Public Capital and Income Distribution: a Marriage of Hicks & Newman-Read*

Yoseph Yilma Getachew
Public Capital and Income Distribution: a Marriage of Hicks & Newman-Read *

Yoseph Yilma Getachew
UNU-MERIT, Maastricht University, The Netherlands

Abstract
This paper examines how publicly provided inputs could affect income distribution. By applying the Newman-Read production function- a generalized Cobb-Douglas production function- to Hicks's idea of the determinant of factor share, such usually complex dynamics remain analytically tractable. The paper shows that whether public capital has an effect on income distribution dynamics depends on its elasticity of substitution to private capital. If the elasticity of substitution of public capital to private capital is greater than unity, then an investment in public capital increases its relative income share and, hence, decreases the private capital share. In such a case, public capital would have a positive impact on income distribution dynamics.

Key words: Income distribution, public capital, economic growth
JEL codes: D31, H54, O41

UNU-MERIT Working Papers
ISSN 1871-9872

Maastricht Economic and social Research and training centre on Innovation and Technology, UNU-MERIT

UNU-MERIT Working Papers intend to disseminate preliminary results of research carried out at the Centre to stimulate discussion on the issues raised.

*I am highly grateful to Thomas Ziesemer and Adriaan van Zon for helpful comments, and, to Francisco Toro for various assistance.
1 Introduction

Does public capital have effect in income distribution? Many argue informally that under certain conditions, in particular if it is targeted at lower income social groups, public capital may reduce inequality. On the other hand, it may aggravate it if only the rich few have access to it. The question remains: how precisely is public capital linked to income distribution dynamics, especially, when it is provided on a non-discriminatory basis? Put differently, how does the income of two individuals who are heterogenous in terms of their initial wealth but similar otherwise, –one is rich and the other is poor–, be affected differently from using a public good in their production functions? In this paper, we propose theoretical answers to these questions.

We begin with the general question of what determines income distribution dynamics, especially, when both private and public inputs are involved in production. Income distribution evolves according to relative private factor income shares. When there are differences in initial endowment among households who are otherwise similar, the dynamics of income distribution depends on the degree to which households are able to exploit their relative initial advantage. The presence of any other (public good-type) inputs (e.g. infrastructure) in production have no effect on income distribution dynamics unless it alters the relative private factor income shares.

Hicks (1932) argues that elasticity of substitution is the only determinant factor for changes in relative factor shares. If the elasticity of substitution of a factor is greater than unity, then an increase in the supply of that factor more rapid than that of the others will increase its relative income share. Of course, if the elasticity of substitution of the factor is less than unity, then the relative share of the factor decreases. If it is equal to unity (the case of Cobb-Douglas), changes in supplies of factors do not have an effect on relative factors shares.

Hicks’s argument provides useful hints for those seeking (informal) solutions to the above problems. Whether public capital has an effect on income distribution dynamics depends on its elasticity of substitution with regard to private capital. If the elasticity of substitution of public

---

2 For a detailed discussion, refer Getachew (2008). See also the example below.
3 In the beginning of 1930s, Sir John Hicks (1932) marked an important advance on identifying the determinant of the distribution of income. In his book, "On The Theory of Wages," Hicks dedicated a chapter, (with an appendix), on analyzing the resultant configuration of relative factor share. He developed a very important concept, which is useful till this time, what he called it "elasticity of substitutions."
capital to private capital, in a production function, is greater than unity, an investment in public capital increases its relative income share, and decreases the private capital income share. Consequently, public capital would have a positive impact on income distribution dynamics. However, if its elasticity of substitution is less than unity, then public capital increases the private capital income share, and hence, public capital would aggravate income inequality. Of course, if the elasticity of substitution is unity, infrastructure investment is neutral to the distribution of income.

We formalize these ideas using a variable elasticity of substitution (VES) production function, which is analytically tractable, and, at the same time, allows some flexibility in the parameters. Note that for a change in factor supply in a production function to change the structure of income distribution, first of all, the elasticity of substitution must be different from unity. However, the standard Cobb-Douglas (CD) production function has no such property. In the CD function, the elasticity of substitution is equal to unity and hence the factor shares are fixed. The analytical tractability of this popular production function comes with the cost of stringent restrictions on factor shares, which makes the production function unsuited for income distribution analysis.

Analysed within the framework of a CD production function, income distribution dynamics are wholly independent of the level or change of the infrastructure inputs used in the production function. For example, suppose that the individual production function is the standard CD function $y_t = A (k_t)'^\alpha (X_t)'^\theta$ where $k_t$ is private capital and $X_t$ is public capital. Assume further that private capital, which initially differs among individuals, is distributed lognormally, i.e., $\ln k_t \sim N(\mu_t, \sigma^2_t)$. Then, an individual’s saving at $t+1$ is $k_{i,t+1} = s y_t = s A (k_t)'^\alpha (X_t)'^\theta$, where $s$ is an exogenous saving rate. Income distribution at $t + 1$ is given by, a long story cut short, $\text{var} (\ln k_{t+1}) = \sigma^2_{t+1} = \alpha^2 \sigma^2_t$. Therefore, in this economy, what matters for income distribution dynamics is neither $X_t$ nor its output elasticity $\theta$ but the predetermined private capital income share $\alpha$.

Therefore, the production function that is used for the study is the

---

4 Although Solow (1957; 1960) argue a Cobb-Douglas production function may do fairly well in tracking observed changes in production, he states in a condition, "as long as no deep distributive meaning is read into the results."

5 An exception to this case is presented in Getachew (2008). Getachew (2008) models public capital, in a CD function, as rival congestible input where its importance varies among households. In that case, in the absence of a perfect credit market, public capital could relax resource constraints of the poor, and brings a disproportional positive impact on the income of the poor, which in turn goes to reducing income inequality.

6 See Appendix A for the aggregation.
less common generalized Cobb-Douglas production function, which is developed by Newman and Read (1961). The Newman-Read production function is a VES production function, but for a certain value of its parameter it contains the standard CD function. As is discussed above, the standard CD production function $y_t = A (h_t)^{\alpha} (G_t)^{\theta}$ imposes strict restrictions on the factor shares and on the elasticity of substitution. Particularly, the relative factor shares, – denoted by $\alpha$ and $\theta$ –, are fixed and the elasticity of substitution is equal to unity. On the other hand, the constant elasticity of substitutions (CES) production function, which is the other standard production function, has a relatively less stringent restrictions on the parameters but may not provide an analytical solution for the problems we pose due to difficulties that may arise during aggregation.

The Newman-Read generalized CD function is both flexible in the values of the parameters and analytically tractable in regard to income distribution studies. Moreover, the production function reduces to the standard CD function for a certain value of its parameter. Newman and Read (1961) developed the generalized CD production function specifically to address the need for less stringent restrictions on factor income shares. They consider a case where the income share of a factor remains invariant to changes in that input itself, with the other factor held constant, but varies with changes of the other factor. In this case, the production function becomes a VES production function where the factor shares vary alongside changes in factors supply.

In the next sections, we present a model of a two-sector economy where one of the sectors uses the Newman and Read (1961) production function. In the model, we suppose an economy, populated by heterogenous agents, consists of two production sectors: human capital accumulation and goods production sector. In the former, human capital is generated using inputs from public and private resources while production technology is characterized by the standard CD function. Moreover, production in the goods sector takes place using both private and public resources, but the production function that is used this time is the generalized CD function of Newman and Read (1961).

Within this setting, we show that a change in the supply of public capital in the goods production sector could affect income distribution dynamics, at least in the short run, although no additional specification are imposed to vary the benefits that accrue from using public capital among different households. As mentioned earlier, the effect of public capital on income distribution dynamics could be negative or positive depending on the elasticity of substitution of public capital to private capital. If the elasticity of substitutions is greater (lesser) than unity,
then public capital decreases (increases) income inequality. In the long run, in the context of a growing economy, the elasticity of substitution is assumed to tend toward unity and, hence, the generalized CD function reduces to the standard CD function. The model thus turns into a standard AK type endogenous growth model in the long term.

Some functions of public capital may also be worth mentioning. In growth literature public capital is usually treated as a factor that enhances productivity and complements the accumulation of private inputs. However, there is no doubt that public capital could also function as a substitutable factor that could relax households’ resource constraints, particularly for poor households’, in the context of imperfect credit markets. Of course, whether the elasticity of substitution between public and private capital is greater or lesser than one depends on the type of public capital made available. But, in general, one of the functions of public capital is to provide opportunities and resources to individuals that they could not otherwise afford. This role of public capital may suggest that it could have a higher elasticity of substitution than private capital as a general proposition. As Hicks noted one of the things upon which elasticity of substitution of factors depends is "...the mere extension of the use of instruments and methods of production from firms where they were previously employed to firms which could not previously afford them" (Hicks 1932, p.120). Moreover, it is possible to mention some instances where public capital has clearly a large elasticity of substitutions, particularly in developing countries. Public education and public health infrastructure, for instance, may be considered good substitutions of private education and private health infrastructure.

The strand of literature related to the present study deals with the relationship between income inequality and public capital. Recently, a growing number of empirical studies try to assess the impact of infrastructure on income inequality. For instance, Calderón and Serven (2004), Calderón and Chong (2004) and Lopez (2004) find that infrastructure (water, sanitary, electricity, etc.) reduce income inequality and enhance economic growth at the same time. World Bank (2003) and Estache (2003) argue infrastructure has a positive and disproportionate impact on growth. OECD (2006) reports that "infrastructure is important for pro-poor growth". Whereas, Ferreira (1995) argues

---

7 Empirical studies show factor shares, particularly that of private inputs (such as labor), show a large movement in the short run whereas they become stable in the long run (e.g. Acemoglu 2003; Revankar 1971).
8 See, for instance, World Bank (1994) and Romp and de haan (2005; 2007).
9 And, this is a question which is more of an empirical.
theoretically, in a model with quite a complex setup, public capital is important for income distribution dynamics. Getachew (2008) attempts to analytically capture their relationship using the popular standard CD function (refer footnote 4).

Another strand of literature related to the present study deals with the dynamics of income inequality and growth within an imperfect credit markets (e.g. Loury (1981), Galor and Zeira (1993), Banerjee and Newman (1993), Piketty (1997), Aghion and Bolton (1997), Aghion and Howit (1998), Aghion, Caroli, and García-Peñalosa (1999), and Benabou (1996; 2000; 2002)). These literatures in general show that in the face of capital market imperfection, income inequality has a definite negative effect on growth. This is because when the credit markets are imperfect, relatively more high-return investment opportunities would be forgone by resource poor households in inequalitarian society than egalitarian ones.

The paper is organized as follows. In section 2 we set up the model, with a brief discussion on the property of the Newman-Read production function. Section 3 is all about the evolution of income distribution and public capital. Section 4 discusses the long run (equilibrium) properties of the model, and section 5 concludes.

2 The Model

2.1 Households

Assume a continuum of heterogenous households, in overlapping generations, \( i \in [0, 1] \). Each household \( i \) consists of an adult of generation \( t \) and a child of generation \( t + 1 \). Population size is thus constant and normalized to be one. Let, at the beginning initial period, each household \( i \) of the initial generation be endowed with an initial human capital \( h_0 \) and a public infrastructure \( G_0 \) which is shared with others. Assume further that the distribution of initial human capital \( \Gamma(h_0) \) takes a lognormal form, \( \ln h_0 \sim N(\mu_0, \sigma_0^2) \).

Agents care about their consumption level and the human capital stock of their children. When young, they accumulate human capital using both private and public resources. Once they become adults, they use their human capital for final goods production. Government taxes income with two fixed flat rate taxes, \( \psi \) and \( \tau \), in order to finance public capital, denoted by \( G_t \) and \( M_t \), in the goods production and human capital accumulation sectors, respectively.\(^{10}\) During their active periods, individuals allocate after tax income between current consumption \( c_t \)

\(^{10}\)Note that in the paper, small and capital letters are used to denote individual and aggregate (average) variables, respectively.
and saving $e_t$ that will be used to educate their children. The latter is incorporated in individuals’ utility functions as the "joy of giving".

In a logarithmic preference, the utility of an individual is defined as

$$\ln c_t + \beta \ln h_{t+1}$$

subject to

$$c_t + e_t = (1 - \tau - \psi)y_t$$

where $y_t$ is income of an individual at time $t$.

The human capital accumulation function of the offspring $h_{t+1}$ is a function of public $M_t$ and parental investment $e_t$. The accumulation function takes the standard CD form, in this sector, with constant returns to scale in factors. Thus, for an individual who is born at time $t$, the human capital at $t+1$ is given by

$$h_{t+1} = B (M_t)^{1-\eta} (e_t)^{\eta}$$

2.2 The Firm

There is an infinite number of competitive small firms. Households own the firms. Production at the firm level occurs using both private and public capital, in a Newman-Read production function setting. Thus, the income of an agent of generation $t$, in the Newman-Read production function, is defined as

$$y_t = A (h_t)^{\alpha} (G_t)^{1-\alpha} \exp (\lambda_t \ln h_t \ln G_t)$$

where $y_t$ and $G_t$ represent output, and public capital, respectively. $\lambda_t$ is a parameter, whose sign is found to be an important determinant of the behavior of the elasticity of substitution. We will discuss its property in detail later on.

The aggregate production function $Y_t$ is computed, simply, by aggregating (4), $Y_t \equiv \int_0^1 y_t \Gamma(h_t),$

$$Y_t = A (G_t)^{1-\alpha} (H_t)^{(\alpha + \lambda_t \ln G_t)} \exp \frac{\sigma^2}{2} (\alpha + \lambda_t \ln G_t) (\alpha + \lambda_t \ln G_t - 1)$$

\footnote{This shuts off the input and the credit markets.}

\footnote{Note that the production function, in (4), reduces to the standard CD function if $\lambda_t$ equal to zero, $\lambda_t = 0$.}
where $H_t$ is the aggregate (average) human capital, which is defined, $H_t \equiv \int_0^1 h_t \Gamma(h_t)$. $\Gamma(h_t)$ is the distribution of wealth at time $t$, which is assumed to take a lognormal distribution, i.e., $\ln h_t \sim N(\mu_t, \sigma_t^2)$.

### 2.2.1 Properties of the Newman-Read Production Function

Although its use is relatively rare in the literature of economic growth, the Newman-Read production function offers a flexible analytical framework for income distribution studies. Empirical studies reveal factor shares show large fluctuations in the short run whereas they show no trend in the long run (e.g. Acemoglu 2003). The Newman-Read generalized CD production function contains a framework that provides a satisfactory approximation to this reality, in particular contrast to the popular CD function. The less popular production function provides a framework for analyzing the short run fluctuations in factor income shares whereas the standard CD function does not. The latter is more appropriate for analyzing long run trends. Whereas, the Newman-Read production function is a general form of the standard CD function that could reduce to it when necessary (e.g., when analyzing long run trends).

According to Solow (1957), a particular functional form adopted for a production function is a matter of no great consequence as far as it possesses a positive partial derivative and the right curvature. For certain values of its parameters, the Newman-Read production function, in (4), satisfies important properties of the neoclassical production function, i.e., it has a positive marginal productivity and a concave curvature:

The first and the second derivative of the production function in (4) are given by

$$ \frac{\partial y_t}{\partial h_t} = \frac{y_t}{h_t} (\alpha + \lambda_t \ln G_t) $$

$$ \frac{\partial y}{\partial G_t} = \frac{y}{G_t} (1 - \alpha + \lambda_t \ln h_t) $$

and

\[ \int_0^1 \left( (\ln h_t)^\alpha \exp(\lambda_t \ln h_t \ln G_t) \right) \Gamma(h_t) \mathrm{d}h_t = (H_t)^{(1+\lambda_t \ln G_t)} \exp \frac{\sigma_t^2}{2} (\alpha + \lambda_t \ln G_t) (\alpha + \lambda_t \ln G_t - 1) \]

(See Appendix A).

\[13\]While aggregating (4), we use the fact that

\[14\]Acemoglu states factor shares of labor show a large movement in the short run whereas they remain stable in the long run. See Figure 1 and 2, which he plots for United State and France, respectively.

\[15\]Recall that elasticity of substitution equals to unity and hence factor shares are fixed in the popular CD function.
\[
\frac{\partial y_t}{\partial h_t \partial h_t} = \frac{y_t}{(h_t)^2} (\alpha + \lambda_t \ln G_t) ((\alpha + \lambda_t \ln G_t) - 1)
\]

\[
\frac{\partial y_t}{\partial G_t \partial G_t} = \frac{y_t}{(G_t)^2} (\lambda_t \ln h_t - \alpha) ((\lambda_t \ln h_t - \alpha) - 1)
\]

**Assumptions 2.2**

1. \(0 < \alpha, \eta < 1; A, G_t, h_t > 1\)

2. \((\ln G_t)^{-1} > -\lambda_t (\alpha)^{-1}; (\ln h_t)^{-1} > \lambda_t (\alpha)^{-1}\) and \((\ln h_t)^{-1} > -\lambda_t (1 - \alpha)^{-1}\)

3. \(\lim_{t \to \infty} \lambda_t = 0, |\lambda_t| < 1\) otherwise; where \(\lambda_t\) is sufficiently large.

Under Assumptions 2.2, the Newman-Read production function in (4) obeys the neoclassical rule in that it has positive marginal productivity and concave curvatures with respect to both private and public capital. Formally,

\[
y_t > 0; \frac{\partial y_t}{\partial h_t} > 0; \frac{\partial y_t}{\partial G_t} > 0; \frac{\partial y_t}{\partial h_t \partial h_t} < 0 \text{ and } \frac{\partial y_t}{\partial G_t \partial G_t} < 0
\]

Moreover, under the above assumptions,

\[
\frac{\partial y_t}{\partial h_t \partial G_t} = \frac{y_t}{G_t h_t} \frac{((1 - \alpha + \lambda_t \ln h_t)(\alpha + \lambda_t \ln G_t) + \lambda_t)}{G_t h_t} > 0
\]

which implies that public capital enhances the marginal productivity of private capital, and vice versa.

The elasticity of substitutions, of the production function, implied by (4), denoted by \(\delta_{hG}\) and \(\delta_{Gh}\), for public and private capital, respectively, are\(^\text{16}\)

\[
\delta_t = \delta_{hG} = \delta_{Gh} = \frac{\frac{\partial y_t}{\partial h_t} \frac{\partial y_t}{\partial G_t}}{y_t \frac{\partial y_t}{\partial G_t \partial h_t}} = \frac{(\alpha + \lambda_t \ln G_t)(1 - \alpha + \lambda_t \ln h_t)}{(\alpha + \lambda_t \ln G_t)(1 - \alpha + \lambda_t \ln h_t) + \lambda_t} > 0
\]

Equation (6) implies that the sign of \(\lambda_t\) determines whether the elasticity of substitution between private and public capital is greater or lesser than, or, equal to unity. That is,

\[
\text{If } \lambda_t \leq 0, \text{ then } \delta_t \geq 1
\]

\(^{16}\text{The original formula was presented by Hicks (1932), Appendix (iii).}\)
2.3 Government

We assume government budget is balanced at all times:

\[ I_t^g = \int_0^1 y_t \psi \Gamma(h_t) = Y_t \psi \]  

(8)

\[ M_t = \int_0^1 y_t \tau \Gamma(h_t) = Y_t \tau \]  

(9)

Thus, the government collects fixed proportional taxes \( \psi \) and \( \tau \) on output \( Y_t \) to finance public investment, while the accumulation of public capital in the goods production sector follows the rule,

\[ G_{t+1} = I_t^g + G_t (1 - \varphi^g) \]  

(10)

where \( G_t, I_t^g \) and \( \varphi^g \) are the public capital stock, public investment and depreciation, respectively.

2.4 Competitive Equilibrium

According to the above description, an individual at period \( t \) solves the following problem, which is derived by substituting (2) and (3) into (1),

\[
\max_{e_t} \ln(1 - \psi) y_t - e_t + \beta \ln B(M_t)^{1-\eta} (e_t)^{\eta} 
\]

(11)

taking as given, \( \psi, \tau, I_t^g, M_t \) and \( G_t \).

The first order condition gives

\[ e_t = a(1 - \tau - \psi)y_t \]  

(12)

where \( a = \frac{\beta \eta}{1 + \beta \eta} \); (12) shows the agent’s optimal saving as the function of his income.

To derive individuals’ human capital accumulation equation, which is associated to their optimal behavior, substitute (9) and (12) into (3), and using (4) and (5), to get,\(^{17}\)

\[ h_{t+1} = B(G_t)^{(1-\alpha)} \left( \tau A(H_t)^{(\alpha+\lambda_t \ln G_t)} \right)^{1-\eta} (h_t)^{\alpha \eta} (a(1 - \tau - \psi)A)^{\eta} \exp(\Xi_t) \]  

(13)

where

\[ \Xi_t = \frac{\sigma^2}{2} ((\alpha + \lambda_t \ln G_t) (\alpha + \lambda_t \ln G_t - 1) (1 - \eta)) + (\eta \lambda_t \ln h_t \ln G_t) \]

\(^{17}\)See Appendix B for the details of the derivation of (13).
According to (13), an individual’s human capital accumulation is determined by the human capital of his parent $h_t$, and aggregate private and public capital stock and initial income distribution, $H_t$, $G_t$, and $\sigma_t^2$, respectively. Income inequality apparently has a negative impact on individuals’ capital accumulation.

3 Public Capital and Income Distribution Dynamics

The following two difference equations that are derived from (13) characterize the evolution of capital accumulation and income distribution in the economy

\[
\mu_{t+1} = E[\ln h_{t+1}] = \alpha \eta \mu_t + \ln BA + \ln (\tau)^{1-\eta} (a(1 - \tau - \psi)^\eta \\
+ (1 - \alpha) \ln G_t + ((\alpha + \lambda_t \ln G_t)(1 - \eta)) \ln H_t + \Xi_t
\]

\[
\sigma_{t+1}^2 = Var[\ln h_{t+1}] = \eta^2 (\alpha + \lambda_t \ln G_t)^2 \sigma_t^2
\]

Under assumptions 2.2, which implies $0 < \eta (\alpha + \lambda_t \ln G_t) < 1$, income inequality will decline through time and ultimately disappear:

\[
\lim_{t \to \infty} \sigma_t^2 = 0
\]

where $\sigma^2$ is equilibrium income distribution. However, the disappearance of income inequality in the steady state should not be confused with a stylized fact. The reason that income inequality disappears in the long run is that, in the model, initial wealth is the only source of heterogeneity between individuals; but agents are otherwise similar (in terms of ability, technology, etc.) Therefore, a diminishing return on net private accumulative factors, $\eta (\alpha + \lambda_t \ln G_t) < 1$, implies resource poor households are more productive than rich ones; consequently, it is inevitable that the poor will catch up with the rich in the long run.

The model thus captures only the short run effect of public capital. The role of public capital in this model influences the speed of convergence of income distribution. Depending on the sign of $\lambda_t$ (and hence the size of $\delta_t$), the public good $G_t$, in the final goods production sector, affects income distribution dynamics negatively or positively during the economy’s transition to its long run equilibrium. Recall from equation (7) that the sign of $\lambda_t$ determines the value of elasticity of substitution $\delta_t$ whether it is greater, less than, or equal to unity, which in turn determines what effect the public capital $G_t$ could have on income distribution dynamics.
If $\lambda_t = 0$, then $\delta_t = 1$,– the Cobb-Douglas case–, there is no relationship between public capital and income distribution. But, if $\lambda_t > 0$, then $\delta_t < 1$, investment in public capital $G_t$ aggravates income disparities, at least in the short run. The reason is, if $\delta_t < 1$, then, an increase in the public capital increases the relative income share of private capital, which ultimately disproportionally benefits the rich who hold much of the production resources. On the other hand, if $\lambda_t < 0$, then $\delta_t > 1$, an increase in the public good $G_t$ decreases the relative income share of private capital. Consequently, income inequality decreases as poor household are able to relax some of their resource constraints through factor substitution.

4 Dynamics of Other Macroeconomic Variables

To determine the dynamics of other macro variables, first aggregate (13) to obtain the equation that characterizes the evolution of aggregate human capital,

$$H_{t+1} = BA (G_t)^{(1-\alpha)} a^\eta (1 - \tau - \psi)^\eta (\tau)^{1-\eta} (H_t)^{(\alpha + \lambda_t \ln G_t)} \exp (\Phi_t (\eta^2 - \eta + 1))$$

(16)

where

$$\Phi_t = \frac{\sigma_t^2}{2} ((\alpha + \lambda \ln G_t) (\alpha + \lambda \ln G_t - 1))$$

Then, derive the dynamic equation for the public capital in the goods production sector by substituting (8) into (10), using (5), and assuming a complete depreciation ($\gamma^g = 1$)

$$G_{t+1} = \psi A (G_t)^{1-\alpha} (H_t)^{(\alpha + \lambda \ln G_t)} \exp (\Phi_t)$$

(17)

At equilibrium $\lim_{t \to \infty} \sigma^2 = 0$, aggregate capital ratio converges to a constant value, using (16) and (17),

$$\frac{H}{G} = \frac{Ba^\eta (1 - \tau - \psi)^\eta (\tau)^{(1-\eta)}}{\psi}$$

(18)

18While aggregating (13), we use the fact that

$$\int_0^1 ((h_t)^{\alpha \eta} \exp (\eta \lambda_t \ln h_t \ln G_t)) \Gamma(h_t) = (H_t)^{\eta (\alpha + \lambda_t \ln G_t)} \exp (\eta^2 \Phi_t)$$

(see appendix A).
From, equation (18), it is apparent that in the long run, the economy behaves as a standard \( AK_t \) model. Aggregate variables will be in a balanced growth path, where \( H, G \) and \( Y \) grow at the same rate.\(^{19}\)

The long run growth rate \( \gamma \) of the economy can easily be computed from the equilibrium value of (16),

\[
\gamma = \ln H_{t+1} - \ln H_t
\]
\[
= \ln B + \ln A + \eta \ln a + \eta \ln (1 - \tau - \psi) + (1 - \eta) \ln \gamma + (\alpha - 1) \ln \frac{H_t}{G_t}
\]
and then substituting (18) into the above equation,

\[
\gamma = \ln A + \alpha \ln B + \alpha \eta \ln a + \alpha \eta \ln (1 - \tau - \psi) + \alpha (1 - \eta) \ln \gamma + (1 - \alpha) \ln \psi
\]

As of the literatures in public capital and economic growth, both taxes relate positively but non-linearly to long term growth (e.g., Barro 1990). The growth maximizing taxes, \( \psi^* \) and \( \tau^* \) are given by

\[
\psi^* = (1 - \alpha) \quad \text{(20)}
\]
\[
\tau^* = (1 - \eta) \alpha \quad \text{(21)}
\]

The growth maximizing tax for public capital in the goods production sector \( \psi^* \) is equal to the share of public capital in the sector (similar to Barro’s (1990) finding) while the growth maximizing tax for public capital in human capital accumulation sector \( \tau^* \) is equal to the share of the public capital in that sector times the long run output elasticity of human capital.

5 Conclusion

It is well understood that public capital is important for economic growth. But, is public capital important for income inequality? It is intuitive that public capital may reduce inequality, particularly if it is targeted at lower income groups. Nonetheless, public capital may aggravate income inequality if only the rich few have access to it. But, as we have shown here, even if it is provided in a non-discriminatory basis, public capital remains important for income distribution dynamics.

The effect of public capital on income distribution is strongly linked to its elasticity of substitution to private capital. If the elasticity of

\(^{19}\)Let the variables without time subscript \( (H, G, Y \text{ and } \sigma^2) \) denote steady state values.
substitution of a given type of public input is greater than unity, then it might have a positive and disproportionate impact on the income of the poor. Particularly, if the credit market is imperfect, the provision of public capital enjoying high elasticity of substitution to private capital, even on a non-discriminatory basis, might help the poor more by relaxing some of their resource constraints. This, in turn, results in an improvement in the distribution of income of the economy.
A Aggregation

With the assumption of a lognormal distribution for individual’s human capital, \( \ln h_t \sim N(\mu_t, \sigma^2_t) \), we have the following relation

\[
\ln E[h_t] = E[\ln h_t] + \frac{\sigma^2_t}{2}
\]

\[
\iff \quad E[\ln h_t] = E[\ln h_t] - \frac{\sigma^2_t}{2} \equiv \ln H_t - \frac{\sigma^2_t}{2} \quad \text{(A1)}
\]

since \( E[h_t] = \int_0^1 h_t \Gamma(h_t) \equiv H_t \).

The fact that \( \int_0^1 \ln y_t(h_t) \Gamma(h_t) = E(\ln y_t) \), where \( \Gamma(h_t) \) is the distribution function of \( (\text{the random variable}) \, h_t \). Then,

\[
Y_t \equiv E[y_t] = E[A(h_t)^{\alpha} (G_t)^{1-\alpha} \exp(\lambda_t \ln h_t \ln G_t)]
\]

\[
= A(G_t)^{1-\alpha} E[(h_t)^{\alpha} \exp(\lambda_t \ln h_t \ln G_t)]
\]

We derive, in equation (5),

\[
Y_t = A(G_t)^{1-\alpha} (H_t)^{(\alpha+\lambda_t \ln G_t)} \exp \frac{\sigma^2_t}{2} (\alpha + \lambda_t \ln G_t) (\alpha + \lambda_t \ln G_t - 1)
\]

by using the above facts.

That is, if \( h_t \) is a lognormal distribution then \( (h_t)^{\alpha} \exp(\lambda_t \ln h_t \ln G_t) \) is also a lognormal distribution since \( \ln (h_t^{\alpha} \exp(\lambda_t \ln h_t \ln G_t)) = (\alpha + \lambda_t \ln G_t) \ln h_t \) is normal. Then, according to (A1),

\[
\ln E[(h_t)^{\alpha} \exp(\lambda_t \ln h_t \ln G_t)]
\]

\[
= E[\ln (h_t)^{\alpha} + (\lambda_t \ln h_t \ln G_t)] + \frac{1}{2} \text{var[\ln (h_t)^{\alpha} + (\lambda_t \ln h_t \ln G_t)]}
\]

\[
= E[(\alpha + \lambda_t \ln G_t) \ln h_t] + \frac{1}{2} \text{var}[(\alpha + \lambda_t \ln G_t) \ln h_t]
\]

\[
= (\alpha + \lambda_t \ln G_t) \left( \ln H_t - \frac{\sigma^2_t}{2} \right) + (\alpha + \lambda_t \ln G_t)^2 \frac{\sigma^2_t}{2}
\]

\[
= (\alpha + \lambda_t \ln G_t) \ln H_t + (\alpha + \lambda_t \ln G_t)(\alpha + \lambda_t \ln G_t - 1) \frac{\sigma^2_t}{2}
\]

Then,

\[
E[(h_t)^{\alpha} \exp(\lambda_t \ln h_t \ln G_t)]
\]

\[
= (H_t)^{(\alpha+\lambda_t \ln G_t)} \exp \frac{\sigma^2_t}{2} (\alpha + \lambda_t \ln G_t) (\alpha + \lambda_t \ln G_t - 1) \quad \text{(A2)}
\]
To derive

\[
E \left[ (h_t)^{\alpha \eta} \exp (\eta \lambda_t \ln h_t \ln G_t) \right] \\
= (H_t)^{\eta (\alpha + \lambda_t \ln G_t)} \exp \left( \frac{\sigma_t^2}{2} \eta^2 \left( \alpha + \lambda_t \ln G_t + \lambda_t \ln G_t - 1 \right) \right)
\]

in (16), follow similar steps as above.

**B Dynamics of Individual Human Capital**

To derive the individual’s human capital accumulation equation, substitute (9) and (12), into (3), to get

\[
h_{t+1} = B (\tau Y_t)^{1-\eta} (a(1 - \tau - \psi)y_t)^{\eta}
\]

Then, substitute (4) and (5) into the above equation

\[
h_{t+1} = B \left( \tau A (G_t)^{1-\alpha} (H_t)^{1-\alpha} \exp \frac{\sigma_t^2}{2} (\alpha + \lambda_t \ln G_t + \lambda_t \ln G_t - 1) \right)^{1-\eta} \\
= \left( \frac{\sigma_t^2}{2} \right) \left( \alpha + \lambda_t \ln G_t + \lambda_t \ln G_t - 1 \right) + (\eta \lambda_t \ln h_t \ln G_t)
\]

After rearranging, we get equation (13) of Section 2.4,

\[
h_{t+1} = B (G_t)^{(1-\alpha)} \left( \tau A (H_t)^{(\alpha + \lambda_t \ln G_t)} \right)^{1-\eta} \left( h_t \right)^{\alpha \eta} \left( a(1 - \tau - \psi)A \right)^{\eta} \exp(\Xi_t)
\]

where

\[
\Xi_t \equiv \frac{\sigma_t^2}{2} \left( \alpha + \lambda_t \ln G_t + \lambda_t \ln G_t - 1 \right) + (\eta \lambda_t \ln h_t \ln G_t)
\]
References


<table>
<thead>
<tr>
<th>Year</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008.1</td>
<td>Science, Technology and Development: Emerging concepts and visions</td>
<td>Luc Soete</td>
</tr>
<tr>
<td>2008.2</td>
<td>Reframing technical change: Livestock Fodder Scarcity Revisited as Innovation Capacity Scarcity. Part 1</td>
<td>Andy Hall, Rasheed Sulaiman V., Mona Dhamankar, Peter Bezkorowajny &amp; Leela Prasad</td>
</tr>
<tr>
<td>2008.5</td>
<td>Is Inter-Firm Labor Mobility a Channel of Knowledge Spillovers? Evidence from a Linked Employer-Employee Panel</td>
<td>Mika Maliranta, Pierre Mohnen &amp; Petri Rouvinen</td>
</tr>
<tr>
<td>2008.6</td>
<td>Financial Constraints and Other Obstacles: Are they a Threat to Innovation Activity?</td>
<td>P. Mohnen, F.C. Palm, S. Schim van der Loeff and A. Tiwari</td>
</tr>
<tr>
<td>2008.7</td>
<td>Knowledge-based productivity in ‘low-tech’ industries: evidence from firms in developing countries</td>
<td>Micheline Goedhuys, Norbert Janz and Pierre Mohnen</td>
</tr>
<tr>
<td>2008.8</td>
<td>The Voyage of the Beagle in Innovation Systems Land. Explorations on Sectors, Innovation, Heterogeneity and Selection</td>
<td>Martin Srholec &amp; Bart Verspagen</td>
</tr>
<tr>
<td>2008.9</td>
<td>Crafting Firm Competencies to Improve Innovative Performance</td>
<td>Boris Lokshin, Anita van Gils &amp; Eva Bauer</td>
</tr>
<tr>
<td>2008.10</td>
<td>The Economics and Psychology of Personality Traits</td>
<td>Lex Borghans, Angela Lee Duckworth, James J. Heckman &amp; Bas ter Weel</td>
</tr>
<tr>
<td>2008.13</td>
<td>Explaining Success and Failure in Development</td>
<td>Adam Szirmai</td>
</tr>
<tr>
<td>2008.15</td>
<td>Productivity effects of innovation, stress and social relations</td>
<td>Rifka Weehuizen, Bulat Sanditov and Robin Cowan</td>
</tr>
<tr>
<td>2008.16</td>
<td>Entrepreneurship and Innovation Strategies in ICT SMEs in Enlarged Europe (EU25)</td>
<td>Kaushalesh Lal and Theo Dunnewijk</td>
</tr>
</tbody>
</table>

2008.18 Multinationals are Multicultural Units: Some Indications from a Cross-Cultural Study by Nantawan Noi Kwanjai & J. Friso den Hertog

2008.19 The Innovativeness of Foreign Firms in China by Branka Urem, Ludovico Alcorta and Tongliang An

2008.20 Beyond the emission market: Kyoto and the international expansion of waste management firms by Ionara Costa, Asel Doranova and Geert-Jan Eenhoorn

2008.21 The ‘making of’ national giants: technology and governments shaping the international expansion of oil companies from Brazil and China by Flavia Carvalho and Andrea Goldstein

2008.22 If the Alliance Fits . . . : Innovation and Network Dynamics by Robin Cowan & Nicolas Jonard

2008.23 Facing the Trial of Internationalizing Clinical Trials to Developing Countries: With Some Evidence from Mexico by Fernando Santiago-Rodriguez


2008.26 New Europe’s Promise for Life Sciences by Sergey Filippov and Kálmán Kalotay

2008.27 A closer look at the relationship between life expectancy and economic growth by Théophile T. Azomahou, Raouf Boucekkine, Bity Diene


2008.29 Worker remittances and government behaviour in the receiving countries by Thomas Ziesemer

2008.30 Strategic motivations for Sino-Western alliances: a comparative analysis of Chinese and Western alliance formation drivers by Tina Saebi & Qinqin Dong


2008.32 Promoting clean technologies: The energy market structure crucially matters by Théophile T. Azomahou, Raouf Boucekkine, Phu Nguyen-Van

2008.33 Local Knowledge Spillovers, Innovation and Economic Performance in Developing Countries: A discussion of alternative specifications by Effie Kesidou and Adam Szirmai
2008.34 *Wage effects of R&D tax incentives: Evidence from the Netherlands* by Boris Lokshin and Pierre Mohnen

2008.35 *Cross-border Investment and Economic Integration: The Case of Guangdong Province and Hong Kong SAR* by Naubahar Shari and Can Huang

2008.36 *Radical versus non-radical inventions* by Wilfred Schoenmakers, Geert Duysters & Wim Vanhaverbeke


2008.38 *The economic impact of AIDS in sub-Saharan Africa* by Théophile T. Azomahou, Raouf Boucekkine, Bity Diene

2008.39 *Further results on bias in dynamic unbalanced panel data models with an application to firm R&D investment* by Boris Lokshin

2008.40 *A multilevel analysis of innovation in developing countries* by Martin Srholec

2008.41 *Experimentation with strategy and the evolution of dynamic capability in the Indian Pharmaceutical Sector* by Suma Athreye, Dinar Kale & Shyama V. Ramani

2008.42 *The Impact of Social Capital on Crime: Evidence from the Netherlands* by I.Semih Akcomak and Bas ter Weel

2008.43 *Portrait of an Odd-Eyed Cat: Cultural Crossing as a Trademark for a Dutch-Thai Strategic Alliance* by Nantawan Noi Kwanjai & J Friso den Hertog

2008.44 *The challenge of measuring innovation in emerging economies’ firms: a proposal of a new set of indicators on innovation* by Luciana Manhães Marins

2008.45 *Intra-firm Technology Transfer and R&D in Foreign Affiliates: Substitutes or Complements? Evidence from Japanese Multinational Firms* by Rene Belderbos, Banri Ito, Ryuhei Wakasugi

2008.46 *To Be or Not to Be at the BOP: A One-North-Many-Souths Model with Subsistence and Luxury Goods* by Adriaan van Zon and Tobias Schmidt

2008.47 *Habit Formation, Information Exchange and the Social Geography of Demand* by Zakaria Babutsidze and Robin Cowan

2008.48 *Agenda Disputes and Strategic Venue Preferences: The Doha Crisis and Europe’s Flight to Regionalism* by Francisco P. Toro

2008.49 *The determinants of the outward foreign direct investment of China and India: Whither the home country?* by Paz Estrella Tolentino

2008.51 Internationalising to create Firm Specific Advantages: Leapfrogging strategies of U.S. Pharmaceutical firms in the 1930s and 1940s & Indian Pharmaceutical firms in the 1990s and 2000s by Suma Athreye and Andrew Godley

2008.52 Internationalization and Technological Catching Up of Emerging Multinationals: A Case Study of China’s Haier Group by Geert Duysters, Jojo Jacob and Charmianne Lemmens

2008.53 India’s Outward Foreign Direct Investments in Steel Industry in a Chinese Comparative Perspective by Nagesh Kumar and Alka Chadha

2008.54 Internationalization Trajectories – a cross country comparison: Are Large Chinese and Indian Companies different? By Fabienne Fortanier and Rob van Tulder

2008.55 Europeanisation strategy of Chinese companies: It’s perils and promises by Sergey Filippov and Tina Saebi

2008.56 Public capital, income distribution and growth by Yoseph Yilma Getachew

2008.57 Growth with Endogenous Migration Hump and the Multiple, Dynamically Interacting Effects of Aid in Poor Developing Countries by Thomas Ziesemer


2008.59 When a good science base is not enough to create competitive industries: Lock-in and inertia in Russian systems of innovation by Rajneesh Narula and Irina Jormanainen

2008.60 Alliance block composition patterns in the microelectronics industry by Geert Duysters & Charmianne Lemmens

2008.61 FDI and Innovation as Drivers of Export Behaviour: Firm-level Evidence from East Asia by Ganeshan Wignaraja

2008.62 Russia’s Emerging Multinationals: Trends and Issues by Sergey Filippov

2008.63 Working remittances, migration, accumulation and growth in poor developing countries by Thomas H.W. Ziesemer

2008.64 The Innovative Performance of Alliance Block Members: Evidence from the Microelectronics Industry by Geert Duysters, Charmianne Lemmens, Wilko Letterie and Wim Vanhaverbeke

2008.65 Formal and informal external linkages and firms’ innovative strategies. A cross-country comparison by Isabel Maria Bodas Freitas, Tommy Clausen, Roberto Fontana and Bart Verspagen

2008.66 The Heterogeneity of MNC’ Subsidiaries and Technology Spillovers: Explaining positive and negative effects in emerging economies by Anabel Marin and Subash Sasidharan
2008.67 EU enlargement and consequences for FDI assisted industrial development by Rajneesh Narula and Christian Bellak

2008.68 Private Capacity and Public Failure: Contours of Livestock Innovation Response Capacity in Kenya by Ekin Keskin, Mirjam Steglich, Jeroen Dijkman and Andy Hall

2008.69 Learning Networks Matter: Challenges to Developing Learning-Based Competence in Mango Production and Post-Harvest in Andhra Pradesh, India by Laxmi Prasad Pant, Helen Hambly Odame, Andy Hall and Rasheed Sulaiman V.

2008.70 Global Migration of the Highly Skilled: A Tentative and Quantitative Approach by Theo Dunnewijk

2008.71 Public Capital and Income Distribution: a Marriage of Hicks & Newman-Read by Yoseph Yilma Getachew