Consistent comparisons of attainment and shortfall inequality: a critical examination

Consistent comparisons of attainment and shortfall inequality: A critical examination*

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Abstract. An inequality measure is 'consistent' if it ranks distributions the same irrespective of whether health quantities are represented in terms of attainment or shortfalls. This consistency property severely restricts the set of admissible inequality measures. We show that, within a more general setting of separate measures for attainments and shortfalls, the consistency property is a combination of two conditions. The first is a compelling rationality condition that says that the attainment measure should rank attainment distributions as the shortfall measure ranks shortfall distributions. The second is an overly demanding condition that says that the attainment measure and the shortfall measure should be identical. By dropping the latter condition, the restrictions on the admissible inequality measures disappear.

Keywords. Health inequality · Attainment inequality · Shortfall inequality · Consistency

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1 Introduction

Clarke et al. (2002) have shown that conclusions drawn by standard inequality measures may differ depending on whether the individual health data are represented by attainments or by shortfalls to an upper bound. Observations like this have led Erreygers (2009a, 2009b), Lambert and Zheng (2011) and Lasso de la Vega and Aristondo (2012), among others, to search for 'consistent' inequality measures, which rank attainment distributions identically as the corresponding shortfall distributions. It has been established that this property—to which we refer as 'strong consistency'—severely restricts the set of admissible inequality measures.

The following example illustrates the limitations imposed by strong consistency. Consider the two-person attainment distributions $x = (10, 70)$ and $y = (30, 90)$. With an upper bound of 100, the corresponding shortfall distributions are $x' = (90, 30)$ and $y' = (70, 10)$, respectively. Relative inequality measures, such as the well-known Gini and Theil measures, decrease under equal absolute additions for all individuals. Hence, each relative inequality measure judges x as more unequal than y, but x' as less unequal than y', and thus violates strong consistency. Moreover, the negative implications of strong consistency go much beyond ruling out the relative inequality measures. For example, Lambert and Zheng (2011) show that the variance is the only absolute and subgroup decomposable inequality measure that satisfies strong consistency.

We will argue that strong consistency is unduly demanding. The matter is important, as an appropriate weakening of the property turns out to impose no a priori restrictions on the set of admissible inequality measures.

First we focus on the implications of strong consistency. The property aims to capture the rationality idea that inequality judgments should not depend on the arbitrary choice of how to represent the basic data. Therefore, strong consistency should not only apply to attainments and shortfalls, but also to other representations. We show that consistency with respect to one additional representation—in terms of 'relative' shortfalls, which measure the proportional (instead of the absolute) increase required to reach the upper bound—already excludes, among others, all relative, absolute and intermediate inequality measures. That is, a direct extension of the logic underlying strong consistency implies the impossibility of inequality measurement using standard methods. This finding encourages scrutinizing the property.

In order to examine strong consistency, we consider a more general setting that allows separate inequality measures for attainments and shortfalls. We show that strong consistency is a combination of two properties, viz., 'weak consistency' and 'uniqueness'. Weak consistency says that the attainment measure should rank attainment distributions in the same way as the shortfall measure ranks the corresponding shortfall distributions. Uniqueness says that the attainment measure and the shortfall measure should be identical. Weak consistency is sufficient to capture the desired rationality requirement, which means that uniqueness is superfluous and strong consistency is too demanding. We show that if weak consistency is imposed without uniqueness, then any inequality measure—without restrictions—can be chosen for either the attainment or the shortfall measure. This choice then fully determines the properties of the other measure.

The next section introduces notation and basic concepts. Section 3 considers the implications of strong consistency and of its underlying logic. Section 4 examines strong consistency in the general setting with separate attainment and shortfall measures. Section 5 concludes.

2 Preliminaries

A distribution is a vector $x = (x_1, x_2, \ldots, x_n)$ in $\mathbb{R}^n_{++}$. The positive real number x_i represents the health quantity—e.g., an attainment or a shortfall—of individual $i = 1, 2, \ldots, n$. For a distribution x, we denote the mean $(x_1 + x_2 + \cdots + x_n)/n$ by $\bar{x}$. We say that a distribution x is non-equal if not all of the entries of x are equal. We write 1_n for the n-vector with a one at each entry.

An inequality measure is a symmetric and strictly Schur-convex function $I : \mathbb{R}^n_{++} \to \mathbb{R}$ that associates with each distribution x in $\mathbb{R}^n_{++}$ an inequality level $I(x)$. Symmetry and strict Schur-convexity ensure that the inequality measure is anonymous (switching individuals' quantities does not change inequality) and satisfies the Pigou-Dalton principle (regressive transfers between individuals increase inequality). These properties are standard in the literature (see, e.g., Cowell, 2000, and Lambert, 2001).

It is common to distinguish inequality measures with respect to their behaviour under equal proportionate and equal absolute increases. We distinguish the following five categories.[1]

[1]Several terms are used in the literature to refer to the super-relative and super-

(i) An inequality measure I is *relative* if $I(x) = I(\lambda x)$ for each distribution x and each $\lambda > 1$.

(ii) An inequality measure I is *absolute* if $I(x) = I(x + \mu 1_n)$ for each distribution x and each $\mu > 0$.

(iii) An inequality measure I is *intermediate* if $I(x + \mu 1_n) < I(x) < I(\lambda x)$ for each non-equal distribution x, each $\mu > 0$ and each $\lambda > 1$.

(iv) An inequality measure I is *super-relative* if $I(x) > I(\lambda x)$ for each non-equal distribution x and each $\lambda > 1$.

(v) An inequality measure I is *super-absolute* if $I(x) < I(x + \mu 1_n)$ for each non-equal distribution x and each $\mu > 0$.

If I is a relative or super-relative inequality measure, then $I(x) > I(x + \mu 1_n)$ for each non-equal distribution x and each $\mu > 0$, and if I is an absolute or super-absolute inequality measure, then $I(x) < I(\lambda x)$ for each non-equal distribution x and each $\lambda > 1$ (see, e.g., Moyes, 1999, Proposition 3.4).

Finally, let b be an upper bound on attainments and let B collect each distribution x in $\mathbb{R}^n_{++}$ such that $x_i < b$ for each individual $i = 1, 2, \ldots, n$. For a distribution x in B, we denote the distribution $(b - x_1, b - x_2, \ldots, b - x_n)$ by $b - x$. If x is an attainment distribution, then $b - x$ is the corresponding shortfall distribution. Conversely, if x is a shortfall distribution, then $b - x$ is the corresponding attainment distribution. The set B may be interpreted as the set of all attainment distributions or, alternatively, as the set of all shortfall distributions.

3 Strong consistency and its implications

To address consistent comparisons of attainment and shortfall inequality, Erreygers (2009a) proposes the 'perfect complementarity' property. Perfect complementarity requires an inequality measure to take the same value for an attainment distribution and its corresponding shortfall distribution. That is, it requires $I(x) = I(b - x)$ for each distribution x in B. For the income-related health inequality setting, which focuses mainly on variants and extensions of the concentration index and concentration curve, Erreygers (2009b) introduced the analogous 'mirror condition'.

absolute categories. For example, Amiel and Cowell (1999) use the terms 'Dalton' and 'anti-Dalton', Kolm (1999) uses 'superintensive' and 'superequal', and Zheng (2007) uses 'extreme rightist' and 'extreme leftist'.

Lambert and Zheng (2011) propose a less demanding consistency property. They require only that an inequality measure's ranking of two attainment distributions coincides with its ranking of the two corresponding shortfall distributions. We focus on the property of Lambert and Zheng and refer to it as strong consistency.

Strong consistency. For all distributions x and y in B, we have

$$I(x) \leq I(y) \quad \text{if and only if} \quad I(b-x) \leq I(b-y).$$

Clearly, each inequality measure that satisfies perfect complementarity also satisfies strong consistency. Therefore, our critique of strong consistency applies also to perfect complementarity (and to the analogous mirror condition).

Strong consistency severely restricts the set of admissible inequality measures. Several contributions to the literature argue that the property suggests an exclusive focus on absolute inequality measures. Lambert and Zheng (2011, Theorem 3) show that among relative, absolute and (a subclass of the) intermediate inequality measures, only absolute inequality measures can satisfy strong consistency. Similarly, Erreygers and Van Ourti (2011a) show that absolute variants of the concentration index satisfy the mirror condition, whereas relative variants do not. This finding also figures prominently in the debate between Wagstaff (2011a, 2011b) and Erreygers and Van Ourti (2011b). Particular absolute inequality measures satisfying perfect complementarity or the mirror condition have been proposed by Erreygers (2009a, 2009b) and Chakravarty et al. (2013).

The following proposition generalizes the incompatibility result of Lambert and Zheng (2011, Theorem 3) to all relative, intermediate,[2] super-relative and super-absolute inequality measures. The proof relies on a simple extension of the two-person example presented in the introduction.

Proposition 1. *There is no relative, intermediate, super-relative or super-absolute inequality measure that satisfies strong consistency.*

[2]Theorem 3 of Lambert and Zheng (2011) covers only the intermediate inequality measures that satisfy Zoli's (1999) 'flexible inequality equivalence' property. Excluded, for example, is the class of intermediate inequality measures introduced by Seidl and Pfingsten (1997). Proposition 1, by contrast, covers all intermediate inequality measures.

Proof. Let b be the upper bound on attainments. Let c, d and e be positive real numbers such that $c + d + e < b$. Consider two attainment distributions $x = (c, c, \ldots, c, c+d)$ and $y = (c+e, c+e, \ldots, c+e, c+d+e)$. The shortfall distributions are $b-x = (b-c, b-c, \ldots, b-c, b-c-d)$ and $b-y = (b-c-e, b-c-e, \ldots, b-c-e, b-c-d-e)$. For each relative, each intermediate and each super-relative inequality measure I, we have $I(x) > I(y)$ and $I(b-x) < I(b-y)$. For each super-absolute inequality measure, we have $I(x) < I(y)$ and $I(b-x) > I(b-y)$. Hence, each relative, each intermediate, each super-relative and each super-absolute inequality measure violates strong consistency. □

Two comments are in order. First, not all absolute inequality measures satisfy strong consistency. Lambert and Zheng (2011, Theorems 4, 5 and 6) identify several demanding additional conditions. Erreygers et al. (2012) provide related results in the income-related health inequality setting. Second, not all inequality measures that satisfy strong consistency are absolute. Lasso de la Vega and Aristondo (2012) and Aristondo and Lasso de la Vega (2014) suggest two procedures to construct an inequality measure that satisfies perfect complementarity.[3] The constructed inequality measure is typically not absolute, but rather forms a compromise between the five categories of relative, absolute, intermediate, super-relative and super-absolute inequality measures. Kjellsson and Gerdtham (2013) and Allanson and Petrie (2014) demonstrate that Wagstaff's (2005) variant of the concentration index—a non-absolute inequality measures that satisfies the mirror condition—is a compromise between the relative and super-absolute categories.[4] The compromise position has been examined further by Kjellsson and Gerdtham (2014).

Proposition 1, together with previous results in the literature, demonstrates that strong consistency rules out many interesting inequality measures. This motivates putting the property under scrutiny. In the next section we will argue that strong consistency is too demanding and that

[3]Both procedures focus simultaneously on the attainment distribution and the corresponding shortfall distribution. The first procedure applies a generalized mean to a given inequality measure's values for the attainment and shortfall distributions (Lasso de la Vega and Aristondo, 2012). The second procedure applies a given inequality measure to the distribution obtained by concatenation of the attainment and shortfall distributions (Aristondo and Lasso de la Vega, 2014).

[4]This is noted also by Erreygers and Van Ourti (2011a, p. 690). However, they regard super-absoluteness, to which they refer as 'inverse-relativeness', as unreasonable.

a proper weakening allows complete freedom in choosing inequality measures. But first we look at the implication of extending the logic of strong consistency to a third representation of the basic data.

Consider a representation of a distribution in terms of 'relative' shortfalls. Whereas the shortfall distribution looks at the absolute amount that has to be added to an attainment to reach the upper bound, the relative shortfall distribution looks at the factor by which the attainment has to be multiplied in order to reach the upper bound. For a distribution x in B, we denote the distribution $(b/x_1, b/x_2, \ldots, b/x_n)$ by b/x. If x is an attainment distribution, then b/x is the corresponding relative shortfall distribution (and vice versa).

The following property applies the logic of strong consistency to relative shortfalls. It requires the inequality rankings of attainment distributions and the corresponding relative shortfall distributions to be the same.

Strong consistency*. For all distributions x and y in B, we have

$$I(x) \leq I(y) \quad \text{if and only if} \quad I(b/x) \leq I(b/y).$$

The next proposition is a counterpart to Proposition 1. The proposition says that strong consistency* excludes all absolute, intermediate, super-relative and super-absolute inequality measures.[5]

Proposition 2. *There is no absolute, intermediate, super-relative or super-absolute inequality measure that satisfies strong consistency*.*

Proof. Let b be the upper bound on attainments. Let $c > 0$, $d > 1$ and $e > 1$ be real numbers such that $cde < b$. Consider two attainment distributions $x = (c, c, \ldots, c, cd)$ and $y = (ce, ce, \ldots, ce, cde)$. The relative shortfall distributions are $b/x = (b/c, b/c, \ldots, b/c, b/(cd))$ and $b/y = (b/(ce), b/(ce), \ldots, b/(ce), c/(cde))$. For each absolute, each intermediate and each super-absolute inequality measure I, we have $I(x) < I(y)$ and $I(b/x) > I(b/y)$. For each super-relative inequality measure I, we have $I(x) > I(y)$ and $I(b/x) < I(b/y)$. Hence, each absolute, each intermediate, each super-relative and each super-absolute inequality measure violates strong consistency*. $\square$

[5] An example of a relative inequality measure that satisfies strong consistency* is $x \mapsto -0.5 + \sum_{i=1}^{n} \bar{x}/(2nx_i)$. This inequality measure is a member of the generalized entropy class: consider equation (5.12a) in Lambert (2001) and set $c = -1$.

The rationality idea underlying strong consistency requires that the inequality ranking should be the same irrespective of whether the basic data is represented in terms of attainments, (absolute) shortfalls, relative shortfalls, or in any other way. That is, if strong consistency is accepted as a compelling rationality requirement, then so should strong consistency*.[6] But as the following immediate implication of Propositions 1 and 2 shows, this leads to the impossibility of inequality measurement using standard methods.

Corollary. *There is no relative, absolute, intermediate, super-relative or super-absolute inequality measure that satisfies both strong consistency and strong consistency*.*

We have looked at the severe implications for inequality measurement of strong consistency and of extending the property's underlying logic. Now we turn to a direct examination of strong consistency.

4 Disentangling strong consistency

We consider a more general setting that allows for separate inequality measures for attainments and shortfalls. Thus, we recognize that a priori one may want different properties for an inequality measure depending on what the numbers to which it applies actually mean (e.g., one may want the Pigou-Dalton principle if the numbers are incomes, but not if the numbers are logged incomes).[7] We denote the attainment inequality measure by I^a and the shortfall inequality measure by I^s. Note that the previous setting with a single measure I for attainments and shortfalls is obtained as a special case by imposing the condition $I^a = I^s\,(= I)$. We discuss this condition (the 'uniqueness' property) at the end of this section.

A pair of inequality measures (I^a, I^s) consistently compares attainment and shortfall inequality if it satisfies the following property.

Weak consistency. For all distributions x and y in B, we have

$$I^a(x) \le I^a(y) \quad \text{if and only if} \quad I^s(b-x) \le I^s(b-y).$$

[6]For our purpose it is irrelevant whether or not relative shortfalls are used in practice. All that matters is that relative shortfalls constitute another way of representing the same basic data.

[7]Marchant (2008) stresses this point in the context of currency unit consistency for bankruptcy rules.

Weak consistency is sufficient to guarantee that inequality judgments do not depend on whether the data are represented in terms of attainments or shortfalls. Moreover, it does the job without the two negative implications of strong consistency that were identified in the previous section. To see this, consider the following proposition (of which we omit the simple proof).

Proposition 3. *A pair of inequality measures (I^a, I^s) satisfies weak consistency if and only if there exists a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ such that*

$$I^a(x) \;=\; f(I^s(b-x)) \qquad \text{for each distribution } x \text{ in } B. \tag{1}$$

First, in contrast to strong consistency (recall Proposition 1 in Section 3), weak consistency imposes no restrictions on the set of admissible inequality measures. Let the attainment measure I^a be any inequality measure (including those excluded by Proposition 1). If the shortfall inequality measure I^s is defined in accordance with equation (1), then the pair (I^a, I^s) satisfies weak consistency. Conversely, weak consistency allows any inequality measure to serve as the shortfall measure I^s, as long as the attainment measure I^a is defined as in equation (1).

As an example, consider the Gini measure,

$$G \;:\; B \;\longrightarrow\; \mathbb{R} \;:\; x \;\longmapsto\; \frac{1}{2n^2\bar{x}} \sum_{i=1}^{n}\sum_{j=1}^{n} |x_i - x_j| \,. \tag{2}$$

The Gini measure is relative and hence excluded by strong consistency. By contrast, weak consistency does allow choosing the Gini measure as, say, the attainment measure I^a, provided that we define the shortfall measure I^s in accordance with equation (1). That is, I^s must be equal to (an increasing transformation of)

$$H \;:\; B \;\longrightarrow\; \mathbb{R} \;:\; x \;\longmapsto\; \frac{1}{2n^2(b-\bar{x})} \sum_{i=1}^{n}\sum_{j=1}^{n} |x_i - x_j| \,. \tag{3}$$

So, B in equation (2) is the set of all attainment distributions, whereas B in equation (3) is the set of all shortfall distributions. The inequality measures G and H each violate strong consistency, but the pair (G, H) satisfies weak consistency.[8]

[8] The measure G is relative, whereas the measure H is super-absolute. This obser-

Second, contrary to strong consistency (recall the corollary in Section 3), weak consistency does not become more restrictive if extended to additional representations of the data. Consider a general 'ψ-representation' that transforms each attainment using a strictly monotonic function $\psi : B \to \mathbb{R}_{++}$. For an attainment distribution x in B, we denote the corresponding 'ψ-distribution' $(\psi(x_1), \psi(x_2), \ldots, \psi(x_n))$ by $\psi(x)$. For example, we have $\psi(x) = b - x$ in the case of shortfalls and $\psi(x) = b/x$ in the case of relative shortfalls. Consider an attainment measure I^a and a measure I^ψ defined by

$$I^a(x) \;=\; f(I^\psi(\psi(x))) \qquad \text{for each distribution } x \text{ in } B,$$

with f a strictly increasing function. We have that I^a ranks attainment distributions in the same way as I^ψ ranks the corresponding ψ-distributions. Clearly, weak consistency can be extended without imposing additional restrictions on inequality measures: the inequality measure for, say, attainments may be chosen freely, and this choice fully determines the inequality measure for each ψ-representation.[9]

We conclude this section by clarifying the relationship between weak consistency and strong consistency. The following property on a pair of inequality measures (I^a, I^s) says that there should be a unique inequality measure to deal with both attainment and shortfall distributions.

Uniqueness. The inequality measures I^a and I^s are identical.

As shown by the next proposition (of which we omit the easy proof), strong consistency coincides with the combination of weak consistency and uniqueness.

Proposition 4. *A pair of inequality measures* (I^a, I^s) *satisfies weak consistency and uniqueness if and only if there exists an inequality measure* $I = I^a = I^s$ *that satisfies strong consistency.*

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vation generalizes to each pair of inequality measures that satisfies weak consistency: if one of the two inequality measures is relative, then the other is super-absolute. Note that, by contrast, some other key properties carry over from one of the two inequality measures to the other: for example, if one of the two inequality measures satisfies the Pigou-Dalton principle, then so does the other, and if one of the two inequality measures satisfies subgroup decomposability (see Lambert, 2001, pp. 111-112, for a formal definition), then so does the other (after a strictly increasing transformation).

[9]It should not be seen as a limitation that the inequality measures for all other representations are fully determined. Indeed, the rationality idea we aim to capture requires exactly that nothing remains to be said after fixing the inequality measure for one particular representation of the data.

Weak consistency already ensures that inequality judgments do not depend on the chosen representation of the data. Therefore, we conclude that uniqueness is superfluous and strong consistency is too demanding.

Given that uniqueness is not required for consistent inequality judgments, the question arises whether there are alternative justifications for the property. A plausible response would be that uniqueness is a property of convenience. It is indeed convenient not having to change the inequality measure if the representation of the data changes. However, this convenience comes at too high a cost: the propositions and corollary in Section 3, for example, demonstrate the severely restricting implications of adding uniqueness to weak consistency.[10]

For an alternative possible justification of uniqueness, consider the following scenario. A practitioner possesses all the data needed for a particular inequality analysis. However, she does not posses the information on whether these data are represented in terms of attainments or in terms of shortfalls. Uniqueness is compelling in this scenario, as it is impossible to determine whether the attainment measure or the shortfall measure should be used. But the scenario is without any practical relevance: precise information on how the data are represented is virtually always available in practice.

The above scenario does, however, present another way of seeing why strong consistency is too demanding. Weak consistency allows use of the information on whether numbers express attainments or shortfalls in making consistent inequality judgments. Strong consistency—by additionally imposing uniqueness—disallows use of this information. But as this information is readily available, a ban on its use is uncalled for.

Interestingly, although the literature does not explicitly discuss uniqueness, some recent studies can be interpreted as being critical of the property. Allanson and Petrie (2013, 2014) and Kjellsson and Gerdtham (2014) stress that a relative inequality measure defined on the domain of attainment distributions expresses fundamentally different inequality judgments than the same relative inequality measure defined on the domain of shortfall distributions. In other words, to describe one's judgments on how to make inequality comparisons it does not suffice to state the chosen inequality measure. Rather, one must state also the particular representation of the data (such as attainments or shortfalls) to which the

[10] As Kolm (1976, p. 420) puts it forcefully in the similar context of consistency with respect to different currency units, "convenience could not be an alibi for endorsing injustice".

inequality measure is applied. This is of course tantamount to a rejection of uniqueness.

5 Conclusion

Consistent comparisons of attainment and shortfall inequality may be implemented using a condition on a single inequality measure, or using a condition on a pair of inequality measures, one measure for attainments and one for shortfalls. The former approach, embodied by strong consistency, severely restricts the set of admissible inequality measures. The latter approach, embodied by weak consistency, does not lead to any restrictions. The literature has taken the former approach, whereas we have advocated the latter.

Strong consistency is equivalent to the combination of weak consistency and uniqueness. The latter property says that the inequality measures used for attainments and shortfalls should be the same. We have argued that the uniqueness property is not compelling. Uniqueness may be regarded as convenient, but this convenience comes at the high cost of ruling out perfectly valid classes of inequality measures.

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