Utility-equivalence of pension security mechanisms
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Abstract. Adequate funding of occupational pension plans is key to benefit security. Across countries different methods of securing funding exist: solvency requirements, a pension guarantee fund, and sponsor support. The key goal of this paper is to investigate the welfare implications to the beneficiary in a hybrid pension scheme. We show that the three security mechanisms can be made utility-equivalent by adjusting the pension contract specifications. The utility-equivalence approach could serve to strengthen the “holistic balance sheet” approach as advised by EIOPA. It enables regulators to compare various pension systems across Europe in a single framework from a utility perspective in stead of a valuation perspective.

Keywords: Pension plans, pension regulation, guarantee systems, power utility, certainty equivalents

JEL: G11, G23
1. Introduction

Many defined benefit pension funds around the globe suffer from funding deficits caused by a combination of volatile equity returns, declining market interest rates and unanticipated improvements in longevity expectations. Protection of pension benefits is the subject of investigation in this paper. We consider solvency requirements, a pension guarantee fund, and sponsor support as possible pension security mechanisms. The key contribution of this paper is to consider under which pension contract specifications these security mechanisms offer equivalent utility to the beneficiary.

The analysis of pension funds is challenging for two reasons. First, the institutional framework is extraordinary. Unlike a commercial insurance enterprise, a pension fund has no external shareholders who bear the residual risk. As a consequence all funding risks are ultimately borne by the pension fund’s beneficiaries, the sponsor\(^1\) or a pension guarantee fund. As such pension funds need to be analyzed differently from other financial institutions. Second, defined benefit pension plans typically face an investment dilemma. On the one hand, pension funds invest in risky assets trying to earn a risk premium. On the other hand, the primary concern is to secure the pension benefits. From this point of view, a pension fund may choose to closely match assets and liabilities, for example by hedging interest rate risks by investing in high grade bonds and fixed income derivatives that replicate the stream of liability payments. Also insurance can be used to safeguard pension benefits, e.g., using longevity risk hedges. A pension fund may however find it difficult to fully implement such a hedging strategy. Due to liquidity constraints, it is usually impossible to invest in assets that exactly replicate the size and nature of its liabilities. This is especially true for defined benefit plans.\(^2\) For defined contribution plans

\(^1\)We define sponsor as the corporation which helps to finance pension accrual of its employees by paying contributions or making occasional lump-sum payments.

\(^2\)Hedging longevity risk is particularly difficult. A recent Joint Forum report estimates that the total global amount of annuity and pension-related longevity risk exposure ranges from USD 15 to 25 trillion. Annually no more than one thousandth is transferred to (re) insurers, or, through longevity swaps, to the broader capital market See BIS (2013).
the liabilities by definition replicate the assets.

Adequate funding of occupational pension plans is key to benefit security. In addition, different methods of securing funding exist in occupational plans. Based on Merton and Bodie (1992), CEIOPS (2008) and Kortleve, Mulder and Pelsser (2011), we identify the following security mechanisms for defined benefit pension plans:

- solvency requirements,
- a pension guarantee fund,
- sponsor support.

Solvency requirements are the additional assets that a pension fund should at least own in excess of the present value of liabilities as a means of a “buffer”. The European Pension Directive, e.g., states that if the pension fund rather than the sponsor bears the risk, or, if the pension fund guarantees a specific level of benefits, then it should hold additional assets above the value of the liabilities.

A pension guarantee fund insures pension benefits, typically, in case the pension fund’s sponsor defaults. In return for this protection the pension guarantee fund receives a fee. Many industrialized countries have such a pension guarantee scheme (e.g., United States, UK, Germany and Japan). For a description of pension guarantee funds see, e.g., Bodie and Merton (1993), Chen (2011) and Broeders and Chen (2013). Sponsor support is defined as the commitment and ability of the sponsor to support its pension fund. That is to continue to pay sufficient contributions to ensure that benefits are paid. CEIOPS (2008) states that sponsor support can also take the form of a claim on the sponsor.

The key goal of this paper is to investigate the welfare implications to the beneficiary for these three different pension security mechanisms. For that we compare certainty equivalents assuming a power utility function. Specifically, we are interested under which conditions the pension security schemes offer equivalent utility. This research is useful for comparing different regulatory regimes. The European Commission (EC, 2011), e.g., has issued a Call for Advice to explore the possibilities for harmonization across different
regulatory regimes. One of the assumptions underlying this Call for Advice is that “Irre-
spective of the security mechanisms used, the level of protection of the scheme members
and beneficiaries should be similar”. Following this call for advice, EIOPA has developed
the concept of a “holistic balance sheet approach” as the way to achieve as much harmo-
nization as possible (EIOPA, 2012).

The holistic balance sheet approach can be used to capture different pension security
mechanisms into a single balance sheet by putting a market consistent value to all available
security mechanisms. In the holistic approach not only the available pension fund’s assets
can be used to provide adequate funding. Other mechanisms providing security may also
be placed on the pension fund’s holistic balance sheet. The holistic balance sheet approach
than assesses whether the pension fund is compliant with overall requirements. Although
theoretically sound, the approach also offers some challenges before it can be made readily
available. It is technically difficult to implement. This is discussed, e.g., in Broeders,

The utility-equivalence approach presented in this paper could serve to strengthen to
the holistic balance sheet approach. It can be used to assess whether different regula-
ory regimes offer similar utility to beneficiaries of pension plans. The utility-equivalence
approach is potentially less complicated from a modeling perspective and is more closely
related to standard asset-liability management tools being used by pension funds.

This paper is organized as follows. Section 2 describes the model setup and 3 intro-
duces the three pension security mechanisms in more detail. In Section 4 we define the
assumptions for a fair contract analysis which we need for the utility-based comparison.
In Section 5 we numerically derive so called fair participation rates. Section 6 focuses on
the utility comparison between the three different mechanisms. Section 7 concludes the
paper.
2. **Model setup**

This paper analyzes the welfare implications for pension beneficiaries under three different pension security mechanisms. In this section, we outline the model setup and introduce the following security mechanisms: solvency requirements, a guarantee fund (PGF) and sponsor support. The paper extends the analysis of Broeders and Chen (2013). In order to ease a utility-based comparison, we make some adjustments to the model assumptions.

Solvency requirements are typically implemented for pension funds that have no external guarantor. The additional assets ensure that the pension fund will be able to meet its liabilities with a high degree of certainty. The amount of additional assets is often defined such that the probability of a funding ratio dropping below threshold $K\%$ is less than $\alpha\%$ over a certain time frame $T$.\(^3\) Providing full insurance through a pension guarantee fund (PGF) implies that the aggregate pension assets will always be worth at least as much as the guaranteed liabilities.\(^4\) A sponsor can also act as an external guarantor. However, unlike the PGF-case, there is a chance that the beneficiaries may not obtain the full guarantee due to insolvency of the sponsor. A sponsor guarantee is usually considered less creditworthy compared to a PGF. External guarantees may provide better downward protection to the beneficiaries. However, this certainty comes at a cost, as a fee must be paid for acquiring external insurance.

2.1. **Contract specification.** We consider a hybrid pension plan in the form of a conditionally indexed defined benefit pension, for a homogeneous group of employees that has to work for another $T$ years.\(^5\) Such a benefit combines a minimum pension income with

\(^3\)This effectively equals the well-known Value-at-Risk (VaR) risk measure on a $T$ year horizon and a confidence level of $\alpha\%$.

\(^4\)In reality, the PGF’s insurance payment is often capped. The PBGC insurance program in the US, e.g., pays pension benefits up to the maximum guaranteed benefit set by law to participants who retire at age 65. Currently the maximum monthly guarantee is USD 4,500 for a 65 year old person. See www.pbgc.gov. We assume that the benefits in our analysis do not exceed this limit.

\(^5\)This way we disregard any intergenerational transfers between different age cohorts.
an extra return if the pension fund’s assets perform well.\textsuperscript{6} It is a hybrid form between a defined benefit and defined contribution plan, see Broeders, Chen and Rijsbergen (2013). The homogeneous group can also be regarded as a representative beneficiary. The assumption of considering a representative beneficiary is justified by the observation in practice that pension funds often take the average beneficiary as a benchmark in contribution and asset allocation decisions.

Let us assume that at time $t_0 = 0$ the hybrid pension scheme is issued to a representative beneficiary who provides an upfront contribution equal to $L$. The pension fund also receives an initial contribution $S_0$ from the sponsor at time $t_0 = 0$. In return, the sponsor receives a claim on the pension fund’s surplus as a compensation for underwriting the pension fund’s downside risk. Consequently, the initial asset value $X_0$ of the pension fund is given by the sum of the contributions from both the beneficiary and the sponsor

$$X_0 = L + S_0.$$ (1)

From now on, we shall denote $L = \alpha X_0$ with $\alpha \in [0, 1]$. We will call $\alpha$ the wealth distribution parameter. It specifies which part of the initial pension fund’s wealth is paid for by the beneficiary. We now describe two possible situations. The payment to the beneficiary at maturity and the payment in case of early termination of the pension contract.

\textbf{2.1.1. Payment at maturity.} We assume that the pension benefits are paid as a lump sum at maturity. The defined benefit can be represented as the initial contributions of the beneficiary accumulated with a (nominal) guaranteed rate of return $\delta$

$$L_T = L e^{\delta T}.$$ (2)

The beneficiary in our hybrid plan is exposed to risk in the following way. If the assets perform well, the beneficiary is entitled to sharing in the pension fund’s surplus. This “bonus” is described by Ballotta, Haberman and Wang (2006) and Kling, Richter and

\textsuperscript{6}This additional return is typically labeled as indexation. It is a periodic adjustment of pension benefits to reflect changes in costs and standards of living.
Ruß (2007a,b). However, in case of a funding shortfall at maturity the benefit is reduced accordingly. Without external support, the pension fund cannot pay out more than its available assets. Combing all three elements, at maturity, the payment to the beneficiary is given by

\[ \psi_L(X_T) := L_T + \beta \alpha \left[ X_T - \frac{L_T}{\alpha} \right]^+ - [L_T - X_T]^+ \] (3)

where \( \beta \) is the rate with which the beneficiary participates in the pension fund’s surplus. In Section 4 we will specify how the participation rate is chosen. The payoff given in (3) is the payoff if there is no premature liquidation of the pension fund. It is a combination of a fixed payment \( L_T \), a call option on the pension fund’s assets to reflect the bonus payment and a short put option on the assets to reflect the potential reduction in the benefit.

2.1.2. Payment at early contract termination. We allow for the possibility that the pension fund is liquidated before maturity. For instance because the pension fund is severely underfunded. The trigger for the premature liquidation of the pension fund may also be a default of the sponsor. Upon premature liquidation time \( \tau \) a rebate payment is paid to the beneficiary and is denoted by \( \Theta_L(\tau) \). For time consistency reasons we assume that this rebate payment accrues at the risk-free rate \( r \) over the remaining time to maturity. This way it is as if the rebate payment is also due at maturity. To sum up, the beneficiary’s contract payoff consists of two parts: the terminal payment and a rebate payment in case of early termination

\[ \tilde{V}_L := \psi_L(X_T) 1_{\{\tau > T\}} + \Theta_L(\tau) e^{r(T-\tau)} 1_{\{\tau \leq T\}}. \] (4)

This ends the definition of the liabilities in the pension contract.

2.2. Asset processes. We now turn to the definition of assets. We need to distinguish between the pension fund’s assets and the sponsor’s assets. We assume that the pension fund has two investment opportunities: a diversified risky asset (the market portfolio) and a risk-free asset. The traded risky asset \( A \) satisfies
\[ dA_t = \mu A_t dt + \sigma A_t dW^1_t, \]

where \( W^1 \) is a standard Brownian motion under the physical probability measure \( P \). The risky asset follows Black-Scholes dynamics with an instantaneous rate of return \( \mu > 0 \) and a constant volatility \( \sigma > 0 \). Also assume the existence of a risk-free asset \( B \) which satisfies

\[ dB_t = rB_t dt \]

for a deterministic risk-free rate \( r \). The pension fund can only trade in these two assets in a self-financing way starting with initial wealth \( x_0 \), which is assumed to be larger than the initial contribution level \( L \). The pension fund’s wealth process is given by the following stochastic differential equation (SDE)

\[ dX_t = X_t(r + \theta(\mu - r))dt + \theta \sigma X_t dW^1_t, \quad X_0 = x_0. \] (5)

Here \( \theta \) denotes the fraction of wealth invested in the risky asset \( A \), while the remainder \((1 - \theta)\) is invested in the risk-free asset \( B \). We assume continuous rebalancing. Pension funds typically follow a rebalancing strategy in which the actual asset allocation fluctuates closely around a given strategic asset allocation, see Bikker, Broeders and de Dreu (2010).

We assume the sponsor’s assets \( C_t \) also follow Black-Scholes dynamics according to

\[ dC_t = \mu_c C_t dt + \sigma_c C_t (\rho dW^1_t + \sqrt{1 - \rho^2} dW^2_t), \] (6)

with instantaneous rate of return \( \mu_c > 0 \) and volatility \( \sigma_c > 0 \). \( W^2_t \) is a Brownian motion independent of \( W^1_t \) under the probability measure \( P \). The sponsor’s asset return correlates with the risky asset’s return with a correlation coefficient \( \rho \).

This ends the definition of the assets.
2.3. **Triggers for early contract termination.** We now consider two triggers for early contract termination: either underfunding of the pension fund or a default of the pension fund’s sponsor. In order to proceed our analysis we need to define the, so-called, first-hitting time in both cases. When the default event is triggered by the underfunding of the pension fund, the default time is the first time the pension fund’s assets $X_t$ breach the following regulatory threshold $\eta Le^{\delta t}$. We use

$$\tau_p := \inf\{t| X_t \leq \eta Le^{\delta t}\}$$

(7)

to denote the first hitting-time, where $\eta < 1$ is a regulatory parameter. By choosing the height of this parameter the regulator can control the strictness of regulation. Furthermore, it is assumed that initially the pension fund is compliant with regulation ($X_0 > \eta L$), as otherwise the pension fund already terminates at inception.

On the other hand, when early contract termination is triggered by the default of the sponsor we use

$$\tau_c = \inf\{t| C_t \leq \phi C_0 e^{gt}\}$$

(8)

to denote this first-hitting time. The threshold level for the sponsor to default is its debt level $\phi C_0$, $\phi \in (0,1)$. Note that $\phi$ needs to be smaller than 1, as otherwise the sponsor defaults at inception. $\phi C_0$ can be considered the initial debt level and for simplicity reasons, we assume $\phi$ is a constant. However, we also allow for the possibility that the debt level increases over time with a constant growth rate $g$.

This ends the definition of the triggers for early contract termination.

3. **Description of security mechanisms**

We consider three different security mechanisms: solvency requirements, a pension guarantee fund and sponsor support. We give a brief description of the technicalities of these security mechanisms below.
3.1. **Solvency requirements.** Under solvency requirements, the early default trigger is a severe level of underfunding of the pension fund. The intervention time at which the pension fund is liquidated is \( \tau_p \). We assume continuous monitoring and prompt corrective action by the regulator.\(^7\) At the default time, the pension fund pays what remains in the pension fund to the beneficiary. The rebate payment to the beneficiary at default time \( \tau_p \) by definition equals \( \eta \) times the guaranteed amount

\[
\Theta_L(\tau_p) = X_{\tau_p} = \eta L e^{\delta \tau_p}.
\] \(^{(9)}\)

3.2. **Pension guarantee fund.** We assume that in this case the pension fund closes a contract with a pension guarantee fund (PGF). The PGF receives an upfront premium from the pension fund and will take over its assets and liabilities should the sponsor default.\(^8\) If the pension fund has a deficit at that point in time, the pension guarantee fund will be liable for this. In line with practice in most countries, we assume the intervention trigger is the sponsor’s default. If the sponsor defaults at \( \tau_c \), the PGF intervenes and makes any necessary payments to the pension fund.

To correctly model the insurance provided by the pension guarantee fund we need to distinguish between two cases: no premature default of the sponsor (\( \tau_c > T \)) and a premature default of the sponsor (\( \tau_c \leq T \)). In the former case, the PGF needs to cover the deficit of the pension fund, if any. Hence, the payoff of the pension insurance at maturity is simply

\[
G_T = \max(L e^{\delta T} - X_T, 0).
\]

This is the amount the PGF would have to pay to cover the deficit in the pension fund, if there is a deficit at maturity. In the latter case, the PGF intervenes immediately at the sponsor default’s time (\( \tau_c \)). It takes over the pension fund’s assets and liabilities and the operation of the pension fund is terminated. We assume that the PGF needs to cover the

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\(^7\)Allowing for a certain waiting time before action is taken would exponentially complicate the analysis.  
\(^8\)In reality, periodic premiums are charged by the pension guarantee fund. Here a single premium is assumed for simplicity and for consistency with the single liability for the representative beneficiary.
deficit immediately at $\tau_c$. The premature payoff of the pension insurance is

$$G_{\tau_c} = \max(Le^{\delta_{\tau_c}} - X_{\tau_c}, 0).$$

This is the amount the PGF would have to pay to cover the pension fund’s deficit, if there is a deficit any time before maturity. A deficit occurs if the pension fund’s assets are insufficient to pay the target guarantee $Le^{\delta_{\tau_c}}$. Given this payout structure we can determine the cost $G$ of the pension insurance paid by the pension fund to the PGF. It is given by

$$G = G_T 1_{\{\tau_c > T\}} + G_{\tau_c} 1_{\{\tau_c \leq T\}}$$

$$= \max(Le^{\delta_T} - X_T, 0) 1_{\{\tau_c > T\}} + \max(Le^{\delta_{\tau_c}} - X_{\tau_c}, 0) 1_{\{\tau_c \leq T\}}.$$

The cost of insurance can be decomposed into two parts: a rainbow down-and-out put option and a rainbow down-and-in put option. Rainbow barrier options are a well-known form of barrier options where the option is written on one underlying asset while the knock-in or knock-out condition is triggered by a second asset.\footnote{These options were firstly analyzed in Heynen and Kat (1994), Zhang (1995) and Carr (1996).} In the case of a pension guarantee fund, the underlying asset is formed by the pension fund’s assets while the knock-out is triggered by the plan sponsor’s assets. Upon default of the sponsor, the pension fund is taken over by the PGF. We have a rainbow down-and-out put option if there is no premature termination of the pension fund, and a rainbow down-and-in put option if there is premature termination at $\tau_c$. The premium for the PGF insurance can be considered as the market value of these rainbow options

$$G_0 = E^* [e^{-rT} G_T 1_{\{\tau_c > T\}}] + E^* [e^{-r_{\tau_c}} G_{\tau_c} 1_{\{\tau_c \leq T\}}]$$

$$= E^* [e^{-rT} (Le^{\delta_T} - X_T)^+ 1_{\{\tau_c > T\}}] + E^* [e^{-r_{\tau_c}} (Le^{\delta_{\tau_c}} - X_{\tau_c})^+ 1_{\{\tau_c \leq T\}}], \quad (10)$$

where $E^*$ denotes the expected value under the risk-neutral measure $P^*$. We are able to obtain a closed-form solution for this premium. We refer the reader to Broeders and Chen (2013) for a detailed derivation.
3.3. **Sponsor support.** We now turn to the case where the sponsor offers protection against the premature default of the pension fund. As in the case of solvency requirements, the trigger is underfunding of the pension fund. Again, the intervention time, $\tau_p$, is the first time the pension fund’s assets breach the regulatory threshold. We assume continuous monitoring by the regulator. If the pension fund defaults at $\tau_p$, it holds assets worth $X_{\tau_p} = \eta L_{\tau_p} \leq L_{\tau_p}$. Or alternatively, the deficit at that point in time equals $Le^{\delta \tau_p} - X_{\tau_p}$.

We make the following two assumption on the actions of the sponsor to cover the deficit

- covering the pension fund’s deficit does not lead to a default of the sponsor,
- if the sponsor is not insolvent but unable to cover the entire pension fund’s deficit, it pays what it has left after paying back its own corporate debt in full.\(^\text{10}\)

We again consider two different default times. A premature default of the pension fund ($\tau_p \leq T$) and a default at maturity ($\tau_p > T$). According to these descriptions, the financial support that the sponsor needs to provide at time $\tau_p \leq T$ is described by

$$
\Phi_c(\tau_p) = (Le^{\delta \tau_p} - X_{\tau_p})1_{\{C_{\tau_p} > \phi C_0 e^{\eta \tau_p} + (Le^{\delta \tau_p} - X_{\tau_p})\}}
+ (C_{\tau_p} - \phi C_0 e^{\eta \tau_p})1_{\{\phi C_0 e^{\eta \tau_p} < C_{\tau_p} \leq \phi C_0 e^{\eta \tau_p} + (Le^{\delta \tau_p} - X_{\tau_p})\}}.
$$

The first term on the right-hand side corresponds to the situation in which the sponsor is able to cover its own outstanding debt and all the pension fund’s deficit. The second term represents its inability to do so: after repaying its own creditors, the sponsor can only pay the remainder to the pension fund.

If default occurs at maturity, it means that the pension fund’s assets $X_T$ have not hit the regulatory threshold $\eta L_T$ and outperformed it during the entire period $[0, T]$, and particularly it holds $X_T > \eta Le^{\delta T} = \eta L_T$. If, furthermore, $X_T \geq L_T$, the pension fund’s assets are sufficient to provide the promised pension payment. Hence, the sponsor does not have to provide the guaranteed amount in this case. On the other hand, if $\eta L_T < X_T \leq L_T$,

\(^{10}\)In our model we assume that pension assets and liabilities are held in a separate legal entity. Therefore it is reasonable to assume that a pension fund’s deficit is less senior compared to regular corporate debt. In a recent decision on Nortel Companies and Lehman Brothers Companies, the UK Supreme Court judged that pension liabilities rank equally with a company’s unsecured creditors (Trinity Term [2013] UKSC 52).
i.e. the pension assets at $T$ exceed the regulatory threshold, but are still lower than the promised pension payments, the sponsor must cover the following deficit

$$
\Phi_c(T) = (L_T - X_T)1\{C_T > \phi C_0 e^{gT} + (L_T - X_T)\}1\{\eta L_T < X_T \leq L_T\} \\
+ (C_T - \phi C_0 e^{gT})1\{\phi C_0 e^{gT} < C_T \leq \phi C_0 e^{gT} + (L_T - X_T)\}1\{\eta L_T < X_T \leq L_T\}. 
$$

(12)

Unlike in the case of a premature default, we do not know the terminal value of the pension fund’s assets ($X_T$). Again, here the sponsor provides either a full or a partial guarantee, depending on its ability to redeem its own debt.

Similar to the case of a pension guarantee fund, we assume that the sponsor gets a fair reward for underwriting the pension fund’s shortfall risk. However, in this case the reward is paid implicitly. The fair “pseudo-premium” $S_{C_0}$ the sponsor would have obtained for providing support is the market value of the sum of the support in case of early termination in (11) and support at maturity in (12). More specifically, we can express this premium as

$$
S_{C_0} = E^*[e^{-r_{p} \tau_p} \Phi_c(\tau_p)1\{\tau_p \leq T\}] + E^*[e^{-r T} \Phi_c(T)1\{\tau_p > T\}],
$$

(13)

under the risk-neutral probability measure $P^*$. The value is calculated explicitly in Broeders and Chen (2013).

4. Fair contract analysis

After the model specification and the description of the security mechanisms, we now need to prepare for the utility analysis. In order to ease a utility-based comparison between the three different security mechanisms, we will make two assumptions about the initial investment and about the fairness of the pension contract. Assumption one is that the initial investment of the beneficiary is the same in all cases. Assumption two is that we adjust the participation rate ($\beta$) to comply with the fair contract principle. We will first elaborate on these assumptions below. Thereafter we show how to calculate the fair participation rate in the three situations we distinguish.
According to our model description, the real initial investment of the beneficiary is not automatically identical under the different security mechanisms. The reason is that the beneficiary needs to pay an insurance premium to either the pension guarantee fund ($G_0$) or to the sponsor ($Sc_0$). The real initial investment of the beneficiary under solvency requirements, PGF support and sponsor support is respectively: $L$, $L + G_0$ and $L + Sc_0$. For a fair utility-bases comparison we need to correct for this. We scale the underlying factors such that the total investment is the same in all three cases. Therefore we introduce assumption 4.1.

**Assumption 4.1.** The real initial investment of the beneficiary is the same under all the three security mechanisms and this amount is denoted by $L$.

Assumption 4.1 implies the following: (i) Under solvency requirements, amount $L$ can be used in full for investing, supplement with the initial contribution of the sponsor, i.e., $X_0 = L + S_0$. (ii) In case of PGF-support, amount $G_0$ is used to pay the insurance premium for the PGF support and $L - G_0$ can be used for investing. Supplemented with the initial contribution of the sponsor, the initial asset value of the pension fund is $L - G_0 + S_0$.

In what follows below, we will provide an elaborate analysis about how to determine this adjusted initial asset value. (iii) In case of sponsor-support the initial asset value of the pension fund equals the beneficiaries’ contribution minus the pseudo premium plus the sponsor’s contribution $L - Sc_0 + S_0$.

There is another issue to address before we can do utility comparisons. Under the PGF and sponsor support, more parameters are involved compared to the solvency requirements. Specifically the dynamics of the sponsor plays a role. To correct for this and to be able to compare the security mechanisms, we will fix all the parameters except the participation rate $\beta$. We make the following assumption.

**Assumption 4.2.** Under all pension security mechanisms, the participation rate is endogenously determined according to the fair contract principle.
Assumption 4.2 states that the participation rate is determined so that the market value of the amount received by the beneficiary coincides with his upfront contribution. We will call this the fair participation rate $\beta^*$. The fair contract principle implies that the market value of the amount received by the beneficiary corresponds to the expected discounted payoff under the risk neutral measure $P^*$, see Grosen and Jørgensen (2002).

4.1. Fair participation rates. In this section, we will show how to determine the fair participation rate $\beta$ for the three pension security mechanisms, following Assumptions 4.1 and 4.2.

Under solvency requirements, we do not need to adjust the initial investment $L$. Assumption 4.2 requires

$$
E^* \left[ e^{-rT} \left( Le^{\delta T} + \beta^* \alpha \left[ X_T - \frac{Le^{\delta T}}{\alpha} \right]^+ - [Le^{\delta T} - X_T]^+ \right) 1_{\{\tau_p > T\}} \right] \\
+ E^* [e^{-r\tau_p} \eta Le^{\delta \tau_p} 1_{\{\tau_p \leq T\}}] = L
$$

The left-hand side of the equation gives the market value of the benefits paid to the beneficiary. Equating this to the initial investment $L$ of the beneficiary allows to determine the fair participation rate $\beta^*$ implicitly.

Under the guarantee provided by the PGF, Assumption 4.1 induces an adjustment of the initial value $L$. This also leads to a change in the initial asset value $X_0$ of the pension fund. We use $\tilde{X}_0$ to denote the adjusted initial asset value

$$
\tilde{X}_0 = \underbrace{L - G(\tilde{X}_0)}_{\text{adjusted initial beneficiaries' contribution}} + S_0.
$$

Solving (15) is not straightforward. The adjusted initial asset value $\tilde{X}_0$ is not explicitly given but defined implicitly by satisfying (15), because the insurance premium $G_0$ paid to the PGF in itself depends on $\tilde{X}_0$. The change in $L$ and in $X_0$ requires a different value for the wealth distribution factor $\alpha$ because $\alpha$ is defined as $L/X_0$. We use $\tilde{\alpha}$ to denote the
adjusted wealth distribution factor which is defined as
\[
\tilde{\alpha} = \frac{L - G(\tilde{X}_0)}{X_0 - G(X_0)} = \frac{L - G(\tilde{X}_0)}{\tilde{X}_0}.
\]

Since the insurance premium \(G(\tilde{X}_0)\) is positive, the new fraction \(\tilde{\alpha}\) is smaller than the original wealth distribution factor \(\alpha\). Based on the adjusted parameters, we can now write the fair contract principle for the PGF case as follows
\[
E^* \left[ e^{-rT} \left( L e^{\delta T} + \tilde{\beta}^* \tilde{\alpha} \left[ \tilde{X}_T - \frac{L e^{\delta T}}{\tilde{\alpha}} \right]^+ \right) \mathbb{1}_{\{\tau_c > T\}} \right] 
+ E^* \left[ e^{-r\tau_c} L e^{\delta \tau_c} e^{\delta \tau_c} \mathbb{1}_{\{\tau_c \leq T\}} \right] = L.
\] (16)

Hereby we have used \(\tilde{X}_T\) to denote the terminal asset value of the pension fund starting with the initial wealth \(\tilde{X}_0\). Accordingly, \(\tilde{\beta}^*\) is the fair participation rate resulting under the PGF-support mechanism. Note that \(\tilde{X}\) is independent of \(\tilde{\beta}\), because the premium of the guarantee \(G(\tilde{X}_0)\) does not depend on \(\tilde{\beta}\). Therefore, using equation (10), we obtain \(\tilde{X}\) implicitly and consequently also \(\tilde{\alpha}\). Then all these parameters are used to determine the fair participation rate \(\tilde{\beta}^*\).

Under sponsor support, we can make similar adjustments to the parameters. The adjusted initial asset value \(\hat{X}_0\) of the pension fund is now implicitly determined by solving
\[
\hat{X}_0 = \frac{L - S c_0(\hat{X}_0) + S_0}{\text{adjusted initial beneficiaries' contribution}} \hat{X}_0.
\]

As a result, the adjusted wealth redistribution factor \(\hat{\alpha}\) is given by
\[
\hat{\alpha} = \frac{L - S c_0(\hat{X}_0)}{\hat{X}_0 - S c_0(\hat{X}_0)} = \frac{L - S c_0(\hat{X}_0)}{\hat{X}_0}.
\]

Finally, the fair participation rate \(\hat{\beta}^*\) under sponsor support results from satisfying the fair contract principle
\[
E^* \left[ e^{-rT} \left( L e^{\delta T} + \hat{\beta}^* \hat{\alpha} \left[ \hat{X}_T - \frac{L e^{\delta T}}{\hat{\alpha}} \right]^+ \right) \mathbb{1}_{\{\tau_p > T\}} \right] 
+ E^* \left[ e^{-r\tau_p} (\eta L e^{\delta \tau_p} + \Phi_c(\tau_p, \hat{X})), \mathbb{1}_{\{\tau_p \leq T\}} \right] = L,
\] (17)
where $\hat{X}_T$ is the terminal asset value of the pension fund starting with the initial wealth $\hat{X}_0$. Similar to the calculation in the PGF case, $\hat{X}_0$, $\hat{\alpha}$ and $\hat{\beta}$ can be determined sequentially. We use (13) to determine $\hat{X}$ and $\hat{\alpha}$. Again a fair-contract principle leads to $\hat{\beta}^\ast$.

5. Numerical derivation of fair participation rates

After making these additional assumptions we are now ready for some numerical analysis. As a first step to the utility analysis in the next section, we start by calculating fair participation rates. We calibrate the parameters as follows. The risk-free rate $r$ is 5%, the guaranteed rate of return in the pension contract $\delta$ is 4.6%, the volatility of equity returns is $\sigma$ is 20%, the volatility of the return on the sponsor’s assets $\sigma_c$ is 33, 3%, the time to maturity $T$ is 15 years, the initial beneficiaries’ contribution $L$ equals 90, the initial sponsor’s contribution $S_0$ equals 10, the regulatory threshold $\eta$ is 90% and the sponsor’s debt ratio $\phi$ is 50%. Table 1 shows the fair participation rate $\beta^\ast$ as a function of the pension fund’s equity allocation $\theta$ and the correlation coefficient $\rho$ between the return on the pension fund’s and the sponsor’s assets. We consider equity allocations ranging from 50 to 90% to test the efficiency of the security mechanisms. Assuming a (very) low equity allocation would not be very insightful as the security mechanisms would only be rarely deployed. We have several observations from this specific set of parameters. The results cannot be generalized.

First, under solvency requirements, the fair participation rate $\beta^\ast$ does not depend on the correlation between the pension fund and the sponsor. This is obvious as the role of the sponsor in the default process is absent in this case.

Second, under solvency requirements, the fair participation rate $\beta^\ast$ decreases for higher equity allocations. This results from the following complex underlying processes. The market value of the beneficiary’s benefits under solvency requirements consists of two components: the market value of the benefits upon natural termination and the market value of the benefits upon premature termination, see (14). The former component can be split into three parts: a) the down-and-out value of the fixed payment, b) the down-and-out surplus call option and c) the short down-and-out put option. Part a) decreases for
\( \rho = 0.25 \quad \rho = 0 \quad \rho = -0.25 \)

<table>
<thead>
<tr>
<th></th>
<th>SR</th>
<th>PGF</th>
<th>SS</th>
<th>PGF</th>
<th>SS</th>
<th>PGF</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>( \beta^* )</td>
<td>( \tilde{\beta}^* )</td>
<td>( \tilde{\beta}^* )</td>
<td>( \hat{\beta}^* )</td>
<td>( \tilde{\beta}^* )</td>
<td>( \hat{\beta}^* )</td>
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</tr>
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<td>58.4%</td>
<td>39.6%</td>
<td>71.0%</td>
<td>37.1%</td>
<td>94.2%</td>
<td>34.7%</td>
</tr>
<tr>
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<td>56.4%</td>
<td>32.8%</td>
<td>66.3%</td>
<td>31.1%</td>
<td>86.8%</td>
<td>29.1%</td>
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<td>26.4%</td>
<td>80.9%</td>
<td>24.9%</td>
</tr>
<tr>
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<td>56.1%</td>
<td>23.6%</td>
<td>60.5%</td>
<td>22.8%</td>
<td>76.2%</td>
<td>21.6%</td>
</tr>
<tr>
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<td>57.3%</td>
<td>20.5%</td>
<td>58.6%</td>
<td>20.0%</td>
<td>72.3%</td>
<td>19.1%</td>
</tr>
</tbody>
</table>

Table 1. Fair participation rates. Parameters: \( \delta = 4.6\% \), \( \sigma = 0.20 \), \( \sigma_c = 0.333 \), \( T = 15 \), \( L = 90 \), \( S_0 = 10 \), \( r = 0.05 \), \( \eta = 0.9 \), \( \phi = 0.5 \). \( \rho \) is the correlation between the pension fund’s and the sponsor’s assets. \( \beta^* \), \( \tilde{\beta}^* \) and \( \hat{\beta}^* \) are the fair participation rates under solvency requirements (SR), a pension guarantee fund (PGF) and sponsor support (SS) respectively.

higher equity allocations, Part b) increases for higher equity allocations and Part c) does not change monotonically for higher equity allocations due to the non-monotonic effect on the value of the short position in the down-and-out put. The latter component increases in the equity allocation, because a premature default becomes more likely to occur for higher risk exposures. Hence, the total market value might increase or decrease in the equity allocation, depending on which part dominates. For the given parameters, the part in charge of the premature termination seems to dominate, i.e., the market value increases in the equity allocation. Therefore, a lower participation rate is required to make the contract fair.

Third, under a PGF, the fair participation rate \( \beta^* \) decreases for higher equity allocations. According to the fair contract principle in (16), the correlation only influences the surplus call option. A higher correlation implies that it is more likely that the pension fund is underfunded when the sponsor defaults. As a consequence, the PGF needs to balance more deficits of the pension fund. Therefore, a higher premium \( G_0 \) and consequently a lower initial real investment for the beneficiary results. On the other hand, a higher correlation
coefficient implies that it is more likely that the pension fund performs well too, when the sponsor’s assets do well. It consequently raises the probability that the surplus option is in-the-money and not yet knocked out at the same time. The first effect would lead to a lower surplus option value while the second effect would cause a higher option value. Altogether, the effect on the surplus option value is ambiguous. For the chosen parameters, the surplus option increases for higher correlations. Therefore, a lower fair participation rate results for a higher correlation coefficient. Only for a positive correlation between pension fund and sponsor ($\rho = 0.25$) and for a very high equity allocation we observe a slight non-monotonic effect of the equity allocation on the fair participation rate in case of PGF-support.

**Fourth**, under sponsor support, again we observe the same effect. The fair participation rate $\beta^*$ decreases for higher equity allocations. In case of sponsor support, all components of the market value of the beneficiary’s benefits are affected by a change in the correlation coefficient (c.f. (17)). A higher correlation makes a double default scenario, in which both the pension fund and the sponsor default simultaneously, more likely. When the pension fund defaults, it is more probable that the sponsor is also underfunded and is unable to provide support. As a result, the sponsor pseudo premium decreases for higher correlations. Consequently, a higher initial investment $\hat{X}_0$ results, which by definition lowers the pension fund’s default probability. The benefit can be decomposed into two parts. (i) The same payments as under solvency requirements but based on the adjusted values for the wealth distribution parameters ($\hat{\alpha}$) and the initial investment ($\hat{X}$). For this part, we adopt the same decomposition we used for the analysis of the effect of the equity allocation under solvency requirements. For default at maturity, the down-and-out value of the fixed payment (part a) is increasing in the correlation. Through the lower default probability and the higher initial investment ($\hat{X}_0$) and wealth distribution parameter ($\hat{\alpha}$, part b) is increasing in the correlation as well. The down-and-out put option behaves not monotonically. For premature defaults, the market value of the remaining part is decreasing in the correlation due to the lower default probability. (ii) The additional payment through the sponsor in case of default. The market value of the second part is
just the additional premium transferred to the sponsor, which decreases in the correlation (see above). Overall, the effect of the correlation coefficient is ambiguous. Here, the market value decreases, which is connected with a higher fair participation rate.

6. Utility-based comparisons

We now have completed all the necessary steps to compare welfare implications for the beneficiary. To start, we concentrate on calculating certainty equivalents. Furthermore, we are interested in discovering under what contract specifications the security mechanisms are equivalent in the perspective of protecting the beneficiary. We assume that the representative beneficiary is risk averse and has the following power utility function

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} \text{ with } \gamma \neq 1,$$

where $\gamma$ is the coefficient of relative risk aversion and $x$ is the terminal wealth of the beneficiary. A higher value of $\gamma$ corresponds to a more risk-averse beneficiary and $\gamma = 0$ gives the special case that the beneficiary is risk-neutral. For ease of comparison, we will derive the certainty equivalents from

$$E[U(x)] = U(CEQ).$$

In what follows, all the results are obtained through simulation. We use 10,000 simulation paths.

6.1. Certainty equivalents. Table 2 compares the certainty equivalents obtained under the three pension security mechanisms for the parameters used in the numerical analysis above. In addition, we assume a return $\mu$ of 8% on the risky assets and the expected return on the sponsor’s assets $\mu_c$ is 10%. We confine our analysis to a correlation coefficient of 0.25 as it reasonable to assume that a company’s equity is positively correlated to the market portfolio. The equity allocation again ranges from 50 to 90% and we consider three levels of relative risk aversion. We observe the following.
First, for all the cases, a more risk-averse beneficiary has a lower utility for the same parameter combinations. This follows from the general fact that the certainty equivalent of a risky payoff $x$ is equal to $E[x]$ for a risk-neutral individual and lower than $E[x]$ for a risk-averse individual. The rationale behind this argument is Jensen’s inequality. Second, solvency requirements always deliver the highest utility for a risk neutral beneficiary. This can be explained as follows. In case of solvency requirements, neither the sponsor nor a PGF balances the pension fund’s deficit. The payoff to the beneficiary is in comparison more risky and therefore delivers the highest expected payment. As a risk-neutral agent benefits from a higher mean but is indifferent about the risk, the certainty equivalent is greatest under solvency requirements. Third, for a moderate allocation to equities ($\theta = 0.5$), insurance through a PGF or the sponsor causes relatively high insurance costs. A low/medium risk-averse beneficiary minds the insurance cost and does not benefit much from the insurance for moderate equity allocations as extreme down turn scenario’s are less likely to occur. Therefore, the beneficiary does not benefit much from the insurance. As a consequence, solvency requirements provide the highest utility. Fourth, for higher equity allocations, insurance provided by the PGF offers the highest utility to the beneficiary. For instance, for $\theta = 0.7$ and $\gamma = 3$, the insurance through PGF out-performs solvency requirements and sponsor support. Note however that for extremely high equity allocation sponsor support outperforms the other two mechanisms.

6.2. Utility-equivalence approach. The analysis in Table 2 can only be used for illustrative purposes. It cannot be used to show that one security mechanism is dominant over the others. Obviously, for different parameters the ranking might be very different. However, our model can be used to show under which assumptions the different security mechanisms offer equivalent utility. For that we allow the pension contract specification to be adjusted. Effectively, it is now possible to make the beneficiary indifferent between the three pension security schemes by, e.g., varying the guaranteed interest rate in the pension contract ($\delta$). This change in $\delta$ results in a new participation rate $\beta$ to make the adjusted contract remain fair.
\begin{table}
\centering
\begin{tabular}{lccccccccc}
\hline
\multicolumn{1}{c}{} & \multicolumn{3}{c}{Solvency} & \multicolumn{3}{c}{PGF} & \multicolumn{3}{c}{Sponsor} \\
\hline
$\theta/\gamma$ & 0  & 3  & 5  & 0  & 3  & 5  & 0  & 3  & 5  \\
\hline
0.5 & 215.4 & 199.8 & 193.5 & 203.7 & 197.6 & 195.2 & 203.0 & 198.3 & 196.2 \\
0.6 & 217.0 & 197.4 & 190.7 & 204.3 & 197.2 & 194.7 & 201.8 & 197.3 & 195.4 \\
0.7 & 219.9 & 195.3 & 188.3 & 204.7 & 196.7 & 194.1 & 200.5 & 196.2 & 194.4 \\
0.8 & 223.4 & 193.9 & 186.7 & 206.7 & 196.7 & 193.8 & 199.3 & 195.3 & 193.7 \\
0.9 & 226.5 & 192.3 & 185.2 & 206.7 & 195.6 & 192.8 & 199.7 & 195.3 & 193.8 \\
\hline
\end{tabular}
\caption{Certainty equivalents given different security mechanisms: solvency requirements, a pension guarantee fund (PGF) and sponsor support. Parameters: $\mu = 0.08$, $\sigma = 0.20$, $\sigma_c = 0.333$, $\mu_c = 0.1$, $T = 15$, $L = 90$, $S_0 = 10$, $r = 0.05$, $\delta = 0.046$, $\eta = 0.9$, $\phi = 0.5$, $\rho = 0.25$. $\theta$ represents the equity allocation and $\gamma$ the coefficient of relative risk aversion.}
\end{table}

Table 3 shows the resulting fair combinations for a risk-neutral beneficiary ($\gamma = 0$).\textsuperscript{11}

The calibration is similar to Table 2. For instance, for an equity allocation of 60\%, a guaranteed interest rate of 4.6\% and a participation rate of 61.2\% under solvency requirements lead to the same utility as a guaranteed rate of 4\% and a participation rate of 94.6\% under a PGF as well as a 3.2\% guaranteed interest rate and a 71.7\% participation rate under sponsor support.

For our specific example we observe the following. For solvency requirements to be equivalent with the offer security mechanisms, the pension fund must offer a high guaranteed interest rate while the surplus participation rate can be lower. In case of a pension guarantee fund, the guaranteed interest rate can be moderate but the surplus participation rate must be relatively high. Finally, in the case of sponsor support, the guaranteed interest rate must be relatively low while the surplus participation rate may be moderate to be equivalent to the other security mechanisms.

\textsuperscript{11}For $\gamma = 3$ or 5 the results are highly comparable and therefore not shown here.
Solvency PGF Sponsor

\[ \theta \delta \beta \]

\[
\begin{array}{cccccc}
\theta & \delta & \beta^* & \delta & \ddot{\beta}^* & \delta & \hat{\beta}^* \\
0.5 & 4.6\% & 62.9\% & 4.0\% & 98.3\% & 3.5\% & 74.0\% \\
0.6 & 4.6\% & 61.2\% & 4.0\% & 94.6\% & 3.2\% & 71.7\% \\
0.7 & 4.6\% & 59.9\% & 4.0\% & 92.5\% & 2.7\% & 70.9\% \\
0.8 & 4.6\% & 58.9\% & 4.0\% & 91.4\% & 2.2\% & 70.4\% \\
0.9 & 4.6\% & 58.1\% & 4.0\% & 91.3\% & 1.7\% & 69.6\% \\
\end{array}
\]

Table 3. Fair combinations of participation rate \( \beta \) and guaranteed return \( \delta \), which lead to the same CEQ's for a risk neutral beneficiary (\( \gamma = 0 \)).

Parameters: \( \mu = 0.08, \sigma = 0.20, \sigma_c = 0.333, \mu_c = 0.1, T = 15, L = 90, S_0 = 10, r = 0.05, \eta = 0.9, \phi = 0.5, \rho = 0.25. \) \( \theta \) represents the equity allocation, \( \delta \) the guaranteed return, \( \beta^*, \ddot{\beta}^* \) and \( \hat{\beta}^* \) are the fair participation rates under solvency requirements, a pension guarantee fund and sponsor support respectively.

Furthermore we observe that the guaranteed interest rate of 4\% under a PGF is independent of the investment policy. At the same time, under PGF support, a very high participation rate results. In contrast, the sponsor support requires a lower guaranteed interest rate and a lower participation rate. The guaranteed interest rate and the participation rate are two pension contract parameters which are revealed to the beneficiary directly. Based on our example, a “naive” beneficiary would probably prefer the PGF scheme over sponsor support because the PGF is able to offer a higher interest rate guarantee combined with a high surplus participation rate. However, there is something hidden behind these two promising parameters. Since PGF provides a full insurance guarantee for underfunding, the premium charge could be relatively high. Hence, the pension fund’s real initial investment \( \bar{X}_0 \) becomes comparably lower. The high guaranteed rate of return and the high participation rate compensate this effect.
7. Conclusion

Adequate funding of occupational pension plans is key to benefit security. The present paper addresses a utility-based comparison between three methods of securing funding that exist in occupational plans - solvency requirements, a pension guarantee fund and sponsor support. These mechanisms are modeled and compared for a general class of hybrid pension schemes. We analyze the welfare implications for pension beneficiaries under these different security mechanisms using a stylized utility function.

To allow for a fair utility based comparison we make two important assumptions. First, the initial contribution of the beneficiary must be the same under all pension security mechanisms. Second, under all three security mechanisms, the surplus participation rate is endogenously determined according to the fair contract principle. In addition the key determinants of the overall utility level are the coefficient of relative risk aversion, the pension fund’s investment policy, the pension contract’s guaranteed interest rate and the correlation between the pension fund’s and sponsor’s assets.

Our model can be used to make the three security mechanisms equivalent by choosing the pension contract specifications. For solvency requirements to be equivalent with the offer security mechanisms, a pension fund must offer a high guaranteed interest rate while the surplus participation rate can be lower. In case of sponsor support the guaranteed interest rate can be relatively low while the surplus participation rate may be moderate to be equivalent to the other security mechanisms. Mostly interestingly, a pension guarantee fund scheme requires a relatively high guaranteed interest rate and at the same time a high surplus participation rate. However, in this case the insurance premium will be high, thereby lowering the pension fund’s real initial investment.

The utility-equivalence presented in this paper could serve to strengthen the holistic balance sheet approach. It can be used to assess whether different regulatory regimes offer similar utility to beneficiaries of pension plans. The utility approach is potentially less
complicated from a modeling perspective and is more closely related to standard asset-liability management tools currently being used by pension funds.
References


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