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Abstract

Problem definition: Streaming platforms such as Spotify are popular media services where content creators may offer their content. Because these platforms operate in a highly competitive market, content creators may leave the platform and join elsewhere. This paper studies conditions under which content creators have no incentives to leave the platform and thus stability can be preserved. **Methodology/results:** We introduce a stylized model for streaming platform situations and associate these situations with a cooperative game. We focus on the (non)emptiness of the core to analyze the stability of the streaming platforms. It turns out that both stable and unstable streaming platforms exist. We show that for streaming platforms operating in a market where users have completely opposite streaming behavior, stability cannot always be preserved. However, in markets where users are more similar in their streaming behavior, stability can be preserved. We further analyze the stability of streaming platforms by means of numerical experiments. Our results indicate that stability of streaming platforms generally is a delicate matter. **Managerial implications:** Streaming platforms are more likely to be stable in markets where users are similar in their streaming behavior. To avoid that content creators leave the platform, it is therefore recommended to focus on particular market segments where these similarities occur.

Keywords: streaming platforms, stability, cooperative game theory, core
JEL classification: C71

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1 Introduction

With more than 400 million active users monthly in 2021, Spotify is one of the most used streaming platforms in the world. Despite its popularity among users, Spotify has been criticized by content creators (artists) who believe that they should earn more from providing their content on the platform. In 2021, a group of content creators held a series of protests in front of Spotify and various famous artists such as Sting, Kate Bush, and Paul McCartney spoke out against too low payments. Because Spotify operates in a highly competitive market, with competitors such as Apple Music, Deezer, and Soundcloud, it is important to take these signals seriously. Otherwise artists, or record labels that hold their rights, may switch to other streaming platforms or decide to collaborate and start a platform by themselves. In other words, it is important for Spotify to allocate the streaming revenues in such a way that stability of the streaming platform is preserved. In this paper, we analyze in what type of situations stability can be preserved.

We introduce a stylized model for streaming platform situations where content creators offer their content to users on a platform. In this model, users pay a fixed premium fee per subscription period to get unlimited access to all content of all creators on the platform. The total revenue of the platform is the product of this premium fee and the number of subscribed users. The streaming platform tries to allocate the total revenue among the content creators in such a way that stability of the platform is preserved.

To define stability, we use concepts from cooperative game theory. We say that a streaming platform is stable if it is able to divide the total revenue in such a way that no group of content creators has incentives to leave the platform. Here, content creators have incentives to leave the platform if they receive less than what they could obtain by starting their own streaming platform. We determine the total revenue of such a streaming platform as follows. First, we identify the streaming matrix of the streaming platform and the reservation prices of all users. The streaming matrix describes the average number of streams of each content creator for each user per subscription period and the reservation price of a user is the maximum amount of money (s)he is willing to pay per stream per period. The latter information can, for instance, be estimated from marketing data. Based on this streaming matrix and the reservation prices of all users, we determine which users are willing to subscribe to the new platform for any given premium fee, taking into account that only the content of the content creators in the group is offered. We assume that the group of content creators charges a premium fee on their own platform that maximizes the total revenue (i.e., that maximizes the product of the premium fee and the number of users that would subscribe to their platform). We introduce a cooperative game that lists the total revenue for each group of content creators and identify the core of this game to analyze the stability of the streaming platform.

By means of two examples, we illustrate that both stable and unstable streaming platforms exist. In particular, we show (in Proposition 1) that for streaming platforms operating in disjoint market segments (i.e., streaming platforms operating in a market where users have completely opposite streaming behavior), stability cannot always be preserved. However, in markets where users are more similar in their streaming behavior, stability can be preserved. We identify two such types of markets (in Proposition 2 and Proposition 3). The first ones are called inclusive markets, where each group of content creators would charge a premium fee on its own streaming platform such that all users subscribe. The second ones are called homogeneous markets, where all users are willing to pay the same amount of money to stay on the platform.

We further analyze the stability of streaming platforms by means of numerical experiments. In particular, we focus on streaming platforms that do not operate in disjoint market segments, inclusive markets, or homogeneous markets. The numerical experiments indicate that stability of streaming platforms generally is a delicate matter. In particular, we observe that streaming platforms switch multiple times from stable to unstable and vice versa when we gradually turn an inclusive market into disjoint market segments.

The remainder of this paper is organized as follows. Section 2 provides a literature overview of the main advancements in the related research disciplines. In Section 3, we introduce streaming platform situations and associated streaming platform games. The stability of streaming platforms is investigated in Section 4 and numerical experiments are presented in Section 5. We conclude this paper in Section 6.

2 Literature review

Our work contributes to the new stream of literature on operations management (OM) problems for music streaming platforms, as well as to the stream of literature on applications of cooperative game theory.

2.1 OM problems for music streaming platforms

One of the first papers in this new stream of literature is Li et al. (2020). This work investigates whether it is beneficial for music streaming platforms to offer both free and subscribed services. Free services come with advertisements (e.g., a short commercial break after a certain number of songs) and/or functionality limitations such as lower sound quality. It is shown that a music streaming platform should offer only subscribed services if the advertisement rate (i.e., the revenue the platform makes from one unit of advertisements) is relatively low. For moderate advertisement rates, it is better to offer both services, while it is optimal to offer only free services for high advertisement rates. A similar problem has been addressed in Amaldoss et al. (2021), except that multiple music streaming platforms are now competing in a market for users and content creators. In such a competitive market, it is never optimal to offer subscribed service only. As suggested in DeValve and Pekeč (2022), music streaming platforms can also offer multiple types of subscribed services that vary in the level of advertisement, where services with more advertisements are charged for a lower price. The authors show that streaming platforms can benefit from offering such a menu of services.

Alaei et al. (2022) is the most recent study in the stream of literature on OM problems for music streaming platforms. In this paper, two well-known allocation rules, namely the pro-rata and user-centric rule, are studied. The pro-rata rule pays content creators proportionally to their share of the overall streaming volume, whereas the user-centric rule divides each user's subscription fee proportionally among the content creators based on the streaming behavior of that user. The authors show that the pro-rata rule is preferable if a streaming platform wants to maximize its revenue. They do so by taking into account that content creators and users can decide to join, but also leave, the platform based on individual considerations.

The four mentioned papers have something in common: they all focus on *optimizing* the performance of streaming platforms (e.g., offering the best menu of services or selecting the allocation rule that maximizes total revenue). Our paper deviates in that regard: we focus on *evaluating* the performance of a streaming platform. That is, we evaluate whether an existing streaming platform is able to divide the total revenue in such a way that no group of content creators has incentives to leave the platform. As such, we contribute to the literature on OM problems for streaming platforms by offering an evaluation method that determines the stability of an already existing streaming platform.

From a modelling perspective, we are not the first ones that incorporate individual considerations of content creators. In particular, in Alaei et al. (2022), content creators may leave the streaming platform if their individual revenue share is smaller than an outside option value. In our paper, we also allow for this, but on top of that allow content creators to leave the platform *collectively* and to start an own streaming platform themselves. We are the first ones that incorporate this group behavior and illustrate that exactly this possibility may make streaming platforms unstable.

2.2 Applications of cooperative game theory

Cooperative game theory has been applied to various domains. In this section, we discuss some of these recent applications in more detail.

A recent application of cooperative game theory to the healthcare sector is Westerink-Duijzer et al. (2020). Here, multiple countries together decide how a limited number of doses of vaccine have to be distributed among the inhabitants of these countries. The authors derive sufficient conditions under which the total return from such a cooperation can be allocated among the countries in a stable way.

Another application domain is the humanitarian sector. Typically, humanitarian supply chains involve many different entities, such as government, military, private, and non-governmental organizations. Well-coordinated interactions between entities can lead to synergies and improved humanitarian outcomes. An example of such a coordinated interaction is discussed in Ergun et al. (2014). The authors study the distribution of costs or benefits from multi-agency coordination and obtain insights about the conditions under which this distribution is stable.

Cooperative game theory has also been applied to the public transport sector. For instance, Algaba et al. (2019) investigates how revenues from an all-in-one travel card should be distributed among participating public transport companies (e.g., metro or bus). The authors introduce two rules that distribute these revenues in a stable way. A collaborative initiative in the transport sector is discussed in Van Zon et al. (2021). The authors consider several transport companies that can pool their fleet to reduce transportation costs and/or pollution. They model a corresponding cooperative game and introduce a row generation algorithm for either determining a core element or concluding that the core is empty.

Bergantiños and Moreno-Ternero (2020) applied cooperative game theory to the television broadcasting industry. They analyze the problem of sharing the revenues from broadcasting sports leagues among participating teams. They introduce two allocation rules: the equal-split rule and the concede-and-divide rule. Both an axiomatic approach and a game-theoretical approach are used to analyze these two rules.

Cooperative game theory has also been applied to the retail sector. For instance, Nip et al. (2022) illustrate how retailers can benefit when coordinating the assortment planning. In particular, each retailer possesses a couple of products and the common objective is to select a subset of those products, to offer to the customers, that maximizes the expected revenue. They show that nonemptiness of the core depends on the behavior of the players outside the coalition. Another example is discussed in Chen and Zhang (2016). The authors show that retailers can benefit by placing joint orders to a supplier. Under the assumption of linear holding and backorder costs, they show nonemptiness of the core. Inspired by shipments for fashion products with long supplier lead times, Özen et al. (2008) illustrate how retailers can benefit by coordinating their orders for delivery into one or more supply nodes, and subsequently reallocating their orders after the demand realization. The authors show that the associated cooperative game has a nonempty core and provide a complete description of this set.

The final application domain we discuss is the service industry. In this industry independent service providers may collaborate by pooling their resources into a joint service system. These service providers may represent diverse organizations such as hospitals that pool beds or maintenance firms that pool repairmen or spare parts. Karsten et al. (2015) illustrates that the costs of such a pooled setting can be allocated in a stable way if resources are pooled completely and service providers are symmetric. Schlicher et al. (2020) shows that stability can always be achieved when optimal pooling of resources is applied.

In summary, cooperative game theory has been applied to several domains for analyzing the stability of collaborations. Our paper contributes to this stream of literature by applying cooperative game theory to the music streaming industry and providing sufficient conditions for stability.

3 The model

3.1 Streaming platform situations

Consider a streaming platform (e.g., Spotify) with a nonempty and finite set N of *content creators* offering their content (e.g., songs). On top of that, there is a nonempty and finite set U of *users* subscribed to this platform. All streams of these users are tracked by a *streaming matrix* $V \in \mathbb{Z}_+^{N \times U}$, where V_{ij} denotes the number of streams of content creator $i \in N$ by user $j \in U$. We assume that each content creator is streamed by at least one user, and each user streams at least one content creator. As a result, $\sum_{j \in U} V_{ij} > 0$ for all $i \in N$, and $\sum_{i \in N} V_{ij} > 0$ for all $j \in U$. Each user $j \in U$ has a *reservation price* $r_j \in \mathbb{R}_{++}$, reflecting the maximum amount of money that this user is willing to pay per stream. Finally, each user pays the same *premium fee* $p^N \in \mathbb{R}_{++}$ to the streaming platform in order to get unlimited access to all content of all creators on the platform. This premium fee is determined by the maximum amount of money that can be charged per user, in such a way that *all* users are willing to stay on the platform. Note that the maximum amount of money that user $j \in U$ is willing to pay to stay on the platform is $r_j \cdot \sum_{i \in N} V_{ij}$, so all users are willing to stay on the platform by charging the minimum of these maximum amounts, i.e.,

$$p^N = \min_{j \in U} \left\{ r_j \cdot \sum_{i \in N} V_{ij} \right\}.$$

Because each user pays this premium fee, the total revenue of the streaming platform is given by $p^N \cdot |U|$. We summarize this *streaming platform situation* as a quadruple $\theta = (N, U, V, r)$.

We now illustrate a streaming platform situation by means of an example. Note that the parameters are not selected to represent reality, but to keep calculations simple and easy to follow.

Example 1. Let $\theta = (N, U, V, r)$ be a streaming platform situation with content creators $N = \{i_1, i_2, i_3\}$, users $U = \{j_1, j_2, j_3\}$, streaming matrix

$$V = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \end{array} \begin{array}{c} \begin{array}{ccc} j_1 & j_2 & j_3 \end{array} \\ \begin{bmatrix} 2 & 2 & 0 \\ 4 & 0 & 4 \\ 6 & 4 & 0 \end{bmatrix} \end{array},$$

and reservation prices $r = (1, 2, 3)$. User j_1 is willing to pay at most $r_{j_1} \cdot \sum_{i \in N} V_{ij_1} = 1 \cdot (2+4+6) = 12$, user j_2 is willing to pay at most $r_{j_2} \cdot \sum_{i \in N} V_{ij_2} = 2 \cdot (2+0+4) = 12$, and user j_3 is willing to pay at most $r_{j_3} \cdot \sum_{i \in N} V_{ij_3} = 3 \cdot (0+4+0) = 12$. Hence, the streaming platform charges a premium fee of $p^N = \min\{12, 12, 12\} = 12$ to each user, leading to a total revenue of $p^N \cdot |U| = 12 \cdot 3 = 36$. △

3.2 Streaming platform games

In a streaming platform situation $\theta = (N, U, V, r)$, a nonempty and strict subset of content creators $S \subset N$ could decide to leave the platform and start its own streaming platform where it offers all content of the creators in S. The nonempty set of users that stream the content of creators in S is given by

$$U^S = \left\{ j \in U \;\middle|\; \sum_{i \in S} V_{ij} > 0 \right\}.$$

The positive maximum amount of money that user $j \in U^S$ would be willing to pay to subscribe to this new streaming platform of S is given by

$$t_j^S = r_j \cdot \sum_{i \in S} V_{ij}.$$

Consequently, for a premium fee $p \in \mathbb{R}_{++}$, the corresponding set of subscribing users is given by

$$Q^S(p) = \left\{ j \in U^S \;\middle|\; t_j^S \geq p \right\}.$$

Note that a higher premium fee never leads to additional subscribers, i.e., $Q^S(p) \subseteq Q^S(p')$ if $p > p'$. The group of content creators charges a premium fee $p \in \mathbb{R}_{++}$ that maximizes the total revenue, i.e., maximizes the premium fee multiplied by the corresponding number of subscribing users, so

$$\max_{p \in \mathbb{R}_{++}} \left\{ p \cdot |Q^S(p)| \right\}.$$

This maximization problem boils down to comparing at most $|U^S|$ local optima with each other. The locations of these local optima correspond to the maximum amount of money that one (or some) of the users in U^S would be willing to pay to subscribe to the platform of S. As a result,

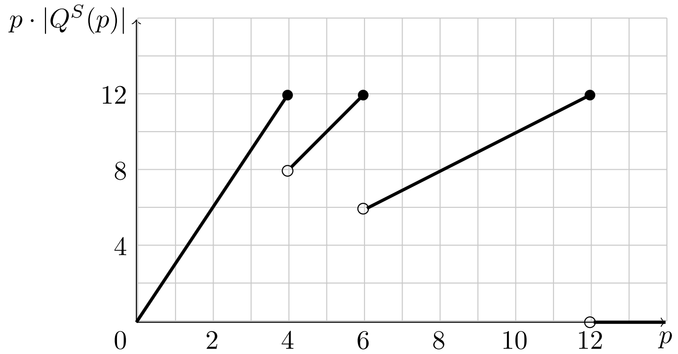
$$\max_{p \in \mathbb{R}_{++}} \left\{ p \cdot |Q^S(p)| \right\} = \max_{j \in U^S} \left\{ t_j^S \cdot |Q^S(t_j^S)| \right\}.$$

The nonempty set of optimal premium fees $P^S \subseteq \mathbb{R}_{++}$ is given by

$$P^S = \operatorname*{argmax}_{p \in \mathbb{R}_{++}} \left\{ p \cdot |Q^S(p)| \right\}.$$

As a result, $P^S \subseteq \{t_j^S \mid j \in U^S\}$. The following example illustrates that the optimal premium fee does not need to be unique.

Example 1 (continued)**.** Consider the group of content creators $S = \{i_1, i_2\}$. Then $U^S = U$, i.e., all users stream the content from the creators in S. The maximum amount of money that users would be willing to pay to subscribe to the streaming platform of S is given by $t_{j_1}^S = r_{j_1} \cdot \sum_{i \in S} V_{ij_1} = 1 \cdot (2+4) = 6$ for user j_1, $t_{j_2}^S = r_{j_2} \cdot \sum_{i \in S} V_{ij_2} = 2 \cdot (2+0) = 4$ for user j_2, and $t_{j_3}^S = r_{j_3} \cdot \sum_{i \in S} V_{ij_3} = 3 \cdot (0+4) = 12$ for user j_3. Hence, for a premium fee p with $0 \leq p \leq 4$, all three users would subscribe to the platform and thus the total revenue equals $p \cdot 3$. For a premium fee p with $4 < p \leq 6$, only users j_1 and j_3 would subscribe to the platform and thus the total revenue equals $p \cdot 2$. For a premium fee p with $6 < p \leq 12$, only user j_3 would subscribe to the platform and thus the total revenue equals $p \cdot 1$. For a premium fee p with $p > 12$, no user is willing to subscribe to the platform and thus the total revenue equals 0. This is summarized in the following figure, where we depict the total revenue $p \cdot |Q^S(p)|$ as a function of the premium fee p.



Hence, the maximum total revenue of 12 is obtained by charging a premium fee from $P^S = \{4, 6, 12\}$. △

Taking into account that each group of content creators could possibly leave and start its own streaming platform, we associate to each streaming platform situation $\theta = (N, U, V, r)$ a *streaming platform game*. This game is a pair (N, v^θ) where the content creators in N take the role as *players* and v^θ assigns to each nonempty *coalition* $S \subseteq N$ the maximum total revenue it would obtain from its own streaming platform, i.e.,

$$v^\theta(S) = \begin{cases} p^N \cdot |U| & \text{if } S = N, \\ \max_{p \in \mathbb{R}_{++}} \{p \cdot |Q^S(p)|\} & \text{if } S \subset N. \end{cases}$$

We conclude this section with an illustrative example of a streaming platform game.

Example 1 (continued)**.** The total revenue of the streaming platform is given by $p^N \cdot |U| = 12 \cdot 3 = 36$ and thus $v^\theta(N) = 36$. For coalition $\{i_1, i_2\}$, the maximum total revenue it would obtain from its own streaming platform is given by $\max_{p \in \mathbb{R}_{++}} \{p \cdot |Q^{\{i_1,i_2\}}(p)|\} = 12$, so $v^\theta(\{i_1, i_2\}) = 12$. The total revenue of each other coalition can be determined in a similar way. The resulting streaming platform game (N, v^θ) is presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_3\}$	$\{i_1, i_2\}$	$\{i_1, i_3\}$	$\{i_2, i_3\}$	$\{i_1, i_2, i_3\}$
$v^\theta(S)$	4	12	12	12	16	24	36

△

4 Stable streaming platforms

4.1 The core

To analyze the stability of streaming platform situations, we focus on the core of the associated streaming platform games. Let $\theta = (N, U, V, r)$ be a streaming platform situation and let (N, v^θ) be the associated streaming platform game. The *core* $C(N, v^\theta)$ consists of all allocations $x \in \mathbb{R}^N_+$ that satisfy the following two conditions:

$$\textit{efficiency:} \quad \sum_{i \in N} x_i = v^\theta(N),$$

$$\textit{coalitional rationality:} \quad \sum_{i \in S} x_i \geq v^\theta(S) \text{ for all nonempty } S \subset N.$$

The efficiency condition states that the total revenue of the streaming platform is fully divided among the content creators. The coalitional rationality condition states that no coalition has an incentive to leave the streaming platform, so each coalition is allocated at least the maximum total revenue it would obtain from its own streaming platform. Because the core is a convex polytope, it consists of zero, one, or infinitely many elements. A streaming platform is *stable* if the core of the associated streaming platform game is nonempty, and *unstable* otherwise. The following two examples illustrate that there exist stable and unstable streaming platforms.

Example 1 (continued)**.** The allocation $x = (6, 16, 14)$ satisfies efficiency because $x_{i_1} + x_{i_2} + x_{i_3} = 36 = v^\theta(N)$, and satisfies coalitional rationality because $x_{i_1} = 6 \geq 4 = v^\theta(\{i_1\})$, $x_{i_2} = 16 \geq 12 = v^\theta(\{i_2\})$, $x_{i_3} = 14 \geq 12 = v^\theta(\{i_3\})$, $x_{i_1} + x_{i_2} = 22 \geq 12 = v^\theta(\{i_1, i_2\})$, $x_{i_1} + x_{i_3} = 20 \geq 16 = v^\theta(\{i_1, i_3\})$, and $x_{i_2} + x_{i_3} = 30 \geq 24 = v^\theta(\{i_2, i_3\})$. Hence, $x \in C(N, v^\theta)$, so $C(N, v^\theta) \neq \emptyset$, and thus the streaming platform is stable. △

Example 2. Let $\theta = (N, U, V, r)$ be a streaming platform situation with content creators $N = \{i_1, i_2\}$, users $U = \{j_1, j_2\}$, streaming matrix

$$V = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \end{array},$$

and reservation prices $r = (1, 1)$. The streaming platform charges a premium fee of $p^N = \min\{1 \cdot 3, 1 \cdot 2\} = 2$ to each user, so that the total revenue is given by $p^N \cdot |U| = 2 \cdot 2 = 4$ and thus $v^\theta(N) = 4$. Content creator i_1 would charge a premium fee of 3 on its own platform, leading to a total revenue of $3 \cdot 1 = 3$, so $v^\theta(\{i_1\}) = 3$. Content creator i_2 would charge a premium fee of 2 on its own platform, leading to a total revenue of $2 \cdot 1 = 2$, so $v^\theta(\{i_2\}) = 2$. The resulting streaming platform game (N, v^θ) is presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_1, i_2\}$
$v^\theta(S)$	3	2	4

Each core allocation $x \in \mathbb{R}^N_+$ fully divides the total revenue of the streaming platform, so $x_{i_1} + x_{i_2} = v^\theta(\{i_1, i_2\}) = 4$, in such a way that $x_{i_1} \geq v^\theta(\{i_1\}) = 3$ and $x_{i_2} \geq v^\theta(\{i_2\}) = 2$. This is impossible, so $C(N, v^\theta) = \emptyset$, and thus the streaming platform is unstable. △

4.2 Stability analysis

In the previous subsection, we made the remarkable observation that streaming platforms are not necessarily stable. This raises the following question:

Which conditions for streaming platform situations lead to (un)stable streaming platforms?

In this subsection, we address this question by providing several sufficient conditions.

Our first condition is inspired by the unstable streaming platform in Example 2. In this example, the content creators are streamed by different users, meaning that the market consists of disjoint segments. Moreover, if the content creators would start their own platform, they charge different premium fees. These two characteristics together lead to an unstable streaming platform in Example 2. In Proposition 1, we show that this is true in general, i.e., we show that disjoint market segments with distinct premium fees always lead to unstable streaming platforms.

Disjoint market segments with distinct premium fees. A streaming platform in situation $\theta = (N, U, V, r)$ operates in *disjoint market segments with distinct premium fees* if there exists a nonempty $S \subset N$ such that the following conditions are satisfied:

(i) $U^S \cap U^{N \setminus S} = \emptyset$;

(ii) $P^S \cap P^{N \setminus S} = \emptyset$.

Proposition 1. *All streaming platforms operating in disjoint market segments with distinct premium fees are unstable.*

Proof. Let $\theta = (N, U, V, r)$ be a streaming platform situation and let nonempty $S \subset N$ be such that $U^S \cap U^{N\setminus S} = \emptyset$ and $P^S \cap P^{N\setminus S} = \emptyset$. In order to prove that the streaming platform is unstable, we first show that $\{U^S, U^{N\setminus S}\}$ forms a partition of U and that $\{Q^S(p), Q^{N\setminus S}(p)\}$ forms a partition of $Q^N(p)$, defined by $Q^N(p) = \{j \in U \mid r_j \cdot \sum_{i\in N} V_{ij} \geq p\}$ for all $p \in \mathbb{R}_{++}$.

For all $j \in U$, we have $\sum_{i\in N} V_{ij} > 0$, so $\sum_{i\in S} V_{ij} > 0$ or $\sum_{i\in N\setminus S} V_{ij} > 0$. As a result, $j \in U^S$ or $j \in U^{N\setminus S}$, so $j \in U^S \cup U^{N\setminus S}$ and thus $U \subseteq U^S \cup U^{N\setminus S}$. By definition we also have $U^S \cup U^{N\setminus S} \subseteq U$, and thus $U^S \cup U^{N\setminus S} = U$. From this, together with the fact that $U^S \cap U^{N\setminus S} = \emptyset$, we conclude that $\{U^S, U^{N\setminus S}\}$ forms a partition of U.

As a result, for all $p \in \mathbb{R}_{++}$, we have

$$\begin{aligned}
Q^N(p) &= \left\{ j \in U \,\middle|\, r_j \cdot \sum_{i\in N} V_{ij} \geq p \right\} \\
&= \left\{ j \in U^S \,\middle|\, r_j \cdot \sum_{i\in N} V_{ij} \geq p \right\} \cup \left\{ j \in U^{N\setminus S} \,\middle|\, r_j \cdot \sum_{i\in N} V_{ij} \geq p \right\} \\
&= \left\{ j \in U^S \,\middle|\, r_j \cdot \sum_{i\in S} V_{ij} \geq p \right\} \cup \left\{ j \in U^{N\setminus S} \,\middle|\, r_j \cdot \sum_{i\in N\setminus S} V_{ij} \geq p \right\} \\
&= Q^S(p) \cup Q^{N\setminus S}(p),
\end{aligned}$$

where the second equality follows from the fact that $\{U^S, U^{N\setminus S}\}$ forms a partition of U, and the third equality follows from $\sum_{i\in N\setminus S} V_{ij} = 0$ for all $j \in U^S$ and $\sum_{i\in S} V_{ij} = 0$ for all $j \in U^{N\setminus S}$. Moreover, because $Q^S(p) \subseteq U^S$, $Q^{N\setminus S}(p) \subseteq U^{N\setminus S}$, and $U^S \cap U^{N\setminus S} = \emptyset$, we also know $Q^S(p) \cap Q^{N\setminus S}(p) = \emptyset$. Consequently, $\{Q^S(p), Q^{N\setminus S}(p)\}$ forms a partition of $Q^N(p)$ for all $p \in \mathbb{R}_{++}$. To conclude,

$$\begin{aligned}
v^\theta(S) + v^\theta(N \setminus S) &= \max_{p\in\mathbb{R}_{++}} \left\{ p \cdot |Q^S(p)| \right\} + \max_{p\in\mathbb{R}_{++}} \left\{ p \cdot |Q^{N\setminus S}(p)| \right\} \\
&> \max_{p\in\mathbb{R}_{++}} \left\{ p \cdot |Q^S(p)| + p \cdot |Q^{N\setminus S}(p)| \right\} \\
&= \max_{p\in\mathbb{R}_{++}} \left\{ p \cdot |Q^N(p)| \right\} \\
&\geq p^N \cdot |U| \\
&= v^\theta(N),
\end{aligned}$$

where the first inequality follows from $P^S \cap P^{N\setminus S} = \emptyset$, the second equality follows from the fact that $\{Q^S(p), Q^{N\setminus S}(p)\}$ forms a partition of $Q^N(p)$ for all $p \in \mathbb{R}_{++}$, and the second inequality holds because $Q^N(p^N) = U$. Hence, $C(N, v^\theta) = \emptyset$ and thus the streaming platform is unstable. □

For streaming platforms operating in disjoint market segments with distinct premium fees, there exists a group of content creators that is not streamed by all users. Therefore, it is not optimal for this group of content creators to charge a premium fee on its own streaming platform such that all users subscribe. In Proposition 2, we show that in the opposite case, namely if each group of content creators charges a premium fee on its own streaming platform such that all users subscribe, then the streaming platform is stable. We refer to such a market as an inclusive market.

Inclusive market. A streaming platform in $\theta = (N, U, V, r)$ operates in an *inclusive market* if for each nonempty $S \subset N$ there exists a $p^S \in P^S$ such that $Q^S(p^S) = U$.

Proposition 2. *All streaming platforms operating in an inclusive market are stable.*

Proof. Let $\theta = (N, U, V, r)$ be a streaming platform situation. Let $j^* \in U$ be such that $r_{j^*} \cdot \sum_{i \in N} V_{ij^*} = p^N$ and define $x \in \mathbb{R}^N_+$ by $x_i = r_{j^*} \cdot V_{ij^*} \cdot |U|$ for all $i \in N$. Then,

$$\sum_{i \in N} x_i = \sum_{i \in N} r_{j^*} \cdot V_{ij^*} \cdot |U| = r_{j^*} \cdot \sum_{i \in N} V_{ij^*} \cdot |U| = p^N \cdot |U| = v^\theta(N).$$

For each nonempty $S \subset N$, let $p^S \in P^S$ be such that $Q^S(p^S) = U$. Then, because the premium fee p^S is such that all users subscribe to the platform of S, we know that $p^S = \min_{j \in U}\{r_j \cdot \sum_{i \in S} V_{ij}\}$. As a result,

$$\begin{aligned}
\sum_{i \in S} x_i = \sum_{i \in S} r_{j^*} \cdot V_{ij^*} \cdot |U| &= r_{j^*} \cdot \sum_{i \in S} V_{ij^*} \cdot |U| \\
&\geq \min_{j \in U} \left\{ r_j \cdot \sum_{i \in S} V_{ij} \right\} \cdot |U| \\
&= p^S \cdot |U| \\
&= \max_{p \in \mathbb{R}_{++}} \{p \cdot |Q^S(p)|\} \\
&= v^\theta(S).
\end{aligned}$$

Hence, $x \in C(N, v^\theta)$ and thus the streaming platform is stable. □

The following example illustrates a streaming platform operating in an inclusive market.

Example 3. Let $\theta = (N, U, V, r)$ be a streaming platform situation with content creators $N = \{i_1, i_2, i_3\}$, users $U = \{j_1, j_2\}$, streaming matrix

$$V = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 3 & 2 \\ 3 & 5 \\ 5 & 3 \end{bmatrix} \end{array},$$

and reservation prices $r = (1, 1)$. The streaming platform charges a premium fee of $p^N = \min\{11, 10\} = 10$. Note that for each nonempty group of content creators $S \subset N$ it is optimal to charge a premium fee on its own platform such that all users subscribe, so there exists a $p^S \in P^S$ with $p^S = \min_{j \in U}\{r_j \cdot \sum_{i \in S} V_{ij}\}$ such that $Q^S(p^S) = U$. Hence, the streaming platform operates in an inclusive market. The sets of optimal premium fees are presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_3\}$	$\{i_1, i_2\}$	$\{i_1, i_3\}$	$\{i_2, i_3\}$	$\{i_1, i_2, i_3\}$
P^S	$\{2\}$	$\{3\}$	$\{3\}$	$\{6\}$	$\{5\}$	$\{8\}$	$\{10\}$

Because the streaming platform operates in an inclusive market, Proposition 2 implies that the streaming platform is stable. The resulting streaming platform game (N, v^θ) is presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_3\}$	$\{i_1, i_2\}$	$\{i_1, i_3\}$	$\{i_2, i_3\}$	$\{i_1, i_2, i_3\}$
$v^\theta(S)$	4	6	6	12	10	16	20

Note that the streaming platform is indeed stable because $(4, 10, 6) \in C(N, v^\theta)$. △

In Example 3, there are only two (types of) users on the streaming platform. In that case, it is in fact sufficient to check whether the streaming platform operates in a semi-inclusive market, which means that for each individual content creator it is optimal to charge a premium fee on its own streaming platform such that all users subscribe. Only that would already guarantee that the streaming platform with two users is stable. This result is formulated in Proposition 3.

Semi-inclusive market. A streaming platform in $\theta = (N, U, V, r)$ operates in a *semi-inclusive market* if for each $i \in N$ there exists a $p^{\{i\}} \in P^{\{i\}}$ such that $Q^{\{i\}}(p^{\{i\}}) = U$.

Proposition 3. *All streaming platforms with two users operating in a semi-inclusive market are stable.*

Proof. Let $\theta = (N, U, V, r)$ with $|U| = 2$ be a streaming platform situation. For each $i \in N$, let $p^{\{i\}} \in P^{\{i\}}$ be such that $Q^{\{i\}}(p^{\{i\}}) = U$. Then, because the premium fee $p^{\{i\}}$ is such that all users subscribe to the platform of i, we know that $p^{\{i\}} = \min_{j \in U}\{r_j \cdot V_{ij}\}$. Moreover, because $p^{\{i\}}$ is an optimal premium fee, we know that

$$\min_{j \in U}\{r_j \cdot V_{ij}\} \cdot 2 \geq \max_{j \in U}\{r_j \cdot V_{ij}\} \cdot |Q^{\{i\}}(\max_{j \in U}\{r_j \cdot V_{ij}\})| \geq \max_{j \in U}\{r_j \cdot V_{ij}\} \cdot 1.$$

As a result, for each nonempty $S \subset N$, we have

$$\min_{j \in U}\{r_j \cdot \sum_{i \in S} V_{ij}\} \cdot 2 \geq \sum_{i \in S} \min_{j \in U}\{r_j \cdot V_{ij}\} \cdot 2 \geq \sum_{i \in S} \max_{j \in U}\{r_j \cdot V_{ij}\} \cdot 1 \geq \max_{j \in U}\{r_j \cdot \sum_{i \in S} V_{ij}\} \cdot 1.$$

This means that there exists an optimal premium fee such that all users subscribe to the platform of S, i.e., there exists a $p^S \in P^S$ such that $Q^S(p^S) = U$. Hence, the streaming platform operates in an inclusive market, so Proposition 2 implies that the streaming platform is stable. □

Note that the streaming platform with two users in Example 3 indeed operates in a semi-inclusive market. If there are more than two (types of) users on the streaming platform, then stability is not guaranteed for semi-inclusive markets. This is illustrated in the following example.

Example 4. Let $\theta = (N, U, V, r)$ be a streaming platform situation with content creators $N = \{i_1, i_2, i_3\}$, users $U = \{j_1, j_2, j_3\}$, streaming matrix

$$V = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \end{array} \begin{array}{c} \begin{array}{ccc} j_1 & j_2 & j_3 \end{array} \\ \begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 6 \\ 12 & 18 & 20 \end{bmatrix} \end{array},$$

and reservation prices $r = (1, 1, 1)$. The streaming platform charges a premium fee of $p^N = \min\{17, 25, 27\} = 17$. Note that for each individual content creator $i \in N$ it is optimal to charge a premium fee on its own platform such that all users subscribe, so there exists a $p^{\{i\}} \in P^{\{i\}}$ with $p^{\{i\}} = \min_{j \in U}\{r_j \cdot V_{ij}\}$ such that $Q^{\{i\}}(p^{\{i\}}) = U$. Hence, the streaming platform operates in a semi-inclusive market. The sets of optimal premium fees are presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_3\}$	$\{i_1, i_2\}$	$\{i_1, i_3\}$	$\{i_2, i_3\}$	$\{i_1, i_2, i_3\}$
P^S	$\{1\}$	$\{4\}$	$\{12, 18\}$	$\{5\}$	$\{20\}$	$\{16\}$	$\{17\}$

The resulting streaming platform game (N, v^θ) is presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_3\}$	$\{i_1, i_2\}$	$\{i_1, i_3\}$	$\{i_2, i_3\}$	$\{i_1, i_2, i_3\}$
$v^\theta(S)$	3	12	36	15	40	48	51

Because $v^\theta(\{i_2\}) + v^\theta(\{i_1, i_3\}) > v(\{i_1, i_2, i_3\})$, we have $C(N, v^\theta) = \emptyset$. Hence, even though the streaming platform operates in a semi-inclusive market, the streaming platform is unstable. △

Another sufficient condition for stability of streaming platforms is a homogeneous market. In such a market, all users are willing to pay the same amount of money to stay on the platform. For equal reservation prices, this boils down to equal total number of streams for all users. In Proposition 4, we show that homogeneous markets always lead to stable streaming platforms.

Homogeneous market. A streaming platform in $\theta = (N, U, V, r)$ operates in a *homogeneous market* if $p^N = r_j \cdot \sum_{i \in N} V_{ij}$ for all $j \in U$.

Proposition 4. *All streaming platforms operating in a homogeneous market are stable.*

Proof. Let $\theta = (N, U, V, r)$ be a streaming platform situation such that $p^N = r_j \cdot \sum_{i \in N} V_{ij}$ for all $j \in U$. Define $x \in \mathbb{R}^N_+$ by $x_i = \sum_{j \in U} r_j \cdot V_{ij}$ for all $i \in N$. Then,

$$\sum_{i \in N} x_i = \sum_{i \in N} \sum_{j \in U} r_j \cdot V_{ij} = \sum_{j \in U} r_j \cdot \sum_{i \in N} V_{ij} = \sum_{j \in U} p^N = p^N \cdot |U| = v^\theta(N).$$

For each nonempty $S \subset N$, we have

$$\begin{aligned} \sum_{i \in S} x_i = \sum_{i \in S} \sum_{j \in U} r_j \cdot V_{ij} = \sum_{j \in U} r_j \cdot \sum_{i \in S} V_{ij} = \sum_{j \in U^S} r_j \cdot \sum_{i \in S} V_{ij} = \sum_{j \in U^S} t_j^S \\ \geq \max_{j \in U^S} \{t_j^S \cdot |Q^S(t_j^S)|\} = \max_{p \in \mathbb{R}_{++}} \{p \cdot |Q^S(p)|\} = v^\theta(S), \end{aligned}$$

where the inequality follows from the fact that the maximum amount of money a new platform could generate is at most the total revenue obtained by charging each user the maximum amount of money it would be willing to pay to subscribe to this new streaming platform. Hence, $x \in C(N, v^\theta)$ and thus the streaming platform is stable. $\square$

We now illustrate that the streaming platform in Example 1 operates in a homogeneous market.

Example 1 (continued)**.** We have $p^N = 12 = r_j \cdot \sum_{i \in N} V_{ij}$ for all $j \in U$, so the streaming platform operates in a homogeneous market. By Proposition 4, the streaming platform is stable, which is in line with our observation that $(6, 16, 14) \in C(N, v^\theta)$. $\triangle$

We would like to remark that homogeneous markets are not necessarily inclusive, and inclusive markets are not necessarily homogeneous. For instance, the stable streaming platform in Example 1 operates in a homogeneous market, but not in an inclusive market, and the stable streaming platform in Example 3 operates in an inclusive market, but not in a homogeneous market. Clearly, there also exist streaming platforms operating in both a homogeneous and inclusive market, including those where all users have equal reservation prices and equal number of streams for each content creator.

As a final observation, note that our stability results are based on streaming similarities of platform users. Instead, we could also focus on streaming similarities of content creators. However, even if the content creators are identical in their value added to the platform, in the sense that the total amount of money that users would be willing to pay is equal for all content creators, then the streaming platform might still be unstable. This is illustrated in the following example.

Example 5. Let $\theta = (N, U, V, r)$ be a streaming platform situation with content creators $N = \{i_1, i_2\}$, users $U = \{j_1, j_2\}$, streaming matrix

$$V = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \end{array},$$

and reservation prices $r = (1, 1)$. Note that the content creators have the same added value to the platform, i.e., $\sum_{j \in U} r_j \cdot V_{i_1 j} = 3 = \sum_{j \in U} r_j \cdot V_{i_2 j}$. The resulting streaming platform game (N, v^θ) is presented in the following table.

S	$\{i_1\}$	$\{i_2\}$	$\{i_1, i_2\}$
$v^\theta(S)$	3	2	4

Because $v^\theta(\{i_1\}) + v^\theta(\{i_2\}) > v^\theta(\{i_1, i_2\})$, we have $C(N, v^\theta) = \emptyset$, so the streaming platfrom is unstable. $\triangle$

5 Numerical analysis

In the previous section, we provided several sufficient conditions that lead to (un)stable streaming platforms. In particular, we showed that streaming platforms operating in an inclusive market are stable. In such a market, each group of content creators would charge a premium fee on its own streaming platform such that all users subscribe. In a situation with only two users, it suffices that each individual content creator charges a premium fee on its own platform such that both users subscribe, which is called a semi-inclusive market. We also showed that streaming platforms operating in a homogeneous market are stable. In such a market, all users are willing to pay the same amount of money to stay on the platform. Inclusive markets, semi-inclusive markets, and homogeneous markets have something in common: they all describe streaming similarities of platform users. The absence of any of these similarities may lead to an unstable streaming platform, which is confirmed by our first proposition: streaming platforms operating in disjoint market segments with distinct premium fees are unstable.

In this section, we further analyze, by means of numerical experiments, the stability of streaming platforms that operate in none of the above discussed type of markets. In particular, we generate convex combinations of disjoint market segments with distinct premium fees and inclusive markets, and discuss when and why these combinations of markets lead to (un)stable streaming platforms.

Let $\theta = (N, U, V, r)$ and $\theta' = (N, U, V', r)$ be two streaming platform situations with content creators $N = \{i_1, i_2\}$, users $U = \{j_1, j_2\}$, streaming matrices

$$V = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 300 & 200 \\ 300 & 200 \end{bmatrix} \end{array} \quad \text{and} \quad V' = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 600 & 0 \\ 0 & 400 \end{bmatrix} \end{array},$$

and reservation prices $r = (1, 1)$. Note that the streaming platform in θ operates in an inclusive market and the streaming platform in θ' operates in disjoint market segments with distinct premium fees. As a result, the streaming platform in θ is stable, while the streaming platform in θ' is unstable.

We now investigate how the stability of a streaming platform changes when we gradually turn the streaming matrix of θ into the one of θ'. Let $\alpha \in \{0, 0.05, 0.1, \ldots, 0.95, 1\}$ and consider the new streaming platform situation $\theta^\alpha = (N, U, V^\alpha, r)$ whose streaming matrix V^α is a convex combination of V and V', i.e.,

$$V^\alpha = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 300 + 300\alpha & 200 - 200\alpha \\ 300 - 300\alpha & 200 + 200\alpha \end{bmatrix} \end{array}.$$

Note that for each $\alpha \in \{0, 0.05, \ldots, 0.95, 1\}$, we have $p^N = 400$ and thus $v^{\theta^\alpha}(N) = 400 \cdot 2 = 800$. Moreover, $V^0 = V$ and $V^1 = V'$, so the streaming platform is stable for $\alpha = 0$, while for $\alpha = 1$ it is unstable. In Figure 1, we illustrate the stability of the streaming platform for each α.

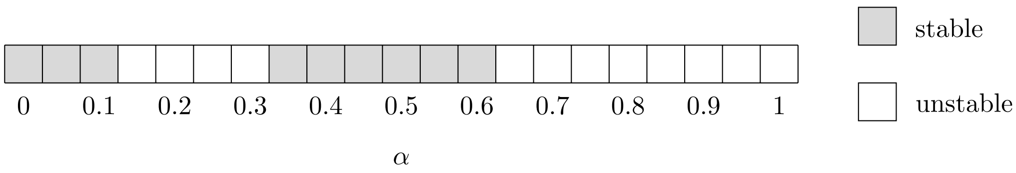


Figure 1: Stability of the streaming platform in θ^α for $\alpha \in \{0, 0.05, 0.1, \ldots, 0.95, 1\}$

Figure 1 shows that stability of the streaming platform in θ^α is not monotonic in α. In fact, the streaming platform switches multiple times from stable to unstable and vice versa when we gradually increase the value of α from 0 to 1. We discuss these transitions in more detail.

The first transition is from $\alpha = 0.1$ to $\alpha = 0.15$, where the streaming platform switches from stable to unstable. To understand this transition, have a closer look at the corresponding streaming matrices

$$V^{0.1} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 330 & 180 \\ 270 & 220 \end{bmatrix} \end{array} \quad \text{and} \quad V^{0.15} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 345 & 170 \\ 255 & 230 \end{bmatrix} \end{array}.$$

For $\alpha = 0.1$, both content creators would still charge a premium fee on their own platform such that both users subscribe. This also holds for $\alpha \in \{0, 0.05\}$, implying that for $\alpha \in \{0, 0.05, 0.1\}$, the streaming platform operates in a (semi-)inclusive market and thus is stable. From $\alpha = 0.15$ onwards, it is no longer optimal for content creator i_1 to charge a premium fee on its own platform such that both users subscribe. Instead, because $r_{j_1} \cdot V^{0.15}_{i_1 j_1} \cdot 1 = 345 > 340 = r_{j_2} \cdot V^{0.15}_{i_1 j_2} \cdot 2$, it is optimal for content creator i_1 to charge a premium fee of 345 such that only user j_1 subscribes. Because $v^{\theta^{0.15}}(\{i_1\}) + v^{\theta^{0.15}}(\{i_2\}) = 345 + 230 \cdot 2 > 800 = v^{\theta^{0.15}}(\{i_1, i_2\})$, we have $C(N, v^{\theta^{0.15}}) = \emptyset$ and thus the streaming platform is unstable.

For $\alpha \in \{0.15, 0.2, 0.25, 0.3\}$, the streaming platform remains unstable. For content creator i_1 it is still optimal to charge a premium fee on its own platform such that only user j_1 subscribes, and for content creator i_2 it is still optimal to charge a premium fee on its own platform such that both users subscribe. However, for content creator i_2, this optimal premium fee is determined by user j_1 instead of user j_2 from $\alpha = 0.2$ onwards. To illustrate this, consider the corresponding streaming matrices

$$V^{0.15} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 345 & 170 \\ 255 & 230 \end{bmatrix} \end{array}, \quad V^{0.2} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 360 & 160 \\ 240 & 240 \end{bmatrix} \end{array}, \quad \text{and} \quad V^{0.25} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 375 & 150 \\ 225 & 250 \end{bmatrix} \end{array}.$$

For $\alpha = 0.15$, the premium fee charged by content creator i_2 on its own platform equals $r_{j_2} \cdot V^{0.15}_{i_2 j_2} = 230$ and is determined by user j_2. For $\alpha = 0.2$, the premium fee equals $r_{j_1} \cdot V^{0.2}_{i_2 j_1} = r_{j_2} \cdot V^{0.2}_{i_2 j_2} = 240$ and is determined by both users. For $\alpha = 0.3$, the fee equals $r_{j_1} \cdot V^{0.25}_{i_2 j_1} = 225$ and is determined by user j_1.

The second transition is from $\alpha = 0.3$ to $\alpha = 0.35$, where the streaming platform switches from unstable to stable. To understand this transition, have a closer look at the corresponding matrices

$$V^{0.3} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 390 & 140 \\ 210 & 260 \end{bmatrix} \end{array} \quad \text{and} \quad V^{0.35} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 & j_2 \end{array} \\ \begin{bmatrix} 405 & 130 \\ 195 & 270 \end{bmatrix} \end{array}.$$

In both situations, it is still optimal for content creator i_1 to charge a premium fee on its own platform such that only user j_1 subscribes. For content creator i_2, it is still optimal to charge a premium fee such that both users subscribe. For $\alpha = 0.3$, we have $v^{\theta^{0.3}}(\{i_1\})+v^{\theta^{0.3}}(\{i_2\}) = 390 + 210 \cdot 2 > 800 = v^{\theta^{0.3}}(\{i_1, i_2\})$, so $C(N, v^{\theta^{0.3}}) = \emptyset$ and thus the streaming platform is unstable. However, for $\alpha = 0.35$, we have $v^{\theta^{0.35}}(\{i_1\}) + v^{\theta^{0.35}}(\{i_2\}) = 405 + 195 \cdot 2 < 800 = v^{\theta^{0.35}}(\{i_1, i_2\})$, so $C(N, v^{\theta^{0.35}}) \neq \emptyset$ and thus the streaming platform is stable. This transition is due to the fact that $v^{\theta^{\alpha}}(\{i_1\})+v^{\theta^{\alpha}}(\{i_2\})$ decreases from 810 to 795 when $\alpha = 0.3$ increases to $\alpha = 0.35$, while $v^{\theta^{\alpha}}(\{i_1, i_2\})$ remains equal to 800.

For $\alpha \in \{0.4, 0.45, 0.5\}$, the streaming platform remains stable because $v^{\theta^{\alpha}}(\{i_1\}) + v^{\theta^{\alpha}}(\{i_2\})$ still decreases in α while $v^{\theta^{\alpha}}(\{i_1, i_2\})$ remains equal to 800. From $\alpha = 0.55$ onwards, it is no longer optimal for content creator i_2 to charge a premium fee on its own platform such that both users subscribe. To illustrate this, consider the corresponding streaming matrices for $\alpha \in \{0.45, 0.5, 0.55\}$

$$V^{0.45} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \overset{\begin{array}{cc} j_1 & j_2 \end{array}}{\begin{bmatrix} 435 & 110 \\ 165 & 290 \end{bmatrix}}, \quad V^{0.5} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \overset{\begin{array}{cc} j_1 & j_2 \end{array}}{\begin{bmatrix} 450 & 100 \\ 150 & 300 \end{bmatrix}}, \quad \text{and} \quad V^{0.55} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \overset{\begin{array}{cc} j_1 & j_2 \end{array}}{\begin{bmatrix} 465 & 90 \\ 135 & 310 \end{bmatrix}}.$$

For $\alpha = 0.45$, content creator i_2 would charge a premium fee of $r_{j_1} \cdot V^{0.45}_{i_2 j_1} = 165$ on its own platform such that both users subscribe. For $\alpha = 0.5$, both premium fees $r_{j_1} \cdot V^{0.5}_{i_2 j_1} = 150$ and $r_{j_2} \cdot V^{0.5}_{i_2 j_2} = 300$ are optimal, leading to a total revenue of 300. For $\alpha = 0.55$, the optimal premium fee equals $r_{j_2} \cdot V^{0.55}_{i_2 j_2} = 310$ such that only user j_2 subscribes. From $\alpha = 0.55$ onwards, $v^{\theta^{\alpha}}(\{i_1\}) + v^{\theta^{\alpha}}(\{i_2\})$ increases in α.

The third transition is from $\alpha = 0.6$ to $\alpha = 0.65$, where the streaming platform switches from stable to unstable. To understand this transition, have a closer look at the corresponding streaming matrices

$$V^{0.60} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \overset{\begin{array}{cc} j_1 & j_2 \end{array}}{\begin{bmatrix} 480 & 80 \\ 120 & 320 \end{bmatrix}} \quad \text{and} \quad V^{0.65} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \overset{\begin{array}{cc} j_1 & j_2 \end{array}}{\begin{bmatrix} 495 & 70 \\ 105 & 330 \end{bmatrix}}.$$

In both situations, it is still optimal for content creator i_1 to charge a premium fee on its own platform such that only user j_1 subscribes. For content creator i_2, it is still optimal to charge a premium fee on its own platform such that only user j_2 subscribes. However, for $\alpha = 0.6$, we have $v^{\theta^{0.6}}(\{i_1\}) + v^{\theta^{0.6}}(\{i_2\}) = 480 + 320 = 800 = v^{\theta^{0.6}}(\{i_1, i_2\})$, so $C(N, v^{\theta^{0.6}}) \neq \emptyset$ and thus the streaming platform is stable. For $\alpha = 0.65$, we have $v^{\theta^{0.65}}(\{i_1\}) + v^{\theta^{0.65}}(\{i_2\}) = 495 + 330 > 800 = v^{\theta^{0.65}}(\{i_1, i_2\})$, so $C(N, v^{\theta^{0.65}}) = \emptyset$ and thus the streaming platform is unstable. From $\alpha = 0.65$ onwards, the streaming platform remains unstable because $v^{\theta^{\alpha}}(\{i_1\}) + v^{\theta^{\alpha}}(\{i_2\})$ still increases in α while $v^{\theta^{\alpha}}(\{i_1, i_2\})$ remains equal to 800.

The analysis of Figure 1 illustrates that stability of streaming platforms is a delicate matter. In particular, streaming platforms operating in between inclusive markets and disjoint market segments could be stable or unstable, depending on the specific realization of the corresponding streaming matrix.

We repeat the experiment above for different total number of streams of user j_1 while the total number of streams of user j_2 remains equal to 400. Denote the total number of streams of user j_1 by $\beta = \sum_{i \in N} V_{ij_1}$. Let $\alpha \in \{0, 0.05, \ldots, 0.95, 1\}$, let $\beta \in \{400, 440, \ldots, 800, 840\}$, and consider the streaming platform situation $\theta^{\alpha,\beta} = (N, U, V^{\alpha,\beta}, r)$ with streaming matrix

$$V^{\alpha,\beta} = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cc} j_1 \qquad & \qquad j_2 \end{array} \\ \begin{bmatrix} \frac{1}{2}\beta(1+\alpha) & 200 - 200\alpha \\ \frac{1}{2}\beta(1-\alpha) & 200 + 200\alpha \end{bmatrix} \end{array}.$$

Note that $V^{\alpha,600} = V^{\alpha}$ for each α. In Figure 2, we illustrate the stability of the streaming platform for each combination of α and β. Note that the row of $\beta = 600$ coincides with Figure 1.

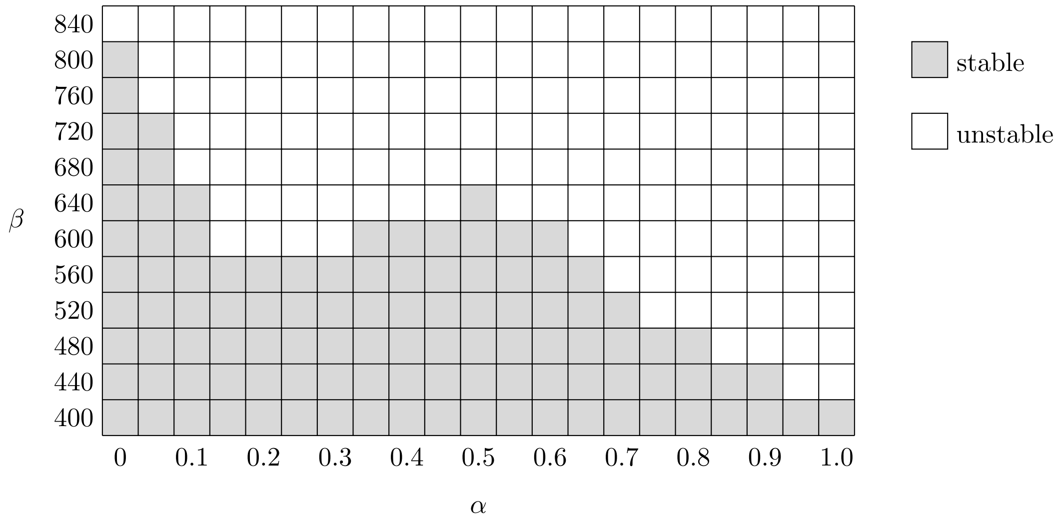


Figure 2: Stability of the streaming platform in $\theta^{\alpha,\beta}$ for various combinations of α and β.

Figure 2 shows that stability of the streaming platform in $\theta^{\alpha,\beta}$ is not necessarily monotonic in α, but is monotonic in β. For $\beta = 400$, the streaming platform operates in a homogeneous market and thus is stable. Monotonicity in β implies that streaming platforms switch from stable to unstable when homogeneous markets are gradually turned into heterogeneous ones.

In summary, this section further analyzed the stability of streaming platforms that do not operate in disjoint market segments, inclusive markets, or homogeneous markets. Numerical experiments illustrate that streaming platforms may switch multiple times from stable to unstable and vice versa when inclusive markets are gradually turned into disjoint market segments. Moreover, streaming platforms switch from stable to unstable when homogeneous markets are gradually turned into heterogeneous ones. In general, these observations indicate that stability of streaming platforms is a delicate matter.

6 Concluding remarks

By using concepts from cooperative game theory, we analyzed the stability of streaming platforms. We identified three sufficient conditions for stability, which all describe streaming similarities of platform users, and one sufficient condition for unstable streaming platforms, which describes opposite streaming behavior of users. Our numerical experiments indicate that stability generally is a delicate matter.

Although our work is inspired by the music streaming platform industry, our model and results apply in a much more general context than for which they are formulated. In particular, stability of any other service bundling format with similar characteristics could be analyzed along the same lines. Examples include museum passepartouts, movie and series platforms, and digital libraries.

The results in this study are established under some assumptions. First of all, we used the concept of the core to define stability of streaming platforms. The core guarantees that content creators are not better off by leaving and starting their own streaming platform, neither individually nor in cooperation with other content creators. In reality, it might be hard or even impossible for some groups of content creators to start their own streaming platform. In such case, the corresponding coalitional rationality constraints for those coalitions could be excluded from the definition of the core. Alternatively, one could define the maximum total revenue of those coalitions as zero in the streaming platform game.

Secondly, to analyze stability of streaming platforms, we only focused on potential revenues, but not on related costs. These costs could be incorporated into our model in the following way: for each coalition, subtract all costs of running its own platform from the revenues in the streaming platform game. We would like to remark that stability is preserved if the streaming platform in the original game is stable and the costs are concave, i.e., the additional costs when a content creator joins a coalition are decreasing in the size of the coalition.

Finally, we assumed that each platform user has a single reservation price reflecting the maximum amount of money that this user is willing to pay per stream. It could be the case that this amount of money depends on the content creator that is streamed. If this information is available, it could easily be incorporated in our model by replacing the vector of reservation prices by a matrix, where each entry indicates the maximum amount of money that a certain user is willing to pay per stream of a certain content creator.

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