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Abstract

This paper analyzes price competition in a duopoly market in which products are both horizontally and vertically differentiated. Firms offer a basic and a premium product to buyers, some of whom are brand loyal. We establish the existence of a unique and symmetric Nash pricing equilibrium. Equilibrium prices are increasing in the degree of horizontal differentiation and the amount of brand loyal customers. The equilibrium price of the basic (premium) good is decreasing (increasing) in the quality difference and profits can increase in costs when this difference is high enough. If the pricing decision is taken at the product (division) level, then there is again a unique (and symmetric) Nash equilibrium. Equilibrium prices and profits are lower than in the centralized case and demand for the basic product is higher when the quality difference is sufficiently large. Welfare is unambiguously lower with decentralized pricing.

Keywords: Vertizontal differentiation, pricing, multiproduct oligopoly.

JEL classification codes: D43, L13.

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1 Introduction

There are many industries that consist of a few firms competing in multiple market segments identified by product quality. Car companies like General Motors and Toyota supply both basic and luxury cars, for example. Similarly, large electronic concerns such as Samsung and Huawei sell standard as well as premium smartphones. The same holds for producers of sunglasses, whiteware, surf gear, pharmaceuticals, air transport services, cleaning products, plastics, coffee machines, cat litters, *et cetera*. As reported in Nocke and Schutz (2018), multiproduct firms account for 91% of total output and 41% of the number of firms with an average four-firm concentration ratio of 35% when measured at the NAICS 5-digit level. Since there are demand linkages between the various quality-segments, a price change in one particular segment often affects overall sales and profit patterns. Moreover, the higher-quality segments are typically characterized by higher prices.

To illustrate, in the period mid 2017 until mid 2018, U.S. citizens spent over $5 billion on dry dog food.[1] The table below lists the four leading dry dog food brands in dollar sales in the period mid 2017 until mid 2018.[2] These four brands are produced by the two major players in the dog food market: Nestlé (39% market share) and Mars (24% market share). The last column of the table presents the price per pound at Walmart for a large-sized bag.[3] Both manufacturers have a low price and a high price brand among the four leading ones.

rank	brand	dollar sales	manufacturer	$/lb
1	Pedigree	603 mln	Mars, Inc.	0.52
2	Purina Dog Chow	457 mln	Nestlé Purina Petcare Co.	0.45
3	Purina One Smartblend	346 mln	Nestlé Purina Petcare Co.	1.08
4	Iams Proactive Health	265 mln	Mars, Inc.	1.08

A critical feature of such quality-segmented markets is that competition has both a horizontal and a vertical dimension. For instance, if a firm raises the price of its premium product, then it is likely to 'lose' customers to rivalling brands as well as to its own lower quality goods. Likewise, if a firm cuts its premium product price, then *ceteris paribus* it steals customers from comparable quality competitors while cannibalizing the sales of its other items. The fact that such multi-product type firms are partly in competition with themselves makes the design of an optimal pricing policy far from trivial.

The purpose of this paper is to study strategic pricing by sellers who are competing "head-to-head" in several quality segments simultaneously. Towards that end, we analyze a price-setting duopoly model of vertizontal product differentiation in which firms offer both a basic and a premium product. Demand for these product types comes from two different sorts of

[1] Source: https://www.statista.com/statistics/253976/pet-food-industry-expenditure-in-the-us/.
[2] Source: https://www.petfoodprocessing.net/articles/12825-state-of-the-us-pet-food-and-treat-industry.
[3] Information retrieved on 20 May 2019.

customers; those who are brand oriented and those who are quality oriented. Brand oriented buyers only choose between the basic and premium product of their preferred supplier. By contrast, quality oriented buyers have a strict preference for a particular quality level and choose between brands only.[4]

Within this framework, we establish the existence of a unique (and symmetric) Nash pricing equilibrium and perform a series of comparative statics exercises on this equilibrium outcome. With regard to the 'horizontal forces', we find that equilibrium prices are increasing in the degree of brand differentiation and decreasing in the number of quality oriented buyers. Regarding the 'vertical forces', we establish that equilibrium prices are increasing in the amount of brand oriented buyers and show that the price of the basic (premium) product is decreasing (increasing) in the quality difference. We furthermore find that equilibrium profits can increase in the production costs of the standard good provided that the difference in quality is sufficiently large.

We also consider the possibility of prices being determined at a more decentralized level. If the pricing decision is taken at the product (division) level, then there is again a unique (and symmetric) Nash equilibrium. The equilibrium outcome is such that both prices and profits are lower than in the centralized case. We furthermore find that the demand for the basic good increases when the difference in quality is sufficiently large and show that decentralized pricing unambiguously implies a welfare loss.

This research is related to the growing literature on strategic firm behavior in multi-product oligopolies. Nocke and Schutz (2018) provides a framework to study multi-product pricing when these goods are horizontally differentiated. With regards to quality differentiation, Champsaur and Rochet (1989) examines what range of qualities profit-maximizing duopolists prefer to offer. More recently, Johnson and Myatt (2003, 2006, 2015) also study competition among firms selling multiple quality-differentiated products to address questions related to pricing, entry and product-line configurations. Perhaps closest to our work is Gilbert and Matutes (1993), which analyzes strategic product line choices by multi-product firms supplying vertizontally differentiated goods.

There are only a few other articles that consider vertizontal product differentiation. Within the context of international trade, Di Comite, Thisse and Vandenbussche (2014) introduces a quadratic representative consumer model with vertizontal preferences to empirically assess firm performance in export markets. Building on Neven and Thisse (1990), Ribeiro (2015) studies price competition between two platforms (*e.g.*, media outlets, clubs) that are vertizontally differentiated. Finally, Li and Peeters (2017) uses a vertizontal differentiation setting to examine strategic quality information disclosure. All these works do not consider multi-product competition, however.

A separate strand of literature considers pricing strategies of firms meeting in more than

[4] At the firm level, therefore, brand oriented buyers are 'captive' and quality oriented buyers are 'non-captive'. See, for instance, Armstrong and Vickers (1993) and Sonderegger (2011).

one market. Bernheim and Whinston (1990), for instance, has shown that such multi-market contact may facilitate collusion. There are quite some industry studies providing empirical evidence of higher prices when firms interact in more than one market, including cement (Jans and Rosenbaum, 1998), hotels (Fernandez and Marin, 1998), telecommunications (Parker and Roller, 1997; Busse, 2000), radio (Waldfogel and Wulf, 2006) and airlines (Evans and Kessides, 1994; Ciliberto and Williams, 2014). Choi and Gerlach (2013) explores multi-market contact in relation to explicit collusion and antitrust enforcement. This study takes account of various degrees of horizontal product differentiation, but does not consider the potential impact of quality heterogeneity.

The next section introduces the model. This model is analyzed in Section 3. Section 4 considers the possibility of decentralized pricing and offers a welfare comparison. Section 5 concludes. All proofs are relegated to the Appendix.

2 A Duopoly Model of Vertizontal Differentiation

Consider a simultaneous-move price-setting duopoly with a low (basic) and a high (premium) quality market segment. In the spirit of Hotelling (1929), let both these submarkets be represented by a unit interval. Quality is homogeneous within each segment and indicated by the quality indices $\beta > 0$ and $\beta + \delta > 0$ for the basic and the premium product, respectively. The additional quality of the premium product is therefore given by $\delta > 0$. Products are positioned at the extremes of their respective segment and, without loss of generality, we assume that firm 1 is located at '0' and firm 2 is located at '1'. Firms' cost structures are identical and the low and the high quality varieties are respectively produced at constant marginal costs c_ℓ and c_h, with $c_h > c_\ell \geq 0$. It is further assumed that $\delta > c_h - c_\ell$, *i.e.*, the difference in product quality exceeds the additional costs of producing the premium product.

The demand side comprises two types of consumers that purchase no more than one product. There is a mass of $2\mu > 0$ buyers who are *brand oriented* and equally divided among both firms. These customers choose between buying a low and a high quality item from a particular brand (*i.e.*, firm 1 or firm 2). They are characterized by a taste parameter y, which is uniformly distributed on an interval $[0, 1]$ and which reflects the willingness to pay for extra quality. Thus, a consumer at the lower bound does not derive utility from more quality, whereas it is maximally valued at the upper extreme.

The remaining consumers are *quality oriented*, meaning they have a strict preference for a given quality and only choose between buying from firm 1 or firm 2 (or do not buy). Both the basic and the premium quality oriented buyers are uniformly distributed on an interval $[0, 1]$ with mass $\mu_\ell > 0$ and $\mu_h > 0$, respectively. Moreover, a customer 'located' at x_j experiences a disutility of $\tau_j \cdot x_j$ when buying from firm 1 and $\tau_j \cdot (1 - x_j)$ when buying from firm 2, where $\tau_j > 0$ for $j = \ell, h$.[5]

[5]Like in Altomonte, Colantone and Pennings (2016), this model therefore allows for asymmetric horizontal

Let us now derive demand for each product type under the assumption that all consumers purchase a product, which effectively requires a sufficiently high value of the parameter β. Consider a brand i oriented buyer 'located' at y_i. This consumer is indifferent between buying the low quality and buying the high quality variety when:

$$\beta + \delta y_i - p_{ih} = \beta - p_{i\ell} \quad \Longleftrightarrow \quad y_i = \tfrac{p_{ih}-p_{i\ell}}{\delta},$$

where p_{ih} and $p_{i\ell}$ are the prices of firm i's high and low quality product, respectively. Likewise, a quality oriented customer 'located' at x_j in the segment $j = \ell, h$ is indifferent between buying from firm 1 and buying from firm 2 when:

$$p_{1j} + \tau_j x_j = p_{2j} + \tau_j(1 - x_j) \quad \Longleftrightarrow \quad x_j = \tfrac{p_{2j}-p_{1j}+\tau_j}{2\tau_j}.$$

Taken together, this gives the following demand for the firms' high and low quality product:

$$\begin{aligned} d_{1h} &= \mu \cdot (1 - \widehat{y}_1) + \mu_h \cdot \widehat{x}_h & \qquad d_{2h} &= \mu \cdot (1 - \widehat{y}_2) + \mu_h \cdot (1 - \widehat{x}_h) \\ d_{1\ell} &= \mu \cdot \widehat{y}_1 + \mu_\ell \cdot \widehat{x}_\ell & \qquad d_{2\ell} &= \mu \cdot \widehat{y}_2 + \mu_\ell \cdot (1 - \widehat{x}_\ell), \end{aligned}$$

where $\widehat{y}_i \equiv \max\left\{ \min\left\{\frac{p_{ih}-p_{i\ell}}{\delta}, 1\right\}, 0\right\}$, $i = 1, 2$ and $\widehat{x}_j \equiv \max\left\{ \min\left\{\frac{p_{2j}-p_{1j}+\tau_j}{2\tau_j}, 1\right\}, 0\right\}$, $j = h, \ell$.

In the ensuing analysis, our focus is on situations where each product type is sold to both brand and quality oriented buyers. The following assumption gives sufficient conditions for this.

Assumption 1. $-(c_h - c_\ell) < \tau_h - \tau_\ell < \delta - (c_h - c_\ell)$.

Assumption 1 effectively provides an upper bound on the difference between τ_h and τ_ℓ. Finally, we suppose that consumer preferences and costs are common knowledge.

3 Equilibrium Pricing and Analysis

In this section, we explore the nature of price competition between the two firms. Using the above demand specification and assuming profit maximization, firm 1 picks prices $p_{1\ell}$ and p_{1h} to maximize

$$\begin{aligned} \pi_1(p_{1\ell}, p_{1h}, p_{2\ell}, p_{2h}) &= (p_{1\ell} - c_\ell) \cdot d_{1\ell} + (p_{1h} - c_h) \cdot d_{1h} \\ &= (p_{1\ell} - c_\ell) \cdot \big(\mu \cdot (\tfrac{p_{1h}-p_{1\ell}}{\delta}) + \mu_\ell \cdot (\tfrac{p_{2\ell}-p_{1\ell}+\tau_\ell}{2\tau_\ell})\big) \\ &\qquad + (p_{1h} - c_h) \cdot \big(\mu \cdot (1 - \tfrac{p_{1h}-p_{1\ell}}{\delta}) + \mu_h \cdot (\tfrac{p_{2h}-p_{1h}+\tau_h}{2\tau_h})\big) \end{aligned}$$

while simultaneously firm 2 sets prices $p_{2\ell}$ and p_{2h} to maximize

$$\begin{aligned} \pi_2(p_{1\ell}, p_{1h}, p_{2\ell}, p_{2h}) &= (p_{2\ell} - c_\ell) \cdot d_{2\ell} + (p_{2h} - c_h) \cdot d_{2h} \\ &= (p_{2\ell} - c_\ell) \cdot \big(\mu \cdot (\tfrac{p_{2h}-p_{2\ell}}{\delta}) + \mu_\ell \cdot (1 - \tfrac{p_{2\ell}-p_{1\ell}+\tau_\ell}{2\tau_\ell})\big) \\ &\qquad + (p_{2h} - c_h) \cdot \big(\mu \cdot (1 - \tfrac{p_{2h}-p_{2\ell}}{\delta}) + \mu_h \cdot (1 - \tfrac{p_{2h}-p_{1h}+\tau_h}{2\tau_h})\big). \end{aligned}$$

product differentiation across market segments.

The next result shows that there is a unique Nash equilibrium and that this equilibrium is symmetric. Moreover, in equilibrium, the price of the premium product exceeds that of the basic product.

Proposition 1. *Under Assumption 1, there is a unique Nash equilibrium. In this equilibrium, firms set prices symmetrically at:*

$$p_\ell^* = c_\ell + \tau_\ell + \tfrac{2\tau_\ell\mu}{4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h}[(c_h - c_\ell)\mu_h + 2(\tau_h - \tau_\ell)\mu_h + 4\tau_h\mu]$$

and

$$p_h^* = c_h + \tau_h + \tfrac{2\tau_h\mu}{4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h}[\delta\mu_\ell - (c_h - c_\ell)\mu_\ell - 2(\tau_h - \tau_\ell)\mu_\ell + 4\tau_\ell\mu].$$

Furthermore, $0 < p_h^* - p_\ell^* < \delta$.[6]

The way in which equilibrium prices are presented here is reminiscent of the equilibrium outcome in a standard version of Hotelling's model of horizontal product differentiation, *i.e.*, $p_j^* = c_j + \tau_j$, $j = \ell, h$. In fact, notice that both prices coincide when the number of brand oriented buyers becomes negligible (*i.e.*, $\mu \downarrow 0$).

Even though one might *a priori* expect firms to set higher prices in the presence of brand oriented buyers, Proposition 1 reveals that vertizontal equilibrium prices may be lower than those in the corresponding horizontal version. The price of the basic product is, for example, lower when there is severe price competition in the premium segment ($\tau_h \downarrow 0$) and relatively low competitive pressure in the other submarket ($\tau_\ell \uparrow c_h - c_\ell$). Likewise, the premium price is lower when there is strong price competition in the standard-quality product market ($\tau_\ell \downarrow 0$) and relatively low competitive pressure in the premium segment ($\tau_h \uparrow \delta - (c_h - c_\ell)$). Intense price competition in one segment may therefore put a downward pressure on prices in the adjacent segment and this 'negative vertical price effect' can dominate the 'positive horizontal price effect'. In other words, the presence of brand oriented buyers can lead to more competitive prices in the segment where price competition is less severe.

We now proceed by studying comparative statics of the equilibrium prices. The next proposition shows that prices are increasing in unit production costs.

Proposition 2. *Under Assumption 1, the equilibrium prices* p_ℓ^* *and* p_h^* *are increasing in the unit production costs* c_ℓ *and* c_h.

A firm's basic and premium price are naturally increasing in the own production costs. Demand for the adjacent quality then rises with a subsequent price increase in that segment. It can, moreover, be shown that the direct effect dominates the indirect effect so that an increase in c_h *ceteris paribus* boosts low quality product sales. An increase in c_ℓ likewise leads to a rising premium product market share, all else equal.

The impact on prices of a change in quality difference (δ), horizontal competitive pressure (τ_ℓ and τ_h) and the number of brand and quality oriented buyers (μ, μ_ℓ and μ_h) is less clear. To explore this in more detail, we impose the following condition:

[6] It can be easily verified that $c_h > c_\ell$ implies $p_\ell^* \geq c_\ell$ and $\delta > c_h - c_\ell$ implies $p_h^* \geq c_h$.

Assumption 2. $-\frac{c_h - c_\ell}{2} < \tau_h - \tau_\ell < \frac{\delta - (c_h - c_\ell)}{2}$.

This assumption is directly comparable to Assumption 1 and restricts the range within which the horizontal differentiation parameters are allowed to vary.

Let us start by exploring the effect of changes in the horizontal dimension of the model. The following result gives the price impact of changes in the horizontal differentiation parameters and the number of low and high quality oriented consumers.

Proposition 3. *Under Assumption 2, the equilibrium prices p_ℓ^* and p_h^* are increasing in the horizontal differentiation parameters τ_ℓ and τ_h and decreasing in the number of low and high quality oriented buyers μ_ℓ and μ_h.*

An increase in the horizontal differentiation parameter τ_j, $j = \ell, h$, leads *ceteris paribus* to an increase of prices in segment j. This, in turn, generates more demand in the other segment with a subsequent price increase. Regarding the number of quality oriented buyers, more customers in, say, the premium segment intensifies competition in that part of the market with lower premium prices resulting. More brand oriented buyers are therefore willing to switch to the high quality product, which in turn makes it less costly to cut prices in the lower segment. As before, the direct effect dominates the indirect effect so that an increase in the number of high (low) quality oriented buyers results in more high (low) quality product sales.

Next, let us turn to the vertical sphere. The next result shows how the equilibrium prices are affected by changes in the number of brand oriented customers and the quality difference.

Proposition 4. *Under Assumption 2, the equilibrium prices p_ℓ^* and p_h^* are increasing in the number of brand oriented consumers μ. Furthermore, p_ℓ^* is decreasing and p_h^* is increasing in the quality difference δ.*

A growing number of brand oriented buyers naturally enhances the incentive to exploit their loyalty through raising prices. The price effect of a change in quality difference is more subtle. As one would expect, an immediate impact of an increase in δ is that more buyers prefer the high quality product. The resulting premium price rise does not fully offset this effect, which is partly due to the presence of competition in the high quality segment. The subsequent loss in brand oriented consumers buying the basic product makes it less costly to cut the basic good price and steal some business in the low quality segment. Overall, however, premium product sales are increasing in the quality difference.

Let us finally turn to equilibrium profits. Profits are not trivial to analyze within the current framework, which is particularly due to the richness in parameters describing the demand side. By imposing symmetry across submarkets, however, we can establish the following supply side effect.

Proposition 5. *Assume $\mu_h = \mu_\ell$ and $\tau_h = \tau_\ell$. Equilibrium profits π_1^* and π_2^* are increasing in c_ℓ and decreasing in c_h if and only if $\delta > 2(c_h - c_\ell)$.*

This result reveals that equilibrium profits can decrease, but may also increase in unit production costs. If the quality difference between the basic and premium product is sufficiently large, then profits are decreasing in c_h and increasing in c_ℓ. The reason is as follows. If δ is high, then a large part of the brand oriented buyers opts for the premium product (*i.e.*, $\widehat{y}_i^* = \frac{p_{ih}^* - p_{i\ell}^*}{\delta}$ is relatively low). The reduced price-cost margin $p_{ih}^* - c_h$ would therefore result in relatively large losses. This negative direct effect is only partly offset by the positive indirect effect, *i.e.*, the rise in the price of the basic good. The latter effect is smaller since only a modest part of the brand oriented buyers prefers the low quality product when the quality difference is high.

In a similar fashion, equilibrium profits increase in c_ℓ. The direct effect is again negative, but relatively small since few brand oriented consumers choose the basic good. By contrast, the positive indirect effect that comes from the increase in the premium price is comparably big because of the large share of brand oriented buyers that picks the high quality product. The combined effect is therefore positive when δ is sufficiently high. A similar logic applies when the difference in quality is sufficiently low, meaning that the majority of brand oriented buyers opts for the basic good (*i.e.*, $\widehat{y}_i^* = \frac{p_{ih}^* - p_{i\ell}^*}{\delta}$ is relatively high). In that case, profits are decreasing in c_ℓ and increasing in c_h.

In sum, firms benefit from a cost decrease in their most popular submarket. Perhaps surprisingly, however, they also benefit from a cost increase in their least popular quality segment.

4 Centralized versus Decentralized Pricing

In the previous section, we analyzed strategic pricing under the assumption that pricing decisions are taken at the firm level. In practice, however, the production of different quality types is frequently organized in separate divisions that themselves determine what price to charge. For such decentralized organisational structures with corresponding incentive mechanisms it is more natural to assume that pricing decisions take place at the product type level. In particular, when division managers have discretion to determine their own prices and bonuses are primarily based on division rather than on company performance, divisions may effectively operate as distinct firms. In this section, we explore how such decentralized pricing affects prices, profits and welfare in comparison to the centralized pricing model of the preceding section.

The next result is comparable to Proposition 1 and shows that there is again a unique (and symmetric) Nash equilibrium when prices are set at the product type level. Akin to the centralized pricing case, all qualities are sold to both brand and quality oriented buyers and the equilibrium price of the premium product is higher.

Proposition 6. *Under Assumption 1 and* $\delta \geq \max\{\tau_\ell, \tau_h\}$*, there is a unique Nash equilib-*

rium. In this equilibrium, firms set prices symmetrically at:

$$p_\ell^\circ = c_\ell + \frac{(2\tau_h\mu+\delta\mu_h)(\delta\mu_\ell+2\mu(c_h-c_\ell))+2\delta\tau_h\mu(\mu_\ell+\mu_h+2\mu)}{(4\tau_\ell\mu+\delta\mu_\ell)(4\tau_h\mu+\delta\mu_h)-4\tau_\ell\tau_h\mu^2}\tau_\ell$$

and

$$p_h^\circ = c_h + \frac{(2\tau_\ell\mu+\delta\mu_\ell)(\delta\mu_h+2\mu[\delta-(c_h-c_\ell)])+2\delta\tau_\ell\mu(\mu_\ell+\mu_h+2\mu)}{(4\tau_\ell\mu+\delta\mu_\ell)(4\tau_h\mu+\delta\mu_h)-4\tau_\ell\tau_h\mu^2}\tau_h.$$

Furthermore, $0 < p_h^\circ - p_\ell^\circ < \delta$.

To avoid humongous expressions, but without renouncing any factor crucial to the understanding of the mechanic forces when comparing decentralized pricing relative to centralized pricing within the context of the vertizontal differentiated market, we restrict our parameters to $\mu_h = \mu_\ell = \mu = 1$ and $\tau_h = \tau_\ell$ and assume $\delta \geq \max\{\tau_\ell, \tau_h\}$ (such that $\widehat{y}^\circ \in (0,1)$). Doing so, we obtain the following result.

Proposition 7. *Prices and profits are lower under decentralized pricing. Moreover, the demand for low quality is larger if and only if* $\delta > 2(c_h - c_\ell)$.

This proposition implies that producer surplus, defined as the sum of profits, is lower. That consumers surplus is higher with decentralized pricing follows from the decrease in prices. Consumers who do not switch product type benefit from the price decrease, whereas those who do switch benefit even more. What happens to overall welfare thus depends on which of these two effects dominates.

To determine the net effect, note that price changes only cause a redistribution of welfare in case of non-switching customers. Any change in total surplus consequently comes from those who switch product type.

Proposition 7 tells us that if $\delta > 2(c_h - c_\ell)$ (alternatively, $\delta < 2(c_h - c_\ell)$), then for each brand there is an interval Y of brand oriented buyers who switch to the low (high) quality product. These switching consumers $y \in Y$ create a welfare gain (loss) by a reduction (increase) in $c_h - c_\ell$ and a welfare loss (gain) of δy. Aggregating these differences over all switching consumers,

$$2\int_Y \delta y - (c_h - c_\ell)\,\mathrm{d}y,$$

gives the overall welfare gain (loss), which leads to the following conclusion.

Proposition 8. *Decentralized pricing unambiguously yields a welfare loss.*

5 Concluding Remarks

In this paper, we consider strategic pricing by sellers who supply multiple quality-variants of their product and compete in the corresponding quality-segments simultaneously. Specifically, we analyzed a duopoly model of vertizontal product differentiation in which firms offer a basic and a premium good. Under the assumption that demand comprises both brand and

quality oriented buyers, we established the existence of a unique (and symmetric) Nash pricing equilibrium. These equilibrium prices are increasing in the degree of brand differentiation as well as in the number of brand oriented buyers. Moreover, equilibrium profits may increase in production costs and the equilibrium price of the basic (premium) product is decreasing (increasing) in the quality difference.

We contrasted these findings with the possibility that pricing decisions are taken decentrally; at the product (division) level. We prove there is again a unique (and symmetric) Nash equilibrium in which both prices and profits are lower than in the centralized case. We furthermore find that demand for the basic product is higher when the quality difference is sufficiently large and show that welfare is unambiguously lower with decentralized pricing.

There are several natural avenues for future research. One is to extend the dimensions of the model (*e.g.*, number of firms or quality segments). This, however, is likely to prove challenging in terms of deriving explicit and meaningful expressions. Another is to use this framework to analyze competition among multi-product and single-product, niche, firms. Finally, one may consider the possibility of price coordination and collusion in case products are vertizontally differentiated.

References

Altomonte C, I Colantone and E Pennings (2016). Heterogeneous firms and asymmetric product differentiation. *Journal of Industrial Economics* 64(4): 835-874.

Armstrong M and J Vickers (1993). Price discrimination, competition and regulation. *Journal of Industrial Economics* 41(4): 335-359.

Bernheim DB and MD Whinston (1990). Multimarket contact and collusive behavior. *RAND Journal of Economics* 21(1): 1-26.

Busse M (2000). Multimarket contact and price coordination in the cellular telephone industry. *Journal of Economics & Management Strategy* 9(3): 287-320.

Champsaur P and J-C Rochet (1989). Multiproduct duopolists. *Econometrica* 57(3): 533-557.

Choi JP and H Gerlach (2013). Multimarket collusion with demand linkages and antitrust enforcement. *Journal of Industrial Economics* 61(4): 987-1022.

Ciliberto F and JW Williams (2014). Does multimarket contact facilitate tacit collusion? Inference on conduct parameters in the airline industry. *RAND Journal of Economics* 45(4): 764-791.

Di Comite F, J-F Thisse and H Vandenbussche (2014). Verti-zontal differentiation in export markets. *Journal of International Economics* 93(1): 50-66.

Evans W and I Kessides (1994). Living by the golden rule: Multimarket contact in the U.S.

airline industry. *Quarterly Journal of Economics* 109(2): 341-366.

Fernandez N and P Marin (1998). Market power and multimarket contact: Some evidence from the Spanish hotel industry. *Journal of Industrial Economics* 46(3): 301-315.

Gilbert RJ and C Matutes (1993). Product line rivalry with brand differentiation. *Journal of Industrial Economics* 41(3): 223-240.

Hotelling H (1929). Stability in competition. *Economic Journal* 39(153): 41-57.

Jans I and D Rosenbaum (1997). Multimarket contact and pricing: Evidence from the U.S. cement industry. *International Journal of Industrial Organization* 15(3): 391-412.

Johnson JP and DP Myatt (2015). The properties of product line prices. *International Journal of Industrial Organization* 43: 182-188.

Johnson JP and DP Myatt (2006). Multiproduct Cournot oligopoly. *RAND Journal of Economics* 37(3): 583-601.

Johnson JP and DP Myatt (2003). Multiproduct quality competition: Fighting brands and product line pruning. *American Economic Review* 93(3): 748-774.

Li X and R Peeters (2017). Rivalry information acquisition and disclosure. *Journal of Economics & Management Strategy* 26(3): 610-623.

Neven DJ and JF Thisse (1990). On quality and variety competition. In: Gabszewicz JJ, JF Richard and L Wolsey (Eds.) *Economic Decision Making: Games, Econometrics and Optimization. Contributions in the Honour of Jacques H. Dréze.* North-Holland, Amsterdam.

Nocke V and N Schutz (2018). Multiproduct-firm oligopoly: An aggregative games approach. Working paper: February 28, 2018.

Parker P and L-H Roller (1997). Collusive conduct in duopolies: Multimarket contact and cross-ownership in the mobile telephone industry. *RAND Journal of Economics* 28(2): 304-322.

Ribeiro VM (2015). Product differentiation in two-sided markets. *Ph.D. Dissertation, Chapter 5.* University of Porto.

Sonderegger S (2011). Market segmentation with nonlinear pricing. *Journal of Industrial Economics* 59(1): 38-62.

Waldfogel J and J Wulf (2006). Measuring the effect of multimarket contact on competition: Evidence from mergers following radio broadcast ownership deregulation. *Contributions to Economic Analysis & Policy* 5(1).

A Proofs

Proof of Proposition 1. The first-order conditions for the two simultaneous maximization problems yield the following system of linear equations:

$$\begin{pmatrix} 4\tau_\ell\mu + 2\delta\mu_\ell & -4\tau_\ell\mu & -\delta\mu_\ell & 0 \\ -4\tau_h\mu & 4\tau_h\mu + 2\delta\mu_h & 0 & -\delta\mu_h \\ -\delta\mu_\ell & 0 & 4\tau_\ell\mu + 2\delta\mu_\ell & -4\tau_\ell\mu \\ 0 & -\delta\mu_h & -4\tau_h\mu & 4\tau_h\mu + 2\delta\mu_h \end{pmatrix} \begin{pmatrix} p_{1\ell} \\ p_{1h} \\ p_{2\ell} \\ p_{2h} \end{pmatrix}$$
$$= \begin{pmatrix} (2\tau_\ell\mu + \delta\mu_\ell)c_\ell - 2\tau_\ell\mu c_h + \delta\tau_\ell\mu_\ell \\ (2\tau_h\mu + \delta\mu_h)c_h - 2\tau_h\mu c_\ell + \delta\tau_h(2\mu + \mu_h) \\ (2\tau_\ell\mu + \delta\mu_\ell)c_\ell - 2\tau_\ell\mu c_h + \delta\tau_\ell\mu_\ell \\ (2\tau_h\mu + \delta\mu_h)c_h - 2\tau_h\mu c_\ell + \delta\tau_h(2\mu + \mu_h) \end{pmatrix}.$$

For both firms, solutions being global maxima of their respective maximization problem is guaranteed by the negative definite Hessians:

$$\begin{pmatrix} -(4\tau_\ell\mu + 2\delta\mu_\ell) & 4\tau_\ell\mu \\ 4\tau_h\mu & -(4\tau_h\mu + 2\delta\mu_h) \end{pmatrix}.$$

Since the matrix of coefficients in the system of linear equations is non-singular, we know that this system has one real solution. Moreover, given that asymmetric equilibria need to come in pairs, this solution must be symmetric. Exploiting this symmetry (that is, assuming $p_{1\ell} = p_{2\ell} = p_\ell$ and $p_{1h} = p_{2h} = p_h$), the above system reduces to

$$\begin{pmatrix} 4\tau_\ell\mu + \delta\mu_\ell & -4\tau_\ell\mu \\ -4\tau_h\mu & 4\tau_h\mu + \delta\mu_h \end{pmatrix} \begin{pmatrix} p_\ell \\ p_h \end{pmatrix} = \begin{pmatrix} (2\tau_\ell\mu + \delta\mu_\ell)c_\ell - 2\tau_\ell\mu c_h + \delta\tau_\ell\mu_\ell \\ (2\tau_h\mu + \delta\mu_h)c_h - 2\tau_h\mu c_\ell + \delta\tau_h(2\mu + \mu_h) \end{pmatrix}.$$

Solving this system gives the solution $(p_\ell^*, p_h^*,)$ as specified in the proposition. Finally, feasibility conditions for the demand being well-specified (i.e., $\widehat{y}_1, \widehat{y}_2 \in (0,1)$ at prices p_ℓ^* and p_h^*) requires $p_h^* > p_\ell^*$ and $p_h^* - p_\ell^* < \delta$. As one can easily verify, Assumption 1 is a sufficient condition for $p_h^* > p_\ell^*$ and $p_h^* - p_\ell^* < \delta$.

Proof of Proposition 2. The first derivative of p_ℓ^* with respect to c_ℓ and c_h is respectively given by:

$$\frac{\partial p_\ell^*}{\partial c_\ell} = \frac{2\mu(\tau_\ell\mu_h + 2\tau_h\mu_\ell) + \delta\mu_\ell\mu_h}{4\mu(\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h} > 0 \quad \text{and} \quad \frac{\partial p_\ell^*}{\partial c_h} = \frac{2\tau_\ell\mu_h\mu}{4\mu(\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h} > 0,$$

and the first derivative of p_h^* with respect to c_ℓ and c_h is respectively given by:

$$\frac{\partial p_h^*}{\partial c_\ell} = \frac{2\tau_h\mu_\ell\mu}{4\mu(\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h} > 0 \quad \text{and} \quad \frac{\partial p_h^*}{\partial c_h} = \frac{2\mu(2\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h}{4\mu(\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h} > 0.$$

Proof of Proposition 3. The first derivative of p_ℓ^* with respect to τ_ℓ, τ_h, μ_ℓ and μ_h is respectively given by:

$$\frac{\partial p_\ell^*}{\partial \tau_\ell} = (4\mu\tau_h\mu_\ell + \delta\mu_\ell\mu_h) \cdot \frac{4\mu(\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h + 2\mu([(c_h - c_\ell) + 2(\tau_h - \tau_\ell)]\mu_h + 4\tau_h\mu)}{(4\mu(\tau_\ell\mu_h + \tau_h\mu_\ell) + \delta\mu_\ell\mu_h)^2} > 0,$$

$$\frac{\partial p_\ell^*}{\partial \tau_h} = 4\tau_\ell \mu_h \mu \cdot \frac{4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h+2\mu([\delta-(c_h-c_\ell)-2(\tau_h-\tau_\ell)]\mu_\ell+4\tau_\ell\mu)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} > 0,$$

$$\frac{\partial p_\ell^*}{\partial \mu_\ell} = -2\tau_\ell\mu(4\tau_h\mu + \delta\mu_h) \cdot \frac{[(c_h-c_\ell)+2(\tau_h-\tau_\ell)]\mu_h+4\tau_h\mu}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} < 0,$$

and

$$\frac{\partial p_\ell^*}{\partial \mu_h} = -8\tau_\ell\tau_h\mu^2 \cdot \frac{[\delta-(c_h-c_\ell-2(\tau_h-\tau_\ell)]\mu_\ell+4\tau_\ell\mu}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} < 0.$$

The first derivative of p_h^* with respect to τ_ℓ, τ_h, μ_ℓ and μ_h is respectively given by:

$$\frac{\partial p_h^*}{\partial \tau_\ell} = 4\tau_h \mu_\ell \mu \cdot \frac{4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h+2\mu([(c_h-c_\ell)+2(\tau_h-\tau_\ell)]\mu_h+4\tau_h\mu)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} > 0,$$

$$\frac{\partial p_h^*}{\partial \tau_h} = (4\tau_\ell \mu_h \mu + \delta\mu_\ell\mu_h) \cdot \frac{4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h+2\mu([\delta-(c_h-c_\ell)-2(\tau_h-\tau_\ell)]\mu_\ell+4\tau_\ell\mu)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} > 0,$$

$$\frac{\partial p_h^*}{\partial \mu_\ell} = -8\tau_\ell\tau_h\mu^2 \cdot \frac{[(c_h-c_\ell)+2(\tau_h-\tau_\ell)]\mu_h+4\mu\tau_h}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} < 0,$$

and

$$\frac{\partial p_h*}{\partial \mu_h} = -2\tau_h\mu(4\tau_\ell\mu + \delta\mu_\ell) \cdot \frac{[\delta-(c_h-c_\ell)-2(\tau_h-\tau_\ell)]\mu_\ell+4\tau_\ell\mu}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} < 0.$$

The terms in squared brackets in the numerator are positive by Assumption 2.

Proof of Proposition 4. The first derivative of p_ℓ^* with respect to μ and δ is respectively given by:

$$\frac{\partial p_\ell^*}{\partial \mu} = 2\tau_\ell \cdot \frac{\delta\mu_\ell\mu_h([(c_h-c_\ell)+2(\tau_h-\tau_\ell)]\mu_h+8\tau_h\mu)+16\tau_h\mu^2(\tau_\ell\mu_h+\tau_h\mu_\ell)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} > 0$$

and

$$\frac{\partial p_\ell^*}{\partial \delta} = -\tau_\ell\mu_h \cdot \frac{2\mu_\ell\mu([(c_h-c_\ell)+2(\tau_h-\tau_\ell)]\mu_h+4\tau_h\mu)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} < 0.$$

The first derivative of p_h^* with respect to μ and δ is respectively given by:

$$\frac{\partial p_h^*}{\partial \mu} = 2\tau_h \cdot \frac{\delta\mu_\ell\mu_h([\delta-(c_h-c_\ell)-2(\tau_h-\tau_\ell)]\mu_\ell+8\tau_\ell\mu)+16\tau_\ell\mu^2(\tau_\ell\mu_h+\tau_h\mu_\ell)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} > 0$$

and

$$\frac{\partial p_h^*}{\partial \delta} = \tau_h\mu_\ell \cdot \frac{2\mu_\ell\mu([(c_h-c_\ell)+2(\tau_h-\tau_\ell)]\mu_h+4\tau_h\mu)}{(4\mu(\tau_\ell\mu_h+\tau_h\mu_\ell)+\delta\mu_\ell\mu_h)^2} < 0.$$

In determining the sign of these derivatives, notice that the term in squared brackets in the numerators is positive by Assumption 2.

Proof of Proposition 5. Setting $\mu_\ell = \mu_h$ and $\tau_\ell = \tau_h$, the first derivative of π^* with respect to c_ℓ and c_h is given by:

$$\frac{\partial \pi^*}{\partial c_\ell} = -\frac{\partial \pi^*}{\partial c_h} = \frac{4\tau_h\mu^2(\delta\mu_h+4\tau_h\mu)[\delta-2(c_h-c_\ell)]}{\delta(\delta\mu_h+8\tau_h\mu)^2}.$$

The term in the squared brackets determines the sign of the derivatives.

Proof of Proposition 6. The first-order conditions for the four simultaneous maximization problems yield the following system of linear equations

$$\begin{pmatrix} 4\tau_\ell\mu + 2\delta\mu_\ell & -2\tau_\ell\mu & -\delta\mu_\ell & 0 \\ -2\tau_h\mu & 4\tau_h\mu + 2\delta\mu_h & 0 & -\delta\mu_h \\ -\delta\mu_\ell & 0 & 4\tau_\ell\mu + 2\delta\mu_\ell & -2\tau_\ell\mu \\ 0 & -\delta\mu_h & -2\tau_h\mu & 4\tau_h\mu + 2\delta\mu_h \end{pmatrix} \begin{pmatrix} p_{1\ell} \\ p_{1h} \\ p_{2\ell} \\ p_{2h} \end{pmatrix}$$
$$= \begin{pmatrix} (2\tau_\ell\mu + \delta\mu_\ell)c_\ell + \delta\tau_\ell\mu_\ell \\ (2\tau_h\mu + \delta\mu_h)c_h + \delta\tau_h(2\mu + \mu_h) \\ (2\tau_\ell\mu + \delta\mu_\ell)c_\ell + \delta\tau_\ell\mu_\ell \\ (2\tau_h\mu + \delta\mu_h)c_h + \delta\tau_h(2\mu + \mu_h) \end{pmatrix}.$$

Since the matrix of coefficients in the system of linear equations is non-singular, we know that this system has one real solution. Moreover, given that asymmetric equilibria need to come in pairs, this solution must be symmetric. Exploiting this symmetry (that is, assuming $p_{1\ell} = p_{2\ell} = p_\ell$ and $p_{1h} = p_{2h} = p_h$), the above system reduces to

$$\begin{pmatrix} 4\tau_\ell\mu + \delta\mu_\ell & -2\tau_\ell\mu \\ -2\tau_h\mu & 4\tau_h\mu + \delta\mu_h \end{pmatrix} \begin{pmatrix} p_\ell \\ p_h \end{pmatrix} = \begin{pmatrix} (2\tau_\ell\mu + \delta\mu_\ell)c_\ell + \delta\tau_\ell\mu_\ell \\ (2\tau_h\mu + \delta\mu_h)c_h + \delta\tau_h(2\mu + \mu_h) \end{pmatrix}.$$

Solving this system gives the solution $(p_\ell^\circ, p_h^\circ,)$ as specified in the proposition. Finally, feasibility conditions for the demand being well-specified (i.e., $\widehat{y}_1, \widehat{y}_2 \in (0,1)$ at prices p_ℓ° and p_h°) requires $p_h^\circ > p_\ell^\circ$ and $p_h^\circ - p_\ell^\circ < \delta$. It can be verified that Assumption 1 and $\delta \geq \max\{\tau_\ell, \tau_h\}$ are a sufficient condition for $p_h^\circ > p_\ell^\circ$ and $p_h^\circ - p_\ell^\circ < \delta$.

Proof of Proposition 7. For $\mu_h = \mu_\ell = \mu = 1$ and $\tau_h = \tau_\ell = \tau$, we have:

$$p_\ell^* = c_\ell + 2\tau - \tfrac{[\delta - 2(c_h - c_\ell)]\tau}{\delta + 8\tau} \qquad p_h^* = c_h + 2\tau + \tfrac{[\delta - 2(c_h - c_\ell)]\tau}{\delta + 8\tau}$$
$$p_\ell^\circ = c_\ell + 2\tfrac{\delta}{\delta + 2\tau}\tau - \tfrac{[\delta - 2(c_h - c_\ell)]\tau}{\delta + 6\tau} \qquad p_h^\circ = c_h + 2\tfrac{\delta}{\delta + 2\tau}\tau + \tfrac{[\delta - 2(c_h - c_\ell)]\tau}{\delta + 6\tau}$$

and

$$\widehat{y}^* = \tfrac{1}{\delta}\Big\{(c_h - c_\ell) + \tfrac{2[\delta - 2(c_h - c_\ell)]\tau}{\delta + 8\tau}\Big\} \qquad \widehat{y}^\circ = \tfrac{1}{\delta}\Big\{(c_h - c_\ell) + \tfrac{2[\delta - 2(c_h - c_\ell)]\tau}{\delta + 6\tau}\Big\}.$$

From this, we obtain the profits

$$\pi^* = 4\tau + \tfrac{(\delta + 4\tau)[\delta - 2(c_h - c_\ell)]^2\tau}{\delta(\delta + 8\tau)^2} \qquad \pi^\circ = 4\tfrac{\delta}{\delta + 2\tau}\tau + \tfrac{(\delta + 2\tau)[\delta - 2(c_h - c_\ell)]^2\tau}{\delta(\delta + 6\tau)^2}.$$

The statement in the proposition follows from a comparison of these values.

Proof of Proposition 8. If $\delta > 2(c_h - c_\ell)$, then $Y = [\widehat{y}^*, \widehat{y}^\circ]$ and the change in welfare is given by

$$2\int_{\widehat{y}^*}^{\widehat{y}^\circ} (c_h - c_\ell) - \delta y \,\mathrm{d}y = -[\delta(\widehat{y}^* + \widehat{y}^\circ) - 2(c_h - c_\ell)] \cdot (\widehat{y}^\circ - \widehat{y}^*).$$

Alternatively, if $\delta < 2(c_h - c_\ell)$, then $Y = [\widehat{y}^\circ, \widehat{y}^*]$ and the change in welfare is given by

$$2\int_{\widehat{y}^\circ}^{\widehat{y}^*} \delta y - (c_h - c_\ell)\,\mathrm{d}y = [\delta(\widehat{y}^* + \widehat{y}^\circ) - 2(c_h - c_\ell)] \cdot (\widehat{y}^* - \widehat{y}^\circ).$$

Both expressions lead to

$$-\tfrac{16\tau^3(\delta+7\tau)[\delta-(c_h-c_\ell)]^2}{\delta(\delta+6\tau)^2(\delta+8\tau)^2},$$

which is negative.