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Abstract. In this paper we integrate two workhorse models in economics: The monopolistic competition model of Dixit and Stiglitz and the search unemployment model of Pissarides. Information and communication technology (ICT) is interpreted as i) technical progress in the matching function of the Pissarides labour market search model where it is increasing the probability of filling a vacancy, and ii) as technical change in the production function of the Dixit-Stiglitz goods market model where it is increasing fixed costs and decreasing variable costs. All effects together, modelled as a permanent once-and-for-all ICT and Internet shock, increase the vacancy/unemployment ratio, decrease the long-run equilibrium unemployment rate, and increase wages. Keywords: ICT, Monopolistic competition, unemployment.

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1. Introduction

In the 1980s labour intermediaries started to use computers in the search process. Profiles of searching workers were entered into computer databases, as were employers’ vacancies. A similar process takes place using the Internet. Public and private intermediaries have set up websites for job search. These measures are expected to improve the chances of employers and workers to find a job for given amounts of time or money invested into a search\(^1\). The OECD (1997) expressed the hope for a higher efficiency through the Internet as follows: “…the PES\(^2\) would need to become more selective in the future and focus its resources on the at-risk groups. This process will be helped by the extension of self-service facilities and the increasing application of information technologies, in particular the Internet. The potential of the Internet could go well beyond the listing of vacancies and job-seekers and hence improving information flows in that appropriate software for searching, matching and screening could be provided free of charge by the PES to anybody wanting to use these facilities. This will further reduce costs in the provision of basic information and matching services and free valuable staff time for in-depth work on identifying at-risk individuals and providing them with early treatment.” Based on this hope, the PESs did set up Internet search systems. The Flemish PES office has developed a large-scale electronic network since 1992, which appears to have increased the number of reported vacancies considerably (OECD 2001b cited in OECD 2001a). The relevant site www.vdab.be has been visited more 180,000 times in June 2000 alone, 140% more than in January 1999. It contains 24,000 vacancies and

\(^1\) See Autor (2001).
\(^2\) PES is Public Employment Services.
more than 60,000 CVs and about 12,000 enterprises have a user code. In addition, private websites play a great role. Similarly, the Portuguese public employment service has both novel and more traditional components. The modern feature is the computerised, comprehensive system of job broking, covering all notified vacancies and unemployed registrants (Addison, 2001). In the Netherlands there is an integrated web-page system for all firms. Whenever a firm introduces a vacancy on its own website it is automatically visible on the website of the whole Public Employment System. The UK Government launched over 1,200 online centres giving public access to computers and the Internet. The Government also announced plans to equip and open a further 1050 UK online centres; and launched a major Department for Education and Employment website - www.worktrain.gov.uk - giving instant online access to 800,000 job and training opportunities across Britain. It also announces that “all 300,000 job vacancies can be searched on the net – at www.employmentservice.gov.uk – 24 hours a day, 365 days a year;” The Australian Job Service has 40-50,000 vacancies open per month. In the summary of OECD (2001b, p.20) it is taken for granted that the Internet enhances the number of matches because of the great improvement of transparency.

These expected improvements of chances to find a job, which caused a huge amount of investment because of the expectation of improving the matching, might be thought of as being unequally distributed when some people do not have access to the Internet. This problem is well understood. The UK government states that UK online centres aim to attract people who may feel that technology is not for them, such as

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3 See Vercammen and Geerts in OECD (2001b)  
4 See Gelderblom (2000).  
5 See (download 04/02/02) http://www.number-10.gov.uk/news.asp?NewsId=1886  
people with basic skills needs, lone parents, people over 60, those with disabilities, people from minority ethnic groups and unemployed people. A recent Department for Education and Employment survey found that 68 per cent of professionals have used the Internet compared with 22 percent of the semi-skilled and unskilled workers. Older people and those from ethnic minorities are also less likely to have access to the Internet. In the Netherlands, among all people searching for (different) employment in the year 2000, 25% used the Internet for their search. This is a strong growth compared to the 19% as of 1999, 10% as of 1998 and 7% as of 1997. Internet is used more the higher the education. For all groups of not working people the Internet is told to be an important channel of search with a strong growth of usage. Working people use it slightly more, 27%. Internet use is lowest among ethnic minorities. Numbers for search of unemployed and for matches related to the Internet are not explicitly given. As working people have a higher percentage (27%) than the average (25%), the use by unemployed people is probably lower. However, as the Internet is characterized as a very important channel for them it cannot be zero. The only group for which the Internet is not used seems to be members of the board of private firms. In the Scandinavian countries 2/3 of all clients of the PES use the self-search procedure of the Internet because an increase in transparency is expected. Unemployed people are the major clients of their PESs.

In sum, there is no doubt that less skilled and unemployed people use the Internet a bit less than others, but it is still important for all groups. In addition, they can benefit from the Internet without direct access when labour intermediaries using the Internet help them in the search process. ‘Assisted intermediation’ for unemployed

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7 See also Koning in OECD (2001b), p.325.
8 See http://www.number-10.gov.uk/news.asp?NewsId=1886
10 See Volkskrant, economie, 5-2-2002, p. 16.
is current practice in, e.g., Germany\textsuperscript{12} and in Flanders.\textsuperscript{13} In the latter case data banks linked to websites allow registered entrepreneurs to collect information about applicants. This applies to 60,000 of the 180,000 jobseekers in Flanders. Even the low-skilled unemployed then use Internet indirectly by having their data in the databank introduced by the PES and screened by the employers without personally touching a keyboard because the PES and the employers are doing this for them. Moreover, the online centres in the \textit{UK} mentioned above are set up exactly with the intention to help these people. In OECD (2001b), training job seekers in the use of Internet – available on terminals of the PES already now - is seen as a future task for the PES. On the one hand this makes clear that nobody is excluded, on the other hand it is clear that low-skilled and unemployed use the Internet less than higher-skilled and employed people; but the unemployed also use the PESs relatively more who in turn can use the Internet to help them.

All of the facts presented so far show several things: (i) a huge amount of money and time have been invested in getting the Internet work in labour intermediation. The investors tell in many places that they expect to improve the transparency and the matching, because otherwise the investments in and costs of using the Internet would not make sense.\textsuperscript{14} (ii) Firms, actual and potential employees use the Internet but employed and higher skilled people use it more intensively. (iii) People with a lower inclination to use the Internet get much help because the Public Employment Services and the firms handle the Internet sites to bring and find their CVs respectively.

\textsuperscript{11} Se Konle-Seidl (2000).
\textsuperscript{12} See http://www.arbeitsamt.de/hst/services/pressearchiv/61_01.html.
\textsuperscript{13} See Vercammen and Geerts in OECD 2001b.
\textsuperscript{14} See in particular OECD (2001b).
In labour market theory this can be captured by technical progress in the matching function of a search model, because here the success probabilities of finding a job or filling a vacancy are modelled. Therefore ICT, increasing these probabilities and shifting the Beveridge curve, should be modelled in the matching and search technology of the labour market, if we want to understand the macroeconomic effects of ICT. All of the results presented below are not in contrast with the empirical literature on shifts in the Beveridge curve and the NAIRU.\textsuperscript{15}

Moreover, firms have also invested into computer facilities, network connectivity and website development in order to ease ordering inputs and selling output\textsuperscript{16}, both of which are implicit parts of the output production functions used in economic theory. The time spent on websites and similar devices as well as the costs of training personnel is an increase in a firm’s fixed costs, whereas the advantages of reduced administration costs are a reduction in variable costs. Some well-known examples\textsuperscript{17} include cost reductions for transfers between bank accounts, processing costs of transactions of British Telecom, automobile producers’ joint exchange to buy components, which are supposed to reduce the costs of making a car. Moreover, in the 1980s computer facilities were at the root of just-in-time production, which also increased fixed costs and decreased variable costs.\textsuperscript{18} When fixed costs are essential, the assumption of perfect competition has to be dropped and an imperfectly competitive market structure has to be assumed.\textsuperscript{19}

\textsuperscript{15} The theoretical result of a decrease in the rate of unemployment derived below is in accordance with the empirical finding that the NAIRU (Non-accelerating inflation rate of unemployment) has decreased during the 1990s (see Meijers 2000 and Autor 2001 for brief summaries of the literature). We do not claim, however, that the entire change in the NAIRU is due to arguments modelled here.


\textsuperscript{17} See The Economist, A thinker’s guide, Business Special, March 30, 2000.

\textsuperscript{18} See Callen, Fader, and Krinsky (2000).

\textsuperscript{19} Meijers (2000) relates the shift to higher fixed and lower variable costs to inflation using a Cournot model.
In this paper we consider these aspects of ICT as once-and-for-all technical change. We investigate the macroeconomic effects of ICT within a framework using the Pissarides (1990) labour market search model and monopolistic competition according to the Dixit-Stiglitz (1977) goods market model. We choose the Dixit-Stiglitz model because it appears to be the most successful imperfect competition goods market model in general equilibrium theory as used in the fields of international trade, endogenous growth, regional economics and macroeconomics. The Pissarides model is one of the most successful in labour market theory and empirics. When examining ICT as a technology of search it is most straightforward to integrate ICT into that labour model, which has an explicit search technology. Among the major labour market models (see Pissarides 1998) the search model is the only one with an explicit search technology.

We investigate in a comparative-static manner how ICT in the goods market and the labour market changes the endogenous variables. This is done in two ways: The comparative-static effect of each change is considered separately and then the effects are considered jointly to see whether or not they work in the same direction. The most important results are that all effects together, modelled as one permanent once-and-for-all ICT and Internet shock, increase the vacancy/ unemployment ratio, decrease the long-run equilibrium unemployment rate and increase wages although rents available for bargaining are reduced by technical progress in the matching function.

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20 It is not necessary to use an endogenous growth model here. Endogenous growth models are preferable when a continuous flow of innovations increasing total factor productivity is considered. This flow, however, is an aggregate from many sectors. When the emphasis is on just one technology one can simplify by using the comparative static manner. In particular, ICT is assumed to have an impact on the matching function but other TFP growth probably does not.

21 It is not a repeated shock as in Mortensen and Pissarides (1994).

22 Of course there is continuous upgrading. The once-and-for-all assumption is simplifying in the sense that we do not have to add the complications of endogenous growth models.

23 As we consider a macroeconomic model with just one skill, we do not analyse skill bias, wage inequality and related issues. See Acemoglu (2000), Jacobebbinghaus and Zwick (2001) and Kaiser (2000) and others on these aspects.
In the following section we merge the Pissarides and the Dixit-Stiglitz model. In section 3 we analyse the existence and uniqueness of the equilibrium of the model and the effects of technical progress in the matching function. In section 4 we consider the effects of technical change, lowering variable but increasing fixed costs of production. In section 5 we summarize the results, as we have partly done in the abstract.

2. The model

*Trade in the labour market*\(^{24}\)

From the Pissarides (1990) model we use the matching function \(m_L = Tm(uL, vL)\), where \(L\) is the labour force, i.e. the total number of employed and unemployed workers, \(u\) is the unemployment rate, \(v\) is the rate of vacancies and \(m_L\) is the number of matches produced by this function. \(T\) is an efficiency parameter or the level of productivity in the matching process. When computers enter the labour intermediation process or job-search websites appear on the Internet, \(T\) is assumed to go up.\(^{25}\) The function is assumed to be increasing in both arguments, concave and linearly homogenous.\(^{26}\) Defining labour market tightness as \(\theta \equiv v/u\), division of the matching function by \(vL\) yields \(q(\theta) = Tm(u/v, 1)\) as the probability (Poisson arrival rate) of a firm to find a worker for a vacancy and \(\theta q(\theta) = Tm/u = Tm(1, v/u)\) as the probability

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\(^{24}\) Subsections are titled as in Pissarides 1990. The search part is explained in greater detail there.

\(^{25}\) An implicit assumption here is that the additional hits from the Internet are not all useless. In this sense, mismatches have to be decreased by the Internet as well and the increase in the number of hits - cleaned for mismatches - have to leave us with an increased number of matches per unit of time.

\(^{26}\) Pissarides (1998, p.167, footnote 15) refers to estimates of the matching function using a Cobb-Douglas functional form, which justifies the assumptions made in the text. Anderson and Burgess (2000) provide similar estimation results but also convincingly argue that their findings - together with job search by the employed - suggest interpreting empirical matching functions as a combination of a
of an unemployed worker to find a job. Both these probabilities are enhanced by a change in ICT. By implication, the expected duration of a vacancy, $1/q(\theta)$, is reduced by technical progress in the matching function and the same holds for the expected time an unemployed worker needs to find a job. We assume that the technical change is neutral. If it were augmenting $uL$ ($vL$), this would mean that it works like having relatively more (less) unemployed people from which the employers can choose than having a greater number of vacancies from which workers can choose. We rather assume that both these effects are equally strong because computer search is equally well accessible for both. Firms can afford computer equipment and workers can use those of public libraries or labour intermediaries, which may even provide some help in using the computer equipment.

A shock is a percentage rate $s$ at which $(1-u)L$ employed workers lose their job by assumption in every period. Therefore $s(1-u)L$ workers go from a job into unemployment every period. On the other hand $\theta q(\theta)uL$ unemployed workers are expected to find a job each period. A labour market steady state equilibrium is defined as a situation where the numbers of workers going into and out of unemployment are equal and expectations turn out to be true, i.e. $s(1-u)L = \theta q(\theta)uL$. All other variables constant, technical progress in the matching function increases the right-hand side of this equation, thus contributing to a quicker process of bringing workers out of unemployment. Solving this equation for $u$ yields the Beveridge or UV curve:

$$u = \frac{s}{s + \theta q(\theta)}, \partial u / \partial s > 0, \partial u / \partial \theta < 0$$ (1)

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structural matching function and a job competition model. We do not include job search by the employed for the mere sake of simplicity.
An increase in $T$, increasing $\theta q(\theta)$, therefore reduces $u$ for a given tightness ratio.

Multiplying equation (1) by $\theta$ yields an equation for the vacancy rate because

$u \theta = uv/u = v$

$$v = \frac{s}{s/\theta + q(\theta)}, \frac{\partial v}{\partial s} > 0, \frac{\partial v}{\partial \theta} > 0$$

Equation (1) and (1’) and their shifts induced by a change in $T$ are drawn in the lower right quadrant of Figure 1. These two results are summarized in the following proposition:

**Proposition 1:** For any given tightness ratio, ICT, interpreted as neutral technical progress in the matching function, decreases the unemployment rate and the rate of vacancies.

There is no rigorous evidence until now, which would give empirical support for this proposition. However, the description in section 1 indicates that firms, people searching for work, the private and Public Employment Services as well as governments expect the Internet to improve the matching process. Jackman, Layard and Pissarides (1989) find an outward shift of the Beveridge curve of the UK for 1968-1987 – a period before the arrival of the Internet. They attribute this to changes in search effectiveness, in particular a more permissive manner of the social security administration, changes in the public attitude towards claiming benefits and in the work ethic. Future econometric work will have to show whether investment and use of the Internet also have an impact. Blanchard and Diamond (1989) suggest that
changes in search behaviour – among other things – will shift the Beveridge curve. One such change may be the use of ICT, in particular the Internet. The authors attribute part of the outward shift of the Beveridge curve, which they find for the USA 1968-84, to an increased geographical dispersion of workers and new jobs. The strength of the Internet - coming up much later - is often claimed to be the bridging of regional distances mentioned there. Coles and Smith (1996) find the same results for the matching function as Blanchard and Diamond (1989) – an elasticity of 0.6 for vacancies and 0.4 for unemployment and an outward shift of the Beveridge curve over time – in a time-series analysis for England and Wales from 1985 – 1993. In their cross-city analysis for March 1987 they find that the position of the curve also depends on the age of the population of the cities, the qualification of the population, a high proportion of manufacturing industry, and wages, which are positively correlated with city size. The interpretation of this latter result is that larger cities have thicker labour markets allowing for better and faster matching, resulting in higher wages, which in turn encourage more intensive search. It is one of the suspicions concerning the Internet in the descriptions above that it will integrate markets, make them thicker, and therefore allow for better and quicker matches. Bleakley and Fuhrer (1997) find one shift of the Beveridge curve towards lower values of $u$ and $v$ taking place around 1987-89 in the USA and they suggest a second shift saying that ‘Indeed the unemployment and vacancy rates from 1995 and 1996 suggest that the Beveridge curve is moving even further inward - to territory not explored since the 1950s.’ They attribute part of this shift to efficiency improvements in the matching function as we do in this model. Clearly, in other countries the shift has been in the opposite direction. They attribute part of this shift to efficiency improvements in the matching function as we do in this model. Clearly, in other countries the shift has been in the opposite direction.27 ICT is only one of the many forces that have an impact on the position of

27 See OECD 2001a, p. 18/19.
the Beveridge curve and therefore other effects can easily outweigh those of ICT. Obviously, the shift is partly due to other effects.

Moreover, observed shifts of single values of $u$ and $v$ are the result not only of a shift in the curves (1) and (1’) but also of i) the consequences of the shifts for the bargaining process; ii) the shift of marginal cost curves for profit maximization of firms; and iii) other changes such as the effects of ICT on fixed and variable costs in the goods market. All of these changes are discussed in the remainder of this paper.

**Government and unemployment benefits**

The government is assumed to pay unemployment benefits $z$ to each unemployed worker. The financing of this is not explicitly treated in Pissarides (1990). We show how this can be modelled to keep Pissarides’ results intact. Total expenditures of the government or unemployment benefits are $zuL$. It will turn out that the incentives are ultimately unchanged if both the employed and the unemployed pay a tax or premium $t$ to finance the unemployment benefits. Revenue then is $tL$. From the balanced budget assumption we make, it follows that $tL = zuL$ and therefore $t = zu$. Workers therefore receive $w-t = w - zu$ and unemployed net benefits are $z - t = z - zu$. As $z$ is considered to be a policy variable, the budget equation determines the value of $t$, whereas $u$ is determined in the general equilibrium part of the model below. On the one hand it will follow from the model below, that a policy of a reduction of the benefit $z$ will decrease unemployment. On the other hand there is the general equilibrium effect that, given the gross benefit $z$, a lower value of the rate of unemployment, $u$, implies a lower unemployment premium $t$.28
Households and workers

Households are assumed to have love-of-variety preferences of the CES type,

\[ y = \left[ \int_{j=0}^{n} c_i^\alpha di \right]^{\frac{1}{\alpha}} \], with \( 0 < \alpha < 1 \), on a continuum of goods with index \( i \), ranging from zero to \( n \), the (integral measure of) the number of firms.\(^{29}\) The market for goods is assumed to have no search frictions. It is well known that this specification of preferences leads to a constant elasticity of the inverse demand function, \( \alpha^{-1} \), and to relative demand of goods independent of the income earned by employed or unemployed persons. If the temporary utility function is discounted and integrated we may get an inter-temporal utility function for which it is well known from endogenous growth theory or the theory of optimal growth that, in the absence of a rate of permanent productivity growth, the steady-state value of consumption will be stationary and the interest rate will equal the discount rate. This seems to be the shortest way to determine the interest rate.\(^{30}\) The problem of a household with an infinite time horizon then is to choose the values of \( c_i \) and of savings such that the choice maximizes

\[ I = \int_{\tau=0}^{\infty} e^{-\rho\tau} \left[ \int_{j=0}^{n} c_i^\alpha di \right]^{\frac{1}{\alpha}} d\tau \] subject to the budget constraint

\[ \dot{W} = I - \int_{j=0}^{n} p_i c_i di + rW \text{ and } W(0) = W_0 \geq 0, \] where \( W \) is current wealth, a dot indicates a time derivative, \( r \) is the interest rate, \( p_i \) is the price of good \( i \), and current non-interest income is \( I = (1-u) w + uz - t \). The assumption here is that a household gets the wage \( w \) with probability \( (1-u) \) and is unemployed and gets benefits \( z \) with probability \( u \), but pays taxes \( t \) in both cases. As the utility function exhibits risk neutrality there are no

\(^{28}\) Policy is discussed more extensively in Pissarides (2000), Chap.9.

\(^{29}\) By implication we only consider the case of a large number of firms in which no strategic behaviour takes place.

complications from the uncertainty. A second interpretation could be that every household is representative in the sense that the same share \(1-u\) of its members is (un-) employed as in the total labour force of the economy.\(^{31}\) In the first interpretation the (ex-post) employed workers lend money to (ex-post) unemployed workers allowing the latter to smooth consumption under the assumption of a perfect capital market. In the second interpretation this happens within the households and lending among identical households must be zero in equilibrium. In the appendix\(^{32}\) we show that the inverse price elasticity is \(\alpha - 1\) and the interest rate in a steady state with a constant number of firms is \(r = \rho\). All results henceforth are steady-state results.

The present value, with discount rate \(r\), of the expected income stream of an unemployed and an employed worker, \(U\) and \(E\) respectively, are: \(U = [z - zu + \theta q(\theta)(E-U)]/r\) and \(E = [w - zu + s(U-E)]/r\). \(E-U\) is the income difference an unemployed worker can gain by finding a job with probability \(\theta q(\theta)\). \(U-E\) is the corresponding loss by a worker from losing his job with probability \(s\). These two equations can be solved for \(E\) and \(U\) explicitly:

\[
U = \frac{(r + s)z + \theta q(\theta)w}{r + s + \theta q(\theta)} \frac{1}{r - zu/r}, \quad E = \frac{sz + [r + \theta q(\theta)]w}{r + s + \theta q(\theta)} \frac{1}{r - zu/r}
\]

\(Firms\)

There is monopolistic competition in the goods market. Each firm produces one of the goods that appear in the utility function. They hire labour in the frictional labour market described above and sell the good to consumers. The present-discounted value

\(^{30}\) Shapiro and Stiglitz (1984, p.435, fn. 5) also follow this procedure.

\(^{31}\) See Pissarides (2000), section 3.4 for this interpretation.
of a vacancy is in terms of real output is \( V = [-\gamma + q(\theta)(J-V)]/r \). It consists of the hiring costs \( \gamma \) and the net return of transferring the vacancy \( V \) into a job with value \( J \), which is expected with probability \( q(\theta) \). We assume that hiring costs are identical for all firms.\(^3\) As the value of the vacancy is zero in equilibrium, we get \( J = \gamma q(\theta) \): the value of a job is equal to the vacant job costs \( \gamma \) multiplied by the expected duration of the vacancy, i.e. expected hiring costs. When considering the firms’ hiring costs we must consider that the occupied job may be separated from the worker again with probability \( s \). The current value of the expected value of a job therefore is \( (r + s)J = (r + s)\gamma/q(\theta) \). These are labour costs in addition to the real wage received by the worker. Labour costs per worker then equal \( w + (r + s)\gamma/q(\theta) \). Technical progress in the matching function then implies the following:

**Proposition 2:** For a given tightness ratio, expected hiring costs and the value of a job are both decreased by technical progress in the matching function because the probability of filling a vacancy, \( q(\theta) \), is increased.

Pissarides (1990) links the above to the neo-classical production function.\(^3\) Here we link it to the model by Dixit and Stiglitz (1977).

Technologies are defined by the production function \( x_i = (l_i - f)/a \), or, solving for labour demand, \( l_i = f + ax_i \), with \( a, f > 0 \). \( l_i \) represents demand for labour and \( x_i \) output per firm to produce good \( i \). \( f \) is the fixed part and \( ax_i \) is the variable part of labour demand and \( 1/a \) is the marginal labour productivity. Due to the fixed costs this

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\(^3\) Appendices are available from the author upon request.

\(^3\) Acemoglu (2001) considers two sectors in which firms have different hiring costs. Rents are different therefore and the firms with higher hiring costs have higher rents and therefore higher wages, i.e. better jobs.
production function generates internal economies of scale, i.e. unit-cost reductions through higher output. As all goods are assumed to be identical in the utility function and in the production technology, their prices and quantities will be the same.

Total labour demand is \( n_l = n(f + ax_i) \). Equating this to employment \((1-u)L\) yields \((1-u)L = n(f + ax_i)\).\(^{35}\) Solving this equation we find the number of firms linked to the rate of unemployment as:

\[
n = (1-u)L/(f + ax_i)
\]  

There is a partial negative relation between the rate of unemployment and the number of firms: The larger the number of firms, the lower the unemployment rate (ceteris paribus), or the lower the unemployment rate the more firms can be in the market.

Present-discounted value of the firm’s expected profits\(^{36}\), which has a current value of zero in every period in equilibrium, are defined in nominal terms as:

\[
\Pi_l = 0^{\infty} e^{-rt} \left\{ p(x_i)x_i - W(f + ax_i) - p\gamma V_l \right\} dt
\]  

\( W \) is the nominal wage rate and real hiring costs for vacancies, \( \gamma V_l \), are made nominal by multiplying their real value with the price. The assumption is that nominal hiring costs are given from the labour market; monopoly pricing then has no impact on the value of hiring costs. The firm maximizes profits as defined in equation (3) through choice of the quantity \( x \) and the number of vacancies \( V_l \) using the dynamic

\(^{34}\) In Pissarides (1990) this leads to the zero-profit condition \( f(k) - (r+\delta)k - w - (r+s)\gamma q(\theta) = 0 \). Here \( f(k) \) is the output per unit of labour and \( \delta \) is the rate of depreciation.

\(^{35}\) Nothing would be changed by setting \( L=1 \). However, it is easier to see where \( L \) has an impact or not if it appears explicitly.
concept of the large firm from Pissarides (2000, chap.3). The dynamics comes from
the fact that the firm can post a number of $V_i$ vacancies, which increase employment
with probability $q(\theta)$ and costs $p\gamma V_i$. On the other hand, the firm looses workers $sl_i$.
The expected change in employment then is

$$\dot{V}_i = q(\theta) V_i - sl_i$$

From $l_i = f + ax_i$ and $dl_i = a dx_i$ we get the corresponding change in
the quantity as

$$\delta = q(\theta) V_i / a - s(f / a + x)$$

The current value Hamiltonian for each firm’s decision problem then is:

$$H = p(x_i) x - W(f + ax_i) - p\gamma V_i + \lambda [q(\theta) V_i / a - s(f / a + x)]$$

The first-order condition for the number of vacancies determines the value of the co-
state variable as marginal hiring costs:

$$\frac{\partial H}{\partial V_i} = -\gamma p + \lambda q(\theta) / a = 0, \text{ or } \lambda = \gamma p a / q(\theta)$$

The other canonical equation is

$$- \frac{\partial H}{\partial x} = -[p' x + p - a W - \lambda s] = \delta - r \lambda$$

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This equation corresponds to equation 3.2 in Pissarides (2000).
Insertion of $\lambda$ from the previous first-order condition and setting its change equal to zero in the steady state, and noticing that the price elasticity $p'x/p=\alpha/l$ this latter first-order condition yields\(^{37}\):

$$p\alpha = a[W + p(r + s)\gamma / q(\theta)]$$

(4)

For technical progress in the matching function, equation (4) implies the following:

**Proposition 3**: For constant wages, technical progress in the matching function decreases marginal costs on the right-hand side of equation (4) because the expected duration of filling a vacancy and therefore expected hiring costs are reduced. The first-order condition then requires decreasing prices or increasing real wages.

In the steady state the change of employment also must be zero and therefore we get the number of vacancies as a function of the quantity produced:

$$V_i = s(f + ax)/q(\theta)$$

The solution for the quantity and the tightness ratio will be derived below.

**Wages**

There are two sorts of rents in Pissarides’ model: there are occupied jobs, indexed $j$, where (i) employed workers do not have to search and therefore have an income rent of $E_j - U$ and (ii) firms do not have to incur hiring costs and therefore have a rent $J_j - V$.

\(^{37}\) Equation (4) corresponds to equation (3.7) in Pissarides (2000).
Bargaining these rents is assumed to determine real wages. This is done by choosing the real wage by maximising the Nash product \((E_j-U)^\beta (J_j-V)^{1-\beta}\) with \(\beta\) as the bargaining power of workers and \(1-\beta\) that of firms, \(V=0, E_j=[w_j-zu + sU]/(r+s), U\) according to the explicit solution given above, and, for \(V = 0\), \(J_j= \gamma/q(\theta)=(\alpha a- w_j)/(r+s)\) where the last equality stems from the solution of (4) for expected hiring costs. \(E, U\) and \(V\) are as in Pissarides (1990). The value for \(J\) differs from Pissarides’ model because we have replaced the neoclassical production function by elements of the Dixit-Stiglitz model: as we have increasing returns on the firm level, the value of an occupied job is the present-discounted value not of the average but rather the marginal profit from a worker gross of hiring costs. The result of the maximisation of the Nash product with respect to the real wage in its general form is identical to that of Pissarides in that workers get a share \(\beta\) of the sum of the rents to be distributed: \(E_j-U = \beta(E_j-U + J_j-V)\). Insertion of the values for \(E_j, U, J_j\) and \(V\) yields the solution for real wages:

\[
w_j = (1 - \beta)(rU + zu) + \beta \frac{\alpha}{a}
\]

Insertion of \(E_j-U = \beta(J_j-V)/(1-\beta)\) from the general form of the bargaining result and \(J=\gamma q(\theta)\) into \(rU = [z - zu + \theta q(\theta)(E-U)]\) yields \(rU = z - zu + \theta \beta \gamma (1-\beta)\). Insertion of \(rU\) into the above wage result yields:

\[
w_j = (1 - \beta)z + \beta \left( \frac{\alpha}{a} + \theta \gamma \right)
\]

\(38\) This result corresponds to equation 1.18 in Pissarides 1990. Note that with \(\beta=1\), the negotiation result would require \(V=J=\gamma q=\gamma (m/v)=0\), which could only hold for \(v=0\) without additional
The last term indicates that workers participate in the hiring costs saved on occupied jobs compared to vacancies. The second but last term is net marginal value product of labour – replacing the output-per-worker \( f(k) - (r + \delta)k \) in Pissarides (1990). The unemployment premium or tax, \( z_u \), has dropped out only in the very last step of the calculation yielding (5). The Pissarides approach is consistent with an explicit financing scheme for the unemployment benefit if both unemployed and employed workers have the same reduction of their gross payments \( w \) and \( z \) respectively. Then the difference of going from a status of unemployed to employed workers is unchanged and all incentives of \( z \) are essentially as in Pissarides’ model.\(^{40}\) Equation (5) essentially has the real wage as a function of the \( v/u = \theta \) ratio, but \( q(\theta) \) and technical change in the matching function do not appear in this equation. This equation is drawn as the BB curve in the upper right quadrant of Figure 1. Technical progress in the matching function then implies the following:

**Proposition 4:** An increase in the matching probability because of technical change in the matching function, \( dT \), will not shift the bargaining curve. ICT has no impact on the curve for the bargained wage.

This model is kept as simple as the basic workhorse models were. We resist the temptation to endogenize the bargaining power parameter or the mark-up, or to

\(^{39}\) This result corresponds to equation 1.19 in Pissarides 1990.

\(^{40}\) In particular, bargaining determines wages according to (5) conditional on the tightness ratio and output. The firm chooses output, \( x \), or employment, \( f + ax \), by profit-maximization for given wages. The intersection of (4’) and (5) then determines wages and the tightness ratio. In a model by Stole and Zwiebel (1996) there is individual bargaining in the firm over employment and wages *simultaneously.*
distinguish between different skills\textsuperscript{41}, or between the parameters for love-of-variety, scale economies and the price elasticity.

3. The Equilibrium Solution: Existence and uniqueness of the model and the effects of technical progress in the matching function

Equations (1)-(5) determine the five variables of the model when goods produced serve as numéraire \( p=1 \): \( u, n, x, \theta \) and \( w \). Insertion of wage per worker from (4) and the number of vacancies into the current profit function contained in (3) allows to solve for the zero-profit\textsuperscript{42} - equilibrium quantity:

\[
x = \frac{\alpha}{a} - \frac{r \gamma}{q(\theta)} f, \quad \partial x / \partial \theta < 0
\]  

(6)

Using equation (6) we can calculate the labour demand per firm as

\[
l_i = f + a x_i = \frac{f}{1 - \alpha + \frac{ar \gamma}{q(\theta)}}, \partial l_i / \partial \theta < 0
\]

Both, output and labour demand depend negatively on hiring costs and the probability \( q(\theta) \) because an increase in the tightness ratio increases expected marginal hiring.

\textsuperscript{41} On the point of skills see Fitzenberger (1999), Chennels and van Reenen (1999) and Kaiser Pohlmeier (2000).
costs. Each firm knows that it will be separated from the worker with probability $s$, resulting in $sl_i$ separations, and can fill a vacancy with probability $q(\theta)$. A flow equilibrium of the firm - allowing the firm to keep the labour demand, which allows producing the profit maximizing output level - then requires that expected separations equal expected hiring, $sl_i = q(\theta)V_i$. The number of vacancies the firm will post to satisfy its labour demand, $l_i$, then is calculated from this equilibrium flow condition as

$$V_i = f[s/q(\theta)]/[1-\alpha + ar\gamma q(\theta)]$$

(7)

The equilibrium output quantity of the model is directly dependent of the labour-market parameters $r$ and $\gamma$ and indirectly on all those having an impact on the tightness ratio stemming from Pissarides’ part of the model (unemployment benefit $z$, hiring costs $\gamma$, unemployment rate $u$, vacancies $v$, separation rate $s$, power parameter $\beta$ and interest $r$). Clearly, this result is due to the fact that the firm part of the Dixit-Stiglitz model is changed by adding hiring costs (per vacancies actually filled) to the wage rate: these terms, the wage and the expected hiring costs constitute marginal costs and therefore have an impact on the quantity, the employment and the vacancies posted.

Using (6) to replace $x$ in (2), we get:

$$n = L(1-u)\frac{1-\alpha + ar\gamma q(\theta)}{f}$$

(2')

42 Note that if the sum of all present-discounted profits is zero, in a steady state with all terms in the profit function constant - except for time in the discount factor – it follows from carrying out the integration that current profits have to be zero.

43 This equation corresponds to equation (3.8) in Pissarides (2000), p.69.
This is a function \( n(\theta) \). If technical progress in the matching function decreases the unemployment rate through a higher tightness ratio, it increases the number of firms. Moreover, a higher tightness ratio increases expected hiring costs, decreases the firm size and therefore increases the number of firms. On the other hand, for a given tightness ratio, technical progress in the matching function decreases hiring costs and therefore increases the number of firms. These last two effects are also working against each other in the solution for the size of the firm in terms of output and employment. Aggregate output can be found by multiplying the solutions for the output and the number of firms, equations (6) and (2’):

\[
x_n = (1-u)L[\alpha/a - r\gamma q(\theta)]
\]

Although there are internal economies of scale on the firm level, aggregate output has constant returns in the size of the economy \( L \), and employment \( L(1-u) \) for a given tightness ratio. An increase in the marginal value product of labour, \( d\alpha/a < 0 \), increases \( nx \) directly because it appears in the numerator but will be shown below to have an indirect impact on the tightness ratio, hiring costs and the unemployment rate \( u \).

Using the result for the number of firms from equation (2’), we can calculate the total number of vacancies from equation (7) as \( vL = nV_i = n(s/q)l_i = (s/q)L(1-u) \). Cancelling \( L \) and dividing by \( (1-u) \) yields \( v/(1-u) = s/q = \theta u/(1-u) \). This equation corresponds to equation (3.14) in Pissarides and can be re-transformed into equation (1) by solving for \( u \).
The number and size of firms and the vacancies posted all depend on the tightness ratio. To solve the system, the next steps serve to get a second equation besides (5) to solve for the wage and the tightness ratio. Dividing (4) by the price and solving for the real wage yields:

\[ w = \frac{\alpha}{a} \frac{r + s}{q(\theta)^{\gamma}} \]  \hspace{2cm} (4')

Larger hiring costs, \((r+s)\gamma q(\theta)\), imply lower wages according to (4') as in Pissarides’ model when interest is given. Here the model resembles Pissarides’ because the zero-profit condition in his model – rewritten in an endnote - implies constant labour costs as long as \(r = f_k - \delta\) and therefore \(k\) are constant. By implication, for a given marginal value product of labour, wages \(w\) always move in the opposite direction of hiring costs, \((r+s)\gamma q(\theta)\), in Pissarides’ model and in ours. Equation (4’) is drawn as a function \(w(\theta)\) in the upper right quadrant of Figure 1, indicated as the MM curve. It is also drawn in the upper left quadrant of Figure 1 with wages as a function of hiring costs. The intersection of lines BB and MM determines the wage and the tightness rate in the upper right quadrant, and hiring costs in the upper left quadrant. Given the rate of tightness thus determined, the solution for the rates of unemployment and vacancies can be found in the lower right quadrant. The tightness ratio then determines the size and number (also via the rate of unemployment) of firms.

INSERT FIGURE 1 OVER HERE
Technical progress in the matching function rotates the MM curve up around its vertical intercept, but leaves the BB curve unchanged and implies the following:

**Proposition 5:** Technical progress in the matching function, \( dT > 0 \), increasing the probability of filling a vacancy, decreases expected hiring costs and marginal costs, and increases tightness and wages. The size of the firms is increased by lower hiring costs. The number of firms is decreased by lower hiring costs and by the decrease in unemployment. The effects of a reduced rate of unemployment and lower hiring costs both increase aggregate output.

Equations (5) and \((4')\) are two functions \( w(\theta) \). As the BB curve is increasing and the MM curve is decreasing, \( \alpha/a > z \) ensures the existence of a unique equilibrium. Thus, there is only one solution or no one at all. Therefore we have a unique or no equilibrium. The fixed cost parameter \( f \) and the size of the economy, \( L \), have no impact on the value of \( v/u = \theta \). If, however, \( az \geq \alpha \) the tightness ratio is zero, there are no vacancies and unemployment is 100% according to equation (2). With no output, \( z \) cannot be paid. Therefore this cannot be an equilibrium situation.

Wages are increased by technical progress in the matching function, because the expected duration of filling a vacancy, and expected hiring costs are reduced. The MM curve rotated upwards because reduced hiring costs imply lower marginal costs. One can see from equation (5) that an increase in the tightness ratio increases wages. It follows from equation \((4')\) that increased wages imply reduced hiring costs.

Technical progress in the matching function increases wages as technical progress in the production function normally does in economic theory.
4. **Comparative static analysis of technical change in the production and matching functions**

Changes in technologies towards computers, Internet connectivity and website technology cause increases in fixed costs - in particular labour costs. Changes in fixed costs, $df > 0$, do not change the tightness ratio and therefore the unemployment and vacancy rates and variable labour costs are unchanged.\(^{44}\) Firm size $x$ is increased\(^ {45}\) and the number of firms is decreased as in Dixit-Stiglitz. Aggregate output, $nx$, is unaffected.

As a consequence of these ICT changes, variable costs of ordering inputs and selling output are reduced. A decrease in marginal costs via $da<0$, shifts up the MM curve according to equation \((4')\) and, to a lesser extent, the BB curve according to equation \((5)\) in Figure 1 as indicated by the arrows drawn. Wages and the tightness ratio, $v/u = \theta$, are increased according to equation \((5)\). This reduces the unemployment rate and increases the rate of vacancies according to equation \((1)\) and \((1')\) and increases hiring costs. The direct effects of the increase in the marginal product of labour are as follows: The size of firms is increased according to equation \((6)\) and the number of firms is decreased according to equation \((2')\) and aggregate output is increased. However, the resulting increase in hiring costs has the opposite effect on each of these variables.

**Proposition 6**: Technical change in the production technology in the form of higher fixed and lower variable costs, $df>0$ and $da<0$, increases wages, the tightness ratio

\(^{44}\) Effects of changes in fixed costs may be quite different in endogenous growth models. See de Groot (2000).
\(^{45}\) Of course, there may be other real world events, such as the shift from industry to services that work towards a decrease of firm size. Here we consider only the effects of ICT in isolation.
and hiring costs, and decreases the unemployment rate but increases the vacancy rate. The size of the firm and aggregate output are increased and the number of firms is decreased by the direct effect of these changes but the increase in hiring costs works in the opposite direction.

Note that technical progress in the matching function and in the production function always shift the MM curve to the right and the BB curves to the left with a stronger net effect of the shift to the right and the UV curve towards lower unemployment rates. This leads us to the following result:

**Proposition 7:** ICT as technical progress in the matching function and in the production function increases the tightness ratio, decreases the unemployment rate of the general equilibrium solution of the model and increases wages. The fall in the unemployment rate implies that the unemployment premium, \( t = zu \), can be reduced if the gross benefit \( z \) is kept constant. Hiring costs are decreased through technical progress in the matching function but increased through technical progress in the production function. The net effect on hiring costs is unclear. The direct effects increase the size of the firm and aggregate output, and decrease the number of firms. The increase in employment increases the number of firms and increases aggregate output - but an increase in hiring costs, which cannot be excluded, would work in the opposite direction. The effect on the vacancy rate is also ambiguous.

The results for hiring costs and the size of firms follow from propositions 5 and 6. The result for aggregate output, \( nx \), follows from the fact that it increases when the

\[\text{See also Barras (1990) on this aspect.}\]
unemployment rate and the marginal cost parameter decreases but decreases by a potential increase in hiring costs. The number of firms, according to equations (2) and (2’) is positively affected by the decrease in the unemployment rate, the increase in the marginal product of labour and negatively by a potential increase in hiring costs.

An increase in the tightness ratio increases the number of vacancies but ICT shifts the vacancy curve to lower values. Vacancies will grow (see appendix for a derivation) if the effect of a change in variable costs, \((-\text{da}/\text{a})\), is sufficiently large, or if either the bargaining power of workers, \(\beta\), or the probability of an unemployed to find a job, \(\theta_q\), are small or the discount rate \(r\) and the separation rate are sufficiently large. In these cases the change in the tightness ratio dominates the change in the position of the \(v\) curve, because all of these changes are favourable for increases in the tightness as opposed to increases in wages.

How does this model differ from perfect competition?

(i) Under perfect competition the fixed costs are zero and there is no product differentiation: \(f=0\) and \(\alpha=1\). Allowing for fixed costs and product differentiation is a gain in realism per se. Whereas changes in fixed costs \(f\) have an impact only on the division of aggregate output into the size and number of firms, an increase in the degree of competition, \(d\alpha>0\), increases aggregate output directly and also indirectly because it shifts up the MM curve and decreases unemployment, but the resulting increase in hiring costs counteracts this effect.

(ii) The introduction of fixed costs and product differentiation has two consequences: a) The number and size of firms is determined; b) Each firm produces only one product that is not produced by any other firm. By implication, each variant of a product will be produced only in one region
or country. This has helped explaining intra-industry trade and regional agglomeration.

(iii) When technical change is treated in the way that is common to industrial organisation literature – decreasing variable costs by increasing fixed costs – one needs a model of imperfect competition because competition cannot be perfect in the presence of fixed costs. This insight was at the heart of endogenous growth theory.

(iv) Changes in fixed costs $f$, and the CES parameter $\alpha$, which is also the degree of competition, produce the same direct effects on the goods market as they do in the Dixit-Stiglitz model, but the latter also changes the size and number of firms via changes in the hiring costs. Similarly, changes in the labour market parameters $s$, $z$, $\beta$, and $\gamma$ produce the same labour market effects as in Pissarides’ model, but the also change the size and number of firms via changes in the hiring costs.

5. Summary and conclusion

Linking Pissarides’ (1990) search theory of unemployment to the Dixit-Stiglitz (1977) model rather than to the neo-classical production function yields a framework in which effects of ICT can easily be identified, which together increase the vacancy/unemployment ratio, decrease the unemployment rate and increase wages. Hiring costs can increase or decrease.

First, ICT serves as technical progress in the matching function of job search. For any given tightness ratio this decreases the rate of unemployment and vacancies. When technical progress reduces expected hiring costs inspite of the increase in the
tightness ratio, there is more room for wages paid to households because marginal costs are lowered. The result of technical progress in the matching function is a higher tightness ratio and a lower rate of equilibrium unemployment. A higher tightness ratio yields higher wages and lower hiring costs.

Second, fixed costs increases as maintaining computers, making Internet connectivity and keeping websites working require trained personnel. This increases the size of firms and decreases the number of firms. Other variables are not affected by a change of the fixed cost parameter.

Third, the shift to higher fixed costs causes lower variable costs for ordering inputs and selling outputs, modelled here as production costs including the process of ordering and selling. This effect increases wages and the tightness ratio and decreases the equilibrium unemployment rate and the unemployment premium as the first effect does.

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Figure 1. The wage bargaining result, BB, and the profit maximising real wage, MM, determine the real wage and the tightness ratio in the upper right quadrant. This implies a solution for the unemployment rate $u$ and vacancies $v$ in the lower right quadrant. Each result for wages implies a result for hiring costs in the upper left quadrant. Technical progress in the matching function shifts the Beveridge curve towards the axes and MM up. The latter effect is supported by a decrease in variable costs.
Appendix: Household’s utility maximization (not for publication)

The Hamiltonian for the dynamic optimisation problem of the household indicated in the text is

\[
H = \left[ \int_{i=0}^{n} c_i^{\alpha} di \right]^{1/\alpha} + \lambda (1-u)w + uz - t - \int_{i=0}^{n} p_i c_i di + rW
\]

\(\lambda\) is a co-state variable. The first-order conditions of dynamic optimisation are the following:

\[
\frac{\partial H}{\partial c_i} = \left[ \int_{i=0}^{n} c_i^{\alpha} di \right]^{1/\alpha} c_i^{-\alpha} - \lambda p_i = 0 \text{ for all goods } i.
\]

\[- \frac{\partial H}{\partial W} = \rho \lambda = -r \lambda\]

The canonical transversality condition \(\lim_{t \to \infty} e^{-\rho t} \lambda = 0\) need not be stated here because the differential equation for the co-state variable implies \(\dot{\lambda} = \rho - r\). For any positive interest rate the transversality condition is redundant. The first-order condition for the goods \(i\) implies that for any two goods \(k\) and \(j\) we get

\[
\frac{c_j}{c_k} = \left( \frac{p_j}{p_k} \right)^{\alpha-1}.
\]

Therefore the inverse partial price elasticity is \(\alpha-1\). Multiplying the first-order conditions for \(c_i\) by \(c_i\) and summing over all goods yields

\[
\left[ \int_{i=0}^{n} c_i^{\alpha} di \right]^{1/\alpha} = \lambda \int_{i=0}^{n} p_i c_i di
\]

As all goods in the model have the same costs and therefore the same prices we set the prices equal to unity using them as numéraire. Quantities then also will be identical because goods enter the utility function symmetrically. The last equation then can be written as \(n^{(1/\alpha)-1} = \lambda > 0\). In any steady state where the number of firms is
constant, the co-state variable $\lambda$ must also be constant and its growth rate $\rho - r$ must also be zero. Therefore $r = \rho$ in a steady state with a constant number of firms.

**Appendix: The comparative static result for vacancies (not for publication)**

The ambiguity concerning the vacancy rate can be explored a bit more. By definition, $v = u\theta$. In growth rates this implies

$$\hat{v} = \hat{u} + \hat{\theta}.$$ 

From equation (1) the growth rate of $u$ can be calculated as $\hat{u} = \epsilon_{u\theta}\hat{\theta} + \epsilon_{uT}\hat{T}$. The expressions before the growth rates are the elasticities of the unemployment rate according to equation (1) with respect to $\theta$ and $T$. Insertion of this latter equation into that for the growth rate of vacancies yields

$$\hat{v} = (\epsilon_{u\theta} + 1)\hat{\theta} + \epsilon_{uT}\hat{T}$$

From equation (1) it is possible to derive $\epsilon_{u\theta} = \epsilon_{mv} / [(s/Tm)+1] > 0$ and $\epsilon_{uT} = -1 / [(s/Tm)+1] < 0$. $\epsilon_{mv} = 1 - \epsilon_{mu}$ is the elasticity of the matching function with respect to vacancies, which equals unity minus the elasticity with respect to unemployment. Blanchard and Diamond (1989) estimate that this elasticity has a value of about 0.6. Therefore $\epsilon_{u\theta}$ must be even smaller. By implication, the growth rate of the tightness ratio has a positive impact on that of vacancies and the growth rate of technology has a negative impact. Next, we try to derive sufficient conditions for the case that
vacancies will grow. Simple manipulation of the formula for the growth rate of $v$
yields the following result:

$$\hat{v} = \hat{e}_{ut} \hat{T} \left( \frac{\hat{e}_{u0} + 1 \hat{\theta}}{\hat{e}_{ut} \hat{T}} + 1 \right)$$

Insertion of the expressions for the elasticities presented above the equation can be
rewritten as follows:

$$\hat{v} = \hat{e}_{ut} \hat{T} \left\{ \left[ \frac{-s}{\hat{T}m} - \hat{e}_{mu} \right] \frac{\hat{\theta}}{\hat{T}} + 1 \right\}$$

As the first elasticity on the right-hand side is negative, the growth rate for vacancies
can only be positive if the term in brackets is negative. This requires a sufficiently
strong growth of the tightness ratio relative to that of $T$. Total differentiation of
equation (8) with respect to $\theta$, $a$ and $T$ after some manipulation yields the following
expression:

$$\frac{\hat{\theta}}{\hat{T}} = \frac{-\frac{da}{a} \alpha q^2 (1 - \beta)}{a \frac{a(r+s)\gamma T m}{dT} + 1} \frac{\hat{T}}{\hat{T} \frac{\hat{e}_{mu} + \frac{\beta \theta q}{(r+s) m T}}{}}$$

As $-da/a$ does appear only in this last fraction and not in the formula for growth of $v$
we can say that a sufficiently large change of $-da/a$, other things constant, will make
sure that vacancies are enhanced. Next, we go on for the case $da=0$. Then the last
equation can be simplified to yield
\[
\hat{\theta} = \frac{1}{\epsilon_{mu} + \frac{\beta \theta q}{(r + s)mT}}
\]

Insertion of this expression into that for the growth rate of \( \nu \) yields:

\[
\hat{\nu} = \epsilon_{uT} \hat{T} \left[ -\frac{s}{Tm} - \epsilon_{mu} \right] \frac{1}{\epsilon_{mu} + \frac{\beta \theta q}{(r + s)mT}} + 1
\]

Putting the term in square brackets on one fraction yields

\[
\hat{\nu} = \epsilon_{uT} \hat{T} \left( \frac{\beta \theta q}{r + s} - s \right) \frac{1}{\epsilon_{mu} + \frac{\beta \theta q}{(r + s)mT}}
\]

As the elasticity before the fraction is negative, vacancies will grow even if \( da=0 \) if (sufficient) \( \beta \theta q/(r+s) - s < 0 \). This means that the growth of the tightness ratio if large enough to increase vacancies if – other things constant - worker power \( \beta \) is sufficiently small, or the probability of an unemployed to find a job, \( \theta q \), is sufficiently small, or the discount rate \( r \) and the separation rate \( s \) are sufficiently large.
List of symbols (not for publication)

\[ \alpha \] marginal revenue, CES parameter
\[ a \] marginal costs
\[ \beta \] bargaining power
\[ \gamma \] hiring costs
\[ f \] fixed costs
\[ L \] country size
\[ n \] number of firms
\[ nx \] aggregate output
\[ r \] interest rate
\[ s \] separation rate
\[ t \] unemployment premium or tax
\[ \theta \] tightness ratio v/u
\[ u \] unemployment rate
\[ v \] vacancy rate
\[ w \] wage
\[ x \] firm size
\[ y \] utility
\[ z \] unemployment benefit