Multiple-steady-state growth models explaining twin-peak empirics?

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Abstract. The explanation of twin peak empirics through multiple-steady-state growth models has one serious implication: Whenever a model generates twin peaks in GDP per capita it also generates twin peaks in other variables. We check for some multiple steady-state models whether or not they have twin peaks in the other variables besides GDP per capita. It turns out that the required twin peaks do not exist for the textbook version of the population trap model but a modified version cannot be dismissed. Stiglitz' (1987) endogenous growth model requires twin peaks in savings per capita, which are not found in the data. Models of human capital accumulation and endogenous population growth are closer to the idea of twin peaks because twin peaks in population growth rates do exist, but human capital accumulation is not exactly distributed in a way twin peaks would require but that distribution rather follows a structure of one peak with one or two stairs depending on the human capital variable.

1. Introduction

Danny Quah and others[2] found twin peaks in the world's distribution of GDP per capita. Oded Galor (1996) did suggest that this phenomenon could be explained using growth models with multiple steady states. This is an interesting suggestion if one accepts the perspective of having one growth model for all countries in the world. In this paper we will work within that perspective. Any theory that explains twin peaks through a multiple steady-state model will imply that there are more variables that should have twin peaks. In this paper we start checking for several models whether or not the corresponding data actually do have these twin peaks in other variables.

[1] I am grateful to Wink Joosten for his careful research assistance in the framework of a regular skills training organized by Joan Muysken. I would like to thank Abraham Garcia and Hugo Hollanders for providing me with their estimates of TFP growth rates and an anonymous referee for useful comments.
[2] See Quah (1993a,b, 1996); the latter for an introductory survey.

We also work on the presumption that there is really a twin-peak structure in the data, although some doubts are in order. When trying to replicate the twin peak result using data from the World Development Indicators 2002 for ten-year intervals, it becomes obvious that the data are there for a different number of countries every year as in Table 1 in the columns titled ‘unrestricted’ below. They are available every year for 97 countries as indicated in the column ‘restricted’.[3] When we reproduce the twin-peak result for all the countries available it turns out that (i) twin peaks did not only develop since 1950s (as in Weissbrich’s (2001) set of 55 countries) but there were always multiple peaks according to both, the restricted and the unrestricted data set (as in Quah (1993b), and (ii) there may be a fairly diluted second peak in 2000.[4] This indicates that the results depend strongly on the chosen sample of countries. Unlike Quah (1993a,b) we did not divide the GDP per capita by the world average values. Therefore we can also see the value of the income of the median and the modus of the distribution: The peak of the distribution (modus) is always at around 1000 1995-dollars; the median is growing in both sets until 1980 and continues to do so in the restricted set, but then goes down from \$1800 to \$1600 in the unrestricted set as poorer countries deliver information. The mean is growing strongly: each ten years it is about \$1500 higher. This is in accordance with the results found by Quah: The poor remain poor, the middle class vanishes, and the rich get richer in an increasingly dispersed manner; thus inequality grows.

Table 1 OVER HERE

2. Twin peak results for multiple steady-state growth models?

In this section we briefly reproduce some multiple steady-state growth models simply in terms of graphs in order to show which of their variables should also generate twin peaks. Then we check whether or not these required twin peaks can be found in the data. We do this for the population trap model, Stiglitz’ (1987) endogenous growth model and the human-capital growth model by Becker, Murphy and Tamura (1990).

[3] The restricted data set always contains those countries for which the data for a certain variable are available for all the years considered.
[4] See Appendix: Figure: GDP per capita (const. 1995 US dollars) – unrestricted data set)

2.1 The population trap model

The standard version of the model is reproduced in basic textbooks[5] on economic development. We give two slightly different versions below. In these two versions, the difference with the textbook version is that for high GDP per capita the GDP growth rate as a function of the GDP per capita is larger than the population growth rate as a function of the GDP per capita. This implies that the highest steady state is not a stable equilibrium but rather allows for permanent growth of the rich countries. This is important from an empirical point of view, because the richest countries are actually growing. Moreover, when the population trap model was developed in the 1950s[6] the dominant growth model was that of Harrod and Domar, which bears much similarity with endogenous growth models of the AK type.[7] Writing down a Harrod-Domar model and making savings and population growth rates dependent on GDP per capita can easily generate the population trap model with some additional assumption on the curvature. We modify the Harrod-Domar growth model by making the savings rate, *s*, and the population growth rate, ε, depend on per–capita income, *y*. The basic assumptions of the model become as follows. The production function is unchanged:

$$Y = aK, \qquad \hat{Y} = \hat{K} \tag{1}$$

A '^'indicates a percentage change or growth rate. The dynamics of capital accumulation, using the equality of savings and investment, is

$$\dot{K} = s(y)Y - \delta K = s(y)aK - \delta K, \qquad \hat{K} = s(y)a - \delta = \hat{Y} \tag{2}$$

Population growth has the inverted u-shape shown in the textbooks if we assume that it is linear-quadratic in per-capita income:

[5] See Todaro 2000 or any other edition.
[6] See for example Nelson 1956.
[7] See Rebelo 1991.

$$\hat{L} = a + by - cy^2 \qquad (3)$$

Equations (2) and (3) are equations for income and population growth as functions of per-capita income. As empirical savings ratios range from zero to 50%, capital productivity is between one and 1/4 and depreciation rates are about 3% of capital and 10% of GDP, the growth rate for income according to equation (2) must be between 0.5 and –0.03. Therefore the graph for equations (2) and (3) may look like that of the textbook or like Figure 1 below. Of course, there is also the possibility that they do not intersect. The crucial point for the existence of several steady states is the concave or S-shaped curvature for the savings ratio, because it must first increase if it is at zero for low values of income and then this increase must be dampened in order to avoid too high savings ratios. The version in figure 1 differs from the textbook version in that it allows for high (low) population growth at low (high) income per capita. A simple concave curvature is shown in Figure 2. Note that also a constant savings function can intersect the population growth curve twice if its value is not too high. The textbook model of the Malthusian trap has three steady states and the one in the middle is unstable. Our examples have two or four steady states with the possibility of permanent growth to the right of the steady state with the highest level of per-capita income. In order to get four steady states it is probably necessary to have an S-shaped savings ratio.

FIGURES 1 AND 2 OVER HERE

The model would predict that there are as many peaks in GDP per capita as there are stable steady states, which would imply two peaks for the first version of the model and one peak for the second version of the model. The Kernel density estimates[8] for GDP per capita in the Appendix show that the case of two peaks is not quite unrealistic although the number of countries in the smaller peaks is not very large. The results in the

[8] See Silverman 1986 or the Appendix in the paper by Weissbrich for an introduction to Kernel density estimates. The essence in comparison to Histograms is that the width of the class and the beginning of the first class are optimized. Moreover, the estimate smoothes over related class sizes, taking into account that the estimate of the optimal class size has of course some variance. Eviews does the routine work.

paper by Weissbrich favours twin peaks. The emphasis here is that *according to the textbook model* or our version with two stable steady states there should not only be (twin) peaks in GDP per capita but also in GDP growth and in population growth. The plot for the distribution of population growth indeed shows twin peaks as well.[9] However, then, according to the textbook version of the model, the GDP growth rate should also have peaks, which we do not find.[10] Should we dismiss the population trap model? Consider the second version of the model in figure 2. The stable steady state corresponds to the big peak of the GDP per capita distribution. If the countries to the right of the unstable steady state have about the same GDP per capita the model is compatible with the basic twin peak idea; whether or not this is temporary depends on the distance between the GDP growth curve and the population growth curve; this distance should not increase with GDP per capita or the second peak be will be dissolved after some time and the observation would be just some sort of random bunching.[11] Alternatively, with no second peak at high incomes the model is compatible with GDP per capita as in the unrestricted data set where there is no second peak. Similarly, there could be (temporary or not) a peak of high population growth around the stable steady state and a constant low population growth to the right of the unstable steady state. In this case GDP growth needs to have only one peak and so would and does growth of the GDP per capita.[12] In short, in this version of the model the second peak is either absent as in the GDP per capita data for the unrestricted data set or it is sort of random bunching, which may or may not be dissolved in the future depending on the details of the curvature of the savings and population growth curve. Ultimately, it would be premature to dismiss the population trap model at least in the second version presented. The textbook version of the model is less plausible.

2.2 Stiglitz' (1987) endogenous growth model

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[9] See Appendix: Figure: Population Growth (annual %). This result was derived in Weissbrich (2001). There is virtually no difference between the restricted and the unrestricted data set here.

[10] See Appendix: Figure: GDP Growth (annual %).

[11] In this case the distribution would not converge to a stable twin peak form. It is the strength of these purely empirical kernel density distributions that they do not prefix convergence results.

[12] See Appendix: Figure: GDP per Capita Growth (annual %).

Stiglitz (1987) elaborates what was indicated in the second figure of Solow (1956), that savings might follow an S-shaped curve. In addition technical progress depends on the capital-labour ratio as in Conlisk (1967). The result can be summarized as in Figure 3.

FIGURE 3 ABOUT HERE

There are two stable steady states: One at a high level of the capital/efficient labour ratio, $k=K/AL$ and one at a low level, which is zero only if the curves do not intersect again at a low level. For countries without technical progress this would yield stationary twin peaks in GDP per capita. If countries have technical progress, there would be twin peaks in GDP per unit of efficient labour. This can be reconciled with the formation of dynamic twin peaks in the data under the assumption that technical change is about zero at the lower steady state and a positive percentage in all countries near the higher steady state.[13] Then the lower peak would be stationary as it is at around constant 1995-$1000. A closer look at the Figure 3 shows that we should also expect twin peaks in the savings ratio. Weissbrich (2001) showed that there is no twin peaks in the savings ratio. In a closed economy setting savings ratios equal investment ratios, but in open economies one needs to add foreign savings to get investment. Therefore we did check for twin peaks in the Gross Capital Formation as a percent of GDP.[14] In the second, very small peak for the year 2000 there are ten countries, which have an investment ratio above 34%. Thus, there is some but weak evidence here. By construction there should also be twin peaks in technical progress. Following the procedure in Young (1995) with and without human capital, constant returns to scale and country and time dummies, neither in the cross section estimate of Hollanders[15] nor in the panel estimates of Garcia[16] for 77 countries with human capital or 91 countries without human capital data for 40 years do we find evidence of twin peaks. We plot those graphs in the Appendix, which are closest to twin peaks. If there is anything like twin peak formation it is just starting and only true if rates

[13] Note that in the Stiglitz model zero technical progress also implies zero savings per capita; however, it is sufficient to introduce a rate of depreciation for technical progress to have a steady state with zero technical progress and positive though small savings rates.
[14] See Appendix: Figure Gross Capital Formation (% of GDP). The unrestricted data set shows this peak a more clearly than the restricted one.
[15] See Hollanders and Ziesemer 1999
[16] See Appendix: Figure TFP growth rates; excel or eviews files are available upon request.

of depreciation of capital are at about 12 or 25 percent. There is no doubt, however, that there is a technological divide: About half of all countries have positive and negative TFP growth rates, but this is not the same as twin peaks as predicted by Stiglitz' model. However, TFP growth captures more than just technical change. It also captures changes in war, civil war, institutions and everything that might enter a residual[17]. In order to focus more on technology it may be useful to focus on an input measure: R&D expenditure per unit of Gross national income.[18] Again we do not see twin peaks but it should be kept in mind that the number of countries for which we have data is small and that there are many which do have no R&D expenditure. These countries might form a second peak. The weak point of the Stiglitz model is the lack of twin peaks in the savings.

2.3. Multiple steady states from human capital formation

When families or societies have to decide how much labour to invest into the number or the education of children, the parents' productivity in educating children is crucial. It is a plausible assumption in a paper by Becker, Murphy and Tamura (1990) that this productivity is larger, if the parent generation has a higher education itself. Then a simple linear specification yields that $H_{t+1} = H_t h_t$, where the first H-term on the right-hand side is the productivity and h is the number of hours; if households feel that they should put in more labour h the more productive they are in terms of H then $h_t = h(H_t), h' > 0$. In this case the function $H_{t+1}(H_t)$ may be S-shaped as in Figure 4. Each economy moves either to the upper or to the lower steady state. Economies which move to the upper (lower) steady state will have a lower (higher) population growth because families with more education choose a lower number of children.[19]

FIGURE 4 OVER HERE

[17] See Islam 1995.
[18] See Appendix: Figure: R&D expenditure/GNI 1992-96.
[19] For similar models see Galor, O. and D.N.Weil (1999).

By implication there should be twin peaks in the value of cumulated human capital and the corresponding flows. The question then is how to proxy these variables. As in most countries education is based on public expenditure on education this might serve as a first proxy. As the Kernel density estimate shows[20], this does not produce twin peaks because what may appear as a second small peak in 1998 does consist of not more than 2 countries.[21] However, this may be due to the inappropriate variable used. When we use years of schooling data, we find again no twin peaks, but in total years of schooling[22] we find a semi-peak similar to the one in population growth and when we look at higher years of schooling we find a peak with two stairs at higher number of years[23]. Compared to the other models, this model's properties are more closely mirrored in the data, because it generates twin peaks in GDP per capita, population growth and – though not exactly so in the years of schooling.

3. Conclusion

The findings in this paper are as follows.

When using a different data set than Quah, who used the Summers-Heston data set with 118 countries, which is larger than our restricted data set but smaller than our unrestricted data set, we find that twin peaks always existed but are fairly diluted for the unrestricted data set in the year 2000.

For the population trap model we find that there are no twin peaks in GDP growth rates. However, for a version of the model with one unstable and one stable steady state is compatible with twin peak empirics, because then there need not be twin peaks in GDP growth rates but twin peaks in GDP per capita and population growth may still occur.

[20] See Appendix: Figure: Public Spending on education, total (% of GDP)
[21] There are no exact criteria here as to what exactly a peak is. The economic intuition suggests that there should be several countries between two minima and they should be there also in the next decennium in order to have some sort of persistence.
[22] See Figure Total years of schooling based on data by Barro and Lee (2000); all available countries.
[23] See Appendix: Figure Higher years of schooling.

For the endogenous growth model by Stiglitz (1987) there is a lack in twin peaks in the savings ratio although twin peaks in R&D expenditure are existent, when a second peak is assumed at the level of zero expenditures on R&D.

For the human capital accumulation models with endogenous population growth there are no twin peaks in the data for public expenditure on education but there is almost a second peak in total years of schooling and there are stairs in the density function for higher years of schooling. Among the models we have investigated so far, this type of models comes closest to the reality of twin peaks in the sense that their other variables have a distribution that is close to twin peaks.

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Table 1: Size of datasets

	unrestricted							*restricted*
	1960	**1961**	**1970**	**1980**	**1990**	**1998**	**2000**	**all years**
	number of observations							
GDP per capita	100		117	140	173		175	97
gross capital formation	84		105	139	165		152	68
population growth	188		191	191	193		194	118
public spending on education	80		110	117	132	115	na	38
GDP growth	na	107	119	138	173		176	97
GDP per capita growth	na	106	118	136	170		173	97
GDP	101		118	142	176		175	97

Source: World Development Indicators

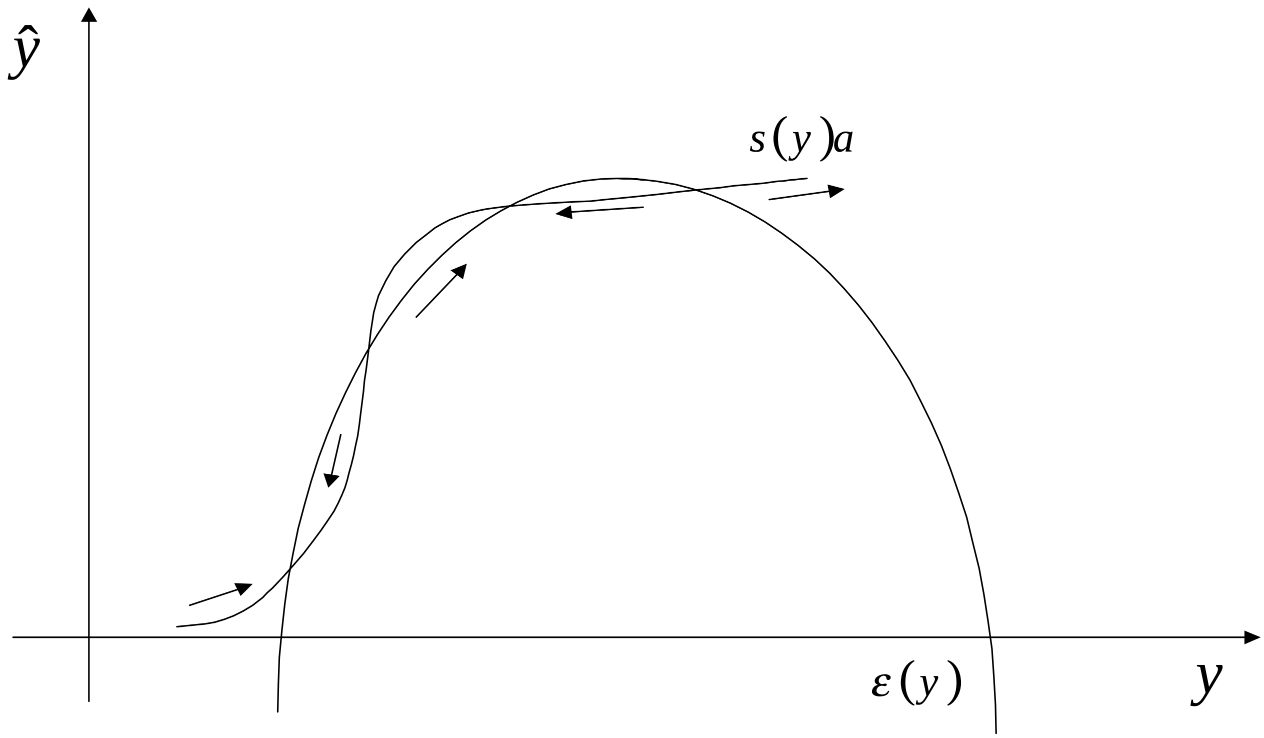


Figure 1. An S-shaped savings ratio and an inverted U-shape of the population growth curve may result in multiple steady states if capital productivity is sufficiently low.

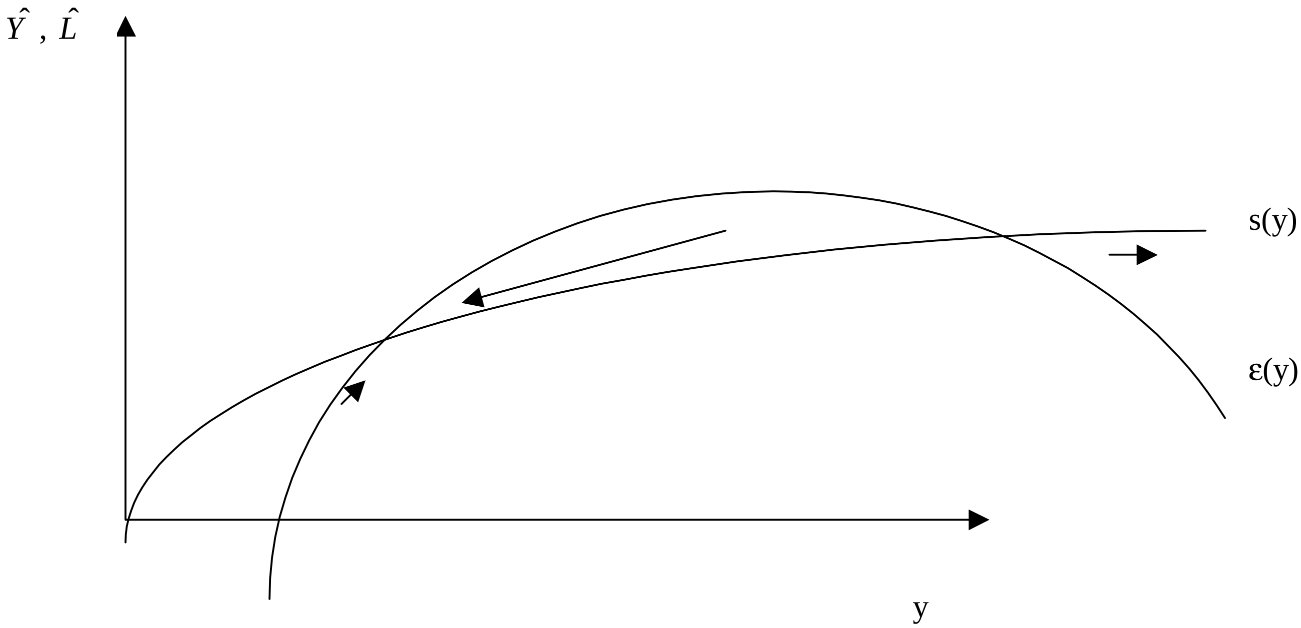


Figure 2. A concave savings ratio and an inversely U-shaped population growth curve may lead to two steady states if capital productivity is too low.

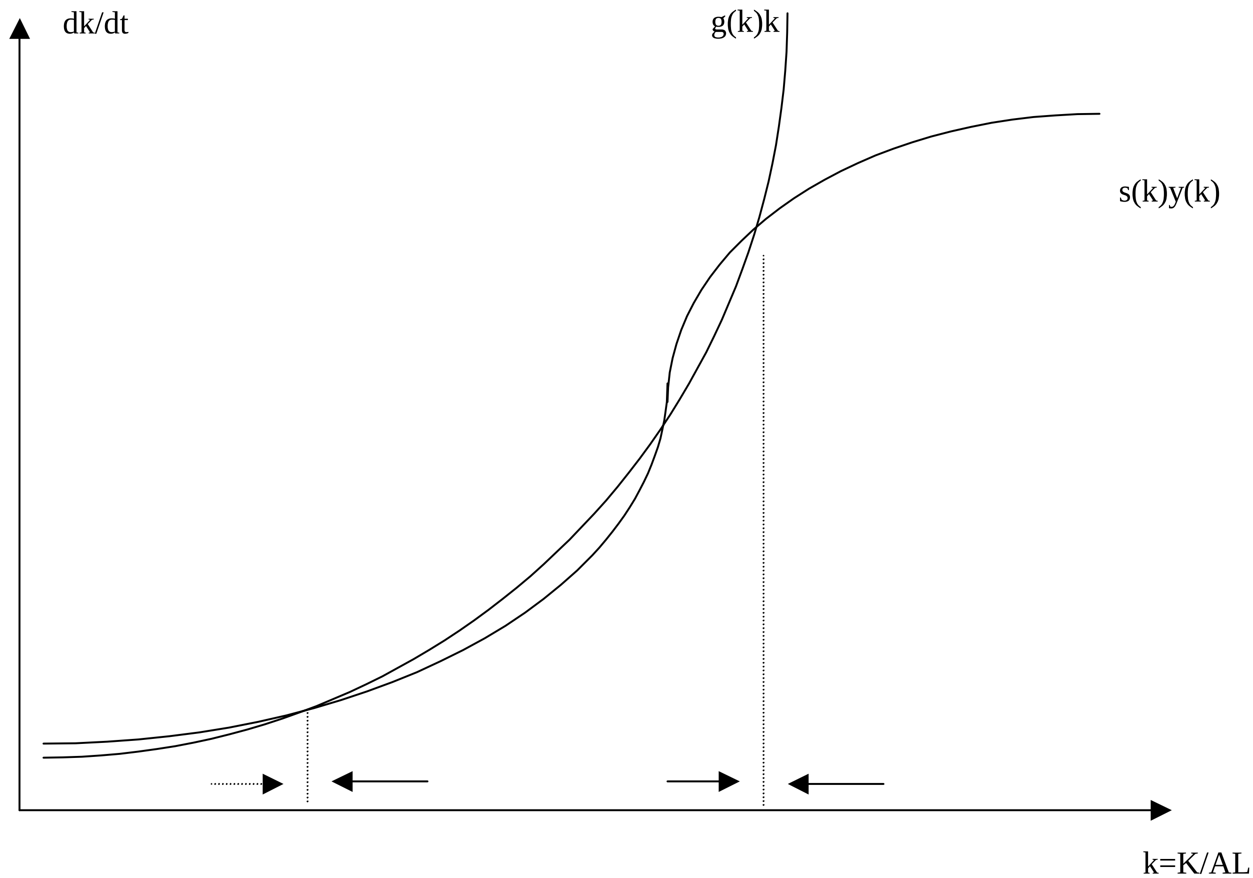


Figure 3. If savings per efficient labour unit, *s(k)y(k)*, is s-shaped and the rate of technical progress *g* is a function of capital per unit of efficient labour, there may be multiple steady states predicting twin peaks in GDP per efficient labour unit, savings, *sy*, and technical progress.

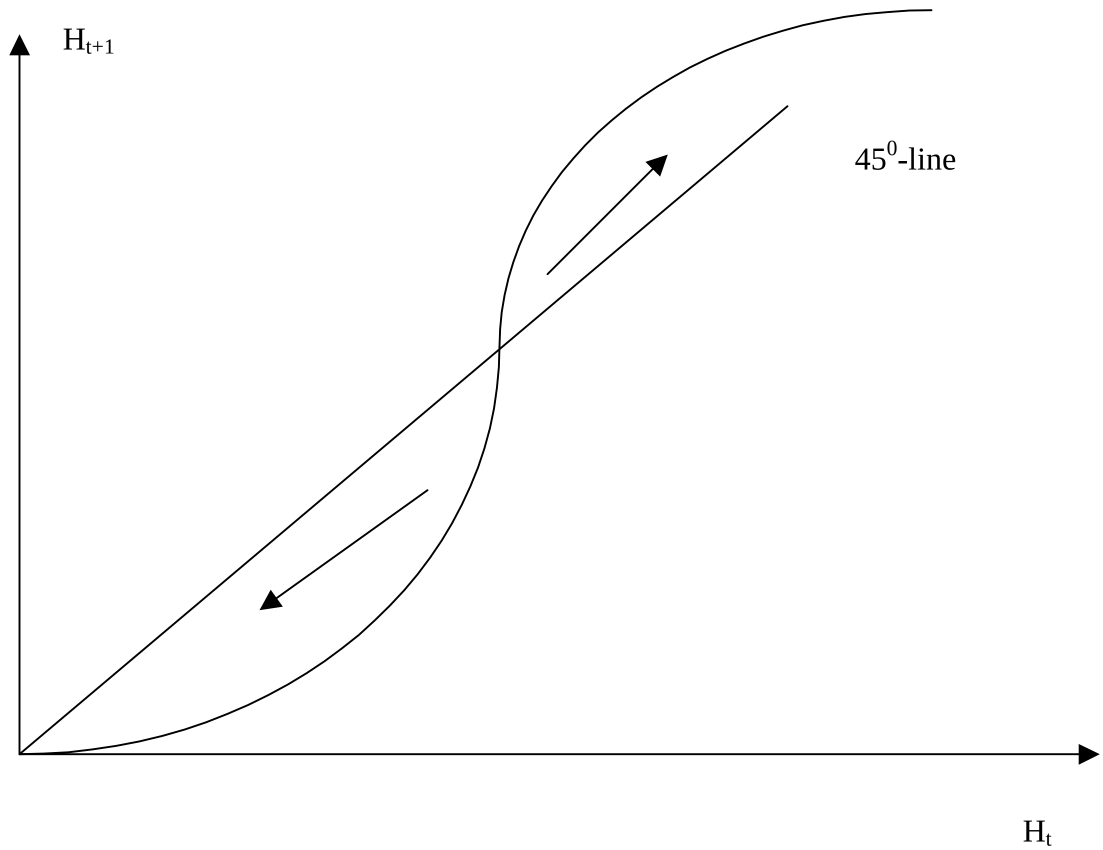


Figure 4. Multiple steady states in human capital formation may occur when higher educated parents invest more time into education of their children.

Figure : GDP Per Capita (constant 1995 US dollars) – *unrestricted dataset*

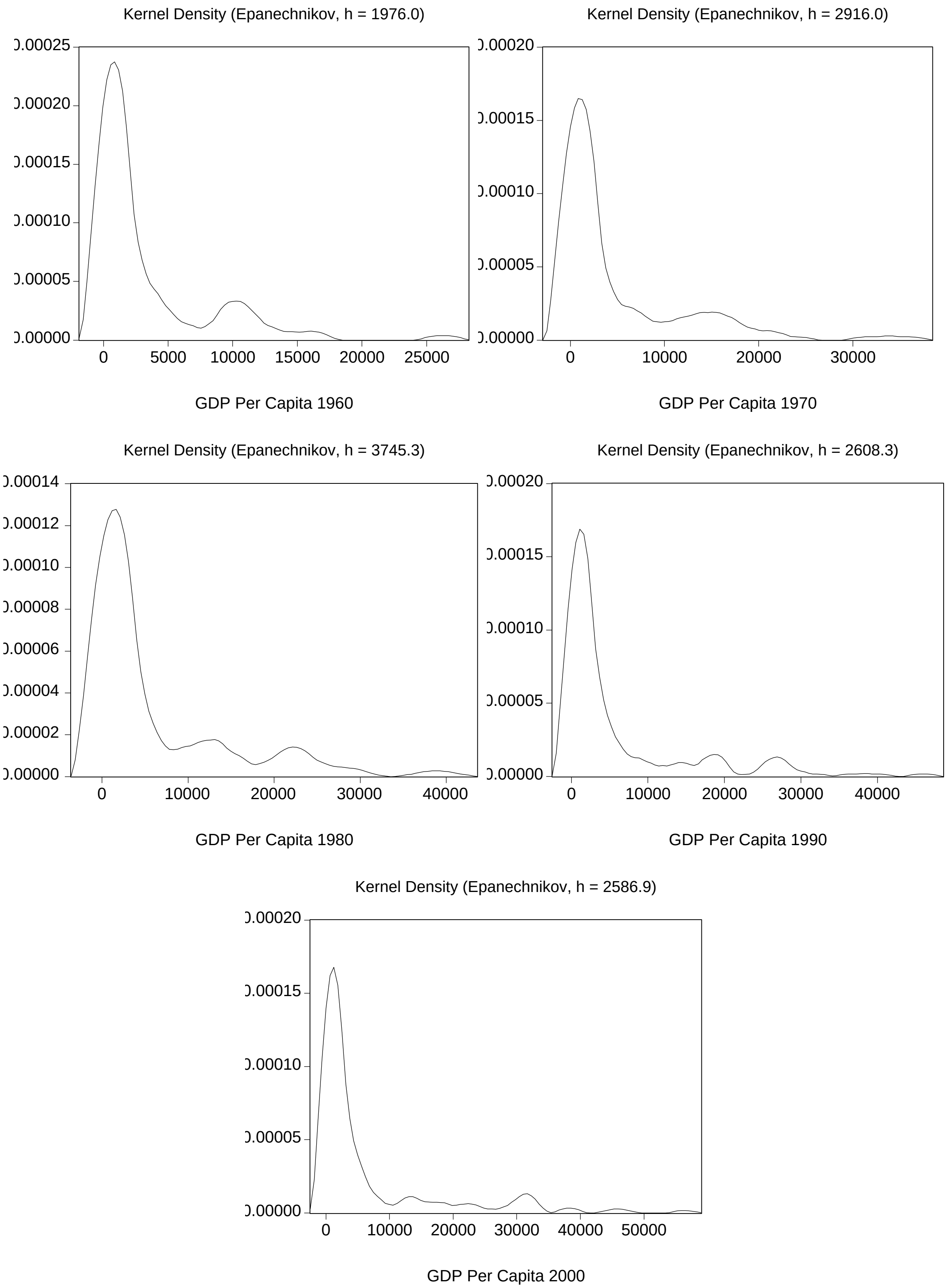


Figure : GDP Per Capita (constant 1995 US dollars) – *restricted dataset*

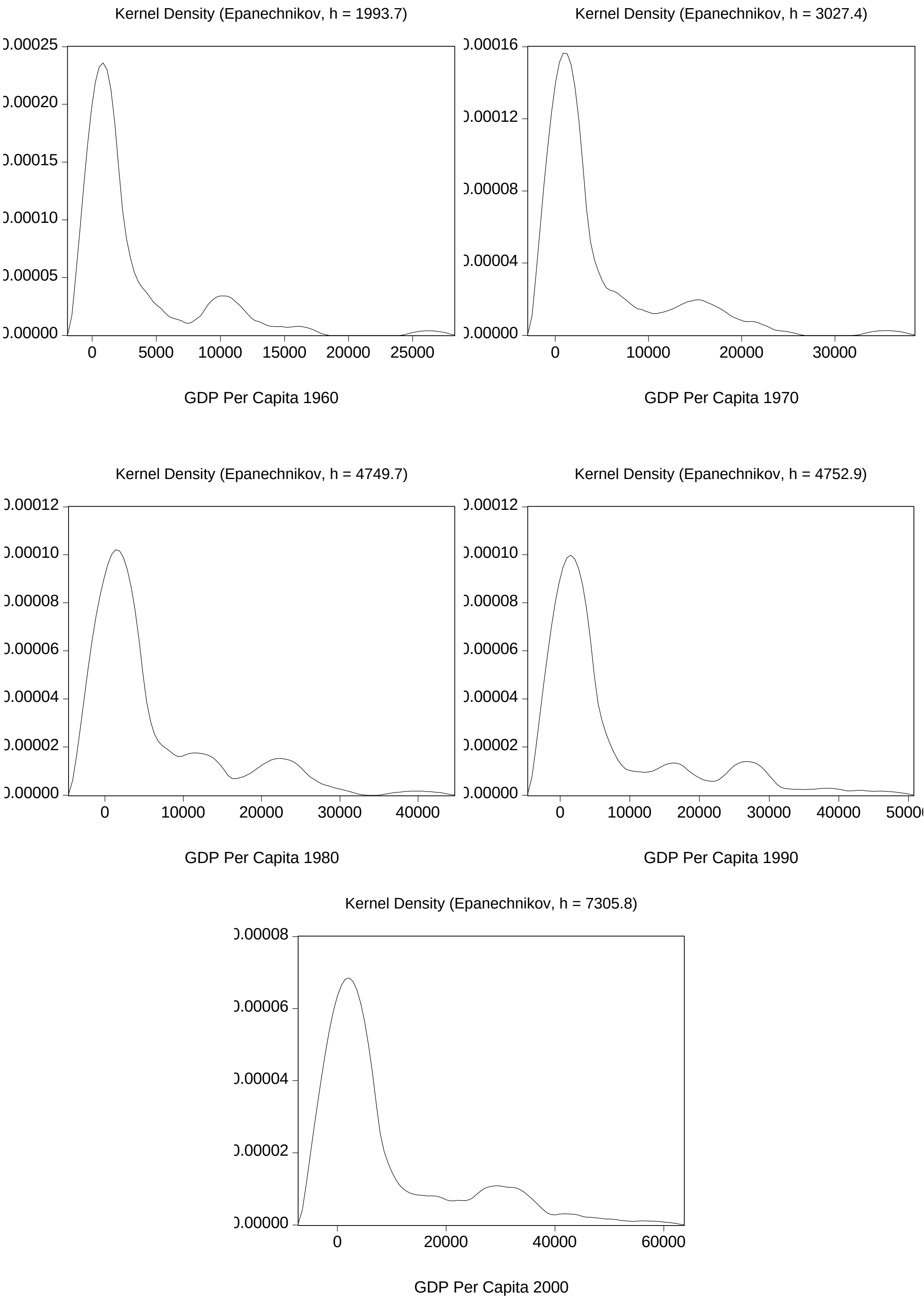


Figure : Population Growth (annual%) – *restricted dataset*

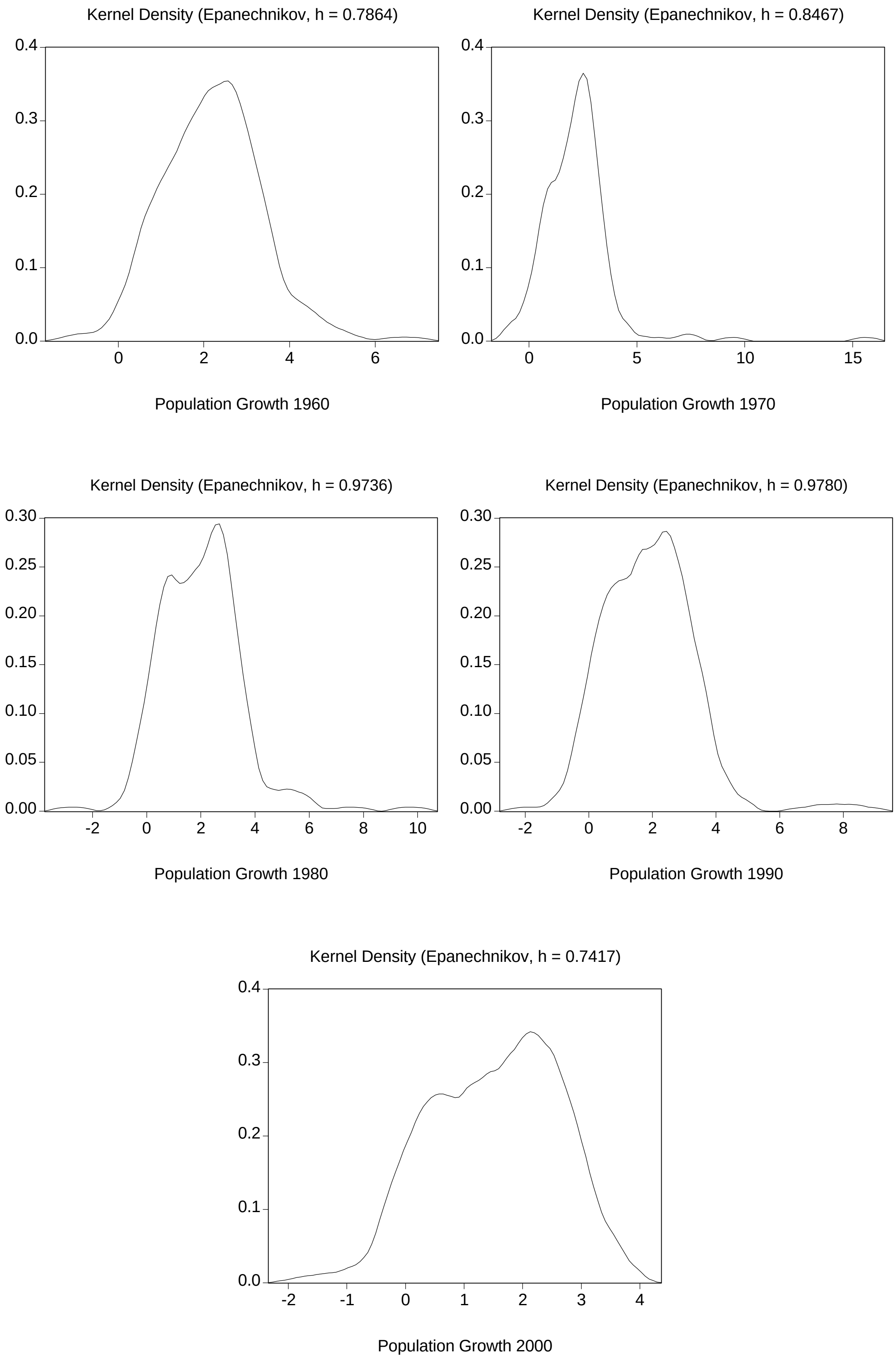


Figure: GDP Growth (annual %) – *restricted dataset*

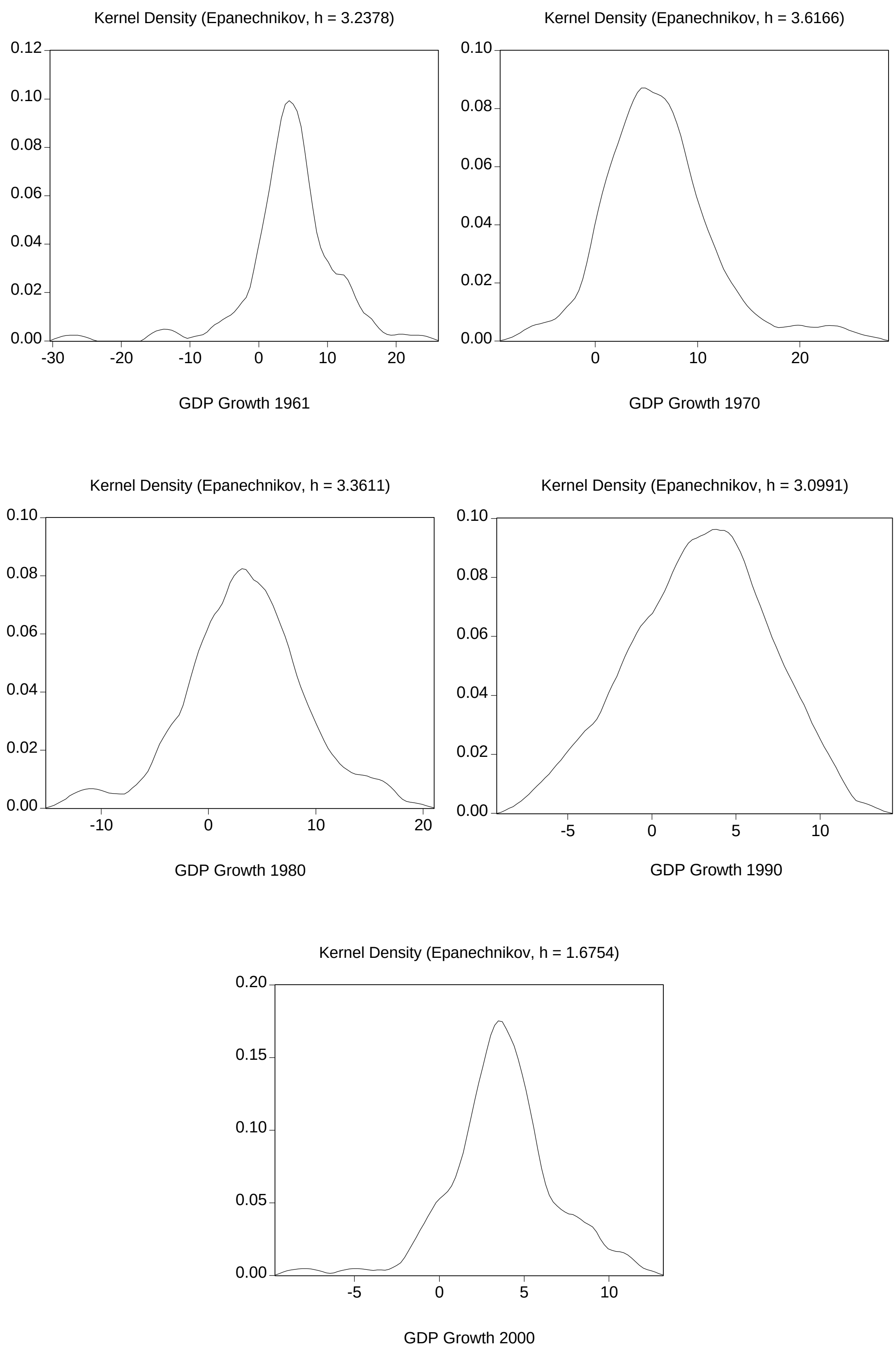


Figure : Gross Capital Formation (% of GDP) – *unrestricted dataset*

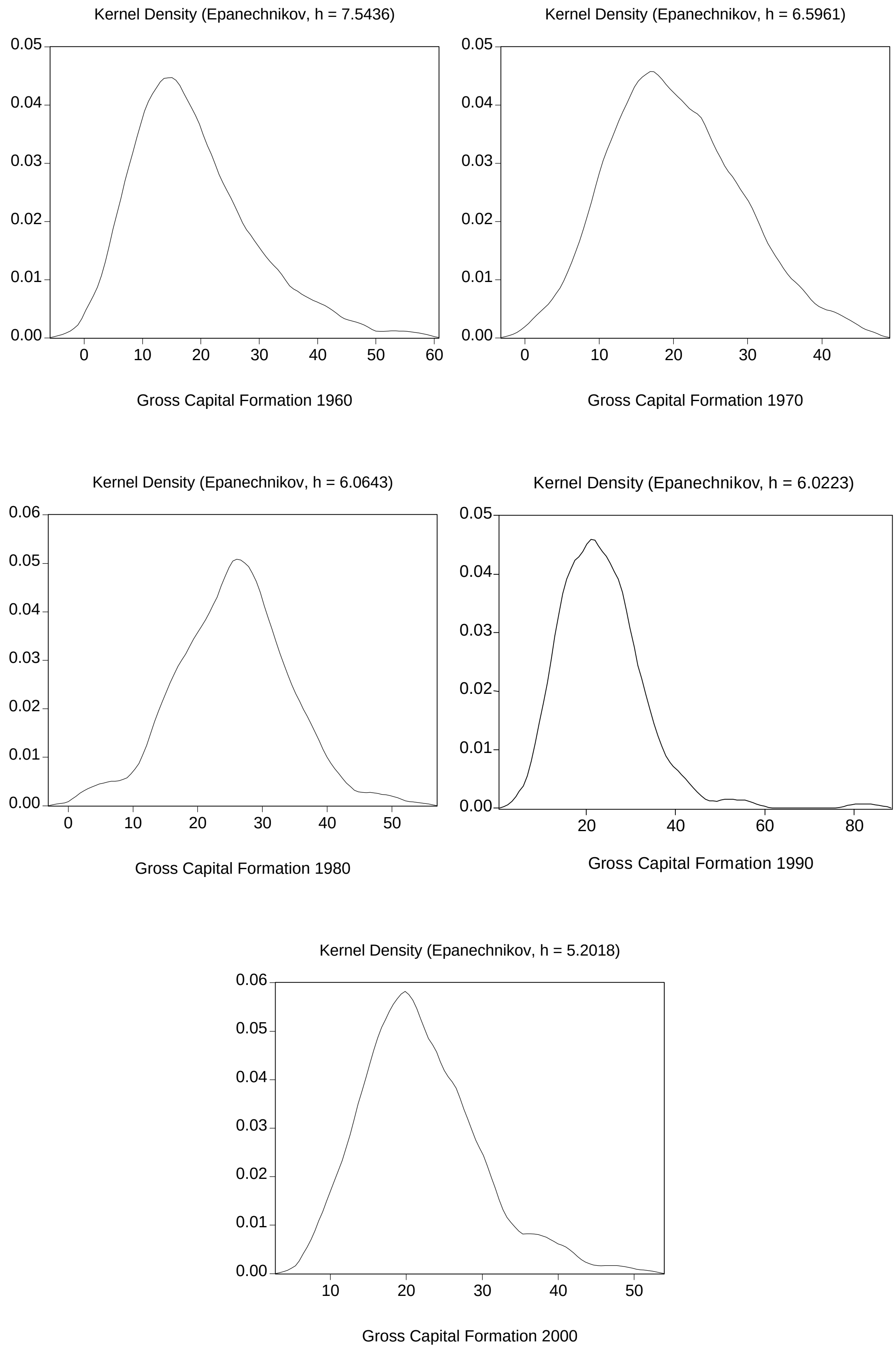
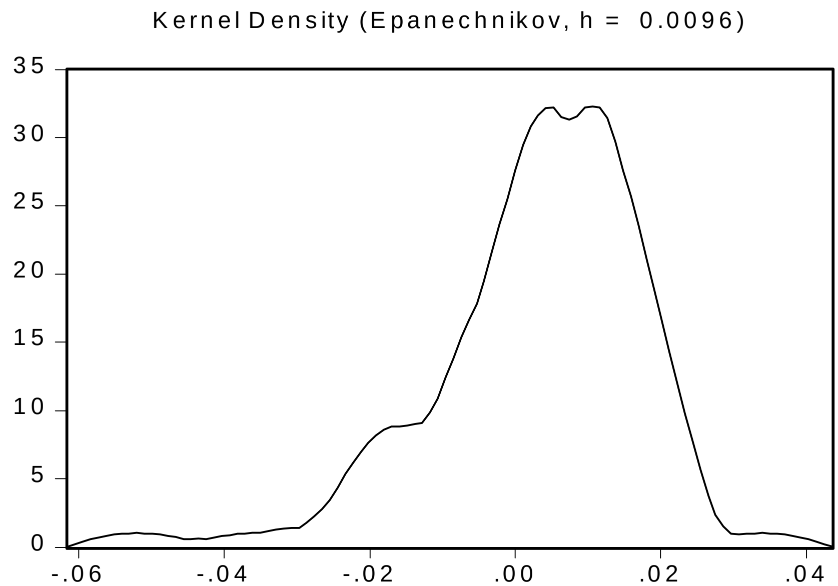
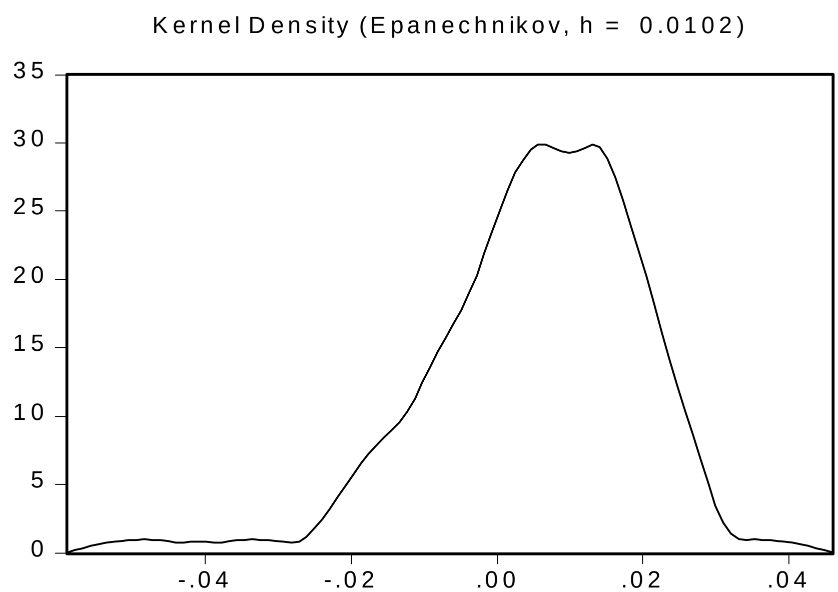


Figure : Kernel density distribution of TFP growth rates



TFP growth rate panel estimates, 77 countries, 12% rate of depreciation



TFP growth rate panel estimates, 77 countries, 25% rate of depreciation

Figure : R&D expenditure/GNI

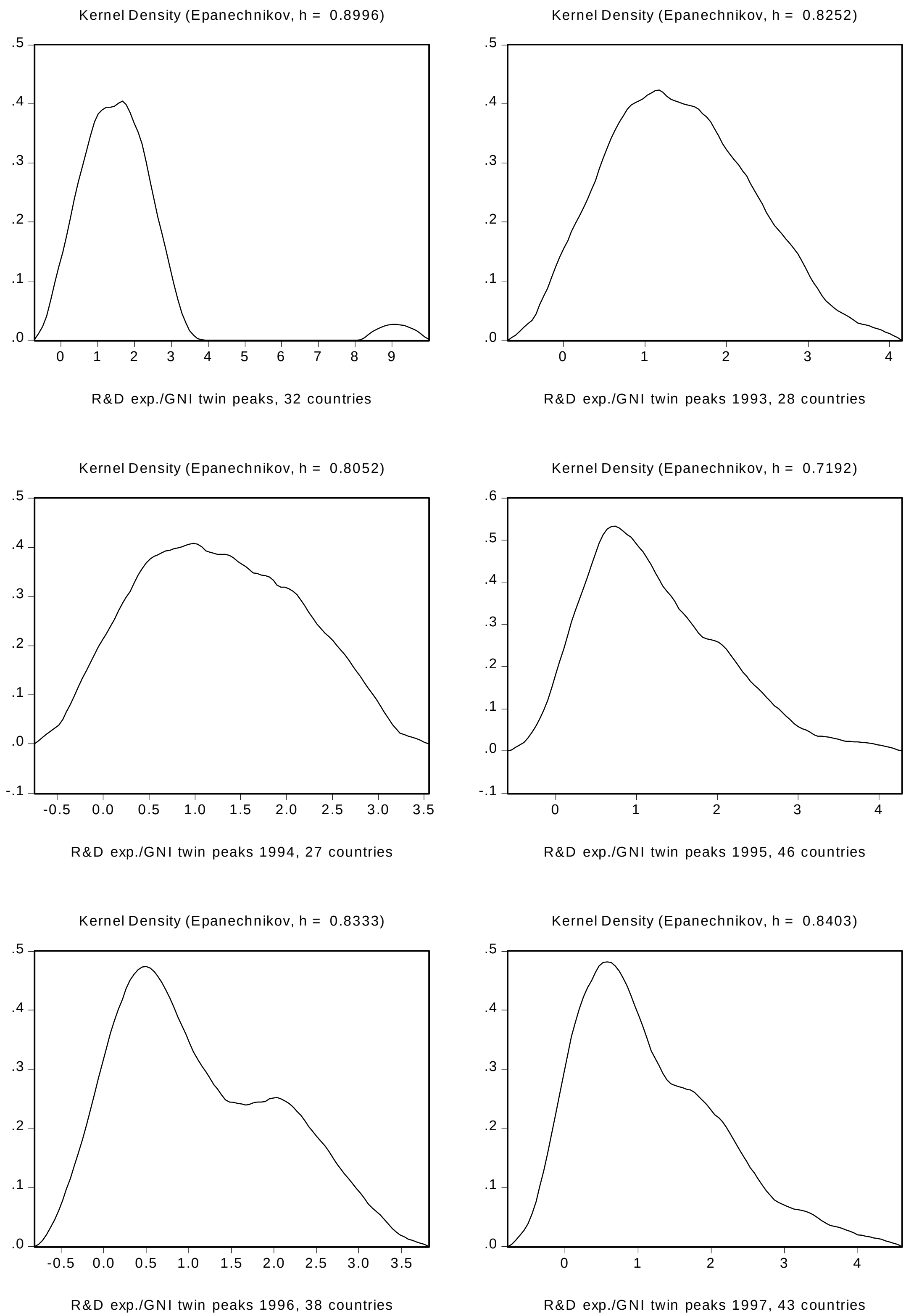


Figure : Public Spending on Education, total (% of GDP) – *restricted dataset*

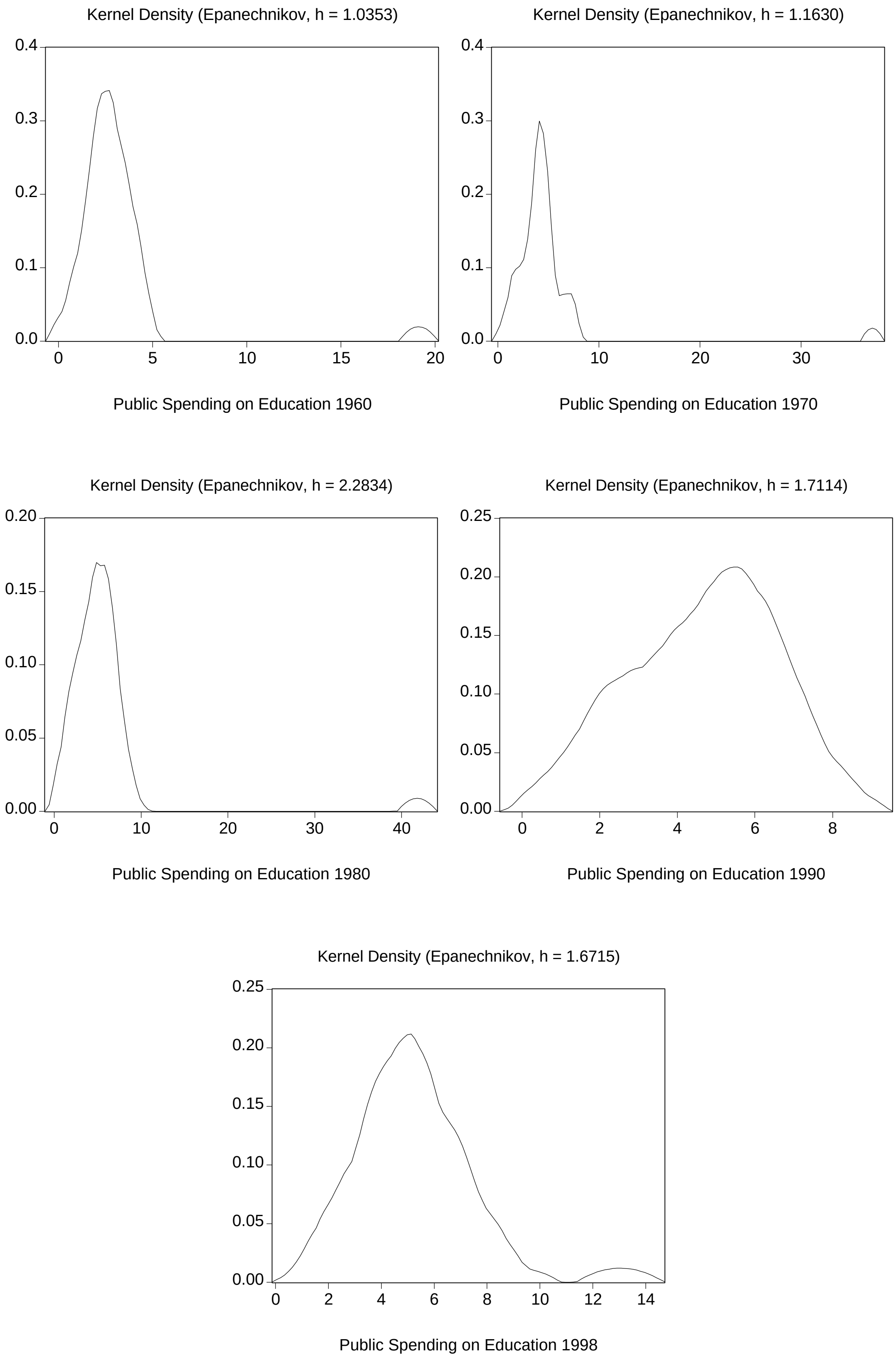


Figure : Total years of Schooling in the labour force 1960-99

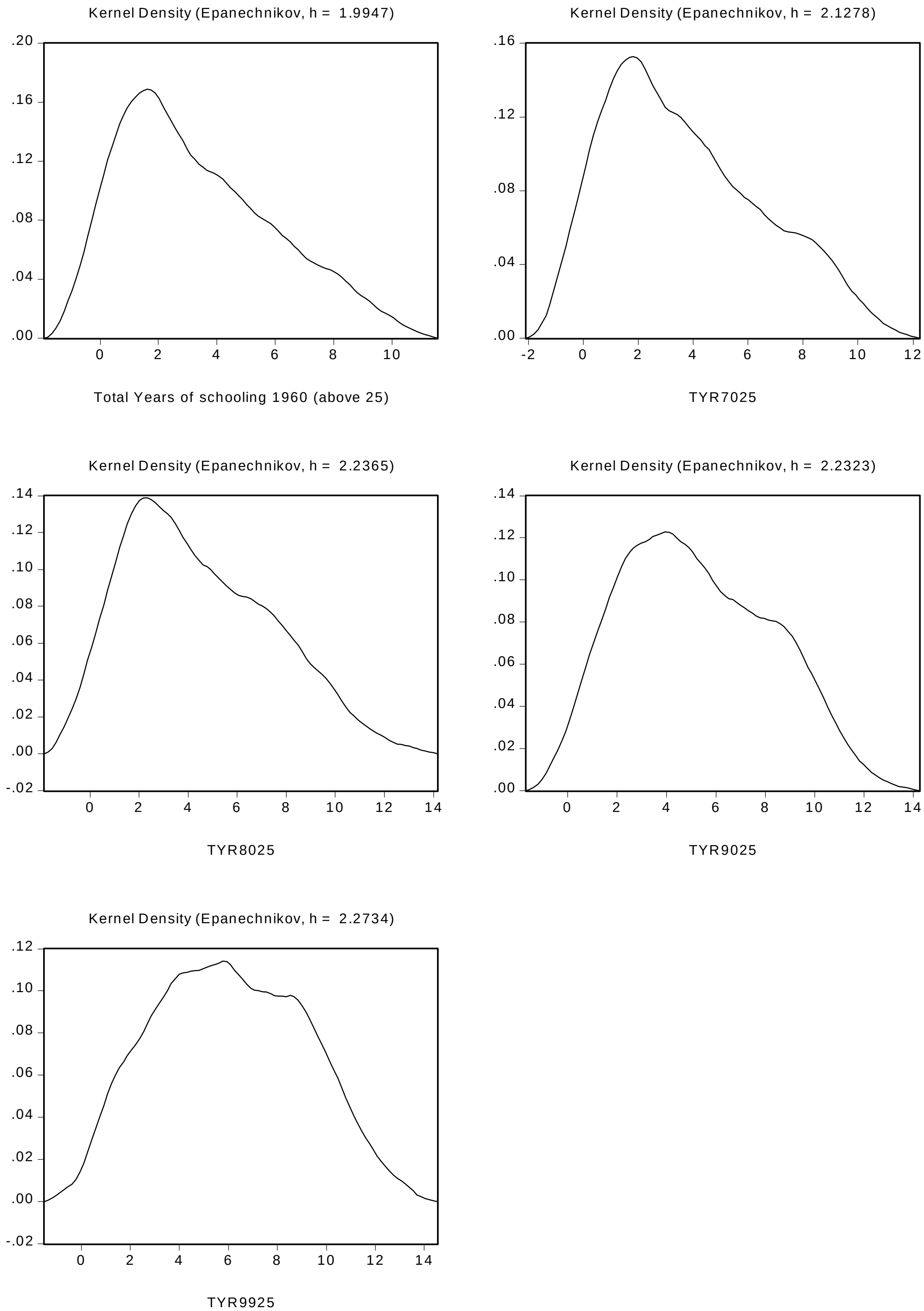


Figure : Higher years of Schooling 1960-99 in the labour force 1960-99