Uniqueness and computation of equilibria in resource allocation games

VALORIZATION

All knowledge is connected to all other knowledge. The fun is in making the connections.

— Arthur Aufderheide

For research to be valuable, one should be able to explain its utilisation and its impact on society. Thus, in this chapter, we discuss the economic and social relevance of this thesis and identify groups for which, in addition to the academic community, these results are of interest. We start with the use of game theory in general, and its shortcomings. Then, we discuss how the field of Algorithmic Game Theory (AGT) arose from these shortcomings, and we discuss some real-life examples in which the AGT-community made a difference. This thesis focusses on the uniqueness and computation of Nash equilibria in resource allocation games, and hence, we also discuss the relevance of these specific type of games and how the results obtained in this thesis can be used in practice. Lastly, we discuss the relevance of this research for the field of Transportation Science, and the steps that are made to collaborate with other communities in order to face the challenges that are arising in network design and autonomous vehicles.

Game theory, also referred to as decision theory, models strategic interaction of multiple rational decision makers. In contrast to the field of classical optimization, game theorists assume that there is no central authority that is making these decisions, but instead, players behave selfishly and aim to maximize their private utility. Applications of this field include a large number of economic and political phenomena and approaches, such as auctions, voting systems, fair division, duopolies, oligopolies and social network formation. Even in biology, game theory has been used as a model to understand many different phenomena, like the stable approximate 1:1 sex ration in most species, animal communication, fighting behavior and territoriality.

Research in game theory usually focusses on equilibria or 'solution concepts', and the most famous of these is the Nash equilibrium. A set of strategies is called a Nash equilibrium when no player can unilaterally deviate from her current strategy and increase her utility. Though game theorists are particularly interested in these Nash equilibria, they do not consider the complexity issues that might arise when one actually wants to compute them, which would be one of the main concerns of any computer scientist. Algorithmic game theory arose due to this discrepancy between game theory and computer science.

Nisan et al. write in their book *Algorithmic Game Theory* [56] that "if an equilibrium concept is not efficiently computable, much of its credibility as a prediction of the behavior of rational agents is lost". We should be able to simulate each decision maker by a machine and as Kamal Jain said: "If your laptop cannot find the equilibrium, neither can the market". Therefore, one of the core activities of the AGT-community is to find efficient algorithms that compute Nash equilibria. Besides computing them, this community also studies the existence of equilibria, the uniqueness of equilibria and the complexity of computing them. Lastly, another popular topic is the convergence of best responses. Given an arbitrary state of the game, people might be able to increase their utility by unilaterally changing their strategy. The best improvement that can be made is called a *best response*. A change of strategy might cause a reaction by other players, and we obtain a sequence of best responses. Does such a sequence converge to a steady state (a Nash equilibrium)? Or will it cycle?

Results on the existence and uniqueness of equilibria are currently used in network design. For example, it is seen in practice that adding a road to an existing network might cause an increased overall journey time, a phenomenon called the Braess paradox. In Seoul, South Korea, congestion was reduced when a motorway was removed as part of a restoration project and in 1990, closing 42nd street in New York improved traffic in that area. When we are able to efficiently compute equilibria, one can identify such roads and close them to reduce the overall journey times. Furthermore, if one knows that best responses will converge to a Nash equilibrium, any set of arbitrary paths chosen by the cars will after some time converge to a steady traffic flow. Thus, applications of being able to efficiently compute equilibria can be found, among others, in transportation science and network design.

In this thesis, I study the uniqueness and complexity of computing equilibria in resource allocation games. These games play a key role in a wide range of applications including traffic networks and telecommunication networks. In particular, we looked at atomic splittable congestion games and multimarket oligopolies. The practical application for multimarket oligopolies seems clear. Given a set of companies and markets, a strategy of a company is to decide how many goods it wants to sell in each market. Atomic splittable congestion games also occur often, but they might be a bit harder to spot. Besides the obvious applications in traffic, we find one in queueing theory: assume we are given a set of M/M/1 queues served in a first-come-first-serve fashion, and a finite set of companies, each sending packets to the queues with some company-specific arrival rate. Each queue has a single server with an exponentially distributed service time. Note that when more packets are assigned to a queue, the average sojourn time will increase. Each company needs to find a fractional distribution of her packets over the queues that minimizes the average sojourn time of her packets.

In Chapter 2 we study the uniqueness of Nash equilibria in strategic games. This property is key to actually predict the outcome of distributed resource allocation: if there are multiple equilibria, it is not clear upfront which equilibrium will be selected by the players. This issue has been raised explicitly by Aumann [5]: "...it is by no means clear how the players would arrive at an equilibrium, why they should play equilibrium strategies, and how a specific equilibrium would be chosen from among the set of all equilibria". In this chapter, we investigate strategy spaces that have the property that Nash equilibria are unique, no matter how the strategy spaces of the different players are interweaved. As a result, we introduced the class of bidirectional flow polymatroids. Such a result can be useful in the design of new games. For example: assume that given a network, each player needs to connect all her nodes in a connected subgraph of this network, e.g., players need to divide their demand over spanning trees of a connected network. Then, whenever this network is known to be generalized series-parallel, equilibria are known to be unique.

In Chapter 3 and Chapter 4 we study the computation of Nash equilibria on networks with only parallel edges or, more generally, strategy spaces where each strategy consists of a single resource. Note that it is already known that equilibria are unique in this setting. The main idea is to construct an equilibrium in a corresponding integral splittable congestion game. This can be done within polynomial time in case cost functions are affine, and within pseudo-polynomial time whenever the cost functions are increasing, non-negative and convex. Furthermore, we showed that multi-market oligopolies with affine price functions and quadratic costs can be modelled as atomic splittable congestion games, and hence, also compute Nash equilibria for this setting. The impact of these results is that whenever a game can be modelled as an atomic splittable congestion game, players are able to compute the unique Nash equilibrium of this game and play accordingly.

In order to make a real change, collaboration with companies as TomTom, Google Maps and the Transportation Science community is necessary. Though the Algorithmic Game Theory community can find exact equilibria in many situations, the models that are used are merely a simplification of reality. Through collaboration and discussions, we should settle on new basic assumptions that make the current models more realistic. Currently, one of the most interesting collaborations in this direction is the Dagstuhl seminar on Dynamic Traffic Flow models in Transportation Science. So far, this seminar has been organized twice, first in Oktober 2016 and a second time in March 2018. This seminar brings together researchers from three different communities: Simulations (SIM), Dynamic Traffic Assignment (DTA) and Algorithmic Game Theory (AGT). Among other points, the seminar initiated a systematic study of the complexity of equilibrium computation for DTA models – which is the core task when resolving dynamic traffic assignment problems. As equilibrium computation and its complexity are one of the core topics of this thesis, I gave a talk at both seminars and in this way contributed to the valuable discussions.