Sovereign Risk and Simple Debt Dynamics: The Case of Brazil and Argentina

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Abstract:
In this paper we develop a simple neoclassical growth model with perfect international capital mobility to analyze the international debt dynamics of developing countries in general and Brazil and Argentina in particular. We show that three different regimes can be distinguished: a stable steady state debtor regime, a stable steady state creditor regime and an unstable regime. A switch from a stable debtor or a stable creditor position to an unstable creditor regime may be a sign of forthcoming trouble. We investigate this issue empirically for Brazil and Argentina over the period 1960-1999. Over the full sample, the evidence suggests that debt dynamics evolved according to the stable debtor case in both countries. Using a rolling regression technique, we find that indeed occasional switches to the unstable regime occurred. In particular, Argentina was in the unstable regime for most of the 1990s way before the Argentine debt crisis erupted.

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1. Introduction

After the hectic Latin-American debt crisis in the early 1980s, sovereign risk appeared to have disappeared from the scene. However, a series of international crises in the late 1990s have brought the issue of sovereign risk back on the agenda. First, the Asia crisis in 1997 hit international financial markets, followed by the Russia crisis in 1998 and the Argentina meltdown in 2001-2002. Moreover, interest rates on Brazilian debt have been rising throughout the first half of 2002 for fears of the populist politician Lula being elected. If so, international investors expect more domestically oriented, protectionist policies and a possible default on Brazil’s international debt. Of course, the rising interest rates on the Brazilian debt considerably contribute to the probability of Brazil defaulting as the debt burden increases.

Unsurprisingly, a large theoretical and empirical literature has emerged on the analysis of sovereign risk. On the theoretical side, a distinction has been made between ability-to-pay models and more modern willingness-to-pay models. The first category focuses on the development of solvency and liquidity ratios to determine the creditworthiness of individual countries. The more modern approach uses concepts from financial contracting theory and asymmetric information problems to investigate in which circumstances a country may choose not to honor its international debts, even if it could afford to do so in principle. Obstfeld and Rogoff (1996) provide an excellent overview of this type of literature. In general, both strands of literature yield a number of potential indicators of forthcoming debt crises. However, empirical studies generally show that forecasting sovereign defaults is far from easy. The performance record of both commercial rating agencies and academic research in this respect is poor, see for example Oetzel, Bettis and Zenner (2001).

In this paper, we return to an older literature that focuses on simple debt dynamics. The model by Amano (1965) is the simplest variant of a model type, which explains the occurrence of debt crises in a neoclassical framework solely due to shocks. In addition, the model generates conditions under which the traditional idea of a debt cycle can be shown to hold or not to hold respectively. Amano (1965) does not go into

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1 For alternative and more complex models of debt cycles we refer to Fischer and Frenkel (1972), Frenkel and Fischer (1974a,b), Onitsuka (1974) and Hori and Stein (1977).
details of the debt cycle but rather emphasizes that a country’s GNP may grow at a rate larger than the natural one if it saves much and foreign investment (credit) income is a large share of GNP. We will use the model to show analytically (i) that the model can show debt crises only in the form of a shock, and (ii) that a dynamic debt process can be derived graphically with several possible outcomes.

These outcomes can be dichotomized as follows. First, a debtor country can be on a stable path towards a steady state. In the final steady state, the country will remain a debtor if its saving ratio is low. However, it can also become a creditor in the steady state if its saving ratio is high enough. Second, the country can be on an unstable path without ending up in a steady state. In that case, the country can become a permanent creditor – again if its saving ratio is high enough. In a sense, we might call this the stable arm of the unstable path. Amano (1965) emphasizes the latter case, in which the country’s GNP is dominated by foreign income and grows at a rate higher than the natural rate. Alternatively, the country can be situated on the unstable arm of the unstable path and experience an exploding debt, inevitably leading to a crisis. We derive each of these cases from a linear differential equation in debt per unit of GNP.

Subsequently, we estimate these equations for Argentina and Brazil respectively both for the whole period 1960-1999 and for shorter periods. Our purpose is to show that a country’s debt dynamics can switch between the stable and unstable paths over time in response to unexpected shocks. Evidence that the country is situated on the unstable path at some point in time potentially contributes to the analysis of its creditworthiness and to the prediction of the possibility of a forthcoming debt crisis. We find evidence of unstable Argentine debt dynamics as early as the late 1980s and the early 1990s.

The paper is structured as follows. In section 2 we present the model, which we use to derive the various debt dynamic processes in section 3. In section 4 we present data and estimation procedure for Brazil and Argentina. We present and discuss the results in section 5. The summary and conclusions follow in section 6.
2. A simple neoclassical growth model with perfect capital mobility

We start from a simple neoclassical growth model with a fixed rate of interest. Due to the assumption of perfect international capital mobility, the domestic interest rate is given and fixed at the level of the world interest rate. In addition, we assume that output \( Y \) is produced by the production factors capital \( K \) and labor \( L \) with labor-augmenting technology \( A \). The production function is linearly homogeneous with positive first and negative second partial derivatives:

\[
Y = F(K,AL) \quad (1)
\]

Profit maximization of the representative firm yields the marginal productivity condition

\[
r = f'(k) \quad \text{with} \quad k \equiv K/AL \quad \text{and} \quad \frac{\dot{K}}{L} = A + \frac{\dot{D}}{L} = g \quad (2)
\]

Since \( r \) is given, the marginal productivity condition determines the capital/efficient-labor ratio. The capital stock \( K \) consists of domestic capital \( W \) and foreign capital or debt \( D \). Conditional on the level of the domestic stock of capital \( W \), the country can choose its desired level of foreign indebtedness \( D \) to make equation (2) hold at any moment. The labor force \( L \) grows at rate \( n \). In the steady state the capital stock \( K \) and output \( Y \) grow at the same constant rate \( g \), keeping \( k \) constant.

We assume goods market equilibrium to hold. In a small open economy under perfect capital mobility, the country then can finance any domestic investment in excess of national savings through the international capital market. Excess investment results in an equal increase in the country’s international indebtedness, as reflected in equation (3):

\[
\dot{D} = \dot{K} - S \quad , \quad (3)
\]

where dots indicate the change in a variable per unit of time. Domestic savings is assumed to depend on the country’s income net of foreign interest payments, with marginal propensity to save equal to \( s \):

\[
S = s(Y - rD) \quad , \quad (4)
\]
Combining equations (2)-(4) yields a differential equation for the change in the country’s external debt per period. Note that these debt dynamics are derived from the goods market equilibrium condition and therefore reflect only the ‘fundamentals’: investment and savings.

\[ B(t) = gK(0)e^{rt} - s[Y(0)e^{rt} - rD] \]  

For a formal treatment of this differential equation we refer to appendix A. In the next section we provide a more intuitive explanation.

3. Debt cycle versus permanent debtor position

First, we slightly rewrite equation (3’). From the marginal productivity condition (2), we see that \( r = \beta Y/K \) where \( \beta \) is the initial capital share.\(^2\) Substitution leads to:

\[ B(t) = K(0)[g - sr / \beta]e^{rt} + srD \]  

Subsequently we express \( D \) in terms of GDP (Y). To this purpose, we first divide both sides of equation (3’’) by \( D \), subtract \( g \) – the steady-state growth rate of \( Y \) – and then multiply by \( D/Y \).\(^3\) The result is

\[ d(t) = k[g - sr / \beta] - [g - sr]d \]  

where \( d \) denotes the ratio of debt over GDP (\( D/Y \)). In equation (5), \( k \) is the capital-output ratio from equation (2). The sign and size of the slope and the intercept depend on the (unobserved) parameters \( g, s, \beta, \) and \( r \) and can be positive or negative. The three possible cases – corresponding to the different stable and unstable paths of the differential equations – are shown in figure 1. In this figure \( d \) is on the horizontal axis, while \( B(t) \) is on the vertical one.

\(^2\) This expression is only used to eliminate \( Y(0) \) in the differential equation.

\(^3\) Alternatively, we may express \( D \) as a percentage of the stock of efficient labor (AL). Equation (5) is unaltered by this transformation, apart from the definition of \( d \), which now is \( D/AL \). Unreported results show that our empirical results are qualitatively the same for both definitions of \( d \). We prefer to use GDP as the denominator as both GDP and D are generally available in USS, whereas AL is only available in real terms. Hyperinflationary periods make conversion much more problematic when AL is used to normalize.
If \( \frac{sr}{\beta} > g > sr \), both the slope and the intercept are negative. The corresponding line in figure 1 is indicated by SC. The stable stationary point (I) is at negative values of \( d \), implying that the country becomes a creditor in the long run. In point I, the ratio of foreign wealth (\( D \)) to GDP remains constant if there are no more shocks. Both \( D \) and \( Y \) grow at the same rate \( g \). Since \( D \) is negative, it implies the country holds positive net foreign assets in the steady state. Suppose the country starts out as a debtor at positive values of \( d \). Subsequently, \( d \) will decrease to converge to the steady state along line SC. It can be easily shown that initially the level of external debt \( D \) will still grow – though at a slower pace than GDP – corresponding to a current account deficit. Over time, the current account deficit is reduced and turns into a surplus with reduction in \( D \). This case is the traditional view on the debt cycle in which every country eventually becomes a creditor (non-debtor).

If \( g > \frac{sr}{\beta} \), and \( g > sr \), the slope remains negative, but the intercept becomes positive. This case is captured by line SD in figure 1. Now, the country converges to a steady state (II) where \( d \) is positive. The economy remains a debtor because of its low savings ratio and/or the low world interest rate. The low savings rate forces the country to externally finance its domestic investment, while the low world interest rate helps to reduce the cost of doing so. In the steady state, \( D \) again grows at the rate \( g \), which implies that the country remains a capital importer. The current account is not balanced in the long run unless the horizontal intercept of the differential equation is at the origin.\(^4\)

If \( g - sr < 0 \), the slope is positive and the vertical intercept is negative. This is the unstable case, labeled U in figure 1. By implication, \( g - sr > 0 \) is a stability condition, but obviously, there is no reason to a priori impose such stability assumption. The stationary but unstable point (III) is at a positive value of \( d \). If \( d(0) \) is less than in the stationary point, the economy starts moving to the region of negative and continuously declining \( d \). In that case, the country will be a creditor in the long run, where GNP grows faster than GDP (see appendix A). This was the point Amano (1965) tried to make.

\(^4\) In the literature it is often assumed that the current account must be balanced in the long run, based on the assumption that creditors will try to increase their welfare by not allowing other countries to borrow permanently (see Cohen 1991). Whether this is the case, however, is an empirical as well as a theoretical question. In our model we do not explicitly impose the restriction of a balanced CA.
A starting point to the right of the stationary point implies that \( D(0) \) sufficiently exceeds \( K(0) \), as shown in appendix A. The country then starts out with more debt than capital and will have negative current wealth for all periods. Clearly this is unsustainable in the longer run. A country on such explosive path will be forced to adjust, one way or the other.\(^5\) As \( D(0) > K(0) \) seems to be an unrealistic case from an empirical point of view, we will interpret explosive debt dynamics as a case of sovereign risk when it appears in our estimates.

In this type of model, a switch from one dynamic process – say the SD line – to another – say the SC line – can only arise due to changes in the underlying parameters of the system, \( g, s, \beta \), and \( r \).\(^6\) As an example, we will discuss a permanent upward jump in the world interest rate. If this happens, the marginal product of capital will increase and the optimal amount of capital \( K \) to be used will fall. For a given national wealth \( W \), a fall in the desired level of capital \( K \) implies that less external debt \( D \) is required. As a result less debt is obtained from the international capital market. Alternatively stated, it implies that more domestic wealth is invested abroad at the higher world interest rate level. Such a decrease in capital inflows or increase in capital outflow – even capital flight – thus can be rooted in the economy’s fundamentals and need not be a purely monetary phenomenon of irrational speculators. Domestically, the resulting lower marginal product of labor leads to a fall in real wages. This is the social side of the crisis.

We can also describe these dynamics in terms of differential equation (5) and Figure 1. Starting from a situation where the country is a long-run debtor (the SD line), a jump in the interest rate leads to both a counter-clock wise rotation (flatter slope) and a downward shift of the SD line. The economy can move from the situation \( g > sr/\beta > sr \) (SD in figure 1) to a situation \( sr/\beta > g > sr \) (SC) or even to a situation \( sr/\beta > sr > g \) (U), depending on the size of the interest rate shock.

\(^5\) Neher (1970) assumes that debt \( d \) grows at the same rate as the capital stock. As we have shown, this is only the case in the steady states of the stable cases. Amano (1965) has treated only the left arm of the unstable case. Our model integrates all these cases and can be seen as a simplified version of the model by Onitsuka (1974).

\(^6\) Of course, there can also be exogenous shocks to \( d \) without changing the prevailing differential equation. This moves the economy along the relevant line in figure 1 rather than changing the position of the line itself.
A similar analysis can be given for other exogenous events changing the slope and intercept of the differential equation, like for instance a jump in the savings rate $s$ or in the rate of population growth $n$ or technical change which in turn changes $g$.

In the empirical part of our paper, we will present evidence on the extent to which Argentina and Brazil respectively have switched between the different regimes of debt dynamics over the period 1960-1999. The empirical equivalent of equation (5) that we use in our estimation is:

$$\delta = \alpha_0 + \alpha_1 d(-1)$$

with $\alpha_0 = k(g - sr/\beta)$, $\alpha_1 = sr-g$. Clearly, the regression coefficients are complex functions of behavioral parameters like the savings rate and exogenous but time-varying variables like the world interest rate. Without additional information on these parameters and variables, we can just report the regression estimates and account for potential time-variation. If more information were available, an extended regression equation could be estimated. For instance, if adequate information on interest rate $r$ could be used, one could (nonlinearly) regress $\delta$ on $d(-1)$, $r$, and $r^*d(-1)$. We leave such exercises for future research. Note also that the explanatory variable in equation (6) is lagged one year. This is due to the switch from the continuous-time derivation of our theoretical model to the discrete-time formulation of the empirical model.

4. Data

For our analysis, we need the stock of external debt for each country and GDP. While GDP is readily available both in current and constant prices, this is not the case for developing country’s stock of external debt. What is available is the time series of gross investments and savings for each country. These are flow variables and the difference between the two can be used as a proxy for the change in a country’s debt position ($D$). In order to arrive at an adequate measure of the level of $D$, we focus on the year 1992. The procedure is as follows, where we use Brazilian data for illustrative purposes. Clearly, the procedure for Argentina is the same.
First, we use the IMF Balance of Payments Statistics\textsuperscript{7} for 1992 to find investment income paid to the rest of the world (9102 million US$) and investment income from abroad (1100 Million US$). The difference (8002 million US$) corresponds to rD in our model. For an estimate of D, an appropriate value for the interest rate r is needed. To obtain an estimate of r, we compute the ratio of investment income paid abroad (9102 million US$) in 1992 to the gross stock of external Brazilian debt in 1992 (about US$ 128, 741 million US$) according to the Global Development Finance database. The resulting interest rate is about 7.07 %. Dividing net interest payments (rD = 8002 million US$) by 0.0707 yields a net foreign debt position of about 113 billion US$ as an initial value for 1992. Subsequently, we cumulatively add (subtract) the gross investment-savings balance for later (previous) years. The resulting debt series is divided by nominal GDP (in US $) to arrive at our series d (and its first difference \(d')\). Appendix B contains the data.

The time paths of Argentine and Brazilian debt/GDP ratios can be found in figure 2. Both countries experienced declining debt ratios in the 1960 and early 1970s, reaching a floor of about 20 percent around 1975. Argentina roughly remains on that level – suggesting a sort of steady state as a permanent debtor – until 1990. Then a gradual rise sets in. In Brazil, debt moves steeply up again after 1975 to a peak in 1984 and a return to approximately the 1975 level around 1990. After 1996, an upward trend to a ratio of 35\% is present again.

5. Empirical Results

We first estimate equation (6) for Brazil and Argentina over the whole sample period, 1961-1999. Table 1 contains the results. The average debt dynamics for Argentina and Brazil appear to be qualitatively and quantitatively similar. For both countries the intercept is positive and (marginally) significant, while the slope is negative. Using standard critical values, the slope coefficients appear significant. However, one may notice that this dynamic specification can be interpreted as a simple Dickey-Fuller unit root test – or equivalently a stability test – of the debt ratio. It is well-known that standard

\textsuperscript{7} When sources are not explicitly mentioned the data are from the World Development Indicators, 1998 or Global Development Finance, 2000.
critical values is invalid in case the dependent variable in fact is nonstationary. The correct 10 percent MacKinnon critical value is 2.60, so that statistically speaking we cannot reject the slope to be zero. Equivalently, we cannot formally reject the debt ratio to be a random walk and behaving as an unstable process. Nevertheless, the results do at least suggest that on average both countries behaved over the period as if they were on the SD line. That is, the regressions predict that both countries over time converge to a steady state external debt to GDP ratio of around 25 percent.

Part of the lack of strong statistical results may come from the short period we are investigating. It is well known that unit-root tests have low power in small samples. Unfortunately, there is little we can do to solve this problem as extending the sample backward is infeasible due to data limitations. Only time can solve the issue. Another explanation for the lack of statistical significance is that the debt dynamics of the two countries under consideration have switched back and forth between the three different regimes (SD, SC, and U). In that case, the average estimate will be biased towards zero and insignificance.

In fact, we are interested exactly in the issue if and when countries switch between regimes and to what extent it contributes to predicting that we are moving into the direction of a crisis (from stable debtor to stable creditor to an unstable situation). Therefore, we would like to identify the sub periods over which different regimes were operative. But small sample problems then become even more overwhelming than in the full sample case. Before presenting at least some evidence of time-variation in the debt dynamics in Brazil and Argentina, we shortly discuss the routes we explored but not used in the end.

First we re-estimated equation (6) with dummy variables (for both intercept and slope). We took economically plausible periods for the sub periods: 1961-1973 (period before the first oil crisis), 1974-1983 (period up till the Latin-American debt crisis), and 1984-1996 (the period between the debt crisis and the Asian crisis). The period 1997-1999 was used as – an admittedly short – benchmark period. As could be expected, results failed to gain significance due to the lack of degrees of freedom. Moreover, we could not be sure we had picked the right timing of regime switches. More sophisticated methods to endogenously determine the timing of structural breaks were inapplicable due
to the short time series. Second, we tried Markov regime-switching and state space estimation with random walk coefficients. Neither approach yielded convergence or interpretable results, again due to the short length of our time series.

Therefore, we turned to a rolling regression approach, where equation (6) is estimated for consecutive windows of seven years. In figure 3 we present the time paths for estimated intercept and slope of the Argentine debt dynamics plus and minus one time the standard deviation of the regression coefficient. Due to the small samples, we don’t expect formal statistical significance. However, when the one standard deviation interval does not include zero – that is, the t-statistic is in excess of unity –, we take this as supportive evidence for a non-zero value of the coefficient. The figure shows that in most 7-year intervals the intercept was positive, while the slope was negative. In these intervals, Argentine debt dynamics apparently can be characterized as consistent with the SD line in figure 1.

Exceptions in the 1970s and 1980s are the sub periods 1967-1973, 1968-1974, and 1978-1984. In these intervals, the intercept becomes negative and the slope positive. This would be the case of the unstable path (U). The first two intervals cover the end of the Bretton Woods system and the subsequent oil crisis, the latter interval includes the contractionary US monetary policy after 1979 with rising interest rates, the global recession in 1980-1981, and the Latin-American debt crisis in 1982-1983. In our view, the fact that the Argentine debt dynamics switch to an unstable time path exactly in these specific periods provides suggestive support for our approach. It is clear, though, that the regression coefficients switched to unstable parameter values only after the shock had occurred. Based on the 1975-1981 results, we would not have predicted the Latin-American debt crisis in 1982-1983. So even if these estimations for the period 1978-84 correctly register the switch to unsustainable debt dynamics after the facts, we do not find predictive power in the 1970s or 1980s. The same appears to be true in the 1990s. From the interval 1991-1997 onward, we document a slightly positive slope and an almost zero

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8 See Bai and Perron (1998) and Perron and Vogelsang (1992) for such methods.
9 Alternatively, we used 6 and 8 year windows. Since the results are qualitatively invariant to such variations in the size of the window, we only report results for the 7-year window regressions.
10 Note, however, that in the periods 1967-73 and 1968-74 the data in the appendix show falling debt/GDP ratios with one or two exceptions and the economy therefore was on the good arm of the unstable debt dynamics.
intercept, suggesting unstable debt dynamics. Since 1992 all observations show increasing debt/GDP ratios, thus indicating that the economy is on the unsustainable arm of the unstable dynamics. However, again this appears to be observable only after the problems – in this case the Asian crisis and its spillover effects – had emerged.

Closer inspection of the Argentine data shows that the above conclusion may be too negative. The regression estimates for the later part of the sample appear to be considerably influenced by the outlier observation of 1990 when a very large decrease in debt occurred. In figure 4, we present similar regression coefficients through time using 7-year windows, excluding the year 1990. Starting with the 1985-1992 window, we now observe increasingly positive slopes and increasingly negative intercepts, pointing to deteriorating debt dynamics long before the Asian crisis. Moreover, the start of the much hailed currency board for the Argentine currency in 1991 has no mitigating effect at all on the worsening debt situation and may even have reinforced it because of a lack of due devaluations. After 1996, the coefficients improve somewhat, though they keep having the wrong sign. We conclude that our regressions do warn for Argentine debt problems long before they surfaced in practice when the outlier observation of 1990 is taken out. On the other hand, we observe strongly increasing uncertainty about the regression coefficients from 1982-1989 onwards as reflected in the large standard error in figure 4. Although the point estimates suggest problems, uncertainty about these point estimates was large and increasing. Moreover, in the period 1985-92 most of the observations show falling debt/GDP ratios. The upshot here is that once the dynamics has the unstable structure of regime U small shocks seem to be enough to move the economy from the good arm to the unsustainable one.

Finally, we turn to the case of Brazil. We present 7-year rolling regressions coefficients in figure 5. Brazil – like Argentina – appears to have been characterized by stable debt dynamics over most of the intervals. Positive intercepts and negative slopes suggest Brazil is in a steady state debtor regime most of the time. Exceptions are the late seventies and early eighties (the windows 1974-1980 through 1977-1983), the late eighties (1984-1990) and late 1990s (1993-1999) where the slope moves towards zero. In the 1990s, problems in Brazil appear much later and weaker than in Argentina. Again, the intervals with unstable debt dynamics are easily identified as periods in which Brazil
suffered from real problems. However, we do not succeed in tracing patterns of instability prior to the occurrence of the problems themselves. On the other hand the trend towards positive slopes may continue and predict problems coming up after 1999.

In general, the data and our empirical results suggest that switches to undesirable and unstable debt dynamics tend to be sudden and short-lived, making them hard to predict. These characteristics make it also difficult to find statistically convincing proof of our model. We do show, however, that the different qualitative regimes of the theoretical model can be observed in reality. Most of the time, this does not allow predicting coming crises. Argentina in the early 1990s appears an exception. When excluding the outlier year 1990, warning signals of coming problems were around already in the late 1980s and early 1990s. Debt dynamics actually deteriorated after the imposition of the currency board in 1991.

6. Conclusion

In this paper we develop a simple neoclassical growth model with perfect international capital mobility to analyze the international debt dynamics of developing countries in general and Brazil and Argentina in particular.

We show that three different regimes can be distinguished: a stable regime where the country in the end always converges to a steady state debtor position, a stable regime where the country ends up in a steady state creditor position and an unstable regime which leads the country either to become a creditor without ever reaching a steady state or to get on an unsustainable path with an exploding debt ratio. Switches between these regimes can be caused only by shocks in the world interest rate or behavioral parameters like the country’s savings rate or rates of population growth and technical progress.

A switch from a stable debtor to a stable creditor or even an unstable creditor regime is a sign of forthcoming trouble. If one would be able to identify such shift in advance, it would contribute to our ability to predict debt crises and to our understanding of such crises.

To investigate this issue empirically, we construct debt ratio series for Brazil and Argentina over the period 1960-1999. Subsequently, we use these series to do the simple debt dynamics regression derived from the theoretical model. Over the full sample, the
evidence suggests that debt dynamics evolved according to the stable debtor case in both countries, although the results lack formal significance. Both the small sample and the possibility of short-lived intermediate switches between regimes over time prevent us from obtaining stronger statistical evidence. Using a rolling regression technique, we find that indeed occasional switches to the unstable regime occurred. The periods in which we find evidence of unstable regimes correspond with well-known problematic episodes in the countries under investigation. In that sense, our model appears a useful tool of the analysis of a country’s debt dynamics. In general, the rolling regression approach has little forecasting capacity, unfortunately. The exception may be Argentina in the early 1990s where we find evidence of unstable debt dynamics before the start of the currency board. According to our results, Argentina was in the unstable regime for most of the 1990s. In practice, Argentine problems only surfaced after 1997. For Brazil, our results show that debt dynamics deteriorated only after 1997, as they did in practice.

A few caveats are in order. In the model, we ignore spreads on LIBOR (EURIBOR) and the prime rate. Moreover, the model does not take into account any features that are typical of developing countries such as imported capital goods, which would link the goods market equation to the balance of payments equation. Including these aspects makes a model much more complicated but has the potential for some improvement. We leave this for future research.
References


Appendix A: Solution of the differential equation

The differential equation has the solution

\[ D(t) = D(0)e^{rt} + K(0)B(e^{rt} - e^{st}) \]  

(A1)

with \( B = \frac{(g-sr/\beta)}{(g-sr)} \). In the long run the larger of the two growth rates will dominate the process:

\[ \lim_{t \to \infty} D(t) = \begin{cases} e^{rt}K(0)B = K(t)B & \text{if} \quad g-sr > 0, \quad \text{or} \\ e^{rt}[D(0) - K(0)B] & \text{if} \quad g-sr < 0 \end{cases} \]  

(A2)

As \( sr/\beta > sr \), we can distinguish four cases, which are interpreted as alternative levels of the savings ratio in Onitsuka (1974):

1. \( g > sr/\beta > sr \) implies \( 1 > B > 0 \) and \( K(t) > D(t) > 0 \)
2. \( sr/\beta > g > sr \) implies \( B < -1 \) and \( K(t) > 0 > D(t) \)
3. \( sr/\beta > sr > g \) implies \( B > 1 \) and \( K(0) > D(0) > 0 > D(t) \)
4. \( sr/\beta > sr > g \) implies \( B > 1 \) and \( D(0) > K(0) > 0 \)

In the first case, the country remains a debtor, but the capital stock is larger than the debt. In the second case the country becomes a creditor and foreign wealth \( D \) is larger than capital \( K \). In the third case, assuming \( D(0) - K(0) < 0 \), \( D \) becomes negative and the country is again a creditor in the long run. In the fourth case, if \( D(0) - K(0) = -W(0) > 0 \) and sufficiently large, \( D \) remains negative, grows at the rate \( sr \) while capital \( K \) grows at rate \( g \). This case if fairly unrealistic because it implies that the country has negative current wealth from the beginning and through eternal times while we see from the data that all countries have positive savings at almost all times.
### Appendix B: Data

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### Table 1  
* A simple debt dynamics regression, 1960-1999

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Figure 1 Three different regimes for debt dynamics
Figure 2a  The Argentine debt ratio (D/Y) over time

Figure 2b  The Brazilian debt ratio (D/Y) over time
Figure 3

* The years in the graphs indicate the last years of the 7-year intervals.
Figure 4

* The years in the graphs indicate the last years of the 7-year intervals.
* The years in the graphs indicate the last years of the 7-year intervals.