10 Skill-biased technological change

On endogenous growth, wage inequality and government intervention

*Hugo Hollander and Bas ter Weel*

Introduction

One of the most prominent observations when analysing labour-market trends in the OECD countries over the past three decades is the fact that labour demand seems to favour more skilled workers, replaces tasks previously performed by unskilled workers, and intensifies inequality. Despite the marked increase in the supply of skilled workers, this shift has caused dramatic declines in unskilled employment rates throughout the OECD, which in some countries and periods has been accompanied by profound shifts in relative wages. The latter trend has been particularly strong in the United States and the United Kingdom and has attracted a lot of attention in the literature.

For example, Katz and Murphy (1992) find for the United States that between 1979 and 1987 the average weekly wage of college graduates with one to five years of experience has increased by some 30 per cent relative to the average weekly earnings of comparable high-school graduates. Acemoglu (2000) finds that in 1971 a worker at the 90th percentile of the wage distribution earned 266 per cent more than a worker at the 10th percentile of the wage distribution did. By 1995, this figure has increased to 366 per cent. Evidence from other countries, brought together by e.g. the OECD (1996), Berman, Bound and Machin (1998), Machin and van Reenen (1998), Berman and Machin (2000) and Hollander and ter Weel (2000), suggests similar strong patterns for the United Kingdom, while in Australia, Austria, Belgium, Canada, Japan, Portugal and Spain wage inequality has also risen but to a lesser extent. The figures for Denmark, France, Germany, Italy, the Netherlands and Sweden are less pronounced and show no strong pattern of rising wage inequality since the 1970s.

In search of explanations for these large shifts in the composition of aggregate labour demand various lines of thought have been developed. The direction of the explanations differs widely from trade with low-wage countries (Wood 1994) and trade between the United States and the European Union countries (Davis and Reeve 1997) to arguing that recent technical change has been in favour of the skilled workers (Krugman 1995 and Acemoglu 2000). According to Berman and Machin (2000) a consensus has now been formed on the technology-based hypothesis, which claims that changing technology under given market conditions is the main
driving force behind the shift in labour demand towards the skilled. However, solutions to the problem are not offered very often and are mostly not considered in a thorough manner. The OECD (1998), e.g. only states that investment in human capital and upgrading of skills will offer a solution to the weak position of low-skilled workers in its member countries without specifying precise action.

This chapter considers in a theoretical fashion the consequences of this ‘skill-biased technical change’ (SBTC) by modelling technical change in such a manner that skilled workers profit relatively more from the technical advancement than unskilled workers do. Building a model to explain and explore SBTC does this. The way of modelling SBTC developed here has recently been applied by Acemoglu (1998) and is referred to as directed technical change. Acemoglu points out that when technical change is endogenous, an increase in the supply of skilled workers increases the market size for skill-complementary technologies and may induce SBTC. A related paper by Caselli (1999) focuses on the substitutability among new technologies. These new technologies (or technological revolutions) are either skill-biased or de-skilling. The former appear if the new skills are more costly to acquire than the skills required by the old type of technology. The latter appear in the opposite case. Kiley (1999) considers SBTC in a setting of increasing product variety. He addresses the question whether an endogenous technology bias can overturn the depressing effect of increases in skilled-labour supply on the relative wage rate. The mechanism he applies is similar to the one used by Acemoglu (1998). Lloyd-Ellis (1999) discusses a skill-biased endogenous growth channel in a model where a certain minimum level of skills is required to implement new ideas and technologies. The rate of absorption of new technologies is fundamental to his findings. When the rate of absorption declines, the rate of technical change and labour-productivity growth falls, which leads under an assumed specific skewed distribution of skills to wage inequality. Finally, Galor and Moav (2000) model SBTC under the assumption that the state of transition brought about by technical change raises the rate of return to ability. In their model, individuals are subject to two opposing effects: erosion and productivity effects. They find wage inequality in the short-run, while the economy converges to a steady-state equilibrium with a positive growth rate of output per worker by a positive endogenous rate of technical change (see also Gould et al. 2000).

These approaches point towards the importance of a complementarity of skill accumulation and technical change, indicating a particular direction of technical change. However, they only use two types of skills: skilled and unskilled workers. Moreover, the models are not able to give general equilibrium properties in a long-run steady-state solution, which are necessary to develop policy measures to solve the problem. Finally, they are not modelled in an endogenous growth setting, hence leaving the growth process undetermined.

The approach developed in this chapter considers a general equilibrium model with heterogeneous labour in a continuum of skills ranging from no skills to a certain maximum level of skills. The heterogeneity of the individuals populating this economy does already exist upon birth, i.e. every individual is born with some
ability or talent, which can be developed during life. This framework is discussed in terms of a steady-state approach to the changing skills profile over time.

Firms in this economy divide their time between investing in knowledge creation to enhance their product and production process (it is assumed that there is only one product in this economy) to become more efficient and in that way obtain a higher level of profits. Firms employ labour and prefer relatively skilled labour to relatively unskilled labour. This provides a positive feedback loop of investment decisions, which gives individuals an incentive to invest in human capital, since process innovations go hand in hand with a higher level of lifetime utility.

The framework considered makes it relatively more complex to model the exact implications of technical change for different groups in society because it steps outside the representative agent framework. Therefore, we focus on steady-state analysis only. However, despite this complexity, the model provides a more realistic picture of the dilemmas and problems governments face to come up with a solution for this persisting problem of inequality.

The remainder of this chapter builds an endogenous growth model to show the effects of SBTC on wages and its consequences on the wage distribution. The plan of this chapter is the following. In the next section, a model of SBTC is built and explored. The section on 'Government policies to reduce the bias in skill accumulation' discusses government policy along the lines of the framework and the conclusions developed in the next section. The chapter ends with some concluding remarks.

**A model of skill-biased technical change**

Consider a closed economy with competitive markets populated by a large number \(N\) of heterogeneous Ramsey consumers. These consumers have standard, discounted, constant elasticity preferences resulting in the following utility function:

\[
U_{t,s} = \frac{C_{t,s}^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t}
\]  

(1)

where the discount rate \(\rho\) and the coefficient of relative risk aversion \(\sigma\) are both strictly positive. \(C_{t,s}\) is defined as the amount of consumption of a person with skill level \(s\) at time \(t\). The skill level \(s\) is assumed to be uniformly distributed over a range \([0, 1]\). Moreover, every individual in this economy has a different skill level, which can be viewed as the ability this individual is born with at time \(t = 0\). Hence, for each skill level there exists only one individual.\(^2\)

Capital accumulation for each individual is defined as net income minus consumption

\[
\dot{K}_{t,s} = (1 - \tau)r_fK_{t,s} + (1 - \tau)w_f(1 - u_{t,s})H_{t,s} - C_{t,s}
\]  

(2)

where \(\tau\) is a capital income tax for both physical and human capital, \(r_f\) is the return on physical capital \(K_{t,s}\), \(w_f\) the return on human capital \(H_{t,s}\), and \(u_{t,s}\) is the amount
of time spent to acquire additional human capital; \((1 - u_{t,s})\) is the amount of time spent to produce output. This budget constraint implies that all wealth generated or all savings is immediately transformed into physical capital.

Accumulation of human capital is assumed to be skill-biased since individuals with a higher skill level profit more from technical progress than individuals with a lower level of skills. Acemoglu (1998) refers to this fact as 'directed technical change' in favour of skilled labour. Equation (3) defines this as follows: individuals with a higher skill level profit exponentially from an increase in technical progress \(A_t\), i.e.

\[
\dot{H}_{t,s} = A_t^\beta u_{t,s} H_{t,s}^\beta B_t^\gamma - \mu H_{t,s}
\]

with \(0 < \beta < 1\); \(\mu\) is the exogenous rate of depreciation of human capital. Throughout this chapter, however, no depreciation of skills or human capital is assumed, i.e. \(\mu = 0\). \(B_t\) is knowledge provided publicly by the government, whereas \(A_t\) is provided privately as a result of the R&D process firms are engaged in, and it is assumed that no public action is taken to directly influence the supply of private knowledge. The stock of public knowledge is enhanced at some costs collected by the government in the form of capital income taxes \(\tau\) (cf. equation (2)).

Now, maximising utility, subject to the budget constraint and human capital accumulation, leads to the following Hamiltonian

\[
\Omega = \frac{C_{t,s}^{1-\sigma}}{1-\sigma} e^{-\rho t} + \lambda_1 (A_t^\beta u_{t,s} H_{t,s}^\beta B_t^\gamma - \mu H_{t,s})
+ \lambda_2 ((1 - \tau) r_t K_{t,s} + (1 - \tau) \omega_t (1 - u_{t,s}) H_{t,s} - C_{t,s})
\]

Solving this problem results in a standard Euler equation for the individual with skill level \(s\)

\[
\dot{C}_{t,s} = \frac{(1 - \tau) r_t - \rho}{\sigma}
\]

where the after-tax return on capital \((1 - \tau) r_t\) is assumed to exceed the discount rate \(\rho\) and a hat over a variable indicates its growth rate. In the steady state the rental rate \(r\) has to be constant because the growth rate of the marginal product of capital equals zero by definition.\(^3\)

Optimising with respect to \(H_{t,s}\) yields the growth rate of human capital which depends both on the growth rates of private and public knowledge accumulation, \(\dot{A}_t\) and \(\dot{B}_t\), respectively

\[
\dot{H}_{t,s} = \frac{s \dot{A}_t + \nu \dot{B}_t}{1 - \beta}
\]

This equation is of central importance to our analysis because it shows that public knowledge accumulation adds an unbiased amount of human capital to the individual’s existing human capital stock. The equation also indicates that private human
capital accumulation is biased and it depends on the skill levels an individual incorporates.

In order to compute the overall level of human capital we must add up all individuals. Since the impact of skills is normalised, Appendix B shows that this leads to

$$H_t = N \int_0^1 H_{t,x} dx = \left( \frac{(1 - \beta)N}{t \hat{A}_t} \right) e^{((\hat{A}_t + \gamma \hat{B}_t)/(1 - \beta))x} - e^{((\gamma \hat{B}_t)/(1 - \beta))x} \tag{6}$$

Equation (6) shows that the steady-state growth rate of the overall level of human capital converges to

$$\lim_{t \to \infty} \hat{H}_t = \frac{\hat{A}_t + \gamma \hat{B}_t}{1 - \beta} \tag{7}$$

which equals the growth rate of human capital of the individual with the highest ability $x = 1$. This result indicates that the skill bias increases because fewer and fewer individuals embody relatively more and more human capital.\(^4\) The recognition of the importance of the notion of knowledge accumulation is challenging not only the traditional focus on the R&D process, but the whole spectrum of scientific and technical activities from invention to diffusion, from basic research to technical mastery. Such a view of technical change rejects the definition of technical capabilities in terms of knowledge or information with the connotation that industrial technology is like a recipe; understood by particular individuals and readily articulable and communicable from one individual to another with the requisite background training and skills. Knowing how to produce a product is as much experienced tacit skill as articulable knowledge. Contrary to the implicit general theory, skills of a skilled worker in the art are not interchangeable: who works with the recipe makes a difference. Therefore, training new workers has become much more expensive when one takes these arguments into account and the human capital employed by firms will increasingly be embodied in less skilled individuals, thereby further increasing the gap between skilled and unskilled workers.\(^5\)

The government collects capital income taxes ($\tau$) on both human and physical capital to finance the accumulation of public knowledge. This can be expressed as follows

$$\hat{B}_t = \tau r, K_t + \tau w_0 \left( N \int_0^1 (1 - u_{t,x}) H_{t,x} dx \right) \tag{8}$$

Public knowledge is invested in the accumulation of human capital as can be observed from equation (3). The intention and main objective of the government is to distribute income and therefore implicitly human capital more equally. However, the collection of additional capital income taxes has both a positive and negative effect. Since the taxes are used to stimulate the level of the available public knowledge, they have a positive effect on overall productivity. On the other hand, additional capital income taxes have a negative effect on the private accumulation of physical and human capital, as shown below.
Firms allocate human capital between final goods production and technology production. Using \((1 - \varphi)\) of the human capital stock available to the firm, they produce output \(Y_t\), using a standard increasing returns-to-scale Cobb–Douglas production function with labour-saving technical change

\[
Y_t = K_t^\alpha \left( A_t N \int_0^1 ((1 - \varphi_t)(1 - u_{t,s})) H_{t,s} ds \right)^{1-\alpha}
\]

(9)

where \(0 < \varphi < 1\) and \(\alpha > 0\). Firms dedicate \(\varphi\) of their available human capital stock to enhance technical progress, i.e.

\[
\dot{A}_t = \left( N \int_0^1 (\varphi_t(1 - u_{t,s})) H_{t,s} ds \right) \delta A_t^\xi
\]

(10)

where \(0 < \delta < 1\), \(0 < \xi < 1\) and \(\delta + \xi < 1\).

Firms then maximise profits according to the following Hamiltonian:

\[
\Pi = K_t^\alpha \left( A_t N \int_0^1 ((1 - \varphi_t)(1 - u_{t,s})) H_{t,s} ds \right)^{1-\alpha} - r_t K_t
\]

\[- w_t N \int_0^1 (1 - u_{t,s}) H_{t,s} ds + \lambda_3 \left( N \int_0^1 (\varphi_t(1 - u_{t,s})) H_{t,s} ds \right) \delta A_t^\xi
\]

Defining \(H_{t,f}\) as

\[
N \int_0^1 (1 - u_s) H_{t,s} ds
\]

and taking partial derivatives with respect to the control variables \(K_t\), \(H_{t,f}\) and \(\varphi\), and the state variable \(A_t\), yields the following expressions for the steady-state growth rate of the physical and human capital stock available to the firm and the stock of private knowledge, respectively:

\[
\dot{K}_t = \frac{(1 + \delta - \xi)}{\delta} \dot{A}_t
\]

(11)

\[
\dot{H}_{t,f} = \frac{(1 - \xi)}{\delta} \dot{A}_t
\]

(12)

and

\[
\dot{K}_t = \dot{C}_t
\]

(13)

The restriction \(\delta + \xi < 1\) ensures that the growth rate of the physical capital stock in equation (11) is positive. An increase in \(\xi\) and \(\delta\), the effectiveness of private knowledge respectively human capital in the production of private knowledge, has a positive effect on the growth rates in equations (11) and (12).
Table 10.1 Investigating equations (14)–(16)

<table>
<thead>
<tr>
<th></th>
<th>$\delta \tilde{K}_t$</th>
<th>$\delta \tilde{H}_t$</th>
<th>$\delta \tilde{A}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta r_t$</td>
<td>$&gt;0$</td>
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<tr>
<td>$\delta \rho$</td>
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<tr>
<td>$\delta \sigma$</td>
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<tr>
<td>$\delta \xi$</td>
<td>$-$</td>
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<td>$\delta \delta$</td>
<td>$-$</td>
<td>$&lt;0$</td>
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Using equations (11)–(13) the outcomes for the growth rates of $K_t$, $H_t$, and $A_t$ can be re-expressed in the parameters of the model. This results in equations (14)–(16):

\[
\tilde{C}_t = \tilde{K}_t = \frac{(1 - \tau)r_t - \rho}{\sigma} \tag{14}
\]

\[
\tilde{H}_t = \frac{(1 - \xi)((1 - \tau)r_t - \rho)}{(1 - \xi + \delta)\sigma} \tag{15}
\]

and

\[
\tilde{A}_t = \frac{\delta((1 - \tau)r_t - \rho)}{(1 - \xi + \delta)\sigma} \tag{16}
\]

From these three equations, and already pointing to the discussion in the next section, it can be observed that capital income taxes have a negative effect on the steady-state growth rate of all three variables. The rationale is intuitively straightforward and consistent with the fact that negative externalities have negative effects on growth rates.

Formally, equations (14)–(16) can be examined by investigating the effects of each of the exogenous parameters in the growth rates of $K_t$, $H_t$, and $A_t$ in the Table 10.1.

In the next section, we explore these results and show how the skill bias due to the bias in private human capital accumulation, as a result of dispersion in abilities, can be reduced by government intervention.

**Government policies to reduce the bias in skill accumulation**

Firm-specific innovations are induced and occur because of the effort of the firm’s research department on the one hand, and public knowledge from the public basin on the other hand. Public knowledge is enhanced by research performed at universities and other research institutes financed by the government through variable $B_t$. Their output in the form of knowledge is often published in scientific journals or transmitted by channels such as conferences. This improves the overall knowledge stock in the economy in an unbiased way and induces innovative activities. As proven above, firms increase labour productivity levels in a skill-biased
manner which in turn lead to higher levels of innovative activities in the research
department and higher levels of production in the manufacturing division.

Equation (5) implies that in the long run or steady state individuals with a higher
ability experience a higher growth rate of human capital. The dispersion in levels
of human capital between high- and low-skilled individuals thus increases over
time. Hence, the need for government intervention to deal with this dispersion is
valid and necessary.

In this section, a partial equilibrium analysis is performed on several variables in
the model. First, it can be shown that technical progress originating in the private
sector leads to an increase in the efficiency of human capital production biased
towards individuals with higher initial abilities. This can be shown by investigating
a case in which there are two individuals with abilities $s_i$ and $s_j$, where $s_i > s_j$.
Using equation (5) the set-up of this problem can be shown by the following
expression

$$\frac{\hat{H}_i}{\hat{H}_j} = \frac{s_i A_i + \gamma B_i}{s_j A_i + \gamma B_i}$$

From this expression for two different levels of skills it can be easily observed that
an increase in the growth rate of private knowledge $A$, leads to a biased increase
in the growth rate of the ratio of human capital, i.e. the ratio $\hat{H}_i/\hat{H}_j$ increases.
This means that the relatively skilled individual (the individual with skill level $s_i$)
profits more from this enhancement in the accumulation of private knowledge than
the relatively unskilled individual (the individual with skill level $s_j$) does. More
formally

$$\frac{\partial (\hat{H}_i/\hat{H}_j)}{\partial A_i} = \frac{(s_i - s_j)\gamma B_i}{(s_j A_i + \gamma B_i)^2} > 0$$

With respect to government intervention, the opposite result can be obtained.
As is defined above, government intervention by means of a capital income tax
hurts relatively skilled labour more than relatively unskilled labour because the
tax is on the income of both physical and human capital income. Physical capital
is assumed to be distributed equally, but human capital not. Therefore, an increase
in public knowledge, financed by an increase in capital income taxes has a positive
effect on the individual with skill level $s_j$ relative to the individual with skill level
$s_i$. Hence, it can be shown in a more formal way, that an increase in the growth
rate of public knowledge leads, ceteris paribus, to a relative increase in the growth
rate of the unskilled individual's human capital

$$\frac{\partial (\hat{H}_i/\hat{H}_j)}{\partial B_i} = \frac{(s_j - s_i)\gamma A_i}{(s_j A_i + \gamma B_i)^2} < 0$$

Focusing on the short run only, these effects can best be studied by in-
level effects of relative skill embodiment instead of growth rates. Ag
two individuals with skill level $s_i$ and $s_j$, where $s_i > s_j$, the relative level of human capital can be expressed in natural logarithms, using equation (5), as follows:

$$\ln \left( \frac{H_{i,t}}{H_{i,s}} \right) = \frac{(s_i - s_j)((1 - \tau)\rho)\delta t}{(1 - \xi + \delta)(1 - \beta)\sigma}$$

This expression leads to the following proposition:

**Proposition 1** Relative dispersion in the relative level of skills as measured by $\ln(H_{i,t}/H_{i,s})$ increases in the short run if:

(a) $\delta$ increases, because an increase in $\delta$ enhances and stimulates the accumulation of private knowledge $A_t$, which in turn enhances human capital accumulation $H_{i,t}$, as can be observed from equations (10) and (12);

(b) $\xi$ increases, because an increase in $\xi$ enhances and stimulates the accumulation of private knowledge $A_t$, as can be observed from equation (10);

(c) $\beta$ increases, since from equation (3) it can be seen that an increase in $\beta$ leads to a higher rate of human capital accumulation. This increased human capital accumulation leads in turn to a faster increase of SBTC through the effects defined in equation (10);

(d) $r_t$ increases, because an increase in the return to physical capital stimulates investment, which enhances the accumulation of skills through the complementarity between skills and physical capital, as can be seen from equation (2);

(e) $\rho$ and $\sigma$ decrease, since present consumption is valued higher than future consumption, following the properties of equation (1). This leads to less need for skills, and hence the private investment in skills will fall. Lower private investment leads to less dispersion over time; and finally and most importantly

(f) $\tau$ decreases. This can be proven by the following expression and is a quite straightforward result, since an increase in capital income taxes decreases the amount invested in the accumulation of private knowledge leading to the skill bias, while leading to an increase in the amount invested in the accumulation of neutral public knowledge, i.e.

$$\frac{\partial \ln(H_{i,t}/H_{i,s})}{\partial \tau} = -\frac{(s_i - s_j)\delta r_t}{(1 - \xi + \delta)(1 - \beta)\sigma} < 0$$

This completes the investigation of Proposition 1.

This result shows that there is to some extent scope for the government to reduce the short-run level effects of SBTC which are prevalent in the private sector. However, as noted above there remains a long-run bias in human capital growth rates in favour of relatively high-skilled individuals.

However, recently, as noted by e.g. Acemoglu (1998) and Muysken and ter Weel (2000), the supply of relatively high-skilled labour increased in a dramatic fashion, while the wage premium or reward for this high-skilled labour increased in an even more dramatic way. This process of so-called directed technical change
in which the direction of technical change is driven by the supply and availability of a particular level of labour has led and is leading to a further process and development of (wage) dispersion resulting in (wage) inequality throughout the OECD countries. Government intervention by means of providing a public threshold level of knowledge embodied in every single individual is therefore becoming increasingly important. The shift in the accumulation of private knowledge can be modelled by dividing the labour force in so-called haves and have-nots, with regard to the level of skills needed to be part of the working labour force, as follows

$$H_{t,s} = A_t^{\text{max}(x,y)} u_{t,s} H_t^B B_t^Y - \mu H_{t,s}$$

which states that individuals have to embody a certain minimum or threshold level of skills to profit from private knowledge accumulation. If their skill level is below this threshold level, they cannot enhance their skills by means of private knowledge accumulation. Hence, there is a strong case for the government to intervene in this process and to provide education and training to guarantee that the (overall) level of skills is sufficiently high. However, another problem of this trend is that it is increasing over time, i.e.

$$\frac{\partial \bar{x}}{\partial t} > 0$$

frustrating government’s efforts to enhance skills.

Government policies to enhance the position of relatively low-skilled workers should therefore be ‘skill-biased’ in the sense that it should aim at improving the position of relatively low-skilled workers. The way the effects of government intervention is modelled in this framework does suggest that efforts made by the government to distribute skills and therefore income more equally among individuals are neutral: every single individual profits, ceteris paribus, in the same manner from \( y B_t \). This knowledge, provided through a ‘public basin’ of knowledge, accessible for every single individual reduces the skill-bias, but only once; over time the bias will again increase. Therefore, government policies have to be effective both in the long run and induce a skill-bias in order for relatively low-skilled individuals, like individuals \( x_j \) in the example above, to profit more from the effort made by the government.

**Concluding remarks**

This chapter has provided a unique framework in which SBTC is explained by means of private investments in knowledge among firms. The investment in knowledge among firms is biased towards relatively skilled individuals because their ability to acquire this knowledge is assumed to be higher. This engine of growth leads to large levels of dispersion in wage income and human capital embodiment. On the other hand, accumulation of public knowledge leads to an unbiased or neutral increase in wage income and human capital embodiment.

This framework stresses the need for continuous government intervention to reduce the ongoing dispersion in human capital formation induced by SBTC.
Moreover, the endogenous decision of individuals to school themselves proves not to be sufficient to deal with SBTC in a comprehensive manner.

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Notes

1 See e.g. Chennells and Van Reenen (1999) and ter Weel (1999) for an overview of more than one hundred studies addressing the causes and consequences of the recent surge in wage inequality.
2 Most models in the literature divide the labour force in a skilled and unskilled segment, following the properties first stated in McDonald and Solow (1985). The results of these approaches are often crowding-out of unskilled labour because unskilled labour and capital are substitutes while skilled labour and capital are assumed to complement each other. The latter induces an engine of growth, which leads to a large dispersion in wages and results in inequality in society.
3 Appendix A provides the proof for these results and properties.
4 Although fewer and fewer individuals will embody relatively more and more human capital, it should be pointed out that no individual will end up holding the entire stock of human capital.
5 Moreover, this trend can also induce a sector bias in technical change, since some sectors might have more resources and scope to invest in knowledge, both codified and tacit, which can lead to large differences in the accumulation of tacit knowledge, inducing an absorbing effect on high-skilled labour from other sectors – cf. Haskel and Slaughter (1998) for one of the initial empirical assessments of the sector bias of technical change.
6 The latter restriction is in line with empirical findings that show that the growth predictions of traditional models of Uzawa (1965) and Romer (1986) with $\delta = \xi = 1$, contradict post-war growth experiences, investigated by Jones (1995b), of the major OECD countries. This is confirmed by the steady-state solution of $g_\alpha$ which shows that the traditional specification $\delta = \xi = 1$ can be ruled out (see equation (C.10) in Appendix C). Furthermore, the restriction that the production of technology exhibits decreasing returns-to-scale is imposed. Jones (1995a) imposes the restriction $\xi < 1$ and shows that this leads to a model in which a balanced growth path is consistent with an increasing number of persons devoted to technology production.
7 See Appendices C and D for full proof.

References


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Appendix A: Consumer optimum

Consumers maximise life-time utility with respect to the budget constraint and their human capital accumulation function. This results in the Hamiltonian $\Omega$, which is defined as:

$$\Omega = \frac{C_{t,s}^{-\sigma} - 1}{1 - \sigma} e^{-\rho t} + \lambda_1 (A_t^s u_{t,s} H_{t,s}^\theta B_t^Y - \mu H_{t,s})$$

$$+ \lambda_2 [(1 - \tau) r_t K_{t,s} + (1 - \tau) \omega_t (1 - u_{t,s}) H_{t,s} - C_{t,s}]$$

The control variables in the Hamiltonian are $C_{t,s}$ and $u_{t,s}$. Taking partial derivatives with respect to the control variables gives us equations (A.1) and (A.2):

$$\frac{\partial \Omega}{\partial C_{t,s}} = C_{t,s}^{-\sigma} e^{-\rho t} - \lambda_2 = 0$$  \hspace{1cm} (A.1)

and

$$\frac{\partial \Omega}{\partial u_s} = \lambda_1 A_t^s H_{t,s}^\theta B_t^Y - \lambda_2 (1 - \tau) \omega_t H_{t,s} = 0$$  \hspace{1cm} (A.2)

Taking partial derivatives with respect to the state variables $H_{t,s}$ and $K_{t,s}$ gives the following pair of differential equations:

$$\dot{\lambda}_1 = -\frac{\partial \Omega}{\partial H_{t,s}} = -\lambda_1 (\beta A_t^s u_{t,s} H_{t,s}^{\beta-1} B_t^Y - \mu) - \lambda_2 (1 - \tau) \omega_t (1 - u_{t,s})$$  \hspace{1cm} (A.3)

and

$$\dot{\lambda}_2 = -\frac{\partial \Omega}{\partial K_{t,s}} = -\lambda_2 (1 - \tau) r_t$$  \hspace{1cm} (A.4)

Dividing equation (A.4) by $\lambda_2$, taking growth rates of equation (A.1) and equating the results gives us the solution for the individual with skill level $s$. This is a standard Euler equation:

$$\hat{C}_{t,s} = \frac{(1 - \tau) r_t - \rho}{\sigma}$$  \hspace{1cm} (A.5)

Dividing equation (A.3) by $\lambda_1$, making use of equation (A.2) and then expressing this in growth rates gives the solution for $\hat{H}_{t,s}$:

$$\hat{H}_{t,s} = \frac{s \hat{A}_t + \gamma \hat{B}_t}{1 - \beta}$$  \hspace{1cm} (A.6)

Dividing equation (A.4) by $\lambda_2$ and expressing equation (A.2) in growth rates results in the growth rates for co-state variables which satisfy:

$$\dot{\lambda}_2 = -(1 - \tau) r_t$$  \hspace{1cm} (A.7)
and
\[ \dot{\lambda}_1 = \dot{\lambda}_2 + \dot{u}_r = -(1 - \tau) r_t + \dot{u}_r \] (A.8)

Dividing equation (A.3) by \( \dot{\lambda}_1 \), making use of equation (A.2) and solving for \( u_{t,s} \), gives the following expression for the amount of time consumers spend on human capital accumulation:
\[ u_{t,s} = \frac{\dot{H}_{t,s} + \mu}{(1 - \beta)(\dot{H}_{t,s} + \mu) - \dot{\lambda}_1 + \mu} \] (A.9)

Making use of equations (A.6), (A.7) and (A.8), and assuming no depreciation of human capital (\( \mu = 0 \)), this expression can be rewritten as
\[ u_{t,s} = \frac{s \dot{A}_t + \gamma \dot{B}_t}{(1 - \beta)(s \dot{A}_t + \gamma \dot{B}_t + (1 - \tau) r_t - \dot{u}_r)} \] (A.10)

Appendix B: Equilibrium level of human capital

From equation (A.6) follows the solution for the level of human capital \( H_{t,s} \):
\[ H_{t,s} = H_0, s e^{(\alpha \dot{A}_t + \gamma \dot{B}_t)/(1 - \beta))t} \] (B.1)

Assuming that all individuals start with the same level of human capital \( H_{0,s} = 1 \), the overall level of human capital can be calculated – by integrating over equation (B.1) for all individuals – as:
\[ H_t = N \int_0^1 H_{t,s} ds = N \int_0^1 e^{(\alpha \dot{A}_t + \gamma \dot{B}_t)/(1 - \beta))t} ds \] (B.2)

Solving this integral and substituting \( s = 0 \) and \( s = 1 \) leads to the solution for \( H_t \):
\[ H_t = \left( \frac{(1 - \beta)N}{\tau \dot{A}_t} \right) \left( e^{(\alpha \dot{A}_t + \gamma \dot{B}_t)/(1 - \beta))t} - e^{(\gamma \dot{B}_t)/(1 - \beta))t} \) \] (B.3)

This expression gives us the overall level of human capital at time \( t \).

Appendix C: Firm optimum

The Hamiltonian for the firm’s profit maximization problem is defined as:
\[ \Pi = K_t^\alpha \left( A_t N \int_0^1 ((1 - \phi_t)(1 - u_{t,s})) H_{t,s} ds \right)^{1-\alpha} - r_t K_t \]
\[ - w_t N \int_0^1 (1 - u_{t,s}) H_{t,s} ds + \lambda_3 \left( N \int_0^1 (\phi_t(1 - u_{t,s})) H_{t,s} ds \right) A_t^\delta \]
The amount of human capital available to the firm can be found by solving:

\[ N \int_0^1 (1 - u_t, s) \hat{H}_{t, s} ds \]

\[ = N \int_0^1 \left( -((1 - \beta) + \beta s) \hat{A}_t - \beta \gamma \hat{B}_t + (1 - \beta)(1 - \tau) r_t \right) \]

\[ + (1 - \beta)(1 - \sigma) \hat{A}_t + \gamma \hat{B}_t + (1 - \beta)(1 - \tau) r_t \]

\[ e^{(t(\hat{A}_t + \gamma \hat{B}_t))/(1 - \beta)u} ds \]

which leads to the following solution

\[ -\beta N \left( e^{(t(\hat{A}_t + \gamma \hat{B}_t))/(1 - \beta)u} - e^{(t \gamma \hat{B}_t)/(1 - \beta)u} \right) \]

\[ + \frac{\hat{A}_t + (1 - \tau) r_t}{1 - \beta} N \]

\[ \left( \sum_{n=1}^{\infty} \left( \frac{(n - 1)!}{\hat{A}_t t^n} \right) \left( e^{(t(\hat{A}_t + \gamma \hat{B}_t))/(1 - \beta)u} \right) \right) \]

It can be shown that this expression has a finite solution (\( n \) is the index of summation). For convenience, this solution is defined as \( H_{t, f} \).

The Hamiltonian can now be rewritten as

\[ \Pi = K_f^\beta (A_t, (1 - \varphi_t) H_{t, f})^{1-\alpha} - r_t K_t - w_t H_{t, f} + \lambda_3 (\varphi_t H_{t, f})^{\delta} \hat{A}_t^\delta \quad (C.1) \]

The control variables are \( K_t, H_{t, f} \) and \( \varphi_t \). Taking partial derivatives with respect to the control variables gives equations (C.2) to (C.4), i.e.

\[ \frac{\partial \Pi}{\partial K_t} = \alpha \left( \frac{(1 - \varphi_t) A_t, H_{t, f}}{K_t} \right)^{1-\alpha} - r_t = 0 \quad (C.2) \]

\[ \frac{\partial \Pi}{\partial H_{t, f}} = (1 - \alpha)(1 - \varphi_t)^{1-\alpha} K_f^\alpha A_t^{1-\alpha} H_{t, f}^{1-\alpha} - w_t + \lambda_3 \delta \hat{A}_t^\delta H_{t, f}^{\delta - 1} = 0 \quad (C.3) \]

and

\[ \frac{\partial \Pi}{\partial \varphi_t} = -(1 - \alpha)(1 - \varphi_t)^{-\alpha} K_f^\alpha A_t^{1-\alpha} H_{t, f}^{1-\alpha} + \lambda_3 \delta \hat{A}_t^\delta H_{t, f}^{\delta} = 0 \quad (C.4) \]

Taking partial derivatives with respect to the state variable \( A_t \) gives the following differential equation for \( \lambda_3 \):

\[ \dot{\lambda}_3 = -\frac{\partial \Pi}{\partial A_t} = -(1 - \alpha)(1 - \varphi_t)^{1-\alpha} K_f^\alpha A_t^{1-\alpha} H_{t, f}^{1-\alpha} - \lambda_3 \delta \hat{A}_t^\delta H_{t, f}^{\delta} \quad (C.5) \]

Furthermore, the change in private knowledge is given by:

\[ \dot{A}_t = (\varphi_t H_{t, f})^\delta \hat{A}_t^\delta \quad (C.6) \]
Dividing equation (C.6) by $A_t$ and taking growth rates, gives the following relation between the growth rates of human capital and private knowledge:

$$
\dot{H}_{t,f} = \left( \frac{1 - \xi}{\delta} \right) \dot{A}_t
$$  \hspace{1cm} (C.7)

Rewriting equation (C.2) in growth rates, and making use of equation (C.7), gives a relation between the growth rates of physical capital and private knowledge:

$$
\dot{K}_t = \left( \frac{1 + \delta - \xi}{\delta} \right) \dot{A}_t
$$  \hspace{1cm} (C.8)

Using (C.4) to rewrite (C.3) in growth rates, and making use of equations (C.7) and (C.8) leads to the conclusion that wages grow with the same rate as private knowledge:

$$
\dot{w}_t = \dot{A}_t
$$  \hspace{1cm} (C.9)

The part of the human capital stock available to firms that is devoted to the production of new knowledge is equal to:

$$
\phi_t = \frac{\delta}{\left( \frac{(1 - \xi)}{\delta} - \delta + \xi \right)}
$$  \hspace{1cm} (C.10)

It can be shown that $0 < \phi_t < 1$.

**Appendix D: Equilibrium growth rate of the physical capital stock**

The accumulation of the stock of physical capital $K_t$ equals the sum of the accumulation of the individual capital stocks:

$$
\dot{K}_t = N \int_0^1 \dot{K}_{t,s} ds
$$  \hspace{1cm} (D.1)

Making use of equations (2), (4), the definition for $H_{t,f}$

$$
H_{t,f} = N \int_0^1 (1 - u_{t,s}) H_{t,s} ds
$$

and the fact that

$$
K_t = N \int_0^1 K_{t,s} ds
$$

equation (D.1) can be rewritten as

$$
\dot{K}_t = (1 - \tau) r_t K_t + (1 - \tau) u_{t,s} H_{t,s} - e^{((1-\tau)(\tau - \rho)/\sigma)t}
$$

The steady-state relations between the growth rates of $K_t$, $H_t$ and $A_t$, as expressed in equations (C.7) and (C.8), leads to the conclusion that the growth rate of the physical capital stock equals that of consumption

$$
\dot{c}_t = \dot{K}_t = \frac{(1 - \tau) r_t - \rho}{\sigma}
$$  \hspace{1cm} (D.2)