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(S,s) Pricing:
Does the Heterogeneity Wipe out the Asymmetry on Micro Level?

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Abstract

In this paper I present a model of asymmetric pricing. Firms here follow the (S,s) pricing rule with different lengths of tails. I use numerical simulations with four-state shocks to detect the link between the present asymmetry in pricing on the micro level and asymmetry in aggregate output movements. This paper investigates whether the asymmetry on firm level can result in asymmetry on the macro level and what is the role of heterogeneity of agents in the process. It looks at two kinds of asymmetries on the aggregate level: (i) asymmetric output responses to positive and negative monetary shocks and (ii) asymmetric responses to shocks during different phases of business cycle. The basic conclusion is that to some extent the first type of asymmetry can be attributed to the asymmetry of adjustment bands and that heterogeneity softens the effect, but the second type of asymmetry is the result of (S,s) pricing behaviour of firms, thus of heterogeneity itself.

Keywords: (S,s) pricing, Asymmetry, Four-state shocks, Heterogeneity.

JEL codes: E31, E37.
(S,s) Pricing:  
Does the Heterogeneity Wipe Out  
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Introduction

Pricing behavior of individual firms has implications for the aggregate price and output movements. The propagation of money supply shocks crucially depends on pricing patterns. If firms in every moment have the frictionless optimal pricing, then it is easy to show that money supply shocks have no real effects.¹ But in real life there are frictions to price adjustment² and usually the actual price does not coincide with the frictionless one. Then, the natural questions arise, whether monetary shocks have real effects and whether the responses to negative and positive monetary shocks are symmetric.

During the last two decades sticky price models have proved to be of great importance. The empirical findings illustrate that prices are not flexible enough to always be at the optimum. The evidence of price stickiness is found in many markets. For example, Stigler and Kindahl (1970) and Carlton (1986) find evidence of price stickiness for industrial goods, Kashyap (1991) finds it for prices quoted in catalogs and Cecchetti (1986) for magazine prices.³ Then, lags and delays in the adjustment of price leave space for monetary policy.

Sticky price models can be divided into two parts: models where the firms follow the time dependent policy of price adjustment and where they follow the state dependent policy. Time-dependent pricing models assume that a firm’s decisions of revising and modifying the existing price are constrained by some time limits. For example, in Fisher (1977) and Taylor (1980) models of staggered pricing firms are allowed to set their prices every other period. In Calvo (1983) the information about the changes in market conjuncture arrives randomly in time. So, decisions

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¹E-mail: babutsidze@merit.unu.edu
²See Akerlof and Yellen (1985), Romer (2001) etc.
³There exist costs of price adjustment.
⁴For a review of these see Wynne (1995). More recent documentation of price stickiness is due to Levy et al. (1997), Blinder et al (1998), Wolman (2000) etc.
about the price changes also follow a random process. All these models feature money non-neutrality. Although these models are not intuitively very appealing, recent studies find support for the time-dependent pricing behavior of the firms.

State-dependent pricing models are more intuitive. The baseline logic here is that firms change prices depending on the state of economy. In this setup firms can change prices every period or leave the price unchanged for tens of periods. The best representation of state-dependence is \((S,s)\) pricing.\(^4\) The \((S,s)\) rule was first introduced by Arrow, Harris and Marschak (1951) for inventory management purposes. Later, Barro (1977) and Sheshinski and Weiss (1977, 1983) also applied it to pricing models. In these models, due to the existence of some kind of adjustment costs, the zone of inaction is created around the optimal price for the firm. As long as the price is inside of the band, it is optimal not to adjust it. When the price crosses any of the inaction bands the adjustment to optimal price is observed.

All these pricing models allow for the heterogeneity of the economic agents, though the strategies and the incentives of all of them are usually assumed to be identical. Heterogeneity comes with the different prices of the producers that are due to the frictions to the price adjustment. If there were no frictions, all the prices would coincide and the behavior of the aggregate variables would be the same as the individual ones, just on the different scales.

Parallel to these considerations, the supporters of the use of representative agent framework in economics, although acknowledging the fact that there is heterogeneity among economic agents, have argued that it does not matter on the aggregate level. They argue that representative agent framework is enough to explain the motion of the aggregate variables, as positive and negative deviations from the optimal price (of a representative agent) perfectly cancel each-other. Furthermore, Caballero (1992) shows that same types of heterogeneity (exactly of a type that \((S,s)\) models assume) cancel some particular types of asymmetries on micro level through the aggregation process. He presents the model where heterogeneous agents that have asymmetric policies result in absolutely symmetric behavior of the aggregate variables.

In this paper I present a model with asymmetric \((S,s)\) bands. Although I take the asymmetry of inaction bands as given, based on a well-documented empirical finding,\(^5\) and basically look at its implications for the output responses to monetary shocks, several justifications to asymmetry are provided in the first section of the paper. One justification is that firms might have asymmetric adjustment costs, the second is that firms might have asymmetric deviation costs. These asymmetries result in an asymmetric distribution function for price deviations from the optimal price.

The aim of the this paper is twofold. The first is to bring together several empirically well-documented facts. I take one empirical finding, asymmetry in pricing on the firm level, and look whether it can explain the other empirical findings, namely two kinds of asymmetries of aggregate output responses to monetary shocks. The first type of asymmetry is in the response of output to positive and negative monetary shocks. The second is in responses to monetary shocks during recessions and booms.

The second is to argue, that the relationship between micro- and macro asymmetry is not as simple task as one might think and that heterogeneity of agents

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\(^4\)See for example Caplin and Spulber (1987) or Caplin and Leahy (1991) etc.

\(^5\)See Tobin (1972), Ball and Mankiw (1994) etc.
plays the important role in the relation. And the role of it is different in different kinds of aggregate asymmetries. For example, for the first type of aggregate asymmetry specified above, which is the type of asymmetry discussed by Caballero (1992), the heterogeneity softens, but not necessarily completely wipes out, the effect of the micro asymmetry on the macro level. While the second type of aggregate asymmetry identified is completely created by heterogeneity of agents, the asymmetry on the micro level does not play a role there.

The methodology is as follows. First, assuming the asymmetry of inaction bands and stochastic changes in optimal price, I derive the density function of the distribution of deviations from the optimal price in economy. Then I proceed with numerical simulations: I shock the optimal price for a firm and look at the resulting density. A measure of the magnitude of responses to shocks is introduced. To examine the model’s implications for the first type of asymmetry I simulate 400 independent periods (200 for each, positive and negative shocks) from initial distribution and compare the resulting two samples. For the second type of asymmetry I generate booms and recessions and then continue with the same procedure as for first type of asymmetry. The main findings are that although the model features both kinds of asymmetries to some extent, micro asymmetry is the reason only for the first one. The first type of asymmetry gets more and more pronounced as shocks become larger, or equivalently, price dispersion reduces. The asymmetry in output responses to monetary policy appears to be very well pronounced, but it exhibits the sign of asymmetry found empirically in only one direction (for positive monetary shocks). The second type asymmetry is not the result of the asymmetric bands, but rather of (S,s) pricing, thus of the heterogeneity of agents, itself.

The rest of the work is organized as follows. In the first section I discuss the importance of asymmetry of inaction bands and provide some justifications for the chosen modeling technique. In the second section the main model is introduced. In the third I describe the simulation methodology and provide the main results. In the fourth section I discuss some interpretations of results, limitations of the approach and some possible extensions. Finally, some general conclusions are drawn.

1 The Importance of Asymmetry

As I have already mentioned, the present work is based on the finding that prices are more rigid downwards (even in almost no inflationary environment), and if they decline, they decline by a higher magnitude relative to price increases. This means that firms’ adjustment policies are asymmetric at microeconomic level. There are two types of asymmetry observed on aggregate level also. One is that the aggregate output has low and high response regimes to the monetary policy. Namely, the output responds to a somewhat lesser extent to positive monetary shocks during the recession than during the normal periods and even lesser than during the booms. Second, the output response is smaller in magnitude when we have positive money supply shocks rather than when we have negative ones.

The asymmetry of microeconomic adjustment policies is quite an old empirically documented fact. Even in the ’70s, economists were talking about the downward

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7 See Cover (1992), Ashworth (1998) etc.
rigidity of prices.\textsuperscript{8} And current research also shows the overwhelming evidence on more frequent price increases than decreases. For example, Borenstein et al. (1997) and Karrenbrock (1991) find the microeconomic asymmetry on gasoline and agricultural products' markets, Jackson (1997) finds it on Bank deposits. To this Chen et al. (2004) add the documentation of the asymmetry in price changes in American supermarket chains. Although I am not aware of a study that documented the difference in magnitudes of price adjustments in the US, Carlton's (1986) results coupled with the well-documented difference in frequency of price adjustment in different directions can be regarded as indirect evidence of this. He shows that there exists positive correlation between the time elapsed after the last change and the average absolute price changes.

There is a better documentation of asymmetry in the frequency and the magnitude of adjustment for European countries. For example, Loupias and Ricart (2004) investigate the pricing behavior of over 1600 French manufacturing firms and find that positive price changes are more frequent than negative ones. They also find that the magnitude of up- and downward price changes are different: they report an average of 3\% for price upgrades in contrast with an average of -5\% for price downgrades. Their findings are supported by another study of French manufacturing firms' behavior by Baudry et al. (2004), who found no evidence of nominal downward rigidity but support the asymmetry in magnitude of changes, although less pronounced (+4\% versus -5\%).

In other European countries the picture is similar. Hoeberichts and Stockman (2004) find more frequent price upgrades than downgrades (the ratio is 1.86) for the manufacturing sector of the Netherlands. They also find supportive evidence for difference in magnitude of changes: +5\% for upgrades as compared to -10\% for downgrades. In neighboring Belgium, Aucremanne and Dhyne (2004) find no differences in the frequency, but in the magnitude of price changes: +6.8\% versus -8.7\%. For Spain, Alvarez and Hernando (2004) find that the ratio of price increases to price decreases is 1.6. With regard to the asymmetry in the magnitude of price changes they report +8.2\% for price increases versus 10.3\% of price decreases. For Portugal, Dias et al. (2004) find no difference in magnitude of changes but a huge contrast in the frequency of price changes in different directions; they report the ratio of positive to negative price changes equal to 2.34.\textsuperscript{9}

Lach and Tsiddon (1992) also find the asymmetry in magnitudes of price deviations for Israel. They examine disaggregated price data of foodstuffs in Israel during 1978-84. Their main conclusion is that the asymmetry is more pronounced during high inflation periods, more precisely when the annual inflation goes above 130\%.\textsuperscript{10}

Of course, these findings are not left without attention. Ball and Mankiw (1994) incorporate the difference in frequency into their model. They do this by introducing the positive drift in inflation process justifying this with some kind of Harrod-Balassa-Samuelson effect due to the faster economic integration and the development of countries. This introduces the asymmetry in price distribution. Although Ball and Mankiw’s (1994) model is able to feature more frequent price

\textsuperscript{8}See Tobin (1972).

\textsuperscript{9}Further evidence on asymmetry for all EU15 countries is provided by Lunnemann and Matha (2004).

\textsuperscript{10}Note that the fact that asymmetry is more visible during high inflation goes in line with Ball and Mankiw (1994) and Tsiddon (1991) models discussed later.
upgrades than downgrades, still the magnitudes of changes on the firms level are equal. Thus, anticipated positive drift in inflation explains only half of the story.

Tsiddon (1991) presents a simple menu cost model for high inflationary environment. He introduces the costs for adjustment that are proportional to the deviation from the optimal price and derives the optimal pricing policy for the representative firm. The author distinguishes between price stickiness and downward rigidity and concludes that the model features the latter. The model exhibits an asymmetry in the following way. According to the optimal pricing policy, during the low inflation periods firms adjust their prices more frequently than during the high inflation periods. This is due to the fact that high inflation increases the uncertainty in future optimal price movements and the optimality is achieved by waiting. A similar result is obtained by Hansen (1999) who derives the dependence of the "first passage time" function on the degree of uncertainty. So, in a sense, Tsiddon’s (1991) model features the difference in the magnitudes of the price adjustment as well as the difference in the frequency of price adjustment.\(^{11}\)

Although the inflation trend assumed in these models is an intuitive device for introducing asymmetry, as it aggravates the effect of a positive shock and mitigates the effect of a negative one, it is not well matched with the empirical findings. For example, Peltzman (2000) shows that asymmetry is very pronounced in the United States in the period 1982-1996, when the positive drift in inflation was measured to be less than 2%. DeLong and Summers (1988) find an asymmetry during the Great Depression period when the price trend was deflationary. All this points to the fact that trend inflation can not explain even the different frequency of price up- and downgrades. Some other factors seem to be in work.

The overwhelming majority of sticky price models, like Ball and Mankiw (1994), Tsiddon (1991) and Bhaskar (2002), take the inaction bands lying on an equal distance from the optimal price. If we take the adjustment cost to be a menu cost\(^{12}\) type, the symmetry is justified: there is no reason why the menu costs can be different for changing the prices in different directions. But the problem is that the adjustment cost is a much wider notion than the menu cost. There are many other factors that can be regarded as the ingredients of the cost of changing price. For example, the psychological factor as seeing the product’s price raising with large jumps can result in loss of consumers and decreasing profits. This can further propagate to firm’s large negative jump in purchases of inputs offending the suppliers. Large discrete downward jumps are rational: this will probably result in "stealing" the buyers from competitors and also hoping to bargain a good discount with a supplier on a larger order due to the increased output. This can be one explanation of why the positive effect on our shopping budget is always notable, more frequently accompanied with changes in shops where we normally go, but rare, while the negative effect is almost negligible, not that much to force us to look for other, cheaper shops. These, and maybe some other, factors seem to be at work underpinning the well-established empirical finding that individual firm’s prices rise with small jumps but frequently, while the decline in prices is rare but large.

\(^{11}\)There are also the examples of the other kinds of asymmetry in price adjustment derived in different setups. See for example Danziger (1988) where asymmetry is due to the discounting of future profits in inflationary environment. There every price spends most of the time being below the optimal one.

\(^{12}\)See for example Mankiw (1985).
The importance of these considerations is outlined in Bowman (2002). The author presents a model of sticky prices without any menu costs. In this model for firms it is optimal not to change prices in response to nominal shocks because doing so increases their profits by expanding the customer base. Then the non-neutrality of the money is obtained without any kind of menu costs. Some other kinds of cost seem to deter firms from adjusting prices.

Also, as documented by Kwapil et al. (2005), firm’s decisions about price upgrades and downgrades depend on different factors. Research on Austrian manufacturing firms shows that changes in wage and intermediate goods’ costs are two of the most important factors for price increases, while changes in competitors’ prices and technological improvements are the main driving factors for price reductions. Furthermore, Loupias et al. (2004) conclude that menu costs are absolutely not important for price changes of manufacturing products. Then, from this point of view, there is absolutely no reason why the costs of price changes in different directions have to be the same.

Although the present work does not concentrate on the derivation of the optimality of asymmetric bands, here I provide further possible explanations and a sketch of possible modeling technique. As I argued before, menu costs and adjustment costs are not exactly the same. So, adjustment costs can be different for movements of price in different directions. I described above some psychological factors that can be at work differing adjustment costs. Then I argue that optimality of asymmetric bands can be derived from the usual monopolist profit maximization problem.\footnote{In Appendix A the solution to the problem is given.}

In principle, the asymmetric adjustment cost is not the only way to get asymmetric bands of adjustment. Similar results can be obtained by assuming the asymmetric profit function. Namely, profit function that is steeper before optimal price and flatter after it. This assumption makes not adjustment, but rather deviation costs asymmetric. To see this, following Hansen’s (1999) notations, define deviation costs as $C_{dev} = \frac{1}{2} \frac{\partial^2 \pi}{\partial p^2} (p - p^*)^2$. Then if a profit function is flatter when $p > p^*$ for the same absolute value of deviation $C_{dev}^{p < p^*} > C_{dev}^{p > p^*}$. Thus, even with symmetric adjustment costs firm’s pricing behavior will feature a longer right tail and a shorter left one.\footnote{Using my notations (see further).}

But even if one finds out the optimal pricing strategy of firms in the economy, this is not enough to characterize the aggregate price and output responses to various shocks to the economy. The link from micro- to macroeconomic asymmetries is very complicated.\footnote{See for example Rátfai (2003).} Microeconomic asymmetry in price adjustment can totally cancel out at the aggregate level, or macroeconomic asymmetry can be introduced by aggregation of the firms with absolutely symmetric microeconomic pricing properties.

As the example of the first kind of link, we can consider Caballero’s (1992) model. Although the model is concerned with asymmetries in job destruction and creation, the logic is directly applicable to asymmetries in price changes. The author presents a model where every firm has the same size of asymmetry in magnitude of adjustment: downgrades (of the number of firm’s employees) are two times larger than upgrades. Caballero shows that in the case of the shock in state, the variable...
following a binomial random walk\textsuperscript{16} aggregation results in an absolute cancellation of the asymmetry. As a result, downgrades are twice as rare as upgrades, that gives the force moving to the opposite direction of the microeconomic adjustment asymmetry with the same magnitude.

As an example of the second kind of link between micro- and macroeconomic asymmetries, consider the model by Bhaskar (2002). The author presents a model with a continuum of sectors and a continuum of firms in each sector. Every firm has an absolutely symmetric price adjustment policy. Firms produce (not perfect) substitutes and the elasticity of substitution is higher between the products of the firms’ belonging to one sector than between the firms’ belonging to different sectors. The setup produces the strategic complementarity in price setting between the firms belonging to one sector. Also, the profit of every firm is higher if it (and every other firm in its sector) has higher prices. Then the model results in multiple equilibria: one equilibrium is when every firm adjusts to the price shocks and the second is when no firm adjusts to the price shocks. The consequence is very intuitive; if the price shock calls for increasing the prices, everyone adjusts to it (if it is sufficiently higher to cover symmetric menu costs), but a negative shock has to be much higher in magnitude in order all the firms to adjust. Thus, the model results in an asymmetry on macroeconomic level without asymmetry in the pricing behavior of an individual firm.

As we have seen, the asymmetry on the micro as well as on the macro level is very well documented in the economic literature. But there is no distinct link identified between these two phenomena. The motivation of the present work is to contribute to this line of research and to model microeconomic asymmetry in a different way than the other researchers do and to try to characterize its links to macroeconomic asymmetries. In the next section I provide the baseline model of the present paper.

\section{The Model}

As shown in the previous section the link between microeconomic and macroeconomic asymmetries is not immediate and obvious. In this section I provide the details of the baseline model of the present work. In the first sub-section I setup the model and formulate the way the microeconomic asymmetry is introduced. In the second section I derive the time-invariant cross-sectional distribution of firms’ prices that is important for further procedure of numerical simulations.

\subsection{Setup of the Model}

I model Chamberlinian monopolistic competition following Dixit and Stiglitz (1977). The economy consists of a continuum of monopolistically competitive firms indexed on \([0;1]\) interval that produce close (but not perfect) substitutes. This form is chosen because in a perfect competition setup positive deviation from the optimal price results in large losses due to loss of entire market share. This is because, in the case of perfect competition, the profit function of the firm is not continuous in own price: it has a discrete jump immediately after the optimal price.\textsuperscript{17} This makes

\textsuperscript{16}This assumption is crucial, as I will discuss in the section 4 of the present work.
\textsuperscript{17}See Akerlof and Yellen (1985).
competitive environment useless for the purposes of this paper.

Consider a monopolistic firm that faces downward sloping demand of a form

\[ Y = \left( \frac{P}{\bar{P}} \right)^\eta \]

where \( P \) is the own price of firm’s product, \( M \) is the money supply per firm, \( \bar{P} \) is the aggregate price. The positivity of monopolistic markup gives the condition \( \eta > 1 \). The firm operates at a constant real marginal costs \( C = b Y^\alpha \), where \( b \) can be interpreted as the real wage per unit of effort (in equilibrium it is constant),\(^{18}\) \( \alpha \) is the inverse of productivity parameter. Then the monopolistic profit maximization problem is

\[
\text{Max} \quad \pi = P Y - P C
\]

with respect to the demand on \( Y \). Assuming symmetry, that the prices of all the goods are equal, the problem results in \( P = \bar{P} \) and gives \( P = k M \), where \( k \) is constant and is equal to \( k = \left( \frac{\eta - 1}{\eta (\eta - 1)} \right)^{\frac{1}{1-\alpha}} \).

Note that in this (no adjustment costs) setup the output of a single firm, and as a consequence of the whole economy, is constant at a value \( k \).

Taking the natural logarithms of the price-money supply relationship, denoting the logarithms by lower case letters, we get

\[ p^* = \ln k + m \]  

Then, it is apparent that \( dp^* = dm \). Thus, the idiosyncratic, mean-zero shocks in money supply would call for no aggregate price changes.

I introduce the variable \( x \) that is the deviation of firm’s actual price from its optimal one, defined as \( x = p - p^* \). Note that unlike most similar papers,\(^{20}\) the negative value of \( x \) means that the actual price is lower and the positive value - that the actual price is higher than the desired price. I make this assumption because of simpler tractability of results of the density function of \( x \) derived in the next sub-section.

I also assume that there is a fixed cost of adjustment that is not necessarily equal for up- and downgrading the price. And there is a cost of being apart from the optimal price. Following Hansen (1999) I assume that this cost is incurred at every moment when \( p \neq p^* \) and can be measured as accumulated flow costs. Note that due to the concavity of the profit function, the cost of being at non-optimum is the second order. Then an entrepreneur makes a decision by comparing the two costs. As long as the deviation cost is sufficiently lower prices do not change. This behavior creates the zone of inaction that is not necessarily symmetric around the optimal price.

2.2 Deriving the Long-run Density

An important thing in the model is to derive the long-run density function of price deviations.\(^{21}\) Define \( f(x) \) as the long-run, time-invariant density function of price

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\(^{18}\)For further details see Akerlof and Yellen (1985).

\(^{19}\)Note that the solution puts stricter requirement on \( \eta \). It requires \( \eta > 1/\alpha \) for the positivity of \( k \).

\(^{20}\)See for example Hansen (1999).

\(^{21}\)This builds on Hansen (1999).
deviations. This function can also be interpreted as the likelihood of having a price deviation equal to \( x \) at any particular moment. For the derivation of the density function I assume that Brownian motion in money supply has very simple properties: it is a mean zero process and at every instant \( dt \) it can change \( x \) by \( dx \) with equal probabilities going up and down. This means that if we are now at \( x \) after one period \( (dt) \) we will be at \( x + dx \) with probability 0.5 and at \( x - dx \) with probability 0.5. Then
\[
f(x) = \frac{1}{2} f(x + dx) + \frac{1}{2} f(x - dx) \quad (2)
\]
as being today at \( x \) means being either at \( x - dx \) or at \( x + dx \) a moment ago. This is a very convenient property. We can rewrite (2) as
\[
(f(x + dx) - f(x)) - (f(x) - f(x - dx)) = 0
\]
Then, division by \( dx \) gives
\[
\frac{f(x + dx) - f(x)}{dx} - \frac{f(x) - f(x - dx)}{dx} = 0 \quad (3)
\]
Note that as \( dx \to 0 \) two parts of left hand side of expression (3) converge to derivatives of \( f(x) \) and then whole left hand side is something like the change in the derivative from point \( x + dx \) to point \( x \). Then the whole expression (3) is equivalent to the second derivative of \( f(x) \) being zero \( \frac{d^2 f(x)}{dx^2} = 0 \).

Now, as \( f(x) \) is a density function, we know that
\[
\int_{-b}^{b} f(x) dx = 1 \quad (4)
\]
where \(-a\) and \( b\) are optimal bands of price adjustment. According to our assumptions it is obvious that \( b > a > 0 \). Thus, price deviation \( x \) is distributed between \(-a\) and \( b\). We also have two boundary conditions \( f(-a) = f(b) = 0 \), by assumption that prices are adjusted immediately as they reach any of the boundaries, thus none of them, in principle, are reached. Then we can split the integral (4) into two parts
\[
\int_{-a}^{0} f(x) dx + \int_{0}^{b} f(x) dx = 1
\]
From the second derivative of \( f(x) \) being zero we know that both of these parts are linear. From the boundary conditions we know their crossing points with \( x \) axis are \( x = -a \) and \( x = b \). Also, note that \( f(x) \) has to reach maximum at \( x = 0 \), because has the highest probability equal to
\[
\frac{1}{2}(f(-dx) + f(b - dx)) + \frac{1}{2}(f(dx) + f(-a + dx)) = f(0) \quad (5)
\]
This is the probability of being either at \(-dx\) or at \( b - dx\) and getting a positive shock plus the probability of being either at \( dx\) or at \(-a + dx\) and getting a negative shock. Then, two straight lines have to cross at \( x = 0 \), otherwise the density function will not be continuous.

All these conditions together imply that \( f(x) \) has a triangle shape with the base \( a + b \) and the height \( 2/(a + b) \) (and it reaches maximum at \( x = 0 \)). This gives us the solution to the problem
\[
f(x) = \begin{cases} 
\frac{2}{a+b} \left( 1 + \frac{x}{a} \right) & \text{if } x < 0 \\
\frac{2}{a+b} \left( 1 - \frac{x}{b} \right) & \text{if } x \geq 0 
\end{cases} \quad (6)
\]

Thus, the resulting density function looks like the one shown on Figure 1.\textsuperscript{22}

The shape of the resulting density function has an interesting implication. Although it is obvious that the right tail of $(S,s)$ is longer than the left one (as $b > a$) (difference in an intensive margin), from the Figure (reference to figure) one can infer that near the upgrading band (near $-a$) there are relatively more firms than near the downgrading band (near $b$) (difference in an extensive margin).\textsuperscript{23} So, the results also emphasize the obscurity of the link between micro- and macro- asymmetry: although price downgrades are higher in magnitude there are fewer firms who want to reduce their prices as a result of a shock. Consequently, it is not obvious that the positive shock in price deviations\textsuperscript{24} will induce the aggregate price level to reduce with higher magnitude than the rise caused by the negative shock of the same magnitude. In fact, there is a chance that these two factors completely cancel out each other and we get the same result as Caballero (1992). This issue is addressed in details in the section 4 of the paper.

3 Simulation and Results

3.1 The Methodology

In this sub-section I provide some basic details of the simulation methodology. Of course, I can not work with the continuum of firms any longer and for simulation purposes I discretize the price deviation space. As I work with price deviations I have to transform the results in terms of price and output responses. Let $x_{0}$ be an initial price deviation for a single firm $x_{0} = p_{0} - p_{0}^{*}$. Then money supply shock of

\textsuperscript{22}Please note here, that the original assumption of discretization of a continuous process, mainly that $x$ can go to only two states, either $x + dx$ or $x - dx$ is not crucial for the form of the density function. If one assumes the four-state shocks, as I do later, it is easy to show that the same shape results. A crucial assumption for the shape is that those two states are reached with the equal probability, which is maintained throughout the whole paper.

\textsuperscript{23}See Klenow and Kryvtsov (2005).

\textsuperscript{24}As shown in the next section a positive shock in price deviations is equivalent to a negative monetary shock.
a magnitude $\epsilon$ is also an optimal price shock of the same magnitude\textsuperscript{25} $p_1^* = p_0^* + \epsilon$. This gives $x_1 = p_1 - p_1^*$. From these identities I get $x_1 = p_1 - p_0^* - \epsilon$. Then it is apparent that a positive shock in money supply transforms into a negative shock in price deviations and vice versa. Intuitively, the immediate rise in optimal price for the firm means that its relative price has lowered. Finally, one can express the evolution of the price of a single firm as

$$p_1 - p_0 = \epsilon + x_1 - x_0$$  \tag{7}$$

So, I track the evolution of every single price in the economy. Then, the evolution of the aggregate price is derived by simply averaging all the prices in the economy.\textsuperscript{26}

For output changes, I proceed with demand functions. Taking natural logarithms of the original demand function and totally differentiating gives

$$dy = \eta(dp - d\bar{p}) + (dm - d\bar{p}) \tag{8}$$

From here it is obvious that the output changes for every single firm depend on the parameter $\eta$. But on the aggregate level, note that by definition $\sum dp = \sum d\bar{p}$, as $n\bar{p} = \sum dp$. So, the first summand in (8) disappears on the aggregate level and we are left with

$$dm = d\bar{p} + d\bar{y} \tag{9}$$

where $\bar{y}$ is a log of aggregate output. So, on the aggregate level the role of price elasticity of demand disappears. Then, to simplify calculations, for aggregate output I proceed with the rearrangement of (9), as I know $dm$ and also $d\bar{p}$.

The next important thing is to correctly choose the number of grid points on the price deviation space. If the price dispersion is very high (many units on the price deviation space grid) and shocks are small, the shift of the density is not significant. Although a relatively very small number of firms adjust their prices, the adjustment for each of them is so large (due to the large grid) that it is enough to change the average price with the magnitude close to monetary shock (as it was small).\textsuperscript{27} And besides, that big dispersion of prices around the optimum is not realistic. That is why I have chosen to carry the simulations with 15 grid points. In order to have the sufficient effect of asymmetry the ratio of lengths of the bands is chosen to be 2. That is, the right tail of the price deviation distribution is twice longer than the left one. Thus, $a = 5$ and $b = 10$.\textsuperscript{28} Further, the price adjustment rule is that a firm has to adjust its price deviation to zero whenever it reaches -5 or 10. So basically these two boundaries are never reached and I am left with the price deviations ranging from -4 to 9.

The simulation is carried out with 3000 firms. The initial distribution of the price deviations is assumed to be of a form derived in previous section. Or, to put in another words, I use the long-run time invariant distribution as the initial one

\textsuperscript{25}As derived in the section 2.
\textsuperscript{26}By symmetry assumption the weights are equal for all the firms.
\textsuperscript{27}These considerations are confirmed by simulations also: I tried to use the grid with 50 units, but any shocks with magnitude up to 4 units produce the results very close to flexible price models.
\textsuperscript{28}One more justification for the choice is that as most of the empirical studies found the price upgrades around 5% and downgrades around 10%, these numbers, as well as the shock integers specified below, can be interpreted as percentages of the price.
too. Actually, this is not an assumption at all, as beginning from any arbitrary distribution of price deviations after sufficient number of idiosyncratic shocks to economy the density converges to the triangular one. In that sense, the long-run density derived in the second section is a very robust feature of the model. And one can say, that the results reported here are independent of the initial distribution of prices.

Random shocks here are of two types: idiosyncratic and aggregate. Idiosyncratic shocks are specific to a firm, they hit every firm in every period. But they are mean-zero across all the firms during every period. That means the sum of all the idiosyncratic shocks across all the firms during every single period is zero. Concerning the form of the shock process, in this paper I use four-state mean zero shocks, which are of a form

\[
\text{Shock} = \begin{cases} 
-k & \text{with } p = 1/4 \\
-l & \text{with } p = 1/4 \\
l & \text{with } p = 1/4 \\
k & \text{with } p = 1/4 
\end{cases}
\]

where \(k \neq l\). This is a novelty for the literature. Most of the papers use the simplest Markov process with two states for the shock process. But, I think, that in reality there is no reason why the idiosyncratic shocks to every firm have to be of the same magnitude, as long as the time is treated as a discrete variable. Just like the direction of shocks, their magnitude can also be different. I argue that the results obtained with the current shock processes are closer to reality compared to binomial Markov chain, which looks more like an approximation of a continuous process. Besides, in section 4, I show that the results of the other papers that are using binary random walks are not robust for other, for example this four-state, shock specifications.

Aggregate shocks are generated very much in a similar way as firm-specific shocks. They basically introduce the correlation in idiosyncratic shocks. I use the same specification as described for the firm-specific shocks in the previous paragraph, but in this case the mean is not zero. So, all in all, four states of the shock are symmetric around some nonzero constant (mean). This, of course, includes the idiosyncratic shocks in itself, the aggregate part of the shock is only the mean. Concerning the size, I use two magnitudes of the aggregate shock (\(j\)) and three pairs of magnitudes of an idiosyncratic shock (\(k;l\)). So, altogether it makes six specifications of the shock process: \((j;k;l) = (1;1;2),(1;1;3),(1;2;3),(2;1;2),(2;1;3),(2;2;3)\).

### 3.2 Results

Here I provide basic results of numerical simulation conducted according to the methodology described in the previous sub-section. As mentioned before, I am basically concerned with the model’s implications for two types of asymmetry: (i) Asymmetric responses to positive and negative shocks during the calm (when the distribution of the price deviations is of initial shape) periods. (ii) Asymmetric responses to monetary shocks during the peaks and bottoms of business cycle. Below I consider them in turn.

---

29 Unlike Caplin and Spulber (1987) where initial distribution is crucial for basic results of their model. Their assumption is that prices are distributed uniformly, but the assumption does not match the empirical facts (see Lach and Tsiddon (1992)).
3.2.1 Asymmetric responses to positive and negative shocks

One very well-documented stylized fact in the current macro literature is that negative and positive money supply shocks have asymmetric effects on the aggregate output. More precisely, the output contraction caused by a negative monetary shock has a much larger magnitude than the output increase caused by a positive monetary shock of the same size. To put in other words, negative shocks reduce output, while positive shocks are inflationary. This observation is made in almost every developed country. For example, Cover (1992) exploits the quarterly data spanning 1951:1-1987:4 and finds a very high degree of asymmetry. He uses three model specifications for the identification of the asymmetry: the one proposed by Barro and Rush (1980), modified specification of Mishkin (1982) and his own. Asymmetry is pronounced in all three models. In Barro-Rush model 73% of a negative monetary shock is passed to output, while the same indicator for positive shocks is only 1% and it is not significant. In the Modified Mishkin model the same indicator is 66% versus 6% (the latter again not significant). In Cover’s original model 96% of negative monetary shock is passed to output, while, although not significant, the passthrough from positive shocks has the wrong sign. From these considerations one can conclude that positive monetary shocks do not have any effect on output and they basically pass to prices while negative shocks are passed to output in large extent. The more recent study of Ravn and Sola (2004) confirms the basic conclusions of Cover (1992) about the existence of asymmetry, but in their case the asymmetry is less pronounced.

The asymmetry to positive and negative monetary shock responses is also found in other parts of the world. Karras (1996) finds asymmetry in 18 European countries. Holmes (1997) adds to this the documentation of asymmetry for France and Italy and Ashworth (1998) for the United Kingdom. Chu and Ratti (1997) find asymmetry in the Japanese economy.

This asymmetry on the aggregate level can be justified in the framework of several different theoretical models. One is just having the upward sloping aggregate supply curve but that has a higher slope above the equilibrium price than below (convex aggregate supply). Second is the collection of the following assumptions: vertical aggregate supply, output function of a form \( Y = \min(AS; AD) \) and sticky prices.\(^3\) In both of these cases, the upward and downward shifts of the aggregate demand have asymmetric effects on output. The third is the “pushing on a string” model, where monetary constructions result in credit rationing and because of that, the contraction of output is aggravated.

In Table 1 the results of simulations are presented. Form there one can easily infer that the present model features asymmetric responses to positive and negative shocks: in all the six cases means and variances of the output responses are higher in the case of the negative shock than in the case of the positive one. In Figure 2 in Appendix B histograms of the response distributions are given. From the histograms the asymmetry is more visible. So, one can conclude that part of the asymmetry in output responses to monetary shocks on aggregate level can be attributed to pricing behavior of individual firms. This kind of pricing behavior seems to be able to produce convex aggregate supply curve mentioned before. So, in the current model the heterogeneity of agents seems to dampen, but not totally wipe out, the micro asymmetry on an aggregate level.

---

\(^3\)See Cover (1992).
Table 1. Share of monetary shock passed to output

<table>
<thead>
<tr>
<th>Shock</th>
<th>(1;1;2)</th>
<th>(1;1;3)</th>
<th>(1;2;3)</th>
<th>(2;1;2)</th>
<th>(1;1;3)</th>
<th>(2;2;3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>POSITIVE</strong></td>
<td>69.9%</td>
<td>60.3%</td>
<td>51.4%</td>
<td>67%</td>
<td>57%</td>
<td>49.8%</td>
</tr>
<tr>
<td></td>
<td>(2.2%)</td>
<td>(2.9%)</td>
<td>(2.7%)</td>
<td>(1%)</td>
<td>(1.4%)</td>
<td>(1.3%)</td>
</tr>
<tr>
<td><strong>NEGATIVE</strong></td>
<td>71.7%</td>
<td>63.1%</td>
<td>55.8%</td>
<td>69.1%</td>
<td>61%</td>
<td>55.1%</td>
</tr>
<tr>
<td></td>
<td>(2.8%)</td>
<td>(3.5%)</td>
<td>(3.2%)</td>
<td>(1.4%)</td>
<td>(1.8%)</td>
<td>(1.7%)</td>
</tr>
</tbody>
</table>

Notes: Figures in the table are averages of two hundred simulations of the output responses to a one period monetary shock to the model beginning from the initial distribution. Standard Errors are given in parenthesis. 

In the first raw show: $j$ - magnitude of the aggregate shock, $k$ and $l$ - two different magnitudes of four-state idiosyncratic shocks.

Although the asymmetry of the responses is not as well pronounced as the earlier discussed empirical studies find, results point to the fact that the model can be calibrated to fit the empirical findings, because changing the magnitude of shocks changes the gap and magnitude between responses. For example, the difference in responses to positive and negative shocks is only 2% in the case of the shocks given in the first column (and the difference is not statistically significant), while the last column shocks result in a difference of more than 5% (that is statistically significant). As changing the magnitude of the shocks can be interpreted as changing the magnitude of price dispersion, the main issue for calibration is to obtain an empirically feasible size of aggregate and idiosyncratic shocks.

Here also note that the asymmetry disappears if the four-state shock process is replaced by a binomial random walk. In this case, the output responses obtained from the model are very close to each other for negative and positive monetary shocks. In this case, the results of the model go in line with the results obtained by Caballero (1992). The perfect symmetry on aggregate level (as shown later) seems to be a property of a binomial random walk shock process. This issue is addressed in more details later in the section 4.

3.2.2 Asymmetric responses during the different phases of the business cycle

The second observation of asymmetric reaction of output to monetary shocks is highlighted by Lo and Piger (2003). They employ a Markov regime-switching model to investigate the asymmetry in output movements after monetary shocks to different directions. Their finding is that there is a very well pronounced time variation in output responses that can be explained by the time varying transition probability model. Basically, they find that the variation can be explained by inclusion in the model of a simple dummy variable indicating whether the economy is in a recession or in a boom. This confirms the authors’ hypothesis that output reaction has two regimes: “low response” and “high response.” In particular, policy actions taken during recessions seem to have larger effects on output than those taken during expansions.

Similar two-regime character of output responses has been found by other researchers and not only for the United States’ economy. For example, Garcia and

31The difference is never statistically significant.
Table 2. Share of positive monetary shock passed to output

<table>
<thead>
<tr>
<th>Shock</th>
<th>(1;1;2)</th>
<th>(1;1;3)</th>
<th>(1;2;3)</th>
<th>(2;1;2)</th>
<th>(1;1;3)</th>
<th>(2;2;3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RECESSION</td>
<td>99.8%</td>
<td>100%</td>
<td>88.9%</td>
<td>88.1%</td>
<td>98.2%</td>
<td>64.7%</td>
</tr>
<tr>
<td></td>
<td>(2.4%)</td>
<td>(3.4%)</td>
<td>(2.7%)</td>
<td>(1%)</td>
<td>(1.7%)</td>
<td>(2%)</td>
</tr>
<tr>
<td>BOOM</td>
<td>30%</td>
<td>21.8%</td>
<td>13.8%</td>
<td>14.6%</td>
<td>10.1%</td>
<td>12.3%</td>
</tr>
<tr>
<td></td>
<td>(3%)</td>
<td>(3.6%)</td>
<td>(4%)</td>
<td>(2.4%)</td>
<td>(2%)</td>
<td>(2%)</td>
</tr>
</tbody>
</table>

Notes: Figures in the table are averages of two hundred simulations. Standard Errors are given in parenthesis. (j;k;l) in the first row show: j - magnitude of the aggregate shock, k and l - two different magnitudes of four-state idiosyncratic shocks. Recessions are generated by giving three (-1;1;2) shocks before the recorded positive one, while booms - by giving three (1;1;2) shocks.


Here I present the results of simulations documenting that the present model features this type of asymmetry on the aggregate level (but unfortunately only for positive monetary shocks). The most interesting thing is that the current model is a kind of hybrid of sticky and flexible price models. Everything depends on the distribution of price deviations and the direction of the monetary shock. For example, if economy is in a boom, that is, it has been hit with several positive shocks, the distribution of price deviations shifts to the left border of (S,s) space. And any further positive monetary shock induces a large number of firms to raise their prices. The model gets closer to flexible price models and the output response is dampened. But this is only for positive monetary shocks. If, in this situation, the economy is hit by a negative monetary shock the distribution will shift to the right and basically no firm will adjust prices. Then, the model gets closer to sticky price models and the whole shock is passed to the output. So, the regime of output responses crucially depends on the direction of the aggregate shock.

In Table 2 I present the simulation results for output responses to positive monetary shocks during the recessions and booms. Hitting with several negative (for recession) or positive (for booms) shocks before the recorded positive shock generates these two states of the economy.

From the table, the difference between the output responses is apparent and very well pronounced, especially for high variance idiosyncratic shocks. In the case of (1;1;3) shock, the positive aggregate shock during the recessions entirely goes to the output, while during the booms two thirds of it are absorbed by prices. Also, in the case of (2;1;3) shocks, the difference in passthrough reaches 88 percentage points.

It is worth mentioning that the statements made in the previous paragraph are valid only in the case of positive monetary shocks. For negative ones, the situation is the mirror image of the one presented previously. In the negative shock case it is absorbed by prices in recessions but passed to the output in booms. But, the point is that this particular kind of heterogeneity of agents is able to produce some type of asymmetry. Stemming from the theoretical considerations above, these results can be derived from any (S,s) pricing model. The asymmetry of the bands is not
required for this result. It is purely due to the shifts of the price deviation density to one of the edges of the distribution. So, asymmetry on the micro level is not the cause of the aggregate output having two regime property, but rather this is due to (S,s) pricing behavior itself. Thus, this kind of aggregate asymmetry is the direct consequence of heterogeneity of agents, no matter whether their micro policies are symmetric or asymmetric.

4 Discussion, Limitations and Extensions

In this part of the paper I discuss the reasons for differences between the model presented hereby and other relevant models in the literature. I will also point to the basic limitations of the model and possible extensions that come to mind.

As reported in the previous section the current model features the asymmetry in output and price responses on the aggregate level. This result contrasts with some other researchers’ results, in particular, asymmetry of responses on negative and positive shocks beginning with time-invariant distribution. For example, Caballero (1992) in his special setting finds that asymmetry in microeconomic adjustment is undone on the macroeconomic level. Even though the up- and downgrades are different in magnitude, the frequency of their occurrence is also different and this fact totally undoes the first effect. In what follows, I investigate the differences between these two models.

The main difference is that asymmetry in adjustment is introduced in different ways. Caballero has symmetric bands but adjustment is not to the center of the distribution for the both sides: if x reaches the lower adjustment band, -2, it is adjusted to be -1. If it reaches the upper adjustment band, 2, it is adjusted to be zero. So, the upgrade is twice as low in magnitude as the downgrade. In my model, every adjustment is made to the optimal level, so, the deviation from the optimum after adjustment is always zero. Rather, the adjustment bands are on a different distance from the optimum.

But this is not the reason for the difference in results. To see this, take the simple example constructed in the first section of Caballero (1992). The shock in the state variable is assumed to follow a simple binomial random walk. With these assumptions the model results in ergodic transition matrix of a following form:

\[
P = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} \\
0 & 1 & 0
\end{pmatrix}
\]

Then it is obvious that \(p(1) = \frac{1}{2}p(-1)\), and asymmetry in microeconomic adjustment is undone by aggregation: although the upgrades are twice less in magnitude, they are twice more probable.

For comparison here I construct the similar simple representation of my model. Assume \(a = 2\) and \(b = 4\) to have the magnitude of downgrades twice as high as that of upgrades. The adjustment rule is that firms adjust to zero immediately as they reach either \(-a\) or \(b\). This assumption, along with the binomial random walk assumption for the shock process, results in an ergodic transition matrix of a form

\[\text{32 Models presented in this paper and in Caballero (1992).}\]
\[
P = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\
0 & \frac{1}{2} & 0 & \frac{1}{2} & 0
\end{pmatrix}
\]

One can show that this matrix leads to \( p(3) = \frac{1}{2}p(-1) \) that is equivalent to having \( p(b) = \frac{1}{2}p(-a) \). This is exactly Caballero’s result and it seems that microeconomic adjustment asymmetry has to be undone on the aggregate level.

But the result is not robust to the change of a shock specification. If we modify it to be of a four-state form \((0;1;2)\), the results change drastically. Now the result is \( p(dn) = 0.74p(up) \), \(^{33}\) and the asymmetry is present even on the aggregate level.\(^{34}\) Results of numerical simulation confirm the crucial role of the random shock process. In Table 1 I reported the results of the simulations conducted for different shock processes. One can obviously see that as the process goes further and further from the binomial random walk, the asymmetry of responses is more and more pronounced. This is due to the fact that, if a firm can have an idiosyncratic shock with the magnitude 2 grid points, the adjustment boundary can be reached also from \( x = -3 \) or \( x = -8 \).\(^{35}\) The relationship between \( p(-3) \) and \( p(8) \) is not the same as the relationship between \( p(-4) \) and \( p(9) \) that was canceling out the inequality of adjustment magnitudes. If the shock process goes further from a binomial random walk and has shocks with magnitude 3, a higher share of asymmetry remains on the aggregate level as the relation between \( p(-2) \) and \( p(7) \) is even further from the relationship between \( p(-4) \) and \( p(9) \).

Note that asymmetry on aggregate level is not the specific feature of this model: Caballero’s findings also disappear if we allow the magnitude of the shock to vary.\(^{36}\) So, we can conclude that as in a more realistic setup, the shock process is much more complicated than a simple binomial random walk, it is not clear that microeconomic asymmetry has to disappear on the aggregate level. Of course, in these two models the importance of asymmetry is significantly reduced on the aggregate level, but it still remains.

One thing also worth noting here is the significant difference in magnitudes of asymmetry after aggregation. In my model 48% of asymmetry remains in work after aggregation (with four-state \((0;1;2)\) shock process). While in Caballero’s model the magnitude is much smaller: 20% or 36%.\(^{37}\) Here the difference in adjustment asymmetry seems to be in work. But I claim that for no inflationary environment there is no reason why firms have to adjust their price levels to different points. It is not clear why firms do want to set prices that differ from the optimum. So, the adjustment asymmetry introduced in this model makes more sense and then, a higher chunk of asymmetry is spilled over the aggregate variables as well.

The model presented in this paper also contrasts with the results of Dotsey and King (2005). In their model output and inflation peaks differ in time; the

\(^{33}\) \( p(dn) \) is the probability of downgrading the price and \( p(up) \) is the probability of upgrading it.

\(^{34}\) Details are given in Appendix C.

\(^{35}\) Note that the density function of the price deviations also changes and is no longer equal to the one given by the equation (6).

\(^{36}\) See Appendix C.

\(^{37}\) Calculation comes from the relation between up- and downgrading of prices: 48% = \((0.74 - 0.5)/0.5\). Similarly for Caballero’s model.
output reaches its highest level well before the inflation peaks. In this model, output and inflation rate peak simultaneously. As I showed before, the model is a hybrid of sticky and flexible price models: the prices here become absolutely flexible immediately as the output reaches its maximum level. This means, that from that point on, the positive monetary shock will be fully absorbed by prices. Then if we assume that the size of the shock does not vary over time, it will be the highest rate of inflation possible and it will be reached together with the output peak.

The reason for this difference is that the setup of the current model is not flexible enough to allow the researcher to incorporate some general equilibrium effects. Because of this, some additional links between the monetary shocks and the output are lost. This makes the results of the current paper more straightforward than of the much more complicated model of Dotsey and King (2005).

The main shortcoming of the model is that it does not have the optimality of the asymmetric bands derived and, in a sense, it lacks micro foundations. This restricts the researcher in looking, for example, at such interesting links as between the magnitude of aggregate output responses to monetary shocks and the uncertainty, or the variance of monetary shocks. Of course, the direct link between this phenomena exists in the model (one can simply compare the percentages of the monetary shocks passed to output with the shocks with different variances), but it is only the partial effect. The increased variance of money supply has to affect the optimal range of the price deviations (See Hansen (1999)), which means that the range of state space has to be changed. For the magnitude of the change, one needs the optimality of asymmetry derived on the micro level.

The current setup investigates the implications of the individual pricing policies for the aggregate output movements and inflation rates. But as noted by Caplin and Leahy (1997) not only the pricing patterns influence the aggregate price level dynamics but there exists also the reverse link. Again, due to the same problem of the lack of micro foundations, this link can not be incorporated in the current model. But as the paper relies on the empirical observation, that the producers indeed have this kind of pricing policy, this issue becomes not very important as rational market players have already accounted for the expected aggregate price movements while deriving their pricing policies.

One more limitation of the approach is that unlike Barro (1977) and Mishkin (1982) I can not identify the anticipated money growth effect on the aggregate output dynamics. All the money shocks here are unanticipated, like Cover (1992). It also seems to be taken into account on the micro level by every individual firm.

Based on the above discussion, the main thing for further research is the derivation of the optimality of asymmetry on the micro level. This will allow the researcher to have a more complete model of the monetary transmission. Then, many more links between microeconomic and macroeconomic asymmetry can be identified. This will help us to have some distinct conclusions for the monetary policy.

One more interesting exercise can be the calibration of the model in order to better match the empirical findings. Although the magnitude of the asymmetry of bands is fitted from empirics, size and dispersion of the shock process can also be fitted from data. Alternatively, one can try to experiment with the shock dispersion in order to get the aggregate asymmetry of a similar size as an observed one and then compare the assumed shock dispersion to the empirically found dispersion. From this prospective, the model can be calibrated to become more ‘history friendly.’38

38See for example Malerba et al. (1999).
This will allow seeing how much of macro asymmetry can be attributed to the asymmetry on the firm level. The reasons of leftover can be searched for in market structure,\textsuperscript{39} credit rationing or some other considerations.

### Conclusion

Individual prices change rarely,\textsuperscript{40} and there is also a staggering in the adjustment since the price changes across the firms differ in time. This behavior is due to some costs involved in the price adjustment process: costs of gathering information about the market conjuncture, costs of loosing the market share, etc. So, the adjustment cost is a wider notion than “menu cost;” the latter is one of the components of the former. Due to the fact that some ingredients of price adjustment costs are asymmetric for price changes in different directions, the adjustment costs, as a whole, are also different for price upgrades and downgrades.

In the current paper I presented the model where individual firms follow asymmetric (S,s) pricing behavior. In particular, price upgrades have smaller magnitudes than price downgrades. This is due to the asymmetry in the adjustment costs mentioned in the previous paragraph. The main concern of the paper was whether the asymmetry in the price adjustment on the firm level spills over the aggregate variables and what is the role of heterogeneity of agents introduced by pricing in the process.

The basic results were derived by numerically simulating the model. But it was analytically shown that the model results in a time-invariant distribution of prices in the economy and due to that fact, unlike some similar papers, the results do not depend on the initial distribution of prices. One more specific character of the current paper is that, unlike the most similar papers, here I did not use simple binomial random walk for the description of shock process. Rather I used four-state mean-zero shocks. This served two purposes. Firstly, with it I showed the results of a related paper are not robust to changes in the shock process, and secondly, I believe, it is closer to reality and gives more reasonable results than the papers that use two-state random walks (for example Caballero (1992)). But it also worth mentioning that the current paper also produces the same results as Caballero (1992) if one uses a binomial random walk.

The paper basically looked at the implications of the asymmetric (S,s) pricing behavior of firms for two kinds of stylized facts about the asymmetry in the aggregate output dynamics. The first is the asymmetric response of output to positive and negative monetary shocks. Here the finding is that in the case of sufficiently high shocks, the model is able to produce statistically significant asymmetry on the aggregate level between responses. In this case, the aggregation of the heterogeneous agents undoes the part, and only the part of the micro asymmetry. The second type of asymmetry is that the aggregate output has low and high response regimes with respect to monetary shocks, depending on whether the economy is in boom or in recession. Although the model is able to produce this kind of effect for positive shocks, the main conclusion is that this is basically not due to the asymmetry on the micro level. In this case, heterogeneity itself creates the asymmetry

\textsuperscript{39}See Bhaskar (2002).

\textsuperscript{40}Although the recent work of Bils and Klenow (2004) finds that prices last twice shorter without changes, than earlier research found.
on aggregate level.

The present approach has its shortcomings, of course. The main one is that although justified theoretically as well as empirically, the optimality of the asymmetry of price adjustment bands is not analytically derived. I take it as the well-documented observed behavior of firms all around the globe. Due to this fact, many interesting links from pricing behavior on the micro level to the aggregate variable dynamics are lost. Stemming from this fact, the main issue for further research is the derivation of optimality of asymmetric (S,s) bands. The model can also be calibrated to fit the empirically found magnitudes of asymmetry and then check the feasibility of the resulted model configuration: the magnitudes of price dispersion and shocks.
Appendices

Appendix A.
Derivation of the optimality of asymmetric bands

(i) Assume Dixit-Stiglitz monopolistic competition. (ii) Define aggregate price level as the simple average of all the prices in the economy. (iii) Assume adjustment cost differential \( C^{adj}_{P(z) > P^*} > C^{adj}_{P(z) < P^*} \). (iv) Abstract from the cost of production and specify the profit function for a firm

\[
\pi(z) = \begin{cases} 
P(z)X(z) & \text{if the firm does not adjust prices} \\
P(x)^*X(z)^* - C^{adj} & \text{if the firm adjusts the prices} 
\end{cases}
\]

(v) Every firm faces a downward sloping demand curve in a form \( X(z) = 2\bar{P} - P(z) \)

Note that the demand curve is designed to capture only the main intuition: it is decreasing in firm’s own price and increasing in other competitor’s prices (of course, here I assume a continuum of small firms that can not individually affect the price index).

Then, the monopolistic profit maximization gives the optimal price, which firms want to adjust to, equal to the aggregate price level. This result simplifies the analysis very much. In this case it is very easy even to derive the values for \( s \) and \( S \). The simple profit equality condition between the profit if firm does not adjust and the profit if firm adjusts gives us

\[
P(z)(2\bar{P} - P(z)) = \bar{P}(2\bar{P} - \bar{P}) - C^{adj}.
\]

The latter results in \( \bar{P} - P_{low} = \sqrt{C^{adj}_{P(z) < P^*}} \) and \( P_{high} - \bar{P} = \sqrt{C^{adj}_{P(z) > P^*}} \). Thus \( S \neq s \).
Appendix B.
Output responses to different monetary shocks

Figure 1. The histograms of output responses to different monetary shocks

Notes: Charts in this figure are histograms of two hundred simulations of the output responses to one period monetary shock to the model beginning from the initial distribution. \((j; k; l)\) in the upper right corner of each chart show: \(j\) - magnitude of the aggregate shock, \(k\) and \(l\) - two different magnitudes of four-state idiosyncratic shocks. The white columns stand for positive monetary shocks. The black - for negative ones.
Appendix C.
Calculation of the relation between the up- and downgrading probability with four-state shock process

C1. In my model

Here I assume that shock in the state variable has the following form:

\[
\text{Shock} = \begin{cases} 
-2 & \text{with } p = 1/4 \\
-1 & \text{with } p = 1/4 \\
1 & \text{with } p = 1/4 \\
2 & \text{with } p = 1/4 
\end{cases}
\]

Then ergodic transition matrix modifies to

\[
P = \begin{pmatrix} 
0 & 3/4 & 1/4 & 0 & 0 \\
1/4 & 1/4 & 1/4 & 1/4 & 0 \\
0 & 1/4 & 1/4 & 1/4 & 1/4 \\
0 & 1/4 & 1/4 & 1/4 & 0 
\end{pmatrix}
\]

Furthermore, here the probabilities of reaching the inaction bands are not proportional to the probabilities of reaching their pre-states. Because the adjustment bands can be reached also from \( x = 0 \) and \( x = 2 \), we also have to keep track of their probabilities. Because of this fact, and because from \( x = -1 \) and \( x = 3 \), the adjustment will be required in the next period with the probability 0.5, up- and downgrading probabilities do become respectively \( p(up) = 1/2p(-1) + 1/4p(0) \) and \( p(dn) = 1/2p(3) + 1/4p(2) \). Then, the given transition matrix results in \( p(dn) = 0.74p(up) \).

C2. In Caballero’s Model

Here I assume the same four-state shock process and modify the transition matrix. But it is not straightforward, because in Caballero’s model the adjustment size is taken in a very ad hoc manner and it is not obvious where \( x \) has to adjust when it receives a shock of size 2 at the position \( x = 1 \). There are two possibilities: either an adjustment is made again to 1 (case a), or an adjustment is made to zero (case b). On the other end of the distribution, things are simpler: upward adjustment is always made to -1. So, here I calculate the dependence of the up- and downgrading probabilities for the both cases.

Transition matrices are

\[
P_a = \begin{pmatrix} 
1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 
\end{pmatrix} \quad \text{and} \quad P_b = \begin{pmatrix} 
1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 
\end{pmatrix}
\]

And again, adjustment probabilities have to be modified to \( p(up) = 1/2p(-1) + 1/4p(0) \) and \( p(dn) = 1/2p(1) + 1/4p(0) \). Then Caballero’s model results in asymmetry on the aggregate level (in both cases): \( p_a(dn) = 0.68p(up) \) and \( p_b(dn) = 0.66p(up) \).
References


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