quandt statistic. Data generating process: ($\omega_t = \gamma_0 + \gamma_1 \ln \frac{S_t}{U_t} + \gamma_2 t + \gamma_3 t^2 + e_t$ with $e_t \rightsquigarrow N(0, \sigma^2)$). The critical values at 10%, 5% and 1% are 10.10, 11.71 and 15.94, respectively, assuming a trimming parameter of 15% at the sample-ends and between break dates.

[c] sig 1%.

[d] sig 5%.

This suggests estimating the Shifts and Twists Model (ST Model from now on) allowing for two breaks and three regimes in the augmented KM model. However, this would require the estimation of 10 parameters with only 40 observations. To save on degrees of freedom and increase efficiency, after inspection of Fig. 1, we reduce the number of parameters to be estimated by restricting the values of γ_1 and γ_3 to return to their initial level in the third regime.

Since the break dates of the parameters are *a priori* unknown, we use the Quandt-statistic that corresponds to the largest Chow-statistic, $Sup - Chow$, measured on the period under scrutiny. We run the Chow-statistic for the stability of the augmented KM Model for all years in the sample and find that the breakdates are 1977 and 1990. As it turns out, the trend parameters in the first and second regime are not significantly different. To gain on efficiency, we therefore re-run the Quandt-statistic for the stability of the augmented

KM Model restricting γ_2 to be stable in the first two regimes as indicated in Eq. (15). The Quandt-statistic for the ST model is equal to 12.33, significant[12] at 5%, and the corresponding break dates are 1977 and 1990. ST Model

$$\omega_{su,t} = \gamma_0 + \gamma_1 \ln\frac{S_t}{U_t} + \gamma_2 t + \gamma_3 t^2 + \left(\gamma_{1T_{77\text{-}90}} \ln\frac{S_t}{U_t} + \gamma_{3T_{77\text{-}90}} t^2\right) D_{T_{77\text{-}90}} \gamma_{2T_{90\text{-}02}} \times t \times D_{T_{90\text{-}02}} + \varepsilon_t \tag{15}$$

where $D_{T_{77\text{-}90}} = \begin{pmatrix} 1 \text{ if } 1977 \leqslant t < 1990 \\ 0 \text{ if } t < 1977 \text{or } t \geqslant 1990 \end{pmatrix}$ and $D_{T_{90\text{-}02}} = \begin{pmatrix} 1\text{if } t \geqslant 1990 \\ 0 \text{ if } t < 1990 \end{pmatrix}$.

The results are reported in Table 2. The fit of the ST Model is better than the KM Model with acceleration of SBTC as indicated by the adjusted R^2 (see Table 2). [13] In the ST Model there are two break dates in the long run relationship between skill premium and relative supply that define three regimes and two states. In the first regime covering the period 1963–1977, the relative demand for skills shifts at an annual rate of 1.7% with a yearly deceleration of 0.099%. In the second regime covering the period 1977–989, the relative demand for skills still shifts at an annual rate of 1.7% but with a yearly acceleration of 0.227%. After 1990, the relative demand for skills shifts at an annual rate of 2.0% with a yearly deceleration of 0.099%. Moreover, in the periods 1963–1976 and 1990–2002 the elasticity of substitution between skill types is relatively large and equal to 2.22. However, between 1977 and 1989, the elasticity of substitution is significantly lower and equal to 1.50.[14]

The results provide strong empirical support for the relevance of twists. The technical changes observed between 1963 and 2002 have altered the distribution of comparative advantage among skilled and unskilled workers differently in the various tasks. The decrease in the elasticity of substitution at the end of the 1970s suggest that skilled workers have become relatively more productive in cognitive tasks whereas unskilled workers have become relatively more productive in manual tasks. In contrast, the increase of the elasticity of substitution indicates a twist in the opposite direction.

3.3. Sources of skill premium growth

To investigate empirically the impact of twists on the skill premium, we use the estimates of the ST Model to derive ex-post predictions of the skill premium. The average annual growth of these ex-post predictions of the skill premium is then decomposed into:

[12] No critial values exist for models with more than 1 break in cointegrated series. Bai and Perron (1998, 2003) present critical values for multiple breaks but for stationary variables. The critical values we used are obtained by bootstrapping. We first simulate the true data generating process as a augmented KM model fed with a random shock, sample size *40* and then, for each of the 10,000 sampling with replacement, calculate the Quandt statistic. Data Generating Process: ($\omega_t = \gamma_0 + \gamma_1 \ln\frac{S_t}{U_t} + \gamma_2 t + \gamma_3 t^2 + e_t$ with $e_t \rightsquigarrow N(0, \sigma^2)$). The critical values at 10%, 5% and 1% are 10.10, 11.71 and 15.94, respectively.

[13] The long run relationship depicted in the ST Model is stationary, $ADF - statistic(\widehat{\varepsilon}_t) = -7.024$ significant at 1%, such that the OLS estimates presented in Table 2 are consistent. Moreover, γ_1, $\gamma_{1T_{77\text{-}90}}$, γ_2 and $\gamma_{3T_{90\text{-}02}}$ estimated by Error Correction Regression are significant which confirms that the coefficients of the ST Model reflect a structural (and not spurious) long run relationship. Moreover, we use Bootstrap estimated standard error (10,000 sampling with replacement) to account for the nonnormal distribution of the OLS estimator in the context of cointegration.

[14] This result seems to be consistent with other empirical results. Acemoglu (2002), using a time-series from 1939 to 1996, finds an elasticity of 1.9, while (Katz and Murphy, 1992) estimate $\sigma = 1.4$ for the period 1963–1987.

Table 3
Elasticity of substitution, output growth and decomposition of labor productivity growth and skill premium growth

	Periods			
	1963:1976	1977:1989	1990:2002	1963:2002
Substitution parameter[a], σ	2.22	1.50	2.22	
Decomposition of:				
Skill premium growth:	0.20	0.97	0.50	0.57
Contribution of:[b]				
ST Model				
Shifts	1.59	3.16	−0.10	1.63
Twists	0.00	−1.26	1.45	0.00
Supply	−1.69	−0.87	−0.96	−1.17
Errors	0.30	−0.06	0.11	0.11
KM Model				
Shifts	2.20	2.20	2.20	2.20
Supply	−2.40	−1.24	−1.37	−1.67
Errors	0.34	−0.01	−0.33	0.04
Labor productivity growth, y_t	0.15	0.18	0.74	0.36
Contribution of:[c]				
Supply, p_t	0.06	−0.03	0.13	0.04
Shifts, b_t	0.09	0.26	0.51	0.29
Twists, σ_t	0.00	−0.05	0.11	0.01

Notes: All figures except the elasticity of substitution are average annual percentage rates.

[a] The elasticity of substitution parameter is derived from the estimates of the ST Model as follows: $\sigma = -1/\widehat{\gamma}_1$ in regime 1 and 3 and $\sigma = -1/(\widehat{\gamma}_1 + \widehat{\gamma}_{1,T_{77\text{–}90}})$ in regime 2. Hats indicate estimates of the ST Model.

[b] The contribution of shifts is the average annual growth rate of: $(\widehat{\gamma}_2 + \widehat{\gamma}_{2,T_{90\text{–}02}} D_{T_{90\text{–}02}})t + (\widehat{\gamma}_3 + \widehat{\gamma}_{3,T_{77\text{–}90}} D_{T_{77\text{–}90}})t^2$. The contribution of supply at constant elasticity of substitution is the average annual growth rate of $\widehat{\gamma}_1 \ln \frac{S_t}{U_t}$. The contribution of twists is: growth rate of $\widehat{\omega}_{\mathrm{su},t}$ less the contribution of shifts and supply. The contribution of errors is the average annual growth rate of the observed series less its estimates.

[c] The contribution of the respective factors are derived using the linear approximation: $\left(\frac{\Delta y}{y} = \frac{1}{y}\frac{\partial g_\sigma}{\partial p}\Delta p + \frac{\partial g_\sigma}{\partial \widehat{b}}\Delta \widehat{b} + \frac{\partial g_\sigma}{\partial \widehat{\sigma}}\Delta \widehat{\sigma}\right)$.

1. the contribution of *shifts*, i.e. the average annual growth rate of $(\widehat{\gamma}_2 + \widehat{\gamma}_{2,T_{90\text{–}02}} D_{T_{90\text{–}02}})t + (\widehat{\gamma}_3 + \widehat{\gamma}_{3,T_{77\text{–}90}} D_{T_{77\text{–}90}})t^2$,
2. the contribution of *supply* at constant elasticity of substitution over time, i.e. the average annual growth rate of $\widehat{\gamma}_1 \ln \frac{S}{U}$ and,
3. the contribution of *twists*, i.e. the average annual growth rate of the ex-post predictions of Eq. (15), i.e. $\widehat{\omega}_{\mathrm{su},t}$, less the contribution of supply and shifts.

Also the contribution of the errors of the model are accounted for as the observed average annual growth less the ex-post predictions average annual growth. This decomposition is reported in Table 3 together with the decomposition corresponding to the KM Model. The decrease in the elasticity of substitution in 1977 has contributed to a narrowing in wage dispersion between 1977 and 1989. However, this narrowing has been offset by the shifts contribution of a magnitude twice as large in that period. Remarkably enough, shifts have had almost no effects on wage dispersion after 1990 whereas twists have contributed to a large wage widening between 1990 and 2002 only partly offset by the increasing skills supply. These empirical figures stem for the importance of twists, that is changes in the elasticity of substitution, in explaining patterns of skill premium over time.

3.4. Sources of labor productivity growth

Proposition 1 indicates that technical change increasing the ease to substitute between labor types will lead to an acceleration in labor productivity growth whereas labor productivity slows down when technical change decreases the elasticity of substitution. We therefore investigate empirical evidence for Proposition 1 and use the estimates of the ST Model to derive ex-post predictions of average labor productivity growth. The predictions of labor productivity growth are derived using Eq. (12) where p_t is the employment share of skilled labor and from Eq. (7) we get:

$$\left(\frac{\widehat{a}_{s,t}}{\widehat{a}_{u,t}}\frac{S_{63}}{U_{63}}\right)^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} = \exp\left(\widehat{\gamma}_0 + \left(\widehat{\gamma}_2 + \widehat{\gamma}_{2,T_{90-02}}D_{T_{90-02}}\right)t + \left(\widehat{\gamma}_3 + \widehat{\gamma}_{3,T_{77-90}}D_{T_{77-90}}\right)t^2\right)$$

$$\widehat{\sigma} = -\frac{1}{\widehat{\gamma}_1 + \widehat{\gamma}_{1,T_{77-90}}D_{T_{77-90}}}$$

$$\widehat{b}_t = \frac{\left(\frac{\hat{a}_{s,t}}{\hat{a}_{u,t}}\frac{S_{63}}{U_{63}}\right)^{\frac{\hat{\sigma}-1}{\hat{\sigma}}}}{1 + \left(\frac{\hat{a}_{s,t}}{\hat{a}_{u,t}}\frac{S_{63}}{U_{63}}\right)^{\frac{\hat{\sigma}-1}{\hat{\sigma}}}}$$

$$\widehat{a}_{u,t} = Y_{63}(1-\widehat{b}_t)^{\frac{\hat{\sigma}}{\hat{\sigma}-1}} \text{ and } \widehat{a}_{s,t} = Y_{63}\widehat{b}_t^{\frac{\hat{\sigma}}{\hat{\sigma}-1}}.$$

The contribution of *supply*, *shifts* and *twists* to average labor productivity growth are reported in Table 3. The decrease of σ after 1977 has contributed to a slowdown of 0.05% points in labor productivity growth between 1977 and 1989 whereas the increase of σ after 1990 has contributed to an acceleration of labor productivity growth of 0.11% points after 1990. The decrease in the elasticity of substitution that occurs in 1977 has contributed to a slowdown in labor productivity throughout the 1980s whereas the increase in the elasticity of substitution in the early 1990s has contributed to speed up labor productivity throughout the 1990s.

Note that the employment share of skilled workers increased between 1963 and 2002 and therefore contributed to labor productivity growth in the first and third regimes since as indicated by Proposition 2 the skill premium was positive in these regimes. However, in the second regime, the skill premium was negative between 1978 and 1981 so that the increase in the employment share of skilled workers has contributed to a slowdown in labor productivity growth.

Moreover, as indicated by Proposition 3, the increase in the relative efficiency of skilled workers has contributed to labor productivity growth between 1963 and 2002 since $\frac{p_t}{1-p_t} = \frac{S_t}{U_t} > \frac{S_0}{U_0}$ and $\mathrm{d}b_t > 0$ for all t.[15]

[15] This result does not depend on the initial relative employment since skilled labor has increased throughout the span 1963–2002.

Finally, we derived a contrafactual series of labor productivity with constant elasticity of 2.2 through 1963–2002. Comparing this contrafactual series with the series with changes in the elasticity of substitution reveals the contribution of the change in the elasticity of substitution in labor productivity growth over time. The predictions of labor productivity in both models are derived using Eq. (11):

$$y_t = g_\sigma(p_t) = \left((\widehat{a}_{\mathrm{u},t}(1-p_t))^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} + (\widehat{a}_{\mathrm{s},t}p_t)^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} \right)^{\frac{\hat{\sigma}}{\hat{\sigma}-1}}$$

where p_t is employment share of skilled labor and $\widehat{\sigma} = -\frac{1}{\hat{\gamma}_1 + \hat{\gamma}_{1,T_{77\text{–}90}} D_{T_{77\text{–}90}}}$ for the series with change in the elasticity of substitution and $\widehat{\sigma} = -\frac{1}{\hat{\gamma}_1}$ for the series without change in σ.

The growth rate of labor productivity predicted by the model with a decrease of σ after 1977 and an increase after 1990 lies in average 0.10% points *below* that of the model with constant σ in the period 1977–1990 and 0.12% *above* after 1990.

Acemoglu (2002) argues that the main difficulty with an acceleration in the gross SBTC, through a time trend or through the capital-skill complementarity (see Krusell et al., 2000 for instance) is that: "It is difficult to imagine how a new and radically more profitable technology will first lead to 25 years of substantially slower growth" (see Acemoglu, 2002, p. 34). The skill-biased technological changes Acemoglu refers to are associated with shifts in the productivity of skilled compared to unskilled workers in favor of the skilled. We argued that technological changes observed in the last decades have not only shifted but also twisted the relative productivity of skilled to unskilled workers. The twists in the distribution of relative productivity are reflected by changes in the magnitude of the elasticity of substitution between both labor types. In this paper, we showed that the decrease in the elasticity of substitution between skill types of labor at the end of the 1970s has contributed to the slowdown in output growth which therefore is reconcilable with an acceleration in SBTC. Moreover, the increase in the elasticity of substitution after 1990 coincides with an acceleration in labor productivity during the 1990s.[16] Hansen (2001) shows that US labor productivity in the manufacturing sector series breaks in 1982 (weak evidence) and in 1994.

4. Conclusion

This paper departs from the standard literature on skill-biased technical change (e.g. Katz and Murphy, 1992) by releasing the implicit assumption that the relative productivity of skilled workers increased proportionally in every task. In the theoretical model we take a closer look at the possible effects of skill-biased technical change in the labor market, by analyzing how skill-biased technical change may affect the productivity of skilled workers relative to unskilled workers in a continuum of tasks. We show that the assignment model developed by Rosen (1978), not only offers a microfoundation for the CES production function, the workhorse model in the SBTC and growth literature, but also reveals a relationship between the elasticity of substitution across workers types and the slope of their productivity schedule across tasks. In this model, skill-biased technical change may lead to

[16] It is widely recognized that US labor productivity slows down in the mid-1970s and speeds up in the second half of the 1990s.

shifts and twists in the productivity schedule of skilled versus unskilled workers. Shifts correspond to increases in the relative efficiency parameters that are commonly associated with skill-biased technical change. Twists reflect changes in the elasticity of substitution between skilled and unskilled workers that have been absent in the skill-biased technical change literature.

Empirical investigation stems for the non-stability of the parameters of an augmented (Katz and Murphy, 1992) model. We show strong empirical evidence that the elasticity of substitution between skilled and unskilled labor has changed over time. This variability of the elasticity of substitution over time is of importance as it (*twist*) explains a significant part of the rise in the skill premium after 1977 but also part of the productivity slowdown observed in the 1970s and 1980s and acceleration in the 1990s as the magnitude of the elasticity of substitution between inputs is directly linked to the growth rate of income per capita as already recognized in the literature on economic growth.

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Appendix. Proof of proposition 1

Proof. We first rearrange Eq. (11) as follows[17]

$$y_t = p_t f_\sigma(p_t) = p_t \frac{Y_0}{S_0}\left((1-b_t)\left(\frac{1-p_t}{1-p_0}\frac{p_0}{p_t}\right)^{\frac{\sigma-1}{\sigma}} + b_t\right)^{\frac{\sigma}{\sigma-1}} \tag{16}$$

Hence, to prove that $\frac{\partial g_\sigma}{\partial \sigma} > 0$ for any $p_t \neq p_0$ it is enough to prove that $\frac{\partial f_\sigma}{\partial \sigma} > 0$ for any $p_t \neq p_0$. Deriving f_σ with respect to σ and rearranging we have:

$$\frac{\partial f_\sigma}{\partial \sigma} = \frac{y_t}{(\sigma-1)^2}\left[\begin{array}{l} \frac{\sigma-1}{\sigma}\frac{(1-b_t)\chi_t^{\frac{\sigma-1}{\sigma}}\ln\chi_t}{(1-b_t)\chi_t^{\frac{\sigma-1}{\sigma}}+b_t} \\ -\ln\left[(1-b_t)\chi_t^{\frac{\sigma-1}{\sigma}}+b_t\right] \end{array}\right] \tag{17}$$

where $\chi_t = \frac{1-p_t}{1-p_0}\frac{p_0}{p_t}$.

[17] Note that (Klump and de La Grandville, 2000): use the functional form:

$$\frac{Y_t}{S_t} = f_\sigma(k_t) = \frac{Y_0}{S_0}\left((1-b_t)\left(\frac{k_t}{k_0}\right)^{\frac{\sigma-1}{\sigma}} + b_t\right)^{\frac{\sigma}{\sigma-1}}$$

where $k_t = \frac{U_t}{S_t} = \frac{1-p_t}{p_t}$ and proved that $\frac{\partial f_\sigma(k_t)}{\partial \sigma} > 0$.

Since $\sigma > 1$, once rearranging we have that $\frac{\partial f_\sigma}{\partial \sigma} \geqslant 0$ if and only if:

$$(1-b_t)\chi_t^{\frac{\sigma-1}{\sigma}} \ln \chi_t^{\frac{\sigma-1}{\sigma}} - \left((1-b_t)\chi_t^{\frac{\sigma-1}{\sigma}} + b_t\right) \ln \left((1-b_t)\chi_t^{\frac{\sigma-1}{\sigma}} + b_t\right) \geqslant 0 \tag{18}$$

We define the function $k(m_t)$ as follows:

$$k(m_t) = (1-b_t)m_t \ln m_t - ((1-b_t)m_t + b_t) \ln((1-b_t)m_t + b_t) \tag{19}$$

where $m_t = \chi_t^{\frac{\sigma-1}{\sigma}}$.

We need to prove that the function k is greater than 0 for all $b_t \in (0, 1)$ and $m_t > 0$. We first note that $k(1) = 0$ for all b_t,[18] $\lim_{m_t} \to 0k(m_t) = -b_t \ln b_t > 0$ for all $b_{t \in (0,\,1)}$. Then we derive k and obtain:

$$k'(m_t) = (1-b_t) \ln \left(\frac{m_t}{(1-b_t)m_t + b_t}\right) \tag{20}$$

From Eq. (20) we see that $k' < 0$ for all b_t on $0 < m_t < 1$, $k' > 0$ for all b_t and $1 < m_t$ and $k' = 0$ for $m_t = 1$. Therefore, the function $k(m_t)$ is monotonic strictly decreasing on $m_t \in (0, 1]$ and monotonic strictly increasing on $m_t \in [1,\infty)$. From this we can conclude that the function $k(m_t)$ is strictly greater than 0 for all b_t and all $m_t = \chi_t^{\frac{\sigma-1}{\sigma}} = (\frac{1-p_t}{1-p_0}\frac{p_0}{p_t})^{\frac{\sigma-1}{\sigma}} \neq 1$ and equal to 0 for $m_t = 1$ (conform $p_t = p_0$). This implies that the inequality represented in Eq. (18) is satisfied and therefore that $\frac{\partial f_\sigma}{\partial \sigma} > 0$ and $\frac{\partial g_\sigma}{\partial \sigma} > 0$ for all $p_t \neq p_0$ and equal to zero for $p_t = p_0$.

Since $\frac{\partial g_\sigma}{\partial \sigma} > 0$ for all $p_t \neq p_0$, the growth in labor productivity slows down as the elasticity of substitution decreases through time. □

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[18] Note that $m_t = 1$ if and only if $p_t = p_0$.

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