Education in times of population ageing

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Contents

1

Introduction

The fear of insufficient (natural) resources to support current and future consumption due to changes in population size and structure has challenged many economists throughout history. The Malthusian trap is probably the most famous early example of the concern that population development might cause poverty and malnutrition. Malthus (1826) predicts that any increase in income will be eaten up by a growing population due to the “passion between the sexes”. By this he means the human reproductive urge, which causes the population to always exist at its biological maximum and to live self-sufficiently with little prospects of income growth leading to a better life. Fortunately, contrary to this pessimistic outlook, Foley (2000) and Ziesemer (2005), among many others, confirm the absence of the Malthusian effect. Whereas Foley (2000) suggests that we are rather close to a Smithian (1776) equilibrium in which a high standard of living is envisioned on the grounds of income growth caused by higher productivity, Ziesemer (2005) confirms strong influence of the population growth problem brought forward by David Ricardo (1817) by estimating Foley’s model for high-, middle-, and low-income countries. Ricardo (1817) suggests that the rent of land increases with population growth because of the higher demand for food. Rapid population growth may lower wages, because of the higher labor supply, which, taken to the extreme, may lead to wages as low as the subsistence level. One proposed way out of this misery was Smith’s (1776) predicted productivity increase. Higher productivity is measured by a higher output per hour, or square meter of land. If it is possible to produce more productively, a given size of arable land is able to support more individuals. The industrial revolution, starting at around 1760, has given rise to mass production by substituting hand labor with machine production. This increased the productivity of workers and led to rises in income. Among others, Ehrlich and Kim (2005) and Galor and

Weil (1993; 1998) developed a model showing the escape of the Malthusian trap with the help of human capital investment.

While it is reassuring that we are not facing the threat of the Malthusian trap, condemning us to live at the existential minimum, the recent development in demographics does raise concern whether the current standard of living can be kept. During the past decades concern has been raised towards the question whether and how the changing age structure in the USA, Europe, China and other countries will change economic behavior. The recent demographic change is caused by increased longevity on the one hand, and lower birth rates on the other (R. Lee, 2003). With increased longevity, a higher share of one's lifetime is spent in retirement as long as the retirement age is constant. Lower birth rates cause a decreasing share of population that will be able to generate output to support the rest of the population. This may result in shortcomings of supporting the non-working members of society. Instead of going into detail about the cause of demographic change, this dissertation looks at its effects on economic growth and education. Consequently, demographic change is conceptualized by analyzing the two major consequences of the transition; the increasing number of retirees and the decreasing workforce. One way to measure these dynamics is to look at the dependency ratio and its development. It indicates how many people need to be supported as opposed to the people who are working. A further increase in the dependency ratio leads to a mismatch of needs and resources in society. A detailed definition is presented in Chapter 2 and a new empirical variable measuring an age-independent dependency ratio is presented in Chapter 3.

Several studies have supported the pessimistic view of diminishing real output per capita and national savings rates due to population ageing within the next years if there is no impact on technical change (Bloom, Canning, & Fink, 2010; Fayissa & Gutema, 2010; Hviding & Mérette, 1998; Muysken & Ziesemer, 2013). Wright, Kinnunen, Lisenkova, and Merette (2014) calculate the loss of per capita output caused by demographic change in absence of technological shifts to be more than 15% within the next 100 years. If the goal is to keep consumption per capita constant or even growing, production per worker needs to increase in order to keep up with the rising number of non-working members of the population. One way to increase production is to keep the hourly output per worker constant and to increase the time spent in production. The longer the production time, the more output can be generated with a constant productivity, implying a higher retirement age or more working hours per year. Another way is to increase the productivity of each worker and, hence, increase output per working-hour. This way, no extra time in production is needed. However, this productivity increase comes at a cost. Workers need more education to learn how to produce more productively, indicating a higher time share devoted to education rather than production. The educational optimum is likely to change in times of ageing. Galasso (2008) analyzes the effect of a higher retirement age in several OECD countries. His results are in favor of a higher retirement age, though, he neglects the possibility of a higher output through higher productivity. This dissertation focuses on the impact of the demographic change on this balance between production and education.

The literature on the interaction of demographic change and schooling is diverse. Through their vintage human capital model with a realistic survival law, Boucekkine, De la Croix, and Licandro (2002) find that an increase in longevity results in longer schooling and a later retirement. The question remains, whether longer schooling is proportionate to the longer life span, or if there is a shift towards a higher or lower share of education. De la Croix and Licandro (1999) show that an increasing life expectancy has a positive effect on the individual time devoted to schooling, but may have a negative effect on participation rates.

In the model presented below, life expectancy, retirement age, fertility, and working hours before retirement are not explicitly dealt with. They are included implicitly in the exogenous growth rates of labor and population. The purpose of this dissertation is to show a simple way to analyze the economy’s reaction to changes in population size and structure without having to spell out the factors mentioned above. Other things constant, an earlier retirement leads to a smaller labor force and a higher dependency ratio. Similarly, living longer without working longer has the same effect on the dependency ratio because population growth is higher because of a lower mortality rate. More children at pre-school age also enhance the dependency ratio. The chosen formulation of the ratio does not require making all these details explicit as microeconomics studies, demographers and pension funds usually do. Countries differ in all these details because of differences in institutions. This why an Uzawa-Lucas growth model with international capital movements, human capital externalities and decreasing returns to schooling time in human capital formation is chosen as base model of this analysis. Using different types of models, such as for instance an OLG model, would alter the analysis in such a way, that the afore-mentioned factors would have to be spelled out explicitly. This would alter the general applicability.

In the beginning population development will be treated as exogenous and will later be endogenized. First, the dependency ratio is introduced as an exogenous measure into the Uzawa-Lucas growth model in Chapter 2. To my knowledge, it is the first time that this type of population dynamics has been introduced in an Uzawa-Lucas growth model with international capital movements. The model will then be estimated for 16 OECD countries. In order to do so, data for several new variables need to be newly constructed to be close to the model and to get a comparable result. This will be done in Chapter 3. Since the countries that are part of the panel in Chapter 3 cannot be classified as small, price-taking economies it is no longer sensible to assume a fixed interest rate. Chapter 4 endogenizes the interest rate of the base model and shows that a fixed interest rate may cause trouble. A new interest rate is estimated on the grounds of the debt of a country to be able to feed it back into the model of Chapter 2, which to my knowledge has not yet been done in this manner. Chapter 5 shows the influence of a changing dependency ratio on the steady state outcome of the base model with an endogenized interest rate. In a second big step, the labor participation rate is endogenized through a corresponding term in the utility function with a Frisch-elasticity in Chapter 6. This makes it possible to analyze the labor to population ratio. It analyzes the response of the newly endogenized growth rate of the labor force to exogenously changing population growth. Lastly, additionally to the labor force participation rate the development of population growth is endogenized in Chapter 8. This entirely endogenized population development shows the endogenous development of the dependency ratio and has not yet been analyzed in the literature. As a policy measure the impact of changes in the efficiency of schooling is analyzed as well as an exogenous shock on the population development function. Chapter 8 also compares the three models of Chapter 2, 6 and 8. The function for population growth development is estimated in Chapter 7, which empirically analyzes population growth with two different approaches. An expression simple enough to be fed into the analyzed Uzawa-Lucas growth model is found to have a similar fit to a model with many explanatory variables. Chapter 9 summarizes the results of the above mentioned innovations.

2

The Impact of the Dependency Ratio

2.1 Introduction

In this chapter the dependency ratio is introduced into a discrete-time Uzawa-Lucas model (Frenkel, Razin, & Yuen, 1996; Lucas, 1988; Uzawa, 1965) with capital movements, decreasing returns to labor in education, and human capital externalities to find out how the economy reacts to the new challenges. Besides allowing for imperfect capital movements, exogenous growth rates of the active population and the total population are introduced. Two optimal shares of education are found. In the lower steady state, the economy faces high participation rates in production with relatively low education, whereas the other steady state is characterized by high schooling. The latter one is stable. The optimal share of education is of particular interest, same as its implied effect on the economy, in particular its level and growth of productivity.

Exogenous growth rates of the active population and the total population are introduced into the discrete-time version of the Uzawa-Lucas model by Frenkel et al. (1996) with international capital movements and non-increasing returns to labor[1] in education. This will show how the long run outcome evolves if there is a discrepancy between the two rates, indicating demographic changes. This discrepancy will be modelled with a change in the dependency ratio which will be numerically altered to see the effect on the allocation and growth of the economy. The economy consists of representative households and firms maximizing their respective utility and profits.

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[1] Uzawa (1965) also used decreasing returns; Lucas (1988) simplified to assuming constant returns in order to allow for an explicit solution for the long-term growth rates in equilibrium and optimum.

The purpose of this chapter is to introduce the base model which, to my knowledge, has not yet been presented in this way. A detailed analysis, interpretation and several extensions are provided in the next chapters. It is structured as follows: Chapter 2.2 introduces the model, Chapter 2.3 analyzes the existence and stability of a steady state by calibration of the model and Chapter 2.4 concludes.

2.2 The Model

The output of the economy is determined by capital, K_t and human capital, H_t. It is formed by a Cobb-Douglas production function, where human capital, $H_t = h_t L_t$, is the number of the members in the active population, L_t, times their respective skill level, h_t. With the total population denoted as N_t, the dependency ratio is defined as the inactive population over the active population, $D_t = \frac{N_t - L_t}{L_t}$. This definition can be rearranged in terms of L_t to see how the active population interacts with the dependency ratio, $L_t = \frac{N_t}{1+D_t}$. For a given population size, the active population decreases if the dependency ratio increases. Replacing this in the previous definition for total human capital gives: $H_t = h_t \frac{N_t}{1+D_t}$. This completes the production function:

$$Y_t = A(K_t)^{1-\alpha} \left((1 - e_t) h_t \frac{N_t}{1+D_t} \right)^{\alpha} \bar{h_t}^{\epsilon} \tag{2.1}$$

The productivity level, A, is assumed to be constant and h_t may grow. The L_t agents in this economy decide to spend their time either in education e_t, or production $(1 - e_t)$. Equivalently, we can think of e_t as the share of the active population in education and $(1 - e_t)$ as the share in production in a given time frame (i.e. one year). By implication, the definition of the variable L_t deviates from the standard labour market definition, where the active population does not include those in education. This is done because classifying them as inactive would ignore the fact that this part of the labor time contributes a period later via human capital to output production, whereas the difference between N and L remains inactive. $N - L$ can be interpreted as the number of old people who will not return to schooling because they are too old. Average human capital $\bar{h}_t^{\epsilon}$ contributes to the productivity of all factors and is modeled after Lucas (1988). As no single person can influence average human capital, the representative optimizing agent takes the externality $\bar{h}_t^{\epsilon}$ as given when deciding on their optimal time spent in education, which leads to a second best solution. Hence, what is called an optimal solution is in fact a second best solution. Through including externalities of human capital formation into the production function for final output there may be two steady states for each growth rate of the dependency ratio. Xie (1994) establishes the possibility of multiple steady states for a large enough external effect of human capital. By implication, it is an empirical question, whether education should be increased or decreased in response to ageing. In general, preferences, technologies, endowments and institutions play a role in this decision. In the Uzawa-Lucas model, there is a special role of a positive human capital externality in final output production, which encourages human capital production from the demand side, but there is also a decreasing marginal effect of more education time in human capital formation, which discourages human capital formation. Which one is stronger is not obvious without theoretical and empirical investigation in a multiple steady state model connecting them to other assumptions on preferences, technologies, endowments of active and inactive population parts, and institutional assumptions. In our model, we do not consider health aspects, education is a private decision, capital markets are perfect up to the interest rate mark up when the debt/GDP ratio increases, the labor market is Walrasian up to the human capital externality on the demand side.

The production function is a simple Cobb-Douglas production function. This was chosen because of its simplicity. Essentially, the Cobb-Douglas function is a special case of the CES production function. It would be possible to introduce a CES function in the model, but this would complicate the analytics in this already rather complex analysis. For this reason a Cobb-Douglas production function was chosen.

The consumer in this economy owns physical and human capital which he supplies to the production sector. The demand for these is determined by the firms which solve a static maximization program:

$$\max_{(1-e_t),K_t} \pi = \mathrm{A}(K_t\,)^{1-\alpha}\left((1-e_t)h_t\frac{N_t}{1+D_t}\right)^{\alpha}\bar{h}_t^{\,\epsilon} - \omega_t(1-e_t)h_t\frac{N_t}{1+D_t} - r_{kt}K_t \qquad (2.2)$$

The first term on the RHS is the output of the firms, the second term is the cost of wages and the third term is the cost of capital.

The first-order conditions for $(1-e_t)$ and K_t are

$$\omega_t = \frac{\alpha Y_t}{(1-e_t)h_t\frac{N_t}{1+D_t}} \qquad (2.3)$$

$$r_{kt} = (1-\alpha)\frac{Y_t}{K_t} \qquad (2.4)$$

In Equation (2.3) the equilibrium consequences of an increase in the dependency ratio are observable. If the dependency ratio increases, ceteris paribus, equilibrium wages will also increase. Alternatively, for given wages and output, either time spent in production goes up (increase in $(1-e_t)$), or individual human capital increases over time. An increase in human capital can be obtained by spending more time in education (see equation (2.6) below). Here the ambivalence of how time should be optimally spent becomes clear.

The consumers' utility is given by an isoelastic utility function, $U_t = \sum_{t=0}^{\infty} \beta^t N_t \frac{c_t^{1-\sigma}}{(1-\sigma)}$, with $0 < \beta < 1$ as the subjective discount factor and $\sigma > 0$ as the intertemporal elasticity of substitution in consumption. The utility function is logarithmic if $\sigma = 1$.

The consumers' budget constraint is

$$N_t c_t + K_{t+1} - (1-\delta_k)K_t = \omega_t(1-e_t)h_t\frac{N_t}{1+D_t} + r_{kt}K_t + B_{t+1} - \left(1 + r\left(\frac{B_t}{Y_t}\right)\right)B_t \qquad (2.5)$$

Where expenses for consumption in period t, $N_t\, c_t$, and savings/investment, $K_{t+1} - (1-\delta_k)K_t$, must equal the income from wages, $\omega_t(1-e_t)\frac{N_t}{1+D_t}h_t$, and capital, $r_{kt}K_t$, plus the borrowings, B_{t+1}, minus the debt service, $(1 + r\left(\frac{B_t}{Y_t}\right))B_t$. The interest rate $r\left(\frac{B_t}{Y_t}\right)$ is an increasing function of the debt to GDP ratio. For now, $r\left(\frac{B_t}{Y_t}\right)$ is assumed to be constant. This assumption will be relaxed in Chapter 4 in which a realistic function of $r\left(\frac{B_t}{Y_t}\right)$ will be estimated. Taking care of the impact of the debt/GDP ratio on the interest rate can be interpreted as a household or government activity. But as there is no other separate government activity we integrate this task into the household decision assuming households take care of their impact on the interest rate. As there is only one household who owns the firm it can also take over this small government task. Moreover, the household cares for the old. As we do not have many heterogeneous households we do not have to decide on which pension model to use. All real-world pension models have been mis-designed and all ran into problems whenever ageing came about. Designing pension fund models is beyond the scope of this paper. The only assumption we make in this regard is that the utility function below has the same per capita consumption for the inactive $N-L$ (old and pre-school children) and the active L (young, working and in school).[2]

Human capital formation is described as

$$h_{t+1} = F\, e_t^{\gamma} h_t + (1-\delta_h)h_t \tag{2.6}$$

F is the knowledge efficiency coefficient, δ_h is the depreciation rate of human capital and γ is the productivity parameter, with $\gamma \leq 1$, to assure diminishing or constant returns to education. A positive but lesser than one exponent on h_t in the human capital production function, allowing for semi-endogenous growth has been considered. It also has been rejected because of empirical complications a semi-endogenous growth model possible implies.

The consumers maximize their utility subject to the budget constraint (2.5) and the development of human capital (2.6) for given initial values K_0, h_0, B_0.

[2] Muysken and Ziesemer (2014) assume that the proportion of consumption of old and young is fixed, but not necessarily unity.

$$\max_{c_t,e_t,B_{t+1},K_{t+1},h_{t+1}} \sum_{t=0}^{\infty} \beta^t \Bigg(N_t \frac{c_t^{1-\sigma}}{(1-\sigma)}$$
$$- \boldsymbol{\mu_t} \left[N_t c_t + K_{t+1} - 1(1-\delta_k)K_t - \omega_t(1-e_t)\frac{N_t}{1+D_t} h_t \right.$$
$$\left. - r_{kt}K_t - B_{t+1} + \left(1 + r\left(\frac{B_t}{Y_t}\right)\right) B_t \right]$$
$$- \boldsymbol{\mu_{ht}} \left[h_{t+1} - F\, e_t^{\gamma}\, h_t - (1-\delta_h)h_t \right] \Bigg)$$

The first order conditions are:

c_t: $\quad c_t^{-\sigma} = \mu_t$ (2.7)

e_t: $\quad \mu_t \omega_t \frac{N_t}{1+D_t} = \mu_{ht} F\, \gamma e_t^{\gamma-1}$ (2.8)

B_{t+1}: $\quad \mu_t = \beta\mu_{t+1}\left(1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)\right) + \beta\mu_{t+1}\frac{B_{t+1}}{Y_{t+1}}\, r'\left(\frac{B_{t+1}}{Y_{t+1}}\right)$ (2.9)

K_{t+t}: $\quad \mu_t = \beta\mu_{t+1}(1 - \delta_k + r_{kt+1})$ (2.10)

h_{t+1}: $\quad \mu_{ht} = \beta\left[\mu_{t+1}\omega_{t+1}(1-e_{t+1})\frac{N_{t+1}}{1+D_{t+1}} + \mu_{ht+1}Fe_{t+1}^{\gamma} + (1-\delta_h)\mu_{ht+1}\right]$ (2.11)

The following transversality conditions must hold[3]:

I. $\lim_{t\to\infty} \beta^t \mu_t K_t = 0$

II. $\lim_{t\to\infty} \beta^t \mu_{ht} h_t = 0$

The above defines a system of 11 equations for 10 endogenous variables: $Y_t, K_t, h_t, e_t, \omega_t, r_{kt}, c_t, B_t, \mu_t,$ and μ_{ht}. Due to Euler's theorem, equations (2.1)-(2.4) are linearly dependent, if (2.2) equals zero. Hence, one of them ((2.2) in this case) can be dropped from the system. This leaves 10 variables and 10 equations. Appendix 2.2 shows that the current account equation holds for the whole economy.

[3] For proof that the utility function has a finite integral, and hence has an interior maximum see Appendix 2.1.

From the 10x10 equation system the rates of return to physical capital, bonds and human capital can be derived.

$$\frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\mu_t}{\beta\,\mu_{t+1}} = R_{Bt+1} = R_{Kt+1} = R_{Ht+1} \tag{2.12a}$$

$$R_{Bt+1} = 1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb}) \tag{2.12b}$$

$$R_{Kt+1} = 1 - \delta_k + (1-\alpha)\frac{Y_{t+1}}{K_{t+1}} \tag{2.12c}$$

$$R_{Ht+1} = (1+g_{\omega})\frac{1+g_N}{1+g_{1+D}} \quad F\gamma e_t^{\gamma-1}\left[1 - e_{t+1} + \frac{1}{\gamma}e_{t+1} + \frac{1-\delta_h}{F\gamma e_{t+1}^{\gamma-1}}\right] \tag{2.12d}$$

The first equation in (2.12a) is derived from (2.7) and the other expressions follow from (12b-d), as equations (2.9), (2.10) and (2.11) are rearranged to equal $\frac{\mu_t}{\beta\,\mu_{t+1}}$. Equation (2.12b) is derived from (2.9) with $\eta_{rb} = \frac{B_{t+1}}{Y_{t+1}}\frac{r'\left(\frac{B_{t+1}}{Y_{t+1}}\right)}{r\left(\frac{B_{t+1}}{Y_{t+1}}\right)}$, (2.12c) is derived from (2.10) where r_{kt+1} is replaced by the expression in (2.4) and equation (2.12d) is derived by (2.11) where μ_{ht} is replaced by the relation in (2.8).

Because $r\left(\frac{B_{t+1}}{Y_{t+1}}\right)$ is assumed to be constant for now $r'\left(\frac{B_{t+1}}{Y_{t+1}}\right) = 0$. This leads to $\eta_{rb} = 0$. This assumption will be relaxed in Chapter 4. For completeness of the model and because it will become crucial in later sections, the η_{rb}term is still carried along and will be set to zero whenever necessary. (2.12b) implies a constant growth rate of c_t for constant interest rate r, which implies constant R_{Kt+1} and R_{Ht+1} in (2.12c) and (2.12d).

Constant R_{Kt+1} implies constancy of $\frac{Y_{t+1}}{K_{t+1}}$ in (2.12c), from which follows that output and capital grow at the same rate. It follows that $\frac{Y_{t+1}}{K_{t+1}} = A\left(\frac{K_{t+1}}{(1-e_{t+1})h_{t+1}\frac{N_{t+1}}{1+D_{t+1}}}\right)^{-\alpha}\bar{h}_{t+1}{}^{\epsilon}$ must be constant. Hence, it can be derived how the growth rates of K_t, e_t and h_t are related. Note that $\bar{h}_{t+1} = h_{t+1}$ since $\bar{h}_{t+1}$ is the average human capital of identical households. This implies: $\frac{Y_{t+1}}{K_{t+1}} = A\left(\frac{K_{t+1}}{(1-e_{t+1})h_{t+1}^{1+\frac{\epsilon}{\alpha}}\frac{N_{t+1}}{1+D_{t+1}}}\right)^{-\alpha}$. Constancy of $\frac{Y_{t+1}}{K_{t+1}}$ implies equality of the growth rates of the numerator and the denominator:

$$1+g_Y = 1+g_K = (1+g_{1-e})(1+g_h)^{1+\frac{\epsilon}{\alpha}}\frac{1+g_N}{1+g_{1+D}} \qquad (2.12c)^{I}$$

$(2.12c)^{I}$ shows the growth rate of capital in relation to the growth rates of time spent in production, $(1-e_t)$, individual human capital, h_t, total population, N_t and the dependency ratio, $(1+D_t)$. An increase in the growth rate of the dependency ratio decreases the growth rate of capital, ceteris paribus. This can, once again, either be offset by increasing the growth rate of time spent in production (which is clearly only a short term measure, as time spent in production cannot exceed 100%), or by increasing the growth rate of human capital, which in turn can be increased by increasing e_t, the share of education (see (2.6)). $(2.12c)^{I}$ shows the trade-off of time spent in the different sectors when keeping g_K constant.

Equation (2.12d) is the central equation to find a steady state expression for e_t. To be able to relate e_t to only exogenous variables, and hence, to be able to determine its steady state value, $(1+g_\omega)$ needs to be eliminated. (2.3) implies:

$$1+g_\omega = \frac{1+g_Y}{(1+g_{1-e})(1+g_h)\frac{1+g_N}{1+g_{1+D}}} \qquad (2.3)^{I}$$

It has been established above that output and capital grow at the same rate, inserting $(2.12c)^{I}$ into $(2.3)^{I}$ leads to:

$$1+g_\omega = (1+g_h)^{\frac{\epsilon}{\alpha}} \qquad (2.3)^{II}$$

Together with (2.6) this shows that g_ω is constant if e_t is constant (i.e. if $g_e = 0$). $(1+g_\omega)$ can be replaced in (2.12d) to relate e_t and e_{t+1} to only exogenous variables:

$$R_{Ht+1} = \left(F\,e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\frac{1+g_N}{1+g_{1+D}}\,F\gamma e_t^{\gamma-1}\left[(1-e_{t+1}) + \frac{1}{\gamma}e_{t+1} + \frac{(1-\delta_h)}{F\gamma e_{t+1}^{\gamma-1}}\right] \qquad (2.12d)^{I}$$

As this expression includes time spent in education of the current and future period, no analysis can be done yet about their behavior in and around the steady state. To do this e_t will be linked to its growth rate. Multiplying $e_t^{\gamma-1}$ into the brackets yields:

$$R_{Ht+1} = \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}} \frac{1+g_N}{1+g_{1+D}}\, F\gamma \left[e_t^{\gamma-1} - e_t^{\gamma}\frac{e_{t+1}}{e_t} + e_t^{\gamma}\frac{1}{\gamma}\frac{e_{t+1}}{e_t} + \left(\frac{e_t}{e_{t+1}}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]$$

Replace $\frac{e_{t+1}}{e_t} = 1 + g_e$:

$$R_{Ht+1} = \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}} \frac{1+g_N}{1+g_{1+D}}\, F\gamma \left[e_t^{\gamma-1} - e_t^{\gamma}(1+g_e) + e_t^{\gamma}\frac{1}{\gamma}(1+g_e) + \left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right] \qquad (2.12d)^{II}$$

With R_{Ht+1} constant, the RHS of (2.12d)^{II} is constant as well. This is the dynamic equation that shows how e_t develops over time, depending on several parameters and exogenous variables. Note that g_N, g_{1+D} and all parameters are exogenous and constant. With help of this equation, the stability of e_t in the steady state can be analyzed. Unfortunately, the above expression cannot be solved for e_t, or g_e analytically. In the next section the model will be calibrated to find and analyze the steady state conditions.

Equation (2.12d)^{II} would be solvable if the parameter γ was set to 1. This would reduce (2.12d)^{II} to

$$R_{Ht+1} = (F\, e_t + (1-\delta_h))^{\frac{\epsilon}{\alpha}} \frac{1+g_N}{1+g_{1+D}}\left[\, F + (1-\delta_h)\right] \qquad (2.12d)^{III}$$

Equation (2.12d)^{II} can be solved for e_t together with (2.12a,b), which will show the steady state value of e_t for given exogenous parameter values.

$$e_t = \frac{1}{F}\left[(1+r)\,\frac{1+g_{1+D}}{(1+g_N)[F+(1-\delta_h)]}\right]^{\frac{\alpha}{\epsilon}} - (1-\delta_h) \qquad (2.12d)^{IV}$$

This is a simple solution, but it does not allow for dynamics in e_t. A graphic representation is shown in Figure 2.3. In the following analysis, γ will be allowed to differ from 1 to show the complexity of multiple steady states.

2.3 Existence and Stability of Multiple Steady States

In this section the relation between g_e and e_t will be analyzed. By calibration of the model two steady states are found. The RHS of the steady state condition (2.12d)$^{\text{II}}$ is constant if either $g_e = 0$ and hence e_t is constant, or if e_t and g_e move in such a way that they offset each other's movements.

Understanding the interactions between g_e and e_t is crucial for finding possible steady states. Since (2.12d)$^{\text{II}}$ can neither be solved for g_e nor for e_t, it needs to be differentiated implicitly to find how the relationship behaves. (2.13) shows the derivative of g_e with respect to e_t.[4]

$$g_e' = -\frac{\frac{\epsilon}{\alpha}\left(F\,e_t^{\gamma}+(1-\delta_h)\right)^{-1}F\gamma\,e_t^{\gamma-1}\left[e_t^{\gamma-1}-e_t^{\gamma}(1+g_e)+e_t^{\gamma}\frac{1}{\gamma}(1+g_e)+\left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]-\left[(\gamma-1)e_t^{\gamma-2}-\gamma e_t^{\gamma-1}(1+g_e)+e_t^{\gamma-1}(1+g_e)\right]}{\left[-e_t^{\gamma}+e_t^{\gamma}\frac{1}{\gamma}+(1-\gamma)(1+g_e)^{-\gamma}\frac{(1-\delta_h)}{F\gamma}\right]} \tag{2.13}$$

For our purposes it proves helpful to do a full analysis of this relation. To find a turning point (maximum, or minimum), $g_e' = 0$ is required:

$$-\frac{\epsilon}{\alpha}\left(F\,e_t^{\gamma}+(1-\delta_h)\right)^{-1}F\gamma\left[e_t^{\gamma-1}+e_t^{\gamma}(1+g_e)\left(\frac{1}{\gamma}-1\right)+\left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]=(1-\gamma)\left(1+g_e-e_t^{-1}\right) \tag{2.13$^{\text{I}}$}$$

This is the most this equation can be simplified without making assumptions about the magnitude of the parameters. Some parameters are set in the literature. The following common values have been applied: $\alpha = 0.6$ and $\delta_h = 0.03$. Lucas (1988) calibrates ϵ to US data for the case $\gamma = 1$. As here $\gamma < 1$ is used, in order to get empirically realistic twice Lucas' assumption is used and ϵ is set to 0.834, which is necessary when the rate of depreciation for human capital is not zero and $\gamma < 1$.[5] Alternatively, I could have set a smaller rate of depreciation and/or a higher γ explained next. F and γ are interdependent through equation (6) for given other parameters and data. In order to find reasonable values for F and γ to fulfill condition (6) for given data, g_h is set to 0.011, close to

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[4] The full derivation is in Appendix 2.3.

[5] An implication of these assumptions is that no indeterminacy result for $\varepsilon > \alpha$ is achieved as the literature does under the assumptions made by Lucas (Benhabib & Perli, 1994; Xie, 1994).

Denison's estimate for the United States (Denison, 1962), also used by Lucas. Finding combinations of values for F and γ, which satisfy equation (6) is done for given e. We set the current value to $\mathrm{e} = 0.334$, the panel mean of 16 OECD countries for the period 1985 to 2010 (see Chapter 3 for details on data). For this purposes, from a set of combinations of values of data for F and γ $\mathrm{F} = 0.055$ and $\gamma = 0.268$ is chosen (see Appendix 2.4 for justification) to ensure greatest possible similarity between the simulation and data analysis in Chapter 3. Whereas Mankiw, Romer, and Weil (1992) assume the same rate of depreciation for physical and human capital, Lucas (1988) clearly prefers a lower one for human capital because of the intergenerational transfer of knowledge. He sets it equal to zero. In the remainder of this dissertation, Mankiw et al. (1992) is followed and the depreciation rate of human capital is set equal to the depreciation rate of physical capital, $\delta_h = \delta_k = 0.03$. In Appendix 2.5 however, we take an intermediate position to show that the choice of the rate of depreciation affects the adequate discount rate and the values of the solution of the model only marginally. We also set $r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb}) = 0.05$ and $g_N = 0.002$, in line with the literature for OECD countries. In the long run it is reasonable to assume that the dependency ratio will be stable. Hence, we set $g_{1+D} = 0$. This assumption will be relaxed later on.

These values are inserted into $(2.13)^{\mathrm{I}}$, which then can be solved for $e_0 = 0.354$. This shows that there is a single value at which $g_e{}' = 0$, which must then be the only maximum, or minimum of the function $(2.12\mathrm{d})^{\mathrm{II}}$. With the given parameters and the assumption $g_e = g_{1+D} = 0$, the roots of function $(2.12\mathrm{d})^{\mathrm{II}}$ are found to be $e_1 = 0.305$ or $e_2 = 0.380$. As $g_e{}'$ is negative for $e_t > e_0$ and positive for $e_t < e_0$, e_0 must be a maximum. According to $(2.12\mathrm{d})^{\mathrm{II}}$ and with the parameters presented above g_e is positive for values below e_2 and above e_1, and negative for values above e_2 and below e_1. This establishes that the function $(2.12\mathrm{d})^{\mathrm{II}}$ must have a maximum at $e_0 = 0.354$ in the $g_e - e$ plane. Figure 1 can then be drawn in the $g_e - e$ plane. Because g_e is the growth rate of e_t, the stability of both

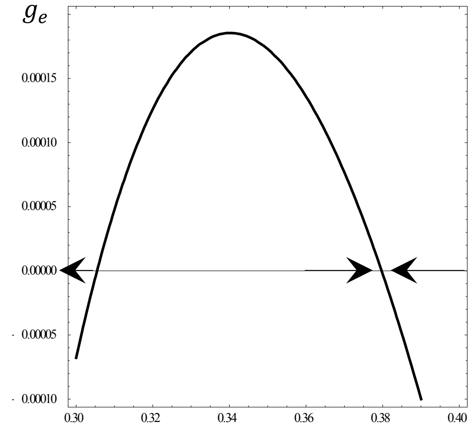


Figure 2.1 – Dynamics in e_t

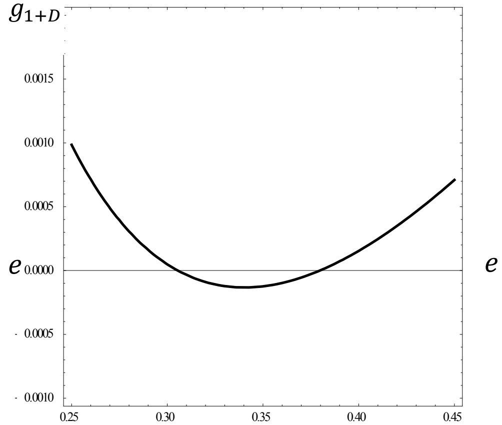


Figure 2.2 – Growth rate of the dependency ratio

steady states can be evaluated. $e_2 = 0.380$ is stable because g_e is positive for values of e_t below e_2 and negative above e_2. $e_1 = 0.305$ is an unstable steady state because g_e is negative below e_1 and positive above e_1. Once out of the steady state, the economy will always return to e_2 if the starting point is to the right of e_1. This shows the existence of multiple steady states with a stable one at $e_2 = 0.380$ for the parameters indicated above. Xie (1994) points out that the existence of multiplicity hinges on the relation between the parameters α and ϵ. According to his research, multiplicity exists if $\alpha < \epsilon$, as is the case in this calibration.

The steady state value of e_t can be interpreted in two different ways. It is either the time share an individual spends in education during his/her time in the active population. $e_2 = 0.380$ can be interpreted as spending 38% of the time in the active population in education. Alternatively, it can be interpreted as the share of the active population that is engaged in education in a given year. $e_2 = 0.380$ can then be interpreted as 38% of the active population are engaged in education and, hence, 62% are in production in the steady state. We prefer the second interpretation, as it gives a more exact indication in times of changing demographics and it is closer to the spirit of the model. Please note that education includes continuous vocational training and on the job training. It is not an indication of the average degree of the population, as education is much more versatile.

Steady State Relations	***From Equation***	***Numerical***
$\mathbf{1}+\boldsymbol{g_Y}=\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\left(\frac{1+g_N}{1+g_{1+D}}\right)$	(2.4) and (2.12c)[I]	$g_Y=0.031$
$\mathbf{1}+\boldsymbol{g_K}=\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\left(\frac{1+g_N}{1+g_{1+D}}\right)$	(2.4) and (2.12c)[I]	$g_K=0.031$
$\mathbf{1}+\boldsymbol{g_h}=F\,e_2^{\gamma}+(1-\delta_h)$	(2.6)	$g_h\;=0.011$
$\mathbf{1}+\boldsymbol{g_\omega}=\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}$	(2.3)[III]	$g_\omega=0.017$
$\mathbf{1}+\boldsymbol{g_\mu}=\left(\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right)^{-1}$	(2.9)	$g_\mu=-0.030$
$\mathbf{1}+\boldsymbol{g_{\mu_h}}=\frac{1}{\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)}\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\frac{1+g_N}{1+g_{1+D}}$	(2.8)	$g_{\mu h}=$ -0.012
$\mathbf{1}+\boldsymbol{g_c}=\left[\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}}$	(2.7) and (2.9)	$g_c=0.029$

Table 2.1 - Steady states

Note to Table: $r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})=0.05, \alpha=0.6, \delta_h=0.03, g_N=0.002, g_{1+D}=0,\ F=0.055,$ $\gamma=0.268,\ \epsilon=0.834, \sigma=1.06, \beta=0.982$ and $e_2=0.380$.

For the transversality conditions to hold, the growth rate of $\beta^t\mu_t K_t$ and $\beta^t\mu_{ht}h_t$ must be negative. With $\beta=0.982$, the growth rate of β^t is -0.018. With $g_\mu=-0.030$ and $g_K=0.031$, the growth rate of the first expression is negative. The growth rate of the second product is also negative because $g_{\mu h}=-0.012$ and $g_h=0.011$. The transversality conditions are, hence, fulfilled.

In the model there are 10 endogenous variables: $Y_t, K_t, h_t, e_t, \omega_t, r_{kt}, c_t, B_t, \mu_t$ and μ_{ht}. Their growth rates need to be determined within the model. The numerical values of the stable steady state will be derived. In addition to values chosen above, the value for σ is set to 1.06[6] and $\beta=0.982$.

The steady state values in Table 2.1 indicate that an increase in the growth rate of the dependency ratio leads to a decrease in the growth rates of output, capital and the shadow price of human capital in the long run, ceteris paribus.

So far, the steady state was analyzed for $g_{1+D}=0$. If this assumption is relaxed, the relation between steady state values for e and g_{1+D} can be

[6] Climate change papers use values between one and two. Denk and Weber (2011) use 2,5 as value for σ. We use 1.06 to ensure stability in the debt dynamics in Section 5.

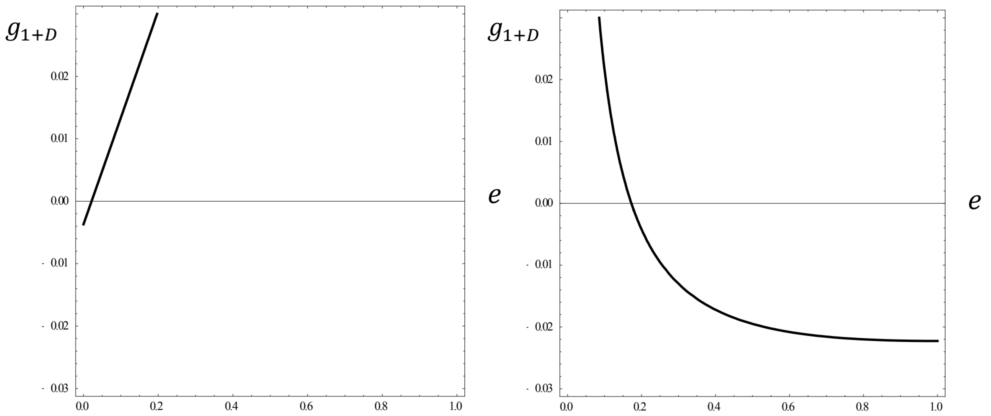
plotted using (2.12d)^{II} as in Figure 2.2 for given $r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})$, g_N, α, δ_h, F, γ and ϵ.

The u-shaped curve of Figure 2.2 reflects the two steady-state time shares of education for each growth rate of the dependency ratio. The active population can either be less educated and spend more time in production to increase output or invest more time in education and be more productive in the future. As has been analyzed in this section, the stable steady states are on the upward sloping part, indicating a higher optimal share of education for a higher growth rate of the dependency ratio if only taking stable equilibria into account.

Xie (1994) sees the reason for multiple steady states in the effects of the externality (ϵ). By disentangling the effects of the externality and the diminishing returns to time spent in human capital formation (γ), it becomes clear that the interaction of these two parameters is vital to the existence of the two steady states. The contribution of both will be shown by setting first only ϵ equal to zero and second only γ equal to one.

Figure 2.3a shows the steady-state relationship between g_{1+D} and e_t with constant returns to time spent in human capital formation ($\gamma = 1$) but with a positive externality and all other parameters as in Table 2.1. By setting γ equal to one, the relationship becomes linear and upward sloping, indicating an unambiguous increase in the share of education if the growth rate of the dependency ratio increases.[7] For a stable dependency ratio the steady state share of education is 0.023, suggesting an unrealistically low share in education when compared with the data considered in detail in the next section. Gruescu (2006) also finds a linear relationship between the growth rate of the dependency ratio and the time shares. In her model she encounters neither the externality effect, nor diminishing returns to education. However, when the more realistic case of decreasing returns is allowed for as we do here, the education policy decision is complicated by

[7] In the calibration, in order for (2.6) to hold, the value of F needs to be adjusted to 0.119 in the same manner as done before (see Appendix 2.4).



$a)\ \gamma = 1,\ \epsilon = 0.834$ $b)\ \epsilon = 0, \gamma = 0.268$

Figure 2.3 – Case Study for $\boldsymbol{\gamma}$ and $\boldsymbol{\epsilon}$

a non-linearity implying multiple steady states. This requires ruling out the policy alternative of increasing labor input in production and decreasing it in education.

Figure 2.3b displays the relationship between g_{1+D} and e_t if the externality effect is set to zero, but diminishing returns to time spent in human capital formation are present. Without the externality effect of human capital, the relationship between the share of education and the growth rate of the dependency ratio is negative. With diminishing returns to time spent in human capital formation and no positive externalities it is not profitable anymore to invest more time in education in times of ageing.

The two effects together create the two steady states that are observed in the model.[8] This is in line with the findings of Xie (1994) and Zhang (2013), who prove the existence and (in-) stability of multiple equilibria, if “the external effect of human capital in goods production is sufficiently large” (Xie, 1994). Finding a realistic combination of the most important parameters is of utmost importance for a good policy decision in the presence of multiple steady states.

[8] Combining the two cases $\epsilon = 0$ and $\gamma = 1$ leads to solution-problems in (2.12d)[II]. There is no variable to ensure equality because only exogenous variables are left.

2.4 Conclusion

This chapter has presented the base model for the analysis of a changing age-independent dependency ratio, which was introduced into a discrete-time Uzawa-Lucas model with international capital movements, human capital externalities and decreasing returns to schooling time in human capital formation. To my knowledge such an age-independent measure has not yet been introduced into this model with diminishing returns to human capital and a human capital externality to show the consequences of demographic development. The economy shows multiple steady states in reaction to demographic changes. Steady state analysis has shown that only a high share in education is associated with a stable steady state. In the neighborhood of the stable steady state it is optimal to spend more time in education when the growth of the active part of the population lags behind that of the inactive part as it is the case in times of ageing. This increases the growth rates of human capital, GDP per capita, wages, and reduces the growth rate of consumption and the debt/GDP ratio. The next chapter will estimate this model empirically to show that these findings and calibration values are in line with the data.

APPENDIX 2.1 – Interior Solution of the Utility Integral

This Appendix shows the proof for the existence of an interior solution of the utility integral for $\sigma > 0$ in steady state. The utility function has a finite integral, and hence has an interior maximum, if the growth rate of discounted utility is smaller than zero ($g_u < 0$). By definition of the growth rate of the individual utility function is $1 + g_u = \frac{u_{t+1}}{u_t} = \beta \frac{c_{t+1}^{1-\sigma}}{c_t^{1-\sigma}} = \beta(1+g_c)^{1-\sigma}$ which is: $g_u = \beta(1+g_c)^{1-\sigma} - 1$

From (2.7)we know that $1 + g_c = \left(\beta(1 + r_{t+1}\,)\right)^{\frac{1}{\sigma}}$ holds in steady state.

$$1 + g_u = \beta\left(\left(\beta(1 + r_{t+1})\right)^{\frac{1}{\sigma}}\right)^{1-\sigma} = \beta^{\frac{1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}} = \left(\frac{\beta}{(1+r_{t+1})}\right)^{\frac{1}{\sigma}} (1 + r_{t+1})^{-1}$$

From $\beta < 1$ and $r_{t+1} > 0$ follows that $\left(\frac{\beta}{(1+r_{t+1})}\right)^{\frac{1}{\sigma}} < 1$ and

$(1 + r_{t+1})^{-1} < 1$. Hence, $\left(\frac{\beta}{(1+r_{t+1})}\right)^{\frac{1}{\sigma}} (1 + r_{t+1})^{-1} < 1$ and

$g_u = \left(\frac{\beta}{(1+r_{t+1})}\right)^{\frac{1}{\sigma}} (1 + r_{t+1})^{-1} - 1 < 0$. This proves the existence of an interior solution of the utility integral for $\sigma > 0$.

APPENDIX 2.2 – Current Account Equation

This appendix checks formally if the current account equation, $B_{t+1} - B_t = C_t + I_t - Y_t + r_t B_t$, holds. Where $C_t = N_t c_t$ and $I_t = K_{t+1} - (1 - \delta_k)K_t$.

To check if this holds, the consumer's and producer's budget constraints must be added. From the firm's side through Euler's theorem we know:

$\pi = 0 \leftrightarrow Y_t = \omega_t(1 - e_t)h_t \frac{N_t}{1+D_t} + r_{kt}K_t$ from Equation (2.2).

The consumer's budget constraint is displayed in Equation (2.5):

$$N_t c_t + K_{t+1} - (1 - \delta_k)K_t = \omega_t(1 - e_t)\frac{N_t}{1+D_t} h_t + r_{kt}K_t + B_{t+1} - (1 + r_t)B_t$$

Adding both up leads to the current account equation displayed above.

APPENDIX 2.3 – Derivation Equation (2.12)

In this appendix, the derivation of Equation (2.13), the derivative of Equation (2.12d)[ll] is shown. Denote the RHS as $J(e, g(e))$ and differentiate implicitly with $g'(e) = -\frac{J_1'(e,g(e))}{J_2'(e,g(e))}$:

Calculate partial derivatives w.r.t. g_e and e_t:

$$\boldsymbol{J_1'(e,g_e)} = \frac{1+g_N}{1+g_{1+D}} F\gamma \left(\frac{\epsilon}{\alpha}\left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}-1} F\gamma\, e_t^{\gamma-1}\left[e_t^{\gamma-1} - e_t^{\gamma}(1+g_e) + e_t^{\gamma}\frac{1}{\gamma}(1+g_e) + \left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right] + \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\left[(\gamma-1)e_t^{\gamma-2} - \gamma e_t^{\gamma-1}(1+g_e) + e_t^{\gamma-1}(1+g_e)\right]\right)$$

$$\boldsymbol{J_2'(e,g_e)} = \frac{1+g_N}{1+g_{1+D}} F\gamma \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\left[-e_t^{\gamma} + e_t^{\gamma}\frac{1}{\gamma} + (1-\gamma)(1+g_e)^{-\gamma}\frac{(1-\delta_h)}{F\gamma}\right]$$

The ratio of the two partial derivatives is the derivative of g_e w.r.t. e_t.

$$\boldsymbol{g_e'} = -\frac{J_1'(e,g(e))}{J_2'(e,g(e))} = -\frac{\frac{\epsilon}{\alpha}\left(F\, e_t^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}-1}F\gamma\, e_t^{\gamma-1}\left[e_t^{\gamma-1}-e_t^{\gamma}(1+g_e)+e_t^{\gamma}\frac{1}{\gamma}(1+g_e)+\left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]}{\left(F\, e_t^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\left[-e_t^{\gamma}+e_t^{\gamma}\frac{1}{\gamma}+(1-\gamma)(1+g_e)^{-\gamma}\frac{(1-\delta_h)}{F\gamma}\right]} - \frac{\left(F\, e_t^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\left[(\gamma-1)e_t^{\gamma-2}-\gamma e_t^{\gamma-1}(1+g_e)+e_t^{\gamma-1}(1+g_e)\right]}{\left(F\, e_t^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\left[-e_t^{\gamma}+e_t^{\gamma}\frac{1}{\gamma}+(1-\gamma)(1+g_e)^{-\gamma}\frac{(1-\delta_h)}{F\gamma}\right]}$$

This can be simplified, by cancelling $\left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}$, which displays the final derivative:

$$g_e' = -\frac{J_1'(e,g(e))}{J_2'(e,g(e))} =$$

$$-\frac{\frac{\epsilon}{\alpha}\left(F\,e_t^{\gamma}+(1-\delta_h)\right)^{-1}F\gamma\,e_t^{\gamma-1}\left[e_t^{\gamma-1}-e_t^{\gamma}\left(1+g_e\right)+e_t^{\gamma}\frac{1}{\gamma}\left(1+g_e\right)+\left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]}{\left[-e_t^{\gamma}+e_t^{\gamma}\frac{1}{\gamma}+(1-\gamma)\left(1+g_e\right)^{-\gamma}\frac{(1-\delta_h)}{F\gamma}\right]} -$$

$$\frac{\left[(\gamma-1)e_t^{\gamma-2}-\gamma e_t^{\gamma-1}\left(1+g_e\right)+e_t^{\gamma-1}\left(1+g_e\right)\right]}{\left[-e_t^{\gamma}+e_t^{\gamma}\frac{1}{\gamma}+(1-\gamma)\left(1+g_e\right)^{-\gamma}\frac{(1-\delta_h)}{F\gamma}\right]}$$

APPENDIX 2.4 – F and γ

In order to find reasonable values for F and γ to fulfill the condition (2.6), g_h is set to 0.011 with δ_h being fixed at 0.03. This can only be done for given e_t. We choose e_t to reflect the panel of the given panel in Chapter 3: $e_t = 0.344$.

Gong, Greiner, and Semmler (2004) suggest estimated values for γ and ϵ which differ substantially from ours. The reason may be because they include an exponent on individual human capital in (2.6), which is smaller than unity, as opposed to 1 in our model. This measurement might change the estimated values for γ and ϵ and leads actually to a semi-endogenous growth model as in Jones (1995). Lower values of ϵ, as in Gong et al. (2004), only have one steady state. As seen in Xie (1994) who establishes that ϵ should be "large enough" to observe two steady states. Chapter 3 shows evidence for two steady states in the data. We pick $F = 0.055$ and $\gamma = 0.268$, because it ensures the closest fit to Figure 3.2.

APPENDIX 2.5 – Different values of δ_h

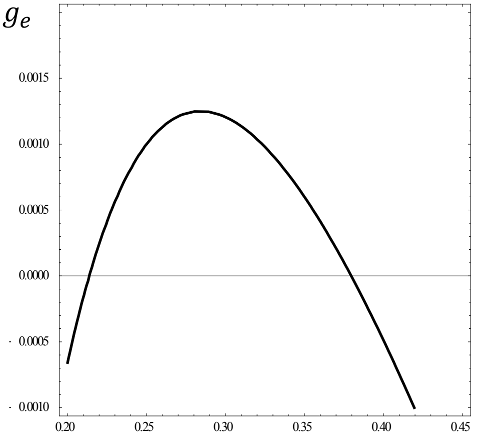


Figure A2.1 – Dynamics in e_t

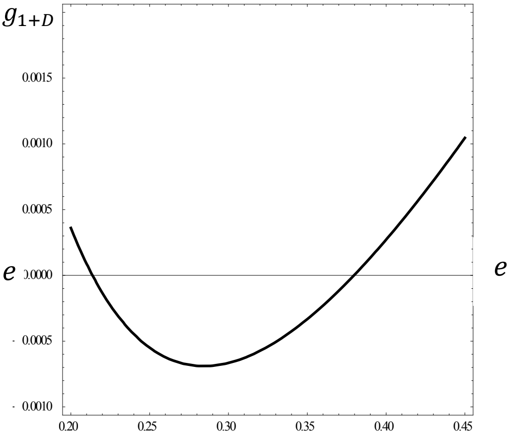


Figure A2.2 – Growth rate of the dependency ratio

This appendix shows that a decrease in the depreciation rate of human capital only marginally affects the results. We set $\delta_h = 0.01$, then according to (2.6), F and γ need to be altered to ensure a close fit to the data, $g_h = 0.011$ for $e = 0.344$ as in the text. Figures A2.1 and A2.2 are the equivalent of Figures 2.1 and 2.2, but are plotted for $\delta_h = 0.01, F = 0.035$ and $\gamma = 0.436$. All other parameters used in the derivation of Figures 2.1 and 2.2 are unchanged.

While the upper (stable) steady state remains unchanged in comparison to the value presented in the chapter, $e = 0.380$, the lower steady state is a little further off $e = 0.214$ as opposed to $e = 0.305$ in the chapter. With $\delta_h = 0.03$ instead of 0.01, the calibrated model is closer to the estimated model of the next chapter at the lower steady state.

3

Empirical Analysis: The Dependency Ratio and Education

3.1 Introduction

Because of the importance of the empirical details for policy under multiple steady states this chapter provides some empirical insights into the relationship between the growth rate of the dependency ratio and the time spent in education as defined in the model of Chapter 2. This chapter will show that the optimal response to a changing dependency ratio, as calculated in Chapter 2 and captured in Figures 2.1 and 2.2, also holds empirically. If this were not the case, different parameter values should have been used.

So far, there is no readily constructed data to display the variable time spent in education, e_t, nor for the growth rate of the dependency ratio, g_{1+D}, as displayed in the model. Consequently, the steady state relation has not yet been estimated to this extent. This chapter will first provide insights into the construction of the new variables (Chapter 3.2) and provide their respective descriptive statistics (Chapter 3.3). Secondly, the steady state relation will be estimated in Chapter 3.4. Chapter 3.5 concludes.

3.2 International Panel Data

The dependency ratio displays the relation of the non-active population to the active population. It gives an indication of how many people in the economy need to be supported by the workforce. The higher the dependency ratio, the more people need to be supported. In many data sources the dependency ratio is typically referred to as the dependence between age groups, indirectly making very specific assumptions about the relation between age and employment. The age-dependency ratio of e.g.

the OECD and the World Development Indicators is defined as the number of people of age 65 and older over the number of people between 15 and 64 years of age. This reveals several assumptions: 1. children under the age of 15 are not working (typically engaged in education), 2. people between the ages 15 and 64 are engaged in production and 3. the retirement age is 65 years. A look at the official retirement ages in OECD countries shows that assuming an universal retirement age at 65 is not accurate enough.

Australia	65,0	**Hungary**	60,0	**Norway**	67,0
Austria	65,0	**Iceland**	67,0	**Poland**	65,0
Belgium	60,0	**Ireland**	65,0	**Portugal**	65,0
Canada	65,0	**Italy**	59,0	**Slovak Republic**	62,0
Czech Republic	61,0	**Japan**	65,0	**Spain**	65,0
Denmark	65,0	**Korea**	60,0	**Sweden**	65,0
Finland	65,0	**Luxembourg**	60,0	**Switzerland**	65,0
France	60,5	**Mexico**	65,0	**Turkey**	44,9
Germany	65,0	**Netherlands**	65,0	**United Kingdom**	65,0
Greece	57,0	**New Zealand**	65,0	**United States**	66,0
OECD average	**63,0**				

Table 3.1 – Official Retirement Ages in the OECD
Source: National officials, OECD calculations and Turner (2007)

Table 3.1 shows that the official retirement ages range between 44.9 years in Turkey and 67 years in Iceland and Norway in the year 2007. The difference of 22.1 years is too big to be neglected. There are many ways in which the ratio of dependency can be redefined. See Quesada and García-Montalvo (1996) for an overview of different definitions. To calculate the dependency ratio as defined in the model of Chapter 2, the active and inactive parts of the population need to be identified. The active population, in line with the model, is divided into two parts; the active workforce ($(1-e_t)L_t$) and people in education (e_tL_t). We include both into the active population because we want to focus on the allocation of time between working and schooling as in the model. The rest is the non-active population, i.e. children before attending school, retirees, unemployed and other people. We have data for 16 countries[9]. For the entire set, data from 1985 to 2011 is used.

The new variable "education" is constructed as follows: It includes any kind of participation in educational programs. Next to the regular school career, this includes vocational training and on-the-job training. To identify the regular students, the OECD iLibrary (Dataset: Students enrolled by age) is used. This includes enrollments in ISCED levels 0-6. Some countries have a dual system of apprenticeships in which students work parts of the week and go to school for the rest. In the available data sets, apprentices are generally counted as full-time students, regardless of their contribution to output. To include their contribution to output in our data, we account 3 days per week in education (60%) and 2 days in production (40%). The 40% of the production side need to be added to the data on the working population and subtracted from educational data. In line with the concept of lifelong learning, many employers provide continuous vocational training (CVT) to their employees. This is usually done in courses that take up a varying amount of time (a couple of hours to several weeks or months per year). Eurostat has conducted a survey in which they track the percentage of time the average employee spends in training during the year. The scope of continuous vocational training has been measured in two years (2005 and 2010). We use the 2005 data as a markup for prior years. The time spent in CVT will be added to education and subtracted from employment. Unfortunately, this data is only available for Europe. We use UK data as estimate for the time spent in CVT in three non-European countries, Australia, USA and Canada.

To calculate the growth rate of the dependency ratio, the variable L_t (the active population) is constructed by adding people in education and in production. Because the variable "time spent in education", e_t, is not measured in hours worked, but in count, the data on the people engaged in

[9] The countries are: Australia, Canada, Denmark, Spain, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Sweden, the UK and the US. We refrain from using compounded data (like EU, or OECD data) because it is impossible to track missing data. Since we are using absolute values, missing data by just one country would make a big difference.

production is also taken as count and not in "hours worked". For this reason the data provided by the World Development indicators was taken. Together with the total population (N_t) the dependency ratio can be calculated with the relation used in the model, $D_t = \frac{N_t - L_t}{L_t}$. To stay in line with the model, we calculate the growth rate of the dependency ratio $(1 + D_t)$ in the same way as in the discrete-time model, to ensure comparability: $\frac{1+D_{t+1}}{1+D_t} = 1 + g_{1+D}$.

3.3 Descriptive Statistics

The average growth rate of the dependency ratio, g_{1+D}, over the whole sample starting 1985 to 2010 for 16 OECD countries is -0.001, with data ranging from -0.062 to 0.059. Temporarily the growth rate of the dependency ratio may be quite low or high, but the panel mean over the last 25 years is close to zero. A temporarily diverging growth rate of the dependency ratio from zero is important to analyze transitions from a younger to an older population, but in the long run, the dependency ratio may be stable.

The variable education has a panel mean of 0.344, ranging between 0.284 and 0.518 for the countries' cross-section mean. The real interest rate, r_t, is taken from the World Development Indicators (WDI) and ranges between -0.025 and 0.146 with a mean of 0.056. In most European countries this data series is only available until the early 2000s. Only for the non-European countries and Italy, the Netherlands and the UK the time range until 2010 is available. It is nevertheless advisable to use WDI data as opposed to e.g. OECD data, because the WDI reports consumer interest rates and is thus closer to the model. The growth rate of the population, g_N, is also taken from the WDI. It ranges between -0.004 and 0.029.

3.4 Empirical Model and Results

As in the theoretical model of Chapter 2, the focus lies on the steady state relation $(2.12\text{d})^{\text{II}}$, which is the equation we would like to estimate. As mentioned in Chapter 2, it is not possible to solve for e_t or its growth rate. Hence, we estimate an approximation in logs. If $(2.12\text{d})^{\text{II}}$ were solvable for

Model 1	$log(e_{it}) = c_0 + c_1 log(e_{\boldsymbol{i(t-1)}}) + c_3 \left(log(e_{\boldsymbol{i(t-1)}})\right)^2 + c_4 \left(log(e_{i(t-1)})\right)^3 + c_5 log(1 + g_{(1+D)i(t-1)}) + c_6 log(1 + g_{(1+D)i(t-1)})^2 + c_7 log(1 + g_{(1+D)i(t-1)})^3 + c_8\, log(1 + g_{Ni(t)})^2 + c_9\, log(1 + g_{Ni(t)})^3 + c_{10} \log(1 + r_{it}) + c_{11} \log(1 + r_{it})^2 + c_{13} \log(1 + r_{it})^3 + \eta_i + \varphi_t + \epsilon_{it}$
Model 2	$log(e_{it}) = c_0 + c_1 log(e_{\boldsymbol{i(t-1)}}) + c_2 \left(log(e_{\boldsymbol{i(t-1)}})\right)^2 + c_3 \left(log(e_{i(t-1)})\right)^3 + c_4 log(1 + g_{(1+D)i(t-1)}) + c_5\, log(1 + g_{Ni(t-1)}) + c_6 \log(1 + r_{it}) + \eta_i + \varphi_t + \epsilon_{it}$
Model 3	$log(e_{it}) = c_0 + c_1 log(e_{i(t-1)}) + c_2 \left(log(e_{i(t-1)})\right)^2 + c_3 \left(log(e_{i(t-1)})\right)^3 + c_4 log(1 + g_{(1+D)i(t-1)}) + c_5\, log(1 + g_{Ni(t-1)}) + \eta_i + \varphi_t + \epsilon_{it}$
Model 4	$log(e_{it}) = c_0 + c_1 log(e_{i(t-1)}) + c_2 \left(log(e_{i(t-1)})\right)^2 + c_3 \left(log(e_{i(t-1)})\right)^3 + c_4 log(1 + g_{(1+D)i(t-1)}) + \eta_i + \varphi_t + \epsilon_{it}$

Table 3.2 – Models to be estimated

either e_t or g_e, the resulting equation would most probably not be linear in the expressions for interest rates, population growth and the dependency ratio. Therefore, linear, quadratic and cubic terms are used in the estimation, because polynomials of the third degree have enough flexibility to capture many forms of non-linearity. To allow for some flexibility, four models have been estimated with stepwise elimination of the most insignificant variables after the first estimation. Their representation is presented in Table 3.2. Model 1 uses the polynomials of the third degree for all variables and Model 2 does so only for the education variable in natural logs and is linear in the logs of the other variables of Equation (2.12d)[II]. Models 3 and 4 are obtained from stepwise elimination of the insignificant variables. Where η_i is the unobservable individual effect and ϵ_{it} is a disturbance term and $c_{0i}$are cross section fixed effects.

Before the models are estimated, unit root tests are executed on the panel for the variables e, g_{1+D}, g_N and r. Four different panel unit root tests are conducted to rule out inconsistencies (Levin, Lin & Chu (LLC), Im, Pesaran and Shin (IPS), Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP)). The detailed results are shown in Appendix 3.1 and indicate, that while the two variables e and $g_{(1+D)}$ show no common unit root for neither test, g_N shows mixed results in which the existence of a unit root cannot be fully rejected. The tests for r show that the existence of a unit root cannot be rejected in any test. Since the tests for neither e nor $g_{(1+D)}$ lead to the conclusion of unit roots in panel, the model will be estimated in levels. The variables g_N and r are used as control variables.

As the data fulfills $T > N$, fully modified OLS (FMOLS) might be the most adequate estimation procedure, assuming cointegration. It would deal with endogeneity, contemporaneous correlation and serial correlation through a data transformation (see Baltagi (2008) chap.12). However, its usage leads to a loss of data because of the data transformation applied by this method, leaving only 9 or even less countries in the sample. Instead, we use the System GMM method.

Because the lagged dependent variable, $\log(e_{i,t-1})$, is correlated with the error, η_i, by definition, the OLS estimator is biased and inconsistent for $N \to \infty$. The within transformation of a fixed effects (FE) model eliminates the time invariant η_i, but the transformation $\left(\log(e_{i,t-1}) - \log(\overline{e_{\iota,t-1}})\right)$ is still correlated with $(\epsilon_{it} - \bar{\epsilon_i})$ with a downward bias with an order of magnitude of $1/T$ (Nickell, 1981). This suggests that the FE estimator becomes consistent only for $T \to \infty$ as we consider a sample with $T = 23$ when the controls are added, the order of magnitude of the bias is $\frac{1}{23}$. Given that the OLS estimator is biased upwards and the FE estimator is biased downwards, we can conclude that a consistent estimator must lie between these two estimates (Bond, 2002). It may be argued, that in macroeconomic analysis with relatively large T as compared to most microeconomic dynamic models, the bias of the FE estimator may not be large. Judson and Owen (1999) show with their Monte Carlo experiments, that the bias can still be up to 20% for an LSDV even if $T = 30$. They promote a bias-corrected LSDV estimator by Kiviet (1995) for which the bias is the smallest in their simulations.

Anderson and Hsiao (1982) suggest first differencing to eliminate η_i in an instrumental variable setting, as an alternative to FE models. First differencing, i.e. transforming the estimates to $\Delta\log(e_{i,t-1}) = \log(e_{i,t-1}) - \log(e_{i,t-2})$ rather than using their levels, aims for minimal dependency between the transformed variable and the lagged level. This way,

$\log(e_{i,t-2})$ can be used as an instrument for $\Delta\log(e_{i,t-1})$ in a 2SLS regression as it is correlated with the estimate, but uncorrelated with the error term, assuming no serial correlation in the errors. Ahn and Schmidt (1995) criticize this approach for not including all available information and suggest including more moment conditions. To avoid asymptotic inefficiency, as it may occur in a 2SLS estimation with $T > 3$, Arellano and Bond (1991) use a two-step GMM estimator, which has been altered with a finite-sample correction by Windmeijer (2005) to increase the reliability of the asymptotic distribution approximations. To address the challenges of a short dynamic panel, Arellano and Bond (1991) and Arellano and Bover (1995) developed dynamic panel estimators that are consistent for "small T, large N" panels for $T \to \infty$, have a linear functional relationship, include a linearly lagged dependent and include not strictly exogenous independent variables. They also incorporate fixed individual effects, heteroskedasticity and autocorrelation within individuals, but not across (Roodman, 2009a).

Arellano and Bover form an estimation method, which is commonly known as "system GMM". They draw from two separate estimation methods: Blundell and Bond (1998) consider the case in which the first differences, $\Delta\log(e_{i,t})$, are uncorrelated with the individual-specific effect in the error term, η_i. If this is the case, the lagged differences of $\log(e_{i,t})$ can be used as instruments in the levels equation. Like this, more instruments can be added to increase efficiency, but the number of instruments should be treated with caution, as too many instruments may increase the bias (see end of this section). Arellano and Bond (1991) take the vice-versa approach in which the differenced equation is instrumented with the (lagged) levels. This estimation method has shown to have a small sample bias. In what is widely referred to as "system GMM" method, both equations mentioned above are estimated simultaneously. The first equation is a within-group estimator, instrumented with the lagged differences, whereas the second estimates differences, instrumented with levels. Blundell and Bond (1998) and Soto (2009) show that this reduces the small sample bias. The advantage is, that time-invariant regressors can be included that would be dropped in difference GMM. The first differences can also be replaced by orthogonal deviations, as in difference GMM. We will use this estimation method for our sample.

In Windmeijer's estimations, the corrected two-step GMM performs better than its robust one-step equivalent in terms of standard errors and bias. Because the estimators are transformed by differencing, these models are also called "difference GMM" models. Arellano and Bover (1995) propose an orthogonal transformation of the estimators in which, as opposed to differencing, fewer observations are lost. With this method, rather than subtracting two subsequent observations magnifying possible gaps in the data, the weighted average of all future available observations is subtracted. This way no extra observations are lost, except for the last in each series and those in the level equation.

For the lack of any other instruments, we will have to rely on the instruments we can draw from within the data set. For the transformed $\log(e_{i,t-1})$ this would be either its lag, $\log(e_{i,t-2})$, or the lagged difference, $\Delta\log(e_{i,t-2})$. Bazzi and Clemens (2009) show that the regression coefficient of the variable and its instrument (its lag in this case) should be high, whereas the instrument as such should be exogenous. The additional explanatory variables, are instrumented with either their current values, or their lags, depending on the assumptions about their correlation with the error term (Bond, 2002). Exogenous variables are instrumented with their current levels, predetermined and endogenous variables with their first exogenous lag and deeper. To test for their endogeneity, the Durbin-Wu-Hausman test is used.

With a moderate time frame and many cross sections, the number of moment conditions may outweigh the number of observations as the difference and system GMM produces a number of instruments that is quadratic in T, which may cause problems in the estimation with a finite sample (Roodman, 2009a). This large number of instruments may also overfit the endogenous variables. Windmeijer (2005) reports that cutting the instruments roughly in half (from 28 to 13) reduced the bias in the two-step estimate by 40% in a difference GMM simulation. This shows that in regard to the numbers of instruments the concept of "more is always better" does not apply, although there are very few guidelines on how many instruments are too many (Ruud, 2000). As rule of thumb it is often stated, that the number of instruments should not outweigh the number of cross sections, counted as observations. We use one instrumental variable per regressor, the second lag for the lagged dependent variable and the first lag for the other regressors, in line with Okui (2009), who suggest this for very short panels. If the regressors and the instruments, both counted as variables, are of the same number the Hansen (1982) – Sargan (1958) J-statistic of overidentifying restrictions will be zero. It will be positive for an overidentified system. As the equations for system and difference GMM are usually overidentified because of the constraints on the coefficients in the two equations the p-value of the J-test statistic for overidentifying restrictions gives an indication of the validity of the instruments. It should, hence, neither be too close to zero or too close to one as this will cast doubt on the underlying moment conditions.

All 4 models are estimated using the orthogonal deviations[10] version of System GMM. The Durbin-Wu-Hausman (DWH) test of endogeneity shows, that for all models $\log\left(1 + g_{(1+D)}\right)$ is endogenous. It will be treated with its lag in the instruments. All instruments are reported in the note to Table 3.3.

After having dropped the other insignificant regressors in Model 1, we find that the squared growth rate of the dependency ratio is just insignificant. If we also drop it, the other variables become insignificant as well and this version of stepwise regression collapses.[11] For the linear Model 2, we find that interest rates are most insignificant. Dropping them leads to Model 3 where the population growth rate is insignificant. Dropping that one also leads to Model 4, in which all variables are significant. The p-values of the J-statistic indicate that it is not too high (or p-values too low) to have a chi-square distribution (Davidson & MacKinnon, 2004), except for the first regression where p is too low, because of a low number of observations. Otherwise we would cast doubt on the instruments or the specification.

[10] Using orthogonal deviations reduces the loss of data, when faced with sporadically missing data (Roodman, 2009a)

[11] Using current or lagged values of the regressors does not improve this result.

	Model 1	Model 2	Model 3	Model 4
$\log(e_{i(t-1)})$	8.657 (7.523)	14.070 (5.883)**	14.869 (5.730)**	15.830 (5.773)***
$(\log(e_{i(t-1)}))^2$	8.542 (7.652)	13.402 (5.992)**	14.741 (5.979)**	15.683 (6.044)**
$(\log(e_{i(t-1)}))^3$	3.019 (2.579)	4.467 2.017)**	5.078 (2.049)**	5.386 (2.076)**
$\log(1+g_{(1+D)(t-1)})$	0.688 (0.437)	0.167 (0.093)*	0.414 (0.121)***	0.429 (0.124)***
$(\log(1+g_{(1+D)i(t-1)}))^2$	-10.949 (7.041)			
$(\log(1+g_{(1+D)i(t-1)}))^3$	-529.620 (240.640)**			
$\log(1+g_{N(t-1)})$	4.340 (5.070)	-0.876 (0.606)	-0.634 (0.748)	
$(\log(1+g_{Ni(t)}))^2$	-1092.066 (711.839)			
$(\log(1+g_{Ni(t)}))^3$	38567.760 (24511.480)			
$\log(1+r_t)$	-0.391 (0.397)	0.145 (0.1363)		
$(\log(1+r_t))^2$	3.853 (9.680)			
$(\log(1+r_t))^3$	13.280 (68.639)			
S.E. of regression	0.018	0.015	0.022	0.023
Instrument Rank	57	51	52	51
J-statistic	20.523	34.923	22.312	21.580
Prob(J-statistic)	0.550	0.039	0.501	0.580
Cross sections	14	14	16	16
Periods	1988-2010	1988-2011	1988-2011	1988-2011
Obs.	158	158	235	235

Table 3.3 – Estimations with Panel Generalized Methods of Moments, Transformations: Orthogonal Deviations

Note to table: In Models 1 and 2 the countries Luxembourg and Portugal are lost because data is not matching. The few observations, compared to the time frame and cross sections available, are due to many asymmetrically missing values.

Instruments: Model 1: $\log(e_{i(t-2)}), (\log(e_{i(t-2)}))^2, (\log(e_{i(t-2)}))^3, \log(1+g_{(1+D)(t-1)}), (\log(1+g_{(1+D)(t-1)}))^2, (\log(1+g_{(1+D)(t-1)}))^3, \log(1+g_{Ni(t-1)}), (\log(1+g_{Ni(t-1)}))^2, (\log(1+g_{Ni(t-1)}))^3, \log(1+r_{t-1}), (\log(1+r_{t-1}))^2, (\log(1+r_{t-1}))^3$

Model 2: $\log(e_{i(t-2)}), (\log(e_{i(t-2)}))^2, (\log(e_{i(t-2)}))^3, \log(1+g_{(1+D)(t-1)}), \log(1+g_{N(t-1)}), \log(1+r_{t-1})$

Model 3: $\log(e_{i(t-2)}), (\log(e_{i(t-2)}))^2, (\log(e_{i(t-2)}))^3, \log(1+g_{(1+D)(t-1)}), \log(1+g_{N(t-1)})$

Model 4: $\log(e_{i(t-2)}), (\log(e_{i(t-2)}))^2, (\log(e_{i(t-2)}))^3, \log(1+g_{(1+D)(t-1)})$

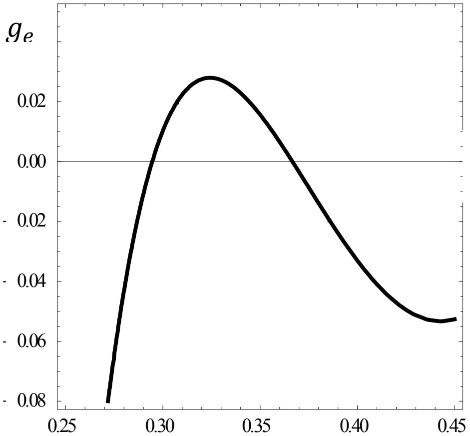


Figure 3.1 – Comparison Fig. 2.1

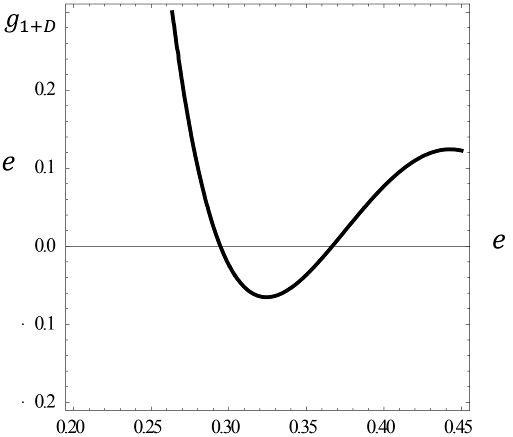


Figure 3.2– Comparison Fig. 2.2

They also indicate that the p-values are not too high (or the J-statistic too low), which would indicate that the over-identifying instruments are ineffective in correcting the bias (Roodman, 2009b). As it is significant in all variables we prefer Model 4 to compare Figure 2.1 and 2.2 in Chapter 2 with the empirical outcomes.

Model 4 gives a specification of[12]:

$$\log(e_t) = 4.532 + 15.830 * \log\left(e_{i(t-1)}\right) + 15.683 * \left(\log\left(e_{i(t-1)}\right)\right)^2 + 5.386 * \left(\log\left(e_{i(t-1)}\right)\right)^3 + 0.429 * \log(1 + g_{1+Dit})$$

For a comparison with Figure 2.1 the model needs to be solved for the growth rate of e_t. With $1 + g_e = \frac{e_{t+1}}{e_t} \leftrightarrow \log(1 + g_e) = \log(e_t) - \log(e_{t-1}) \approx g_e$ this becomes:

[12] The constant is the average of country-specific intercepts, not reported in Table 4 (see Greene (2008) for the procedure of calculation). Bun and Windmeijer (2010) state that the variance of the fixed effect should be similar to the variance of the regression. A simple check shows that the ratio in our case is $\frac{(0.0226)^2}{(0.0223)^2} = 1.0271$ for Model 4, which is close to unity, which was used in the Monte Carlo simulation studies underlying the validity of System GMM.

$$\log(1+g_e) = 4.532 + (15.830-1) * \log\left(e_{i(t-1)}\right) + 15.683 * \left(\log\left(e_{i(t-1)}\right)\right)^2 + 5.386 * \left(\log\left(e_{i(t-1)}\right)\right)^3 + 0.429 * \log(1+g_{1+Dit}) \quad (3.1)$$

Figure 3.1 compares to Figure 2.1 with $\log(1+g_e) \approx g_e$ on the vertical axis. Up until $e_t \approx 0.45$, it has a similar shape as Figure 2.1. It has a (local) maximum at $e_t = 0.324$. The calibrated model of Chapter 2 shows a similar maximum at $e_t =$ 0.354. The steady state values of the estimation are e_1 =0.294 and e_2 =0.367 comparing to the very similar ones of the calibration in Chapter 2, $e_1 = 0.305$ or $e_2 = 0.380$. This approximately confirms the calibrated values by estimation, with impacts of interest rates and population growth rates close to zero. If both of these were significant instead, interest rates would shift the curve up and population growth rates would shift it down. If we use Model 2 instead, the steady-state values are $e_1 = 0.263$ and $e_2 = 0.359$, which are very similar again.

Only few data points have values of e_t higher than 0.45, they mostly belong to Ireland and to a lesser extent to Spain. This is an indication that the upward trend above educational shares of 45% per cent may not be relevant.

To show the relation to Figure 2.2, g_e is set to zero and Equation (3.1) is solved for $g_{(1+D)t}$.

$$g_{(1+D)t} = \frac{4.532+(15.830-1)*\log\left(e_{i(t-1)}\right)+15.683*\left(\log\left(e_{i(t-1)}\right)\right)^2+5.386*\left(\log\left(e_{i(t-1)}\right)\right)^3}{-0.429}$$

This is plotted in the $g_{(1+D)} - e_t$ plane. Figure 3.2 relates to Figure 2.2, with $g_{(1+D)t}$ on the vertical axis. Up until $e_t \approx 0.45$, this has a similar shape as Figure 2, with a minimum at $e = 0.324$ and the same roots as Figure 3.1, e_1 =0.294 and e_2 =0.367. The panel of the 16 countries shows a similar structure as the simulation of the model in Figure 2.1 and 2.2. This shows, as the first major result of this section, that the choice of parameters is in line with Lucas (1988). Specifically F, γ and ϵ of the model were close to the mark, whereas other models, without externalities are not.[13] If our choices for parameters values leading to Figures 2.1 and 2.2 had not been realistic

[13] See Figure 2.3b for graphical representation of the case without externality.

Figures 2.1 and 3.1 as well as Figures 2.2 and 3.2 would differ much more from each other. Before getting this empirical support we could not have excluded the possibility that the empirical figures might have looked like those of Figures 2.3 a or b.

In the theoretical framework, we were forced to do two case studies of either setting $g_e = 0$, or $g_{1+D} = 0$ for analytical reasons. The empirical model allows us to include both cases of the theoretical framework. The second major finding of this section is that whereas an increase in the growth rate of population shifts the $g_e - e$ curve in Figure 3.1 downward, an increase in the growth of the dependency ratio and the interest rate shift the curve upward leading to a faster second best growth rate of e_t in the transition to the steady state and a higher steady state value. By implication, the second-best policy response to ageing is more education for more human capital and productivity growth.[14] This partial result holds for a constant interest rate, which will be treated as endogenous next.

3.5 Conclusion

In Chapter 3, the model of Chapter 2 has been estimated for 16 OECD countries. To ensure comparability, the variables "time share of education", e_t, and the "growth rate of the dependency ratio", g_{1+D}, have been newly constructed to be close to the variables of the Uzawa-Lucas growth model developed in Chapter 2. The analysis of this chapter has shown two major results. Firstly, the 16 analyzed OECD countries do follow the predicted pattern with two steady states very close to those of the model. This justifies the parameter choice of Chapter 2. Secondly, the empirical model gives better insights into the theoretical model because the analytical restrictions of the model in Chapter two can be avoided.

To analyze the full dynamics of this model it is important to note that the model of Chapter 2 is analyzed for small open economies that, by definition, act as price takers in the bond market. This assumption seems

[14] Seeing the threat of an increasing g_{1+D}, pension funds in countries like the Sweden, Netherlands and Germany have announced to pay lower pensions per working year. In response to that announcement, people have started working more years. With more years, longer education time becomes profitable. In countries with less ageing or more immigration this may be different.

worrisome when analyzing countries like the US, France, or Germany. In the next chapter this assumption will be relaxed to a flexible interest rate.

APPENDIX 3.1 – Unit Root Tests

Four different unit root tests are conducted, namely: Levin, Lin & Chu (LLC); Im, Pesaran and Shin (IPS); Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP). All tests test for an AR(1) relationship of the form $e_{it} = c_{0i} + c_{1i} * e_{i(t-1)} + \epsilon_i$, where c_i are assumed to differ across countries for all but the Levin, Lin and Chu test. These four tests can be subdivided into two categories. The ADF-Fisher and PP-Fisher tests both rely on the individual unit root rests of Dickey-Fuller and Phillips-Perron respectively and combine the individual p-values of these tests. This makes them less reliable in panel, because the cross section dimension is neglected and valuable information may be lost (R. P. Smith & Fuertes, 2010). In contrast, LLC and IPS are panel unit root tests, which are both less likely to commit a type-II error because cross-sectional information (not cross-sectional dependence) is taken into account (R. P. Smith & Fuertes, 2010). In the limit they lead to normally distributed statistics. The difference between LLC and IPS is that IPS does allow for some heterogeneity in the cross sections, meaning that it allows for some non-stationary processes, whereas LLC does not. The Im, Pesaran and Shin test also includes further lags of e_t based on the optimum lag found by the Akaike Info Criterion.

	Levin, Lin & Chu t	**Im, Pesaran and Shin W-stat**	**ADF - Fisher Chi-square**	**PP - Fisher Chi-square**
$\log(e_{it})$	0.0001	0.0693	0.0442	0.0388
$\log(e_{it})^2$	0.0011	0.1608	0.0499	0.0402
$\log(e_{it})^3$	0.0008	0.1348	0.0443	0.0440
$\log(1 + g_{(1+D)it})$	0.0000	0.0000	0.0000	0.0000
$\log(1 + g_{(1+D)it})^2$	0.0000	0.0000	0.0000	0.0000
$\log(1 + g_{(1+D)it})^3$	0.0000	0.0000	0.0000	0.0000
$\log(1 + g_{Nit})$	0.1371	0.0184	0.0209	0.3857
$\log(1 + g_{Nit})^2$	0.1711	0.0456	0.0083	0.6024
$\log(1 + g_{Nit})^3$	0.0028	0.0010	0.0002	0.4682
$\log(1 + r_{it})$	0.1678	0.6611	0.7446	0.6379
$\log(1 + r_{it})^2$	0.0017	0.0152	0.0652	0.0609
$\log(1 + r_{it})^3$	0.0001	0.0009	0.0042	0.0011

Table A3.1: Reported are the p-values of the different tests. Null hypothesis: common unit root

* Probabilities are computed assuming asymptotic normality

Table A3.1 reports the results for all variable transformations used in the four models. It shows that there is no clear sign of a unit root for $\log(e_t)$, whereas there is some indication of a unit root for its square and cube. There is no indication of a unit root for any transformation of the growth rate of the dependency ratio. The growth rate of population and the interest rate show mixed results.

4

Debt Dynamics

4.1 Introduction

In Chapters 2 and 3 one of the assumptions has been a zero elasticity of the interest rate with respect to the debt ratio, $\eta_{rb} = 0$, and thereby a fixed interest rate given by the world market because a small and open economy has been considered. This assumption was made in order to postpone the discussion of feedback effects of changes in e on those of the debt/GDP ratio and the interest rate r, which would have an effect on the $g_e - e$ relation according to equation (2.12d)[II] and Figure 2.1, although the estimated model shows that this effect is insignificant and therefore at best small. If the elasticity is assumed to be zero in some sub-sections, this feedback effect on the interest rate is interrupted and partial equilibrium analysis gives adequate results. In this chapter this assumption will be loosened to a flexible interest rate for several reasons. First, price-takership taken literally requires countries to be infinitesimally small, which they never are; even if there is no attempt to exploit an impact on the interest rate, a country's impact on the world market interest rate cannot be exactly zero. Second, EU countries have understood that each country's cumulated foreign and government deficits will increase the interest rate for all of them because of the size effect just explained and, therefore, it is rational to consciously limit government and foreign debt as modeled in Bardhan (1967) by a higher interest rate at higher debt/GDP ratios. Third, building on the first two points raised here, if a country's debt is large, the risk of debt service rescheduling, moratoria or repudiation is also large, as was shown by Eaton and Gersovitz (1981); more empirical support was obtained by Edwards (1984) and S. H. Lee (1991); (1993). We will follow this line of reasoning in our modelling strategy as others, cited below, did. The main objective is to show the existence of a steady state of the previously developed model in the presence of an endogenous interest rate for the reasons given above. It will first be shown how debt behaves under a fixed and given interest in Chapter 4.2 in order to illustrate the stability problem. In a second step, the model will be extended to include an endogenous interest rate depending on the magnitude of debt, which has not been done in an Uzawa-Lucas model so far. This will be done in Chapter 4.3.1. Third, the interest-debt relation will be estimated in Chapter 4.3.2 which, to our knowledge, has also not been done yet. Chapter 4.4 concludes. In Chapter 5, the new found interest rate relation will be introduced in the model of Chapter 2 and the effect of ageing on all steady-state variables of the model will be derived.

4.2 Fixed Interest Rate

This sub-section illustrates the stability problem in the presence of foreign debt. The special case of no effect of debt on the interest rate, $\eta_{rb} = 0$, is instructive here. In the model economy the current account equation must hold at all times. It states that next period's debt/lending must balance the deficit/surplus in production after consumption, saving and previous interest payments have been accounted for.

$$B_{t+1} = N_t c_t + K_{t+1} - (1 - \delta_k)K_t - Y_t + \left(1 + r\left(\frac{B_t}{Y_t}\right)\right)B_t \tag{4.1}$$

Equation (4.1) is the equivalent to equation (2.5) with ω_t and r_{Kt} at their equilibrium values of equations (2.3) and (2.4) and, hence, with $\omega_t(1 - e_t)h_t \frac{N_t}{1+D_t} + r_{kt}K_t = Y_t$.

Because the previously determined relations hold, K_{t+1} and K_t can be replaced by $K_{t+1} = \frac{1-\alpha}{r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})+\delta_k} Y_{t+1}$ from Equation (2.12c) and (2.12b) with $R_{Ht+1} = R_{Bt+1} = 1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1 + \eta_{rb})$ for their respective periods. The interest rate is still given and constant in this section, requiring the assumption $\eta_{rb} = 0$. If the debt to GDP ratio is defined as $b_t = \frac{B_t}{Y_t}$, then (4.1) can be rewritten as:

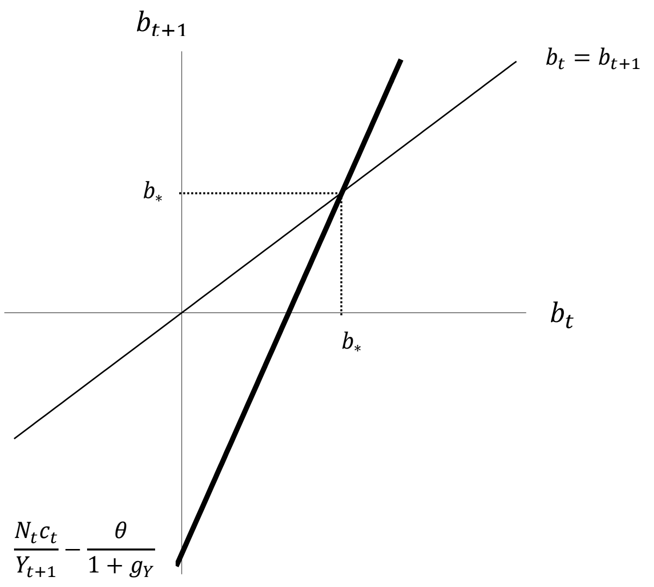


Figure 4.1 – Fixed interest rate

$$b_{t+1} = \frac{N_t c_t}{Y_{t+1}} - \frac{\theta}{1+g_Y} + \frac{1+r}{1+g_Y} b_t \qquad (4.1)^{\mathrm{I}}$$

Where $\theta = (1-\delta_k)\frac{1-\alpha}{r(1+\eta_{rb})+\delta_k} + 1 - (1+g_Y)\frac{1-\alpha}{r(1+\eta_{rb})+\delta_k}$ and is constant in steady-state.

Equation $(4.1)^{\mathrm{I}}$ describes the development of the debt-to-GDP ratio. For the previously discussed special values of the interest rate and the constant steady state growth rates of output and consumption, the slope of $(4.1)^{\mathrm{I}}$ is constant and equal to 1.018. The intercept is $\frac{N_t c_t}{Y_{t+1}} - \frac{\theta}{1+g_Y}$.

The marginal propensity to consume is constant as $N_t c_t$ grows at the rate $(1+g_N)(1+g_c) = 1.031$ which is the same rate as that of the denominator, $1 + g_Y = 1.031$. The last term of the intercept, $\frac{\theta}{1+g_Y}$, contains only exogenous variables and parameters together with the growth rate of output which is constant in steady state. These special assumptions for only this sub-section boil down to a partial equilibrium analysis and imply a stable average propensity to consume and a stable intercept. Because the slope is larger than unity, b_* is an unstable steady state value, indicating that any deviation from the steady state value cannot lead back to b_* if all other parameters are unchanged. It leads to ever increasing debt as described by Blanchard (1983) using the Cass-Koopmans model. Without extra assumptions about the magnitude of the marginal propensity to consume, no conclusions about the value of b_* can be drawn. One special assumption would be the replacement of the dynamic process by a jump to b_*, similar to the jump of consumption per capita onto the saddle-point stable trajectory in the Cass- Koopmans model. For higher consumption rates this would imply lower debt-GDP ratios, because with higher consumption there is less money for debt services; the current account surplus would be lower. However, in general the dynamics in b show an unstable steady state because of the parameter choices. For different parameter choices, especially σ and β, the average propensity to consume is not constant anymore. If $\sigma = 2$ and $\beta = 0.97$, the growth rate of consumption is 0.009 instead of 0.029, which leads to a decreasing average propensity to consume as the numerator grows with $(1 + g_N)(1 + g_c) = 1.011$ and the denominator is stable with $1 + g_Y = 1.031$. The intercept of the debt to GDP development function thus becomes more and more negative causing an outward shift of the debt to GDP line with no steady state. By implication, a jump onto a certain value of the debt/GDP ratio cannot be a general solution to the stability problem and more adjustment is needed. If the interest rate increases in r, removing the zero-elasticity assumption, the slope increases with increasing debt/GDP ratio, which would aggravate the stability problem. θ would increase, moving the line to the left, which would also destabilize the process. Adjustment then must mainly come from decreasing the consumption share, because changes of education shares e and effects from e on g_Y are small. One solution to stabilize the debt dynamics is to introduce a debt-dependent interest rate. This will be done next.

4.3 Endogenized Interest Rate

Chapter 4.1 has shown the solution to the model with a constant interest rate to be an unstable steady state for one specific parameter set. With a constant interest rate it would have been possible to keep borrowing to finance infinite consumption, implying that no utility maximum exists. However, an interest rate depending on the debt to GDP ratio avoids this problem of infinite borrowing and stabilizes the unstable debt process to act as a borrowing restriction as assumed in the literature (Bhandari, Haque, & Turnovsky, 1990; Hamada, 1969; Philippopoulos, 1991). Therefore, to analyze the dynamics around the steady state it is useful to assume a non-fixed interest rate which depends on the debt of the country. In this case of a model extension, the interest rate depends on the world interest rate $\bar{r}$ and the spread of the interest rate which in turn depends on debt over GDP. The new interest rate is thus: $r(b_t) = \bar{r} + r_s(b_t)$. To our knowledge this is the first time that an empirically founded endogenous interest rate has been introduced to an Uzawa-Lucas model. In this subsection we will first understand how the creditor chooses the lending rate for a specific country with the new debt-dependent interest rate and then find an expression for $r(b_t)$ that holds empirically which will then be introduced into the model at hand.

4.3.1 The Creditor Chooses the Lending Rate

So far, the exact function for $r(b_t)$ has not been specified. This will be done in Chapter 4.3.2. This section focuses on the side of the creditor to get an idea of how the creditor chooses the country specific lending rate. The creditor is assumed to maximize his profit in a competitive market. His profit is the revenue he gets, $r_c B$, times the repayment probability $p(b)$ minus the cost. It is assumed that $p(b)$ is decreasing in b since it is more difficult to repay if the debt/GDP ratio rises. The creditors costs are the world market interest rate times the debt issued, $\bar{r}B$. Because the payback probability of the country may be less than one under a temptation to repudiate, the interest charged, r_c, may be higher than the world interest rate. The creditor then maximizes:

$$\max_B \quad p(b) r_c \, B - \bar{r} B$$

Where $p(b)$ is the probability of debt repayment, $\bar{r}$ is the world interest rate and r_c is the lending rate of the creditor. The first-order condition is $p(b)\, r_c + p'(b) r_c \frac{B}{Y} - \bar{r} = 0$ which can be solved for the lending rate of the creditor:

$$r_c = \frac{\bar{r}}{p(b)[1+\eta_{pb}]}$$

with $b\frac{p'(b)}{p(b)} = \eta_{pb} < 0$ as the elasticity of the repayment probability to debt per GDP. The lending rate formula shows the supply schedule for debt if the creditor maximizes profit in a competitive market. If debt per GDP increases, the probability of payment decreases and hence the optimal lending rate increases, given a constant elasticity η_{bp}. Subtracting the world market interest rate from the lending rate yields

$$r_c - \bar{r} = \frac{\bar{r}}{p(b)[1+\eta_{pb}]} - \bar{r}$$

The spread is equal to the right-hand side. In case of a competitive creditor's market, the profits covering some fixed costs are expected to be zero. Then it follows from the definition of non-negative profits, that

$$r_c - \frac{\bar{r}}{p(b)[1+\eta_{pb}]} > 0$$

The left-hand side of the spread formula is even larger under positive profits.

Under costless perfect information the creditors may also take the debt dynamics (4.1)[l] into account. However, in practice country studies are expensive and it is cheaper to know only the value of the debt/GDP ratio as sufficient information in our model, which is sufficiently simple to allow linking it to the Uzawa-Lucas growth model. If r_t is a suitable function of b_t, it is constant in the steady state with constant b. This section has shown how the creditor chooses a country specific lending rate. The next section empirically finds the expression for this.

4.3.2 Estimation of the Interest-Debt Relation

So far $r(b_t)$ was not yet specified. This section will estimate the interest rate dependent on the countries' debt. For the analysis, the same data set with the same time frame as in Chapter 3 has been chosen. The basic idea in this section and the related literature is that interest rates increase with the debt/GDP ratio either because the country has a large market share in the capital market Bardhan (1967) or because higher debt relative to the potential to pay the debt service increases the risk of financial problems. Whereas this is widely accepted, the second and third derivatives of the relation are discussed much less and often imposed by mere assumption. Theoretically, Stiglitz and Weiss (1981) balance the risk compensation against the increase of the probability being unable to pay, leading to credit rationing. In the model of the previous section, the interest rate is linear in that of the world market, but the probability to pay debt service has a negative relation with the debt/GDP ratio. Second order conditions are more likely to be fulfilled if the second derivative is also negative in particular for low values of the interest rate. But this is less necessary under higher interest rates, and the Stiglitz-Weiss argument would say that the second-derivative should be decreasing when debt/GDP is high. Thus, second derivatives may be changing and third derivatives are likely to increase them from negative to positive. In order to solve our endogenous growth model we need an estimate for the same countries for which we have considered education data. We chose a cubic specification because it is very flexible and allows for changing signs of derivatives. Assume a positive transformation of

$$\log[1+r_t] = c_1 + c_2\log[1+\bar{r}] + c_3\log[2+b_t] + c_4(\log[2+b_t])^2 + c_5(\log[2+b_t])^3$$

which may balance the consumption reduction effect and the partial instability of b(b(-1)) dynamics. Adding the quadratic and cubic term is an empirical decision for a non-linear effect. In order to not lose any of the already scarce data, the measurements have been adapted in the following way. We chose to regress on $\log(1+r_t)$ instead of $\log(r_t)$, because r_t ranges between -0.025 and 0.146 and some of the data would be lost if $\log(r_t)$ was employed. For the same reason $\log(2+b_t)$ has been used in the model as b_t ranges between -1.6 and 1.7.

For the variable b_t we choose the measure “Net foreign debt” from the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). It accounts for a country’s assets held within and outside of its borders per GDP, where a positive value indicates a borrowing position of the country and a negative value a lending position. This measurement is very similar to the variable b_t of the model. The interest rate, r, is taken from the World Development Indicators (as above).

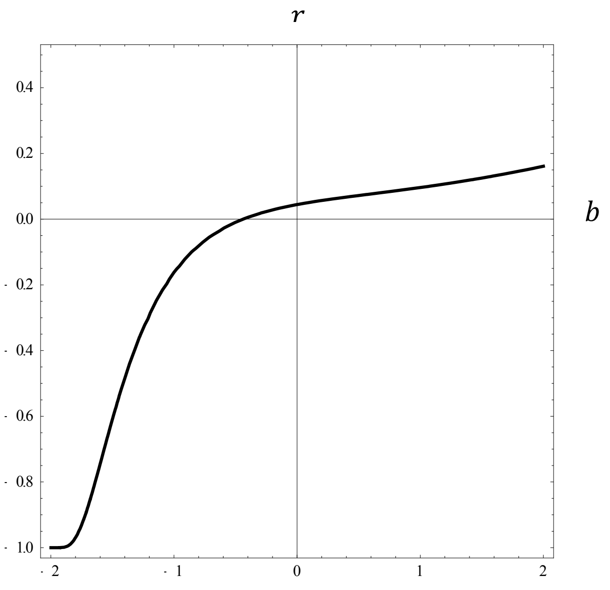


Figure 4.2 - Development $\boldsymbol{r(b_t)}$

$\bar{r}$ is approximated by the average interest rate of the United States, r_{USA}, which is 1.05. This way, the part of the movement of the world interest rate which is not caused by debt movements of the specific country is accounted for by the movements in the US interest rate.

We apply Fully Modified Ordinary Least Square (FMOLS) estimation method to the data. Phillips and Hansen (1990) first propose the FMOLS estimator as a consistent estimator for samples with high endogeneity and serial correlation. This cointegrated panel estimator is also consistent for small samples with contemporaneous correlation of the residuals (Pedroni, 2004). The high heterogeneity across the different countries in this sample and the relatively small panel would otherwise most likely lead to a biased result. This avoided using fixed effects.

Testing for cointegration between the interest rate and debt yields mixed results. We conduct the Pedroni (1999) test for cointegration in panel data, which is Engle-Granger (1987) based. Eight of the 11 tests are significant, leading to a rejection of the null hypothesis of “no cointegration” in all but three cases. Because the other tests with the same hypothesis show very low p- values and the number of observations is low, cointegration between the two variables can be assumed.[15] For detailed test statistics refer to Appendix 4.1.

The estimated equation is reported below as Equation (4.2). With the truncated-uniform kernel[16], the regression shows the highest adjusted R² and the most significant estimates. The estimation has an adjusted R² of 0.1196 and a DW statistics of 0.6035. Both values are not satisfactory, but given the scarcity of the available data, this is the best possible. Standard errors are reported in parentheses below the estimate. *** indicates the significance level of 1%.

$$\log[1+r_t] = \underset{(0.002)^{***}}{-0.204} + \underset{(0.041)^{***}}{0.566}\log[1+\bar{r}] + \underset{(0.038)^{***}}{0.605}\log[2+b_t]$$
$$\underset{(0.049)^{***}}{-0.564}(\log[2+b_t])^2 + \underset{(0.059)^{***}}{0.214}(\log[2+b_t])^3 \qquad (4.2)$$

With this specification the development of the interest rate with respect to the debt to GDP ratio is

$$r(b_t) = 0.815exp^{-0.564\log[2+b_t]^2+0.214\log[2+b_t]^3}(2+b_t)^{0.605}(1+\bar{r})^{0.566} - 1 \qquad (4.2)^{\text{I}}$$

which is displayed in Figure 4.2.

4.4 Conclusion

This chapter has analyzed the debt-sector solution of the model of Chapter 2 with a fixed interest rate. This has led to an unstable steady state debt-value, which may lead to infinite borrowing. As a model extension and to ensure stability and non-exploding debt, a debt-dependent interest rate has been introduced to show how the creditor chooses the lending rate. In a final step, the newly introduced interest rate function has been estimated to cope endogenously with increasing debt. To our knowledge this was the first try to estimate such an interest rate function that is simple enough to be able to be introduced into the Uzawa-Lucas growth model.

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[15] The variable $\log(2+b_t)$ shows the existence of a unit root with and without a trend added. The variable $\log(1+r_t)$ shows a unit root if no trend is added, but with an added trend there is mixed evidence.

[16] Estimating the long-run covariances using the Bartlett kernel with automatic bandwidth choice of Newey-West or Andrews yields very similar results.

The next chapter will introduce this newly found interest rate function into the Uzawa-Lucas growth model.

APPENDIX 4.1 – Unit Root Tests

The existence of unit roots in the variables $\log(1 + r_{it})$ and $\log(2 + b_{it})$ is essential for cointegration. The results are reported in Table A4.1 and suggest the existence of unit roots for both variables. For more details on the test please refer to Appendix 3.1.

	Levin, Lin & Chu t	Im, Pesaran and Shin W-stat	ADF - Fisher Chi-square	PP - Fisher Chi-square
$\log(1 + r_{it})$	0.1230	0.6609	0.8047	0.6224
$\log(2 + b_{it})$	0.9232	0.9692	0.9070	0.7278

Table A4.1 – Unit Root Tests

The Fully Modifies Ordinary Least Square estimator of equation (4.2) requires cointegration of the regressors. The following shows the test results of the Pedroni Residual Cointegration Test (Pedroni, 1999) between the dependent variable $\log(1 + r_t)$ and $\log(2 + b)$. Pedroni's cointegration tests allow for heterogeneous intercepts as well as trends across the cross sections (countries). Table A4.2 presents the p-values for the Pedroni Residual Cointegration test, with the $\nu -$ statistic, $\rho -$statistic, the Phillips-Perron statistic and the Augmented Dickey-Fuller statistic. For details about the specifics, please refer to Appendix 3.1. The weighted and unweighted results are presented. All four test statistics test for a common AR in the coefficients. The results show a rather clear result with 7 of the 8 measures rejecting the null of "no cointegration", which leads to the conclusion that they are indeed cointegrated.

	Statistic	Wheighted Statistic
Panel v-Statistic	0.0157	0.1045
Panel rho-Statistic	0.0774	0.0438
Panel PP-Statistic	0.0285	0.0174
Panel ADF-Statistic	0.0030	0.0409

Table A4.2 – Cointegration Tests

	Prob.
Group rho-Statistic	0.4404
Group PP-Statistic	0.0243
Group ADF-Statistic	0.2827

Table A4.3 – Cointegration Tests

The results reported in Table A4.3 show that there is mixed evidence for individual AR coefficients in the between dimension. The tests show low power because of the low data availability making the assumption of cointegration plausible.

5

When the Dependency Ratio Changes

5.1 Introduction

Whereas Chapter 4.1 has shown that a fixed interest rate may lead to debt explosion, Chapter 5 will introduce the endogenized interest rate developed in Chapter 4.3.2 into the model of Chapter 2 to show the existence of a steady state if large open countries are assumed. To our knowledge this has not yet been attempted. Chapter 5.1 will introduce the new interest rate and discuss the existence of a steady state. In Chapter 5.2 the influence of the growth rate of the dependency ratio will be analyzed. Chapter 5.3 will deal with the stability of the steady state with the new interest rate.

5.2 Steady State Existence

This section introduces the new interest rate of Equation (4.2)[I] into the debt sector of the model, which was analyzed in Chapter 4. The three main variables for determining this balance are the debt to GDP ratio, b_t, time spent in education, e_t and the marginal propensity to consume, $X_t = \frac{N_t c_t}{Y_t}$. First, we analyze Equation (4.1)[I], in which the debt development can be tracked, then the dynamics in X_t are analyzed. The analysis is finalized by reintroducing Equation (2.12d)[II].

Implementing $\frac{N_t c_t}{Y_t} = X_t$ as the marginal propensity to consume in Equation (4.1)[I] and solving for $\frac{b_{t+1}}{b_t} = (1 + g_b)$, it must hold that

$$(1 + g_b) = \frac{1}{1+g_y}\left(\frac{1}{b_t}X_t - \frac{1+\frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}\left((1-\delta_k)-(1+g_y)\right)}{b_t} + 1 + r(b_t)\right) \qquad (5.1)$$

In steady state, $g_b = 0$ and (5.1) can be solved for X_t. This gives the steady state relation for X_t, the marginal propensity to consume.

$$X_t = \left(1 + g_Y - \left(1 + r(b_t)\right)\right) b_t + \frac{(1-\alpha)(1+g_y)}{r(b_t)(1+\eta_{rb})+\delta_k}\left(\frac{(1-\delta_k)}{1+g_Y} - 1\right) + 1 \qquad (5.1)^{\text{l}}$$

Where $1 + g_Y = \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\left(\frac{1+g_N}{1+g_{1+D}}\right)$ and η_{rb} is the elasticity of the interest rate with respect to the debt to GDP ratio that is derived from equation $(4.2)^{\text{l}}$ with $\eta_{rb} = \frac{B_{t+1}}{Y_{t+1}} \frac{r'\left(\frac{B_{t+1}}{Y_{t+1}}\right)}{r\left(\frac{B_{t+1}}{Y_{t+1}}\right)}$ [17].

$$\eta_{rb} = b_t\left(\frac{0.494 exp^{-0.564\text{Log}[2+b_t]^2+0.214\text{Log}[2+b_t]^3}(1+\bar{\text{r}})^{0.566}}{(2+b_t)^{0.395}} + 0.815(2+b_t)^{0.605} exp^{-0.564\text{Log}[2+b_t]^2+0.214\text{Log}[2+b_t]^3}(1+\bar{\text{r}})^{0.566}\left(-\frac{1.127\text{Log}[2+b_t]}{2+b_t} + \frac{0.642\text{Log}[2+b_t]^2}{2+b_t}\right)\right) * 1\Big/\left(-1 + 0.815(2+b_t)^{0.605} exp^{-0.564\text{Log}[2+b_t]^2+0.214\text{Log}[2+b_t]^3}(1+\bar{\text{r}})^{0.566}\right)$$

Equation $(5.1)^{\text{l}}$ is plotted in the in the $b - X$ plane in the left-hand panel of Figure 5.1 for the steady state value $e = 0.380$ and the previously stated parameter set.

For $(4.1)^{\text{l}}$, the dynamic equation describing b_t, to hold, not only the dynamics in e_t are important, but also those of the average propensity to consume. With the previously used definition $\frac{N_t c_t}{Y_t} = X_t$, $1 + g_X$ is by definition

$$1 + g_X = \frac{(1+g_N)(1+g_c)}{1+g_Y} = \frac{1+g_N}{\left(F\, e_t^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\left(\frac{1+g_N}{1+g_{1+D}}\right)}\left[\beta\left(1 + r(b_t)(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}} \qquad (5.2)$$

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[17] For better overview this expression is not explicitly plugged into Equation $(5.1)^{\text{l}}$.

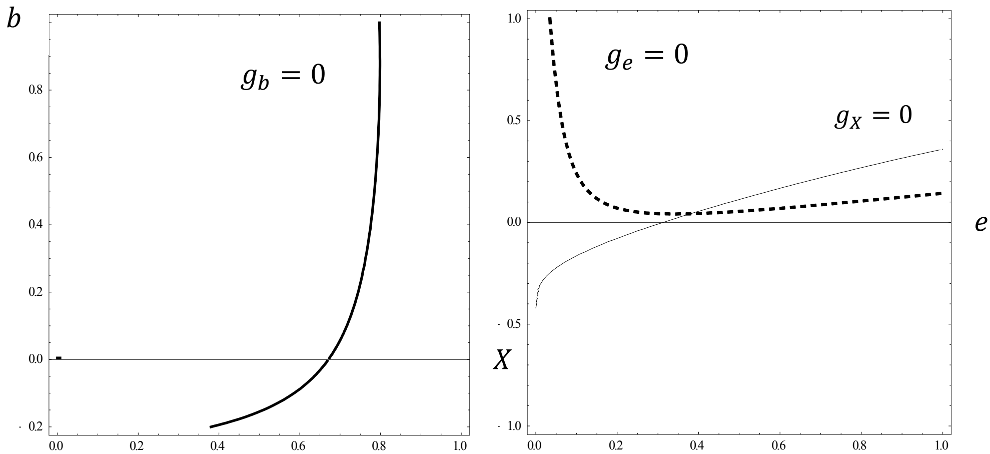

Figure 5.1 – Steady states in $\boldsymbol{b_t}$, $\boldsymbol{X_t}$ and $\boldsymbol{e_t}$

Where $1 + g\_y$ and $(1 + g_c)$ are their steady state relations as reported in Table 2.1. In steady state $g_X = 0$, (5.2) thus becomes:

$$\frac{1}{\beta}\left(\left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\left(\frac{1}{1+g_{1+D}}\right)\right)^{\sigma} = \left(1 + r(b_t)(1+\eta_{rb})\right) \qquad (5.2)^{\text{I}}$$

As $r(b_t)$ and η_{rb} take the previously discussed forms, this cannot be solved analytically for either b_t or e_t and hence is plotted as $g_X = 0$ line in Figure 5.1 for the known parameters.

The dynamics of g_e are given by equation (2.12d)$^{\text{II}}$, where $R_{Ht+1} = 1 + r(b_t)(1+\eta_{rb})$ from (12b$^{\text{I}}$). In steady state $g_e = 0$, then together with the estimation for $r(b_t)$, (2.12d)$^{\text{II}}$ becomes

$$1 + r(b_t)(1+\eta_{rb}) = (F\, e_t^{\gamma} + (1-\delta_h))^{\frac{\epsilon}{\alpha}} \frac{1+g_N}{1+g_{1+D}}\, F\gamma \left[e_t^{\gamma-1} - e_t^{\gamma} + e_t^{\gamma}\frac{1}{\gamma} + \frac{(1-\delta_h)}{F\gamma}\right] \qquad (5.3)$$

Equation (5.3) shows all steady state combinations of b_t and e_t for the given parameter set with $\bar{r}$ set to 0.05, the average over the sample. It cannot be solved analytically for either b, or e. To analyze the relationship further, it has been plotted in Figure 5.1 for the parameters assumed in Table 2.1 in the $b - e$ plane. The $g_e = 0$ - line in the right panel of Figure 5.1 represents all steady state pairs of b and e. The values for education and the marginal propensity to consume are constant where the two lines,

$g_e = 0$ and $g_X = 0$ intersect. At this point, both variables are at their steady state value, which is at $e = 0.380$ and $b = 0.042$. Plugging these values into Equation (5.1)' implies an X_t of 0.464 and an r of 0.05 if the values are plugged into Equation (4.2)'.

This concludes the analysis of a steady state in the debt sector. This subsection has shown that, contrary to a fixed interest rate, a flexible interest rate that increases with the debt value does lead to a plausible solution for the steady state.

5.3 The Influence of the Growth Rate of the Dependency Ratio

So far, we have set $g_{1+D} = 0$. This section analyzes in what happens if the growth rate of the dependency ratio changes. For this we exemplarily calculate five steady-state scenarios. Since the data of g_{1+D} ranges between -0.062 and 0.059 with an average of -0.001, see Chapter 3.3, the scenarios are chosen for the values -0.06, -0.03, 0, 0.03 and 0.06. From (5.2)' it follows that for given b, a higher g_{1+D} shifts the $g_X = 0$-line to the right. From (5.3) it follows that for given e_t, the $g_e = 0$ –line shifts down. The latter move dampens the increase in e_* from the first, and the former dampens the fall in b_*. Whether these dampening effects even dominate can only be answered numerically. The same holds for X_t because the $g_b = 0$-line was drawn for steady-state values b_* and e_*.

$\boldsymbol{g_{1+D}}$	$\boldsymbol{e_t}$	$\boldsymbol{b_t}$	$\boldsymbol{X_t}$	$\boldsymbol{r_t}$	$\boldsymbol{1 + g_c}$
-0.06	0.307	0.722	0.394	0.083	1.059
-0.03	0.342	0.351	0.411	0.065	1.043
0	0.380	0.042	0.464	0.050	1.029
0.03	0.425	-0.150	0.522	0.033	1.014
0.06	0.477	-0.274	0.736	0.021	1.003

Table 5.1 – Summary of Influence of Changes in $\boldsymbol{g_{1+D}}$

Table 5.1 summarizes the results. With an increasing growth rate of the dependency ratio, the optimal time spent in education increases substantially in order to compensate the lack of labor by more technical change. The optimal debt to GDP ratio decreases, indicating the need for different optimal practices when expecting ageing, and the interest rate decreases. The marginal propensity for current consumption increases because of ageing and the growth rate of consumption decreases. Hence, the optimal reaction to ageing and the implied higher consumption is more education at the cost of consumption growth, leading to less credit demand and a lower interest rate, implying a lower burden of debt service.

5.4 Stability with Endogenous Interest Rate: Tentative Notes

In Chapters 5.1-5.3 the existence of a steady state in the case of an endogenized interest rate has been analyzed. This section will deal with the behavior of the economy out of the steady state and will analyze partial aspects of the stability of the system.

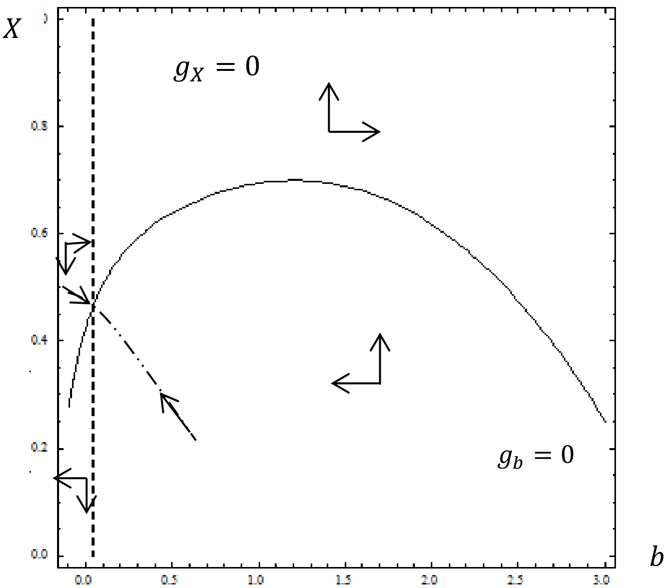

Figure 5.2 – Partial Stability in X_t and b_t

There are three mutually interacting variables that influence the dynamics: e_t, b_t and X_t. The central equations are (2.12d)^{II} for the dynamics in e_t, (4.1)^{II} for the dynamics in b_t and (5.2) for the dynamics in X_t. Figure 2.1 shows the dynamics in e_t if r is a function of b_t and b_t is constant. For given b_t it has already been established, that the steady state value $e_* = 0.380$ is stable.

Figure 5.2 shows the partial stability in X_t and b_t. The vertical, dashed, line represents Equation (5.2) for $g_X = 0$ and the hump shaped, solid, line represents Equation (4.1)$^{\text{II}}$ for $g_b = 0$. Where both lines intersect, for given e_t, the sub-system is stable in the saddle-point sense. If b_t is to the right (left) of the $g_X = 0$-line in Figure 5.2, the larger (smaller) value of b_t implies a positive (negative) growth rate of X_t in Equation (5.2). This is shown in the vertical movement-arrows. If X_t is larger (smaller) than its steady state value in (4.1)$^{\text{II}}$, g_b becomes positive (negative). This is captured in the horizontal arrows above and below the $g_b = 0$-line. The arrows indicate that there is a saddle-point-stable trajectory. If the economy starts on this trajectory with given e_t, X_t and b_t, it ends up in steady state. If the economy started to the right of this trajectory, debt and consumption would keep growing into the upper right part of the figure which violates the transversality condition. If the economy started on the left, it would move towards the origin, indicating that less is consumed than possible. This is also not optimal. Hence, the only optimal solution is to jump right on the saddle-point-stable trajectory if not into the steady state itself.

This shows the partial stability of the system for all three variables. To achieve stability X_t has to control b_t directly as in Figure 5.2 and via b_t also e_t indirectly as in Figure 2.1. Unfortunately, the impossibility to solve (2.12d)$^{\text{II}}$ for g_e, or e_t makes it impossible to show stability via simulation.

To show the complexity of the stability of the whole system of three equations, we start with the original steady-state values: $e = 0.379741,\ b = 0.041806,\ X = 0.463633$. One question now is how the dynamics behave, if the starting value of b_t is increased.

b_t is increased from $b_t = 0.041806$ to $b_t = 0.045$. This influences the growth rate of e_t and leads to a new partial preliminarily stable e_t through Equation (2.12d)$^{\text{II}}$. Figure 5.3 shows the relation of Figure 2.1 with the original $b_t = 0.0418063$ (dashed, lower line) and with $b = 0.045$ (solid, upper line). The new stable value of e_* is 0.4234549. This new e_t influences the lines in Figure 5.2, which leads to a downward rotation of the $g_b = 0$ line and a right shift of the $g_X = 0$ line, as displayed in Figure 5.4.

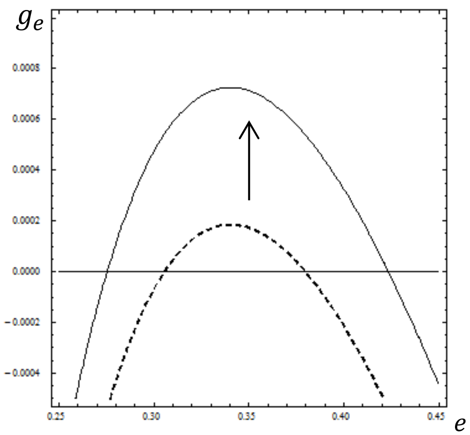

Figure 5.3 – Partial Stability

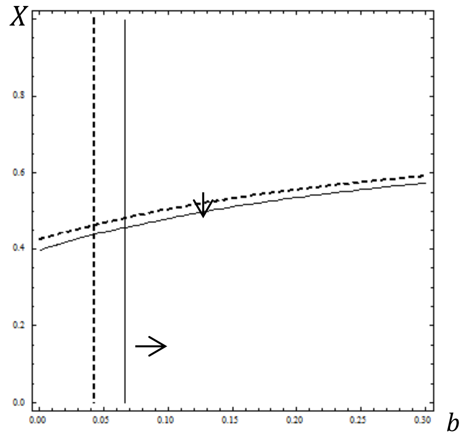

Figure 5.4 – Partial Stability

The downward movement of the $g_b = 0$-line can be explained as follows:

In Equation (4.1)$^{\text{II}}$, g_b is set to zero and solved for X_t, as shown in Equation (4.1)$^{\text{III}}$.

$$X_t = (1+g_Y)\left(b_t - \frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}\right) + \frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}(1-\delta_k) - b_t\big(1+r(b_t)\big) + 1 \tag{4.1$^{\text{III}}$}$$

Where $r(b_t)$ and η_{rb} take the forms as explained in Chapter 4.3.2, and $(1+g_Y)$ is at its steady state relation, as presented in Table 2.1. If e_t increases, then the growth rate of output, $(1+g_Y)$, increases as well. Hence, for given b_t, $X_t$increases if $b_t > \frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}$ and X_t decreases if $b_t < \frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}$. This shows that if e_t is increased the $g_b = 0$ line rotates around some b_t, where $b_t = \frac{1-\alpha}{r(1+\eta_{rb})+\delta_k}$. Around the steady state $b_t \approx 0.04$, hence, $\eta \approx 0.06, r \approx 0.05$ and the parameters are $\alpha = 0.3, \delta_k = 0.03$. This leads to $\frac{1-\alpha}{r(1+\eta_{rb})+\delta_k} \approx 8.78 \gg b_t$. Hence, for given b_t, $X_t$is smaller than before and the $g_b = 0$ curve shifts down. This is true around the steady state and may differ for a lot larger b_t.

As a didactic, heuristic exercise, we conduct a numeric example to show the partial dynamics in e_t, b_t and X_t. This is displayed in form of a table in which, the first column is the starting value of e_t. We start with the steady state value of $e_t = 0.379741$. This value is plugged into Equation (5.2) for $g_X = 0$. As Equation (5.2) only depends on e_t and b_t, we can calculate the corresponding b_t value. With Equation (4.1)^{II}, the value of X_t is determined with the previously determined e_t and b_t. The value for b_t is then plugged into the $g_e = 0$ function, Equation (2.12d)^{II}. With this, the new e_t is determined. In steady state is the same as the one we started with. This is displayed in the first row. The corresponding $r(1+\eta)$ is displayed in the last column.

We start the simulation in the second row. A higher value for b_t is chosen. The corresponding X_t and e_t values are determined. The determined e_t will be plugged back into (5.2) and is the starting value of row 3 and so on.

$\boldsymbol{e_1}$	$\boldsymbol{b}$	$\boldsymbol{X}$	$\boldsymbol{e_2}$	$\boldsymbol{r(1+\eta)}$
0.379741	0.041806	0.463633	0.379741	0.05
0.379741	0.042	0.463795	0.383329	0.050025
0.383329	0.043873	0.463107	0.410427	0.050263
0.410427	0.059292	0.459354	0.538811	0.052198
0.538811	0.128446	0.445548	0.926139	0.060291
0.926139	0.308596	0.423480	1.877946	

Table 5.2 – Partial Stability calculations, higher than steady state $\boldsymbol{b_t}$

$e = 1.877946$ is already out of the scope of our model. This shows that with a slight increase in b, e and b increase and X decreases, leading out of steady

state. In Table 5.3 b_t will be decreased to 0.041. The first row is again the steady state. Here we also reach the limit of the model quite quickly but at the other end of the spectrum then before.

$\boldsymbol{e_1}$	$\boldsymbol{b}$	$\boldsymbol{X}$	$\boldsymbol{e_2}$	$\boldsymbol{r(1+\eta)}$
0.379741	0.041806	0.463633	0.379741	0.05
0.379741	0.041	0.462957	0.359868	0.049897
0.359868	0.030243	0.466682	Not solvable	

Table 5.3 – Partial Stability calculations, lower than steady state $\boldsymbol{b_t}$

This exercise has shown how the partial solutions for the variables e_t, b_t and X_t behave if b_t alone moves out of steady state and the other two variables do not adjust to land on the saddle-path-stable trajectory. By implication, adjustment of X_t has to steer b_t directly as in Figure 5.2 and that of e_t indirectly via b_t in Figure 2.1. Unfortunately, a simulation exercise showing the stable saddle path trajectory is not feasible because equation (2.12d)^{II} cannot be solved for g_e or e_t.

5.5 Conclusion

The estimated debt-dependent interest rate of Chapter 4 has been introduced into the Uzawa-Lucas growth model with exogenous population dynamics. The chapter shows how the three main variables of the resulting dynamic system, namely, time spent in education, debt and the marginal propensity to consume, change with demographic developments. It has been shown that an exogenous increase in the growth rate of the dependency ratio increases steady state time spent in education and the marginal propensity to consume, while it decreases the debt to GDP ratio, the interest rate and consumption growth. A higher growth of human capital is the optimal compensation for the availability of less labor time per person in the population.

A natural next step is to introduce more information on the growth rate of the dependency ratio. This would allow treating ageing as an out-of-steady-state, temporary, but still long-term phenomenon. This is done stepwise in the next chapters. Chapter 6 analyzes the model if the development of one part of the dependency ratio, the active population, is endogenized.

6

Endogenized Labor Supply

6.1 Introduction

The ultimate goal of this thesis is to endogenize population dynamics in the discrete time Uzawa-Lucas growth model to find the optimal response to recent demographic changes in the economy. After analyzing the response of the economy to exogenous changes of the dependency ratio with a static and exogenously given labor share, it is the next logical step to endogenize the choice of labor force participation. The question is how the agents in the economy will allocate their time if no exogenous restrictions on the time spent in the active population are set in times of ageing. An increase of the dependency ratio in the previous model was characterized by a larger share of people being inactive as opposed to actively involved in either education or production. For this analysis it was irrelevant whether a higher dependency ratio was caused by a smaller active population, a larger inactive population, or both. This made it easier to analyze the dynamics between the active and inactive population, i.e. a change of the dependency ratio.

This chapter takes the previous model one step further as the size of the active population becomes subject to personal choice. As before, there are two ways of interpretation. Individually, this can be interpreted as the time each individual wants to spend either in work or education, as opposed to retirement. On a macroeconomic level, this can be interpreted as the possibility for the economy as a whole to choose the size of the active and inactive shares of the population. In this chapter the latter is preferred. The aim is to find the optimal reaction of the growth of the active population, L_t, to an ageing population. To my knowledge, this has not been examined before in this framework. The model will be calibrated to the parameters defined in the previous chapters to ensure comparability. With these parameters, the agents in the economy will spend more time in education and the endogenized growth rate of the active population will be lower, leading to a higher dependency ratio than analyzed in Chapter 2. When bigger countries are analyzed the debt-dependent interest rate, found in Chapter 4, needs to be introduced. A shrinking population growth rate, as is the case in the recent demographic developments, leads to a lower than proportionate shrinking of the labor force, indicating that the people tend to work more as the population shrinks, which leads to a decrease of the dependency ratio. This chapter's model is an intermediate step to finding the response of the economy to an aging population. Keep in mind that this chapter's model cannot model a change in the population structure that is apparent in ageing, but can only alter the population size, as population growth is still exogenous.

Chapter 6.2 introduces the new utility function with a Frisch-elasticity on labor supply. Chapter 6.3 analyses the existence and stability of multiple steady states. Chapter 6.4 introduces the debt-dependent interest rate as found in Chapter 4. Chapter 6.5 analyses the response to changes in the population growth rate and Chapter 6.6 concludes.

6.2 The Model

The endogenization of the choice of participation in the active population[18] is conducted by introducing the relation between the active population and the entire population, L_t/N_t, in the utility function. In the light of the previous model, this is the inverse of the dependency ratio, $1 + D_t = \frac{N_t}{L_t}$. It has a Frisch-elasticity of labor supply, ϑ, which captures the substitution effect of a change in the share of the active population. Being involved in the active population, or dedicating a high share of the population to the active population, has a negative effect on utility. On the other hand, being inactive, or involving few resources in the active population, will lead to low output which will result in low consumption and, hence, low utility. Hence, there must be an optimal labor supply.

[18] Usually it is interpreted as "labor force participation". This interpretation is not feasible in this model as the active population is split into the actual labor force and people in education.

Consumers are assumed to maximize their utility subject to consumption, time spent in education and labor supply.

$$U_t = \sum_{t=0}^{\infty} \beta^t N_t \left(\frac{c_t^{1-\sigma}}{(1-\sigma)} - \xi \frac{\left(\frac{L_t}{N_t}\right)^{1+\vartheta}}{1+\vartheta} \right)$$

The expression above shows the utility function of the entire population, where $0 < \beta < 1$ is the subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution, ϑ is the Frisch elasticity parameter, ξ is a parameter which measures the disutility of participation in the active population, N_t is the size of the entire population, c_t is individual consumption and L_t is the size of the active population.

Similar to the model of Chapter 2, the economy consists of output-producing firms and labor- and capital-supplying consumers. Output is formed by a Cobb-Douglas production function and is determined by physical capital, K_t, and human capital, $H_t = h_t L_t$. Human capital is formed by individual human capital, h_t, and the number of people in the active population, L_t. The agents in the active population decide between spending their time in production $(1 - e_t)$ for immediate output generation and education, e_t, to increase their productivity for later production. A human capital externality is added, modelled after Lucas (1988), to include the influence of the average skill level, $\bar{h}_t^{\epsilon}$, on the economy. This forms the production function:

$$Y_t = A(K_t)^{1-\alpha}\big((1 - e_t)h_t L_t\big)^{\alpha}\bar{h}_t^{\ \epsilon} \tag{6.1}$$

The demand for physical and human capital, provided by consumers, is determined in the firm sector in which firms maximize their profits.

$$\max_{(1-e_t),K_t} \pi = A(K_t)^{1-\alpha}\big((1 - e_t)h_t L_t\big)^{\alpha}\bar{h}_t^{\ \epsilon} - \omega_t(1 - e_t)h_t L_t - r_{kt}K_t$$

$$\omega_t = \frac{\alpha Y_t}{(1-e_t)h_t L_t} \tag{6.2}$$

$$r_{kt} = (1 - \alpha)\frac{Y_t}{K_t} \tag{6.3}$$

Equations (6.2) and (6.3) represent the optimal wages of labor and rental rates for capital. Consumers face the well-known budget constraint.

$$N_t c_t + K_{t+1} - (1-\delta_k)K_t = \omega_t(1-e_t)h_t L_t + r_{kt}K_t + B_{t+1} - \left(1 + r\left(\frac{B_t}{Y_t}\right)\right)B_t \tag{6.4}$$

The RHS of Equation (6.4) represents the total income which is the income from labor, $\omega_t(1-e_t)h_t L_t$, the income from capital rent, $r_{kt}K_t$, and the borrowed money from outside the economy's borders minus the interest and re-payments, $B_{t+1} - \left(1 + r\left(\frac{B_t}{Y_t}\right)\right)B_t$. $r\left(\frac{B_t}{Y_t}\right)$ is the debt dependent interest rate. For the sake of the argument it is assumed to be given and constant in the first part of the analysis and will be endogenized later on in Chapter 6.4. The LHS of Equation (6.4) represents the spending on consumption, $N_t c_t$, and capital investment/savings, $K_{t+1} - (1-\delta_k)K_t$. For the budget constraint to hold, both sides must be equal at all times.

Human capital formation is determined by the time share spent in education, e_t, with diminishing or constant returns to scale. Equation (6.5) shows how human capital is formed with the productivity parameter, $\gamma \leq 1$, the knowledge efficiency coefficient, F, and depreciation of human capital, δ_h.

$$h_{t+1} = F\, e_t^{\gamma} h_t + (1-\delta_h)h_t \tag{6.5}$$

Consumers maximize their utility subject to their budget constraint, Equation (6.4), and the human capital formation function, Equation (6.5). Compared to the previous model of Chapter 2, the consumers do not only maximize their consumption and share of time devoted to education, they also decide on the size of the active population. The maximization program for the consumers is thus:

$$\max_{c_t, e_t, L_t, B_{t+1}, K_{t+1}, h_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(N_t \left(\frac{c_t^{1-\sigma}}{(1-\sigma)} - \xi \frac{\left(\frac{L_t}{N_t}\right)^{1+\vartheta}}{1+\vartheta} \right) \right.$$
$$- \boldsymbol{\mu_t} \left[N_t c_t + K_{t+1} - (1-\delta_k)K_t - \omega_t(1-e_t)h_t L_t - r_{kt}K_t \right.$$
$$\left. - B_{t+1} + \left(1 + r\left(\frac{B_t}{Y_t}\right)\right) B_t \right]$$
$$\left. - \boldsymbol{\mu_{ht}} \left[h_{t+1} - F\, e_t^{\gamma}\, h_t - (1-\delta_h) h_t \right] \right)$$

The first order conditions are:

c_t: $\quad c_t^{-\sigma} = \mu_t \qquad (6.6)$

e_t: $\quad \mu_t \omega_t L_t = \mu_{ht} F\, \gamma e_t^{\gamma-1} \qquad (6.7)$

L_t: $\quad \xi N_t^{-\vartheta} L_t^{\vartheta} = \mu_t \omega_t (1-e_t) h_t \qquad (6.8)$

B_{t+1}: $\quad \mu_t = \beta \mu_{t+1} \left(1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)\right) + \beta \mu_{t+1} \frac{B_{t+1}}{Y_{t+1}}\, r'\left(\frac{B_{t+1}}{Y_{t+1}}\right) \qquad (6.9)$

K_{t+1}: $\quad \mu_t = \beta \mu_{t+1}(1 - \delta_k + r_{kt+1}) \qquad (6.10)$

h_{t+1}: $\mu_{ht} = \beta\left[\mu_{t+1}\omega_{t+1}(1-e_{t+1})L_{t+1} + \mu_{ht+1} F e_{t+1}^{\gamma} + (1-\delta_h)\mu_{ht+1}\right] \qquad (6.11)$

The following transversality conditions hold[19]:

I. $\lim_{t\to\infty} \beta^t \mu_t K_t = 0$
II. $\lim_{t\to\infty} \beta^t \mu_{ht} h_t = 0$

The first order conditions are similar to those of the previous model with an exogenous development of the dependency ratio. One obvious difference is

[19] For proof that the utility function has a finite integral, and hence has an interior maximum has been shown in Appendix 2.1. The same logic applies here.

the new first order condition with respect to L_t, Equation (6.8), in which the size of L_t is linked to the population size, N_t, labor income, $\omega_t(1-e_t)h_t$, and the co-state variable μ_t.

The above defines a system of 11 equations for 11 endogenous variables, $Y_t,\ K_t,\ L_t,\ h_t,\ e_t, \omega_t, r_{kt}, c_t, B_t, \mu_t$, and μ_{ht}. The rates of return to physical capital, bonds, human capital and, now additionally, future labor input can be derived. They are displayed in Equations (6.12a-e).

$$\frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\mu_t}{\beta\ \mu_{t+1}} = R_{Bt+1} = R_{Kt+1} = R_{Ht+1} = R_{Lt+1} \tag{6.12a}$$

$$R_{Bt+1} = 1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb}) \tag{6.12b}$$

$$R_{Kt+1} = 1 - \delta_k + (1-\alpha)\frac{Y_{t+1}}{K_{t+1}} \tag{6.12c}$$

$$R_{Ht+1} = (1+g_\omega)(1+g_L)\ \ F\gamma e_t^{\gamma-1}\left[1 - e_{t+1} + \frac{1}{\gamma}e_{t+1} + \frac{1-\delta_h}{F\gamma e_{t+1}^{\gamma-1}}\right] \tag{6.12d}$$

$$R_{Lt+1} = \frac{(1+g_N)^{\vartheta}(1+g_\omega)(1+g_{1-e})(1+g_h)}{\beta(1+g_L)^{\vartheta}} \tag{6.12e}$$

The first part of (6.12a) is derived from (6.6), where the other equivalencies on the right hand side follow from (6.12b-e), by rearranging equations (6.9), (6.10) and (6.11) to equal $\frac{\mu_t}{\beta\ \mu_{t+1}}$. Equation (6.12b) follows straight forwardly from (6.9) with $\eta_{rb} = \frac{B_{t+1}}{Y_{t+1}}\frac{r'\left(\frac{B_{t+1}}{Y_{t+1}}\right)}{r\left(\frac{B_{t+1}}{Y_{t+1}}\right)}$; (6.12c) is derived from (6.10) where r_{kt+1} is replaced by the expression in (6.3) and equation (6.12d) is derived from (6.11) where μ_{ht} is replaced by the relation in (6.7). The new relation, (6.12e), for the rate of return of the active population size is derived from (6.8).

If $r\left(\frac{B_t}{Y_t}\right)$ is assumed to be constant (an assumption that will be relaxed in Chapter 6.4) this leads to a constant growth rate of c_t through (12a, b). From this follow constant R_{Kt+1} and R_{Ht+1} and R_{Lt+1} in (6.12c), (6.12d) and (6.12e) respectively. Constancy of $\frac{Y_{t+1}}{K_{t+1}}$ follows directly from constant R_{Kt+1}

in (6.12c), which can be expressed as: $\frac{Y_{t+1}}{K_{t+1}} = A\left(\frac{K_{t+1}}{(1-e_{t+1})h_{t+1}^{1+\frac{\epsilon}{\alpha}}L_{t+1}}\right)^{-\alpha}$. This implies equality of the growth rates of the numerator and the denominator:

$$1 + g_Y = 1 + g_K = (1 + g_{1-e})(1 + g_h)^{1+\frac{\epsilon}{\alpha}}(1 + g_L) \tag{6.12c)^{I}}$$

Equation (6.12c)^{I} shows that the growth rates of output and capital depend on the change of time spent in production, the growth rate of human capital, and the growth rate of labor supply.

The growth rate of wages is crucial to determine the rates of return to labor and human capital. Wages are determined in the firm sector where Equation (6.2) implies:

$$1 + g_\omega = \frac{1+g_Y}{(1+g_{1-e})(1+g_h)(1+g_L)} \qquad (6.2)^{I}$$

Inserting (6.12c)^{I} into (6.2)^{I} leads to:

$$1 + g_\omega = (1 + g_h)^{\frac{\epsilon}{\alpha}} \qquad (6.2)^{II}$$

Together with the human capital formation function, Equation (6.5), this shows that g_ω is constant if e_t is constant (i.e. if $g_e = 0$). Equation (6.2)^{II} shows that the development of wages per labor efficiency unit depends solely on that of individual human capital. Wage growth per labor efficiency unit is only positive if the externalities are positive.

The growth rate of the active population, $(1 + g_L)$, can be derived from Equation (6.12e). With the expression in Equation (6.12b), R_{Lt+1} can be replaced by $1 + r\left(\frac{B_t}{Y_t}\right)(1 + \eta_{rb})$. Plugging Equation (6.2)^{II} in Equation (6.12e), and thus eliminating $(1 + g_\omega)$ leads to

$$(1 + g_L) = (1 + g_N)\left(\frac{(1+g_h)^{1+\frac{\epsilon}{\alpha}}(1+g_{1-e})}{\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}} \qquad (6.12e)^{I}$$

As long as the interest rate and the population growth rate are exogenous, the steady state growth rate of the labor force depends on the endogenous development of individual human capital (and through this on time spent in education) and on the growth rate of time spent in production, $(1-e_t)$. Because the time spent in production, $(1-e_t)$, can by assumption not exceed 1, its growth rate cannot be stable at any other value than zero. Equation (6.12e)[I] shows that if the choice of the size of the labor force is endogenized through negative utility associated with labor supply, the steady state growth rate of the labor force is determined by time spent in education. If time spent in education is stable, so is $(1+g_L)$. The interest rate and the population growth rate will be endogenized later on, in Chapter 6.4 and Chapter 8 respectively.

In order to find the optimal time spent in education, the dynamics in e_t need to be derived. This is done by relating e_t and e_{t+1} to only exogenous variables. The main equation used is Equation (6.12d). To show how the rate of return of human capital relates to time spent in education only, $(1+g_\omega)$ can be replaced in Equation (6.12d) using Equation (6.2)[II] and Equation (6.5).

$$R_{Ht+1} = \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon}{\alpha}} (1+g_L)\, F\gamma e_t^{\gamma-1} \left[(1-e_{t+1}) + \frac{1}{\gamma} e_{t+1} + \frac{(1-\delta_h)}{F\gamma e_{t+1}^{\gamma-1}}\right] \qquad (6.12\text{d})^{\text{I}}$$

Next, the endogenous variables g_L and g_h are replaced successively. With Equation (6.12e)[I] replacing the growth rate of the labor force, $(1+g_L)$, Equation (6.12d)[I] becomes

$$R_{Ht+1} = (1+g_h)^{\frac{\epsilon}{\alpha}}(1+g_N)(1+g_h)^{\left(\frac{1}{\vartheta}+\frac{\epsilon}{\vartheta\alpha}\right)}(1+g_{1-e})^{\frac{1}{\vartheta}} \left(\frac{1}{\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}} F\gamma e_t^{\gamma-1}\left[(1-e_{t+1}) + \frac{1}{\gamma} e_{t+1} + \frac{(1-\delta_h)}{F\gamma e_{t+1}^{\gamma-1}}\right]$$

Replacing $(1+g_h) = F\, e_t^{\gamma} + (1-\delta_h)$ and $(1+g_{1-e}) = \frac{1-e_{t+1}}{1-e_t}$ yields:

$$R_{Ht+1} = \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon(1+\vartheta)+\alpha}{\vartheta\alpha}} (1+g_N) \left(\frac{1-e_{t+1}}{1-e_t}\right)^{\frac{1}{\vartheta}} \left(\frac{1}{\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}} F\gamma \left[e_t^{\gamma-1} - e_t^{\gamma}\frac{e_{t+1}}{e_t} + e_t^{\gamma}\frac{1}{\gamma}\frac{e_{t+1}}{e_t} + \left(\frac{e_t}{e_{t+1}}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]$$

(6.12d)^{II}

This relates e_t and e_{t+1} to only exogenous variables and parameters. $\frac{e_{t+1}}{e_t}$ can then be replaced by $1+g_e$. Since $1+g_e = \frac{e_{t+1}}{e_t} \leftrightarrow e_{t+1} = (1+g_e)e_t$, the expression $\left(\frac{1-e_{t+1}}{1-e_t}\right)^{\frac{1}{\vartheta}}$ thus becomes $\left(\frac{1-(1+g_e)e_t}{1-e_t}\right)^{\frac{1}{\vartheta}}$. This leads to:

$$R_{Ht+1} = \left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{\frac{\epsilon(1+\vartheta)+\alpha}{\vartheta\alpha}} (1+g_N) \left(\frac{1-(1+g_e)e_t}{1-e_t}\right)^{\frac{1}{\vartheta}} \left(\frac{1}{\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}} F\gamma \left[e_t^{\gamma-1} - e_t^{\gamma}(1+g_e) + e_t^{\gamma}\frac{1}{\gamma}(1+g_e) + \left(\frac{1}{1+g_e}\right)^{\gamma-1}\frac{(1-\delta_h)}{F\gamma}\right]$$

(6.12d)^{III}

With R_{Ht+1} constant and equal to $1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})$ because of the relations in Equations (6.12b) and (6.12a), the RHS of Equation (6.12d)^{III} is constant as well. This is the dynamic equation that determines how e_t develops over time, depending on several parameters and exogenous variables. With help of this equation, the stability of e_t in the steady state can be analyzed, see Chapter 6.3. Unfortunately, the above expression cannot be solved for e_t, or g_e analytically.

The inclusion of the labor force participation decision changed the dynamics in the system such that steady state e_t now additionally depends on the growth rate of the population and the Frisch elasticity, as can be seen when comparing Equation (2.12d)^{II} and Equation (6.12d)^{III}. The next section deals with the existence and stability of multiple steady states.

6.3 Existence and Stability of Multiple Steady States

Because of the $\left(\frac{1-(1+g_e)e_t}{1-e_t}\right)^{\frac{1}{\vartheta}}$ term in Equation (6.12d)^{III}, the full analysis of the dynamic equation in e_t, is much more complicated than the equivalent of Chapter 2.3. Therefore, we refrain from a full analytical analysis of Equation (6.12d)^{III} and plot the function with the help of Mathematica, which is done by iteration in Figure 6.1. Equation (6.12d)^{III} is plotted in the $g_e - e$ plane for the same parameter set used in finding the steady state relations in Chapter 2[20]. For a discussion of these, please refer to Chapter 2. Additionally, the value of the Frisch parameter, ϑ, is set to 3 in line with Peterman (2014), who estimates the macro Frisch parameter to be between 2.9 and 3.1.

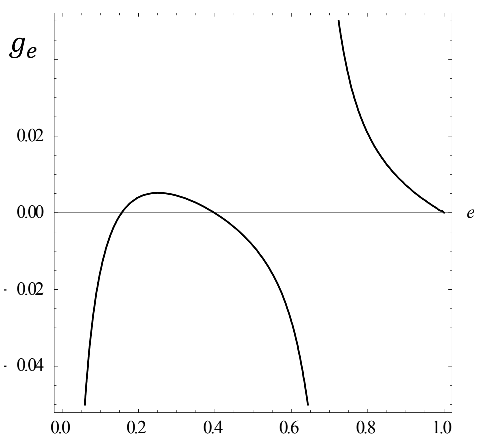

Figure 6.1– Dynamics in e_t

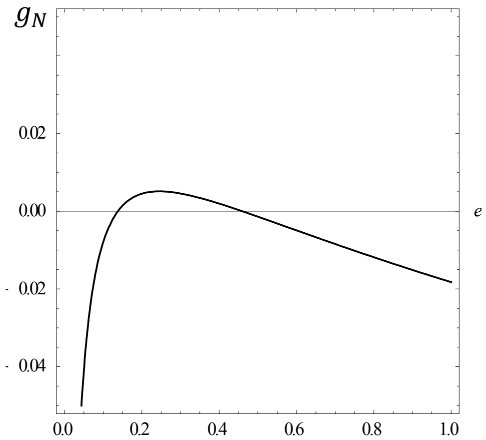

Figure 6.2 – Population Growth Rate

Figure 6.1 shows that with these parameter values there are two steady states[21]. With an exogenous interest rate of $r = 0.05$ the stable steady state value of e_* is 0.399. The interpretation is twofold again. $e_* = 0.399$ can be interpreted such that individuals choose to spend 39.9% of their active time in education, or that 39.9% of the active population are engaged in

[20]The parameter values are: $F = 0.055, g_N = 0.002, r = 0.05, \alpha = 0.6, \gamma = 0.268, \delta_h = 0.03, \epsilon = 0.834, \beta = 0.982, \sigma = 1.06$ and $\delta_k = 0.03$.

[21]Benhabib and Perli (1994) also find two steady states under certain parameter choices if the labor choice is endogenous. The two models are not directly comparable as Benhabib and Perli use slightly different physical and human capital production functions.

education. This slightly higher than the calibrated steady state value of the model in Chapter 2 for exogenous g_L, which was $e_* = 0.380$. This leads to the conclusion that with the same parameter values as before, being able to choose the active share of the population will lead to a higher educational share.

The lower, unstable, steady state of e_* is 0.156 which is significantly lower than that of the previous model, which was $e_1 = 0.305$. This indicates that if the economy started at a value of e_t at a little bit higher than 0.156, it would still be able to reach the higher steady state, whereas this was not possible in the previous model with an entirely exogenous development of the dependency ratio. As has been shown in Chapter 3.3, the empirical range for e_t is between 0.284 and 0.518. The lower steady state of $e_* = 0.156$ is outside the indicated data range. The switch in sign in Figure 6.1 around $e_t = 0.700$ indicates that if the economy were to start higher than $e_t = 0.700$ it would move towards full employment in education. This cannot be optimal, because no resources would be devoted to production, which would lead to no output and hence no consumption[22]. Given the empirical background, an educational share of roughly 70% or higher does not seem plausible. Mathematically, the implausible steady state of $e_t = 1$ is not defined.

Figure 6.2 shows Equation (6.12d)$^{\mathrm{III}}$ in the $g_N - e$ plane for $g_e = 0$ to show the influence of the population growth rate on e_t. The line represents the steady state values of e_t for different values of g_N. It shows that if the economy is in the empirically significant range, of e_t between 0.284 and 0.518, the relation between g_N and e_t is negative. This indicates that a lower population growth rate will lead to more education and less time in production. Note that this is only true if the interest rate is kept constant. A discussion of a change of the population growth rate with an endogenous interest rate follows in Chapter 6.4 and Chapter 6.5.

With the value of $e_* = 0.399$ and the given parameters, the growth rate of the labor force can be determined through equation (6.12e)$^{\mathrm{II}}$. It follows that:

[22] This is the case if the possibility of an entirely debt financed economy is excluded.

Steady State Relations	***From Equation***	***Numerical***
$\mathbf{1}+\boldsymbol{g_h}=F\,e^{\gamma}+(1-\delta_h)$	(6.6)	$\boldsymbol{g_h}\ =\mathbf{0.011}$
$\mathbf{1}+\boldsymbol{g_L}=(1+g_N)(1+g_h)^{\left(\frac{1}{\vartheta}+\frac{\epsilon}{\vartheta\alpha}\right)}\left(\frac{1}{\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}}$	(6.12e)[III]	$\boldsymbol{g_L}=\mathbf{0.00186}$
$\mathbf{1}+\boldsymbol{g_Y}=\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}(1+g_L)$	(6.4) and (6.12c)[I]	$\boldsymbol{g_Y}\ \ =\mathbf{0.024}$
$\mathbf{1}+\boldsymbol{g_K}=\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}(1+g_L)$	(6.12c)[I]	$\boldsymbol{g_K}\ \ =\mathbf{0.024}$
$\mathbf{1}+\boldsymbol{g_\omega}=\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}$	(6.3)[III]	$\boldsymbol{g_\omega}\ \ =\mathbf{0.009}$
$\mathbf{1}+\boldsymbol{g_\mu}=\left(\beta(1+r)\right)^{-1}$	(6.9)	$\boldsymbol{g_\mu}=-\mathbf{0.030}$
$\mathbf{1}+\boldsymbol{g_{\mu_h}}=\frac{1}{\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)}\left(F\,e_2^{\gamma}+(1-\delta_h)\right)^{\frac{\epsilon}{\alpha}}\ (1+g_L)$	(6.8)	$\boldsymbol{g_{\mu h}}=-\mathbf{0.019}$
$\mathbf{1}+\boldsymbol{g_c}=\left[\beta\left(1+r\left(\frac{B_t}{Y_t}\right)(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}}$	(6.7) and (6.9)	$\boldsymbol{g_c}\ \ =\mathbf{0.029}$

Table 6.1 – Steady States

Note to table: In steady state: $\boldsymbol{g_e}=\boldsymbol{g_{1-e}}=\mathbf{0}$ and $\boldsymbol{e}=\mathbf{0.399}$, parameter values are: $\boldsymbol{r}\left(\frac{\boldsymbol{B_{t+1}}}{\boldsymbol{Y_{t+1}}}\right)(\mathbf{1}+\boldsymbol{\eta_{rb}})=\mathbf{0.05},\boldsymbol{\alpha}=\mathbf{0.6},\boldsymbol{\delta_h}=\mathbf{0.03},\boldsymbol{g_N}=\mathbf{0.002},\ \boldsymbol{F}=\mathbf{0.055},\boldsymbol{\gamma}=\mathbf{0.268},\ \boldsymbol{\epsilon}=\mathbf{0.834},\boldsymbol{\sigma}=\mathbf{1.06},\boldsymbol{\beta}=\mathbf{0.982}$
For the transversality conditions to hold, the growth rate of $\beta^t\mu_tK_t$ and $\beta^t\mu_{ht}h_t$ must be negative. With $\beta=0.982$, the growth rate of β^t is -0.018 With $g_\mu=-0.030$ and $g_K=0.024$, the growth rate of the first expression is negative. The growth rate of the second product is also negative because $g_{\mu h}=-0.019$ and $g_h=0.011$. The transversality conditions are, hence, fulfilled.

$1+g_L=1.00186$. In the light of the previous model, this would imply a positive growth rate of the dependency ratio: $1+g_{1+D}=\frac{1+g_N}{1+g_L}=\frac{1.002}{1.00186}=1.00014$.

The steady state values are summarized in Table 6.1. The positive growth rate of the dependency ratio is the reason for the deviating steady state values from Chapter 2. In the steady state calculations of Chapter 2 g_{1+D} was set to zero.

6.4 Debt Dynamics

The above analysis has been conducted for a constant and given interest rate, which can be assumed for a small open economy. This assumption will now be relaxed as it is unlikely to be fulfilled for most countries analyzed in the sample, such as the US, Germany and France. In the risk mark-up interpretation it is also valid at high debt levels of small countries.

Therefore, to analyze the dynamics it is useful to assume a non-fixed interest rate which depends on the debt of the country[23]. An estimation of the debt-dependent interest rate has been conducted in Chapter 4.3.2. The same estimation will be used for this chapter's model, for details refer to Chapter 4.

$$r(b_t) = 0.815exp^{-0.564\log[2+b_t]^2+0.214\log[2+b_t]^3}(2+b_t)^{0.605}(1+\bar{r})^{0.566} - 1 \qquad (6.13)$$

Where $\boldsymbol{b_t} = \frac{\boldsymbol{B_t}}{\boldsymbol{Y_t}}$ and $\bar{\boldsymbol{r}}$ is the world interest rate, approximated by the US interest rate, which is on average 0.05. The elasticity of the interest rate with respect to the debt to GDP ratio, $\boldsymbol{\eta_{rb}}$, is derived from Equation (6.13) with $\boldsymbol{\eta_{rb}} = \boldsymbol{b_t}\frac{\boldsymbol{r'(b_t)}}{\boldsymbol{r(b_t)}}$.

$$\eta_{rb} = 20 * b_t\left(\frac{0.507exp^{-0.564\mathrm{Log}[2+b_t]^2+0.214\mathrm{Log}[2+b_t]^3}}{(2+b_t)^{0.396}} + 0.838(2+b_t)^{0.605}exp^{-0.564\mathrm{Log}[2+b_t]^2+0.214\mathrm{Log}[2+b_t]^3}\left(-\frac{1.127\mathrm{Log}[2+b_t]}{2+b_t}+\frac{0.642\mathrm{Log}[2+b_t]^2}{2+b_t}\right)\right)$$

The debt sector is characterized by the budget constraint:

$$B_{t+1} = N_t c_t + K_{t+1} - (1-\delta_k)K_t - Y_t + \left(1 + r\left(\frac{B_t}{Y_t}\right)\right)B_t \qquad (6.14)$$

Equation (6.14) is the equivalent to Equation (6.4) with ω_t and r_{Kt} at their equilibrium values shown in Equations (6.2) and (6.3) and, hence, with $\omega_t(1-e_t)h_tL_t + r_{kt}K_t = Y_t$.

Because the previously determined relations hold, K_{t+1} and K_t can be replaced by $K_{t+1} = \frac{1-\alpha}{r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})+\delta_k} Y_{t+1}$ from Equations (6.12c) and (6.12b) with $R_{Ht+1} = R_{Bt+1} = 1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})$ for their respective periods.

[23] Chapter 4.2 has shown that a fixed interest rate may lead to infinite consumption through infinite borrowing. The treatment for this problem was an interest rate depending on the debt to GDP ratio to stabilize the unstable debt process.

$$b_{t+1} = \frac{N_t c_t}{Y_{t+1}} - \frac{\theta}{1+g_Y} + \frac{1+r(b_t)(1+\eta_{rb})}{1+g_Y} b_t \qquad (6.14)^I$$

Where $\theta = (1-\delta_k)\frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k} + 1 - (1+g_Y)\frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}$ and b_t is constant in steady-state. Setting $\frac{N_t c_t}{Y_t} = X_t$ in Equation (6.14)^I together with Equation (6.12b) it must hold that

$$(1+g_b) = \frac{1}{1+g_y}\left(\frac{1}{b_t}X_t - \frac{1+\frac{1-\alpha}{r(b_t)(1+\eta_{rb})+\delta_k}\left((1-\delta_k)-(1+g_y)\right)}{b_t} + 1 + r(b_t)\right) \qquad (6.15)$$

In steady state, $g_b = 0$ and Equation (6.15) solved for X_t becomes

$$X_t = \left(1+g_Y - \left(1+r(b_t)\right)\right) b_t + \frac{(1-\alpha)(1+g_y)}{r(b_t)(1+\eta_{rb})+\delta_k}\left(\frac{(1-\delta_k)}{1+g_Y} - 1\right) + 1 \qquad (6.15)^I$$

Equations (6.15) and (6.15)^I are the equivalent of Equations (5.1) and (5.1)^I in Chapter 5. This chapter's growth rate of output is

$$1+g_Y = \left(F\,e_2^{\gamma} + (1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}(1+g_L)$$

with $1+g_L = (1+g_N)\left(F\,e_2^{\gamma} + (1-\delta_h)\right)^{\left(\frac{1}{\vartheta}+\frac{\epsilon}{\vartheta\alpha}\right)}\left(\frac{1}{\beta\left(1+r(b_t)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}}$.

Defining $\frac{N_t c_t}{Y_t} = X_t$ as the marginal propensity to consume, then $1+g_X$ by definition is

$$1+g_X = \frac{(1+g_N)(1+g_c)}{1+g_Y} = \frac{1+g_N}{\left(F\,e_t^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}(1+g_L)}\left[\beta\left(1+r(b_t)(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}} \qquad (6.16)$$

Where $1+g_y$ and $1+g_c$ are their respective steady state relations. In steady state $g_X = 0$, with the expression for $r(b_t)$ of Equation (6.13), Equation (6.16) becomes:

$$\frac{1}{\beta}\left(\frac{1+g_N}{\left(F\,e_t^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}(1+g_L)}\right)^{\sigma} = \left(1+r(b_t)(1+\eta_{rb})\right) \qquad (6.16)^I$$

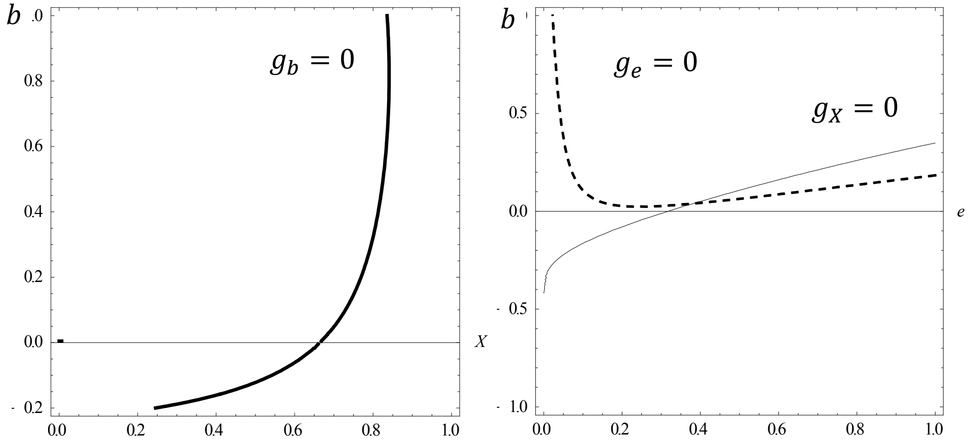

Figure 6.3 – Steady states in b_t, X_t and e_t

With the steady state expression for $\mathbf{1} + \boldsymbol{g_L}$ of Equation (6.12e)$^{\mathrm{I}}$, Equation (6.16)$^{\mathrm{I}}$ becomes

$$\beta^{\frac{\sigma}{\vartheta}-1}\left(\left(F\, e_t^{\gamma} + (1-\delta_h)\right)^{-\left(1+\frac{1}{\vartheta}\right)\left(1+\frac{\epsilon}{\alpha}\right)}\right)^{\sigma} = \left(1 + r(b_t)(1+\eta_{rb})\right)^{1-\frac{\sigma}{\vartheta}} \qquad (6.16)^{\mathrm{II}}$$

Equations (6.12d)$^{\mathrm{III}}$, (6.15) and (6.16) form the dynamic system for the debt sector. To illustrate the solution, Equation (6.15)$^{\mathrm{II}}$ is plotted in the $X - b$ plane in the left panel of Figure 6.3 and Equations (6.12d)$^{\mathrm{III}}$ and (6.16)$^{\mathrm{II}}$ are plotted in the right panel of Figure 6.3 in the $e - b$ plane. Their intersection denotes the steady state $e_* = 0.380$ and $b_* = 0.038$. With these values steady state $X_* = 0.694$ can be derived through equation (6.15)$^{\mathrm{II}}$. The value for e_* differs from the steady state value presented in Chapter 6.3, where $e_* = 0.399$. This is due to a change in the interest rate because of a lower debt. The interest rate associated with a value of $b_* = 0.038$, as above, is r =0.0496 instead of $r = 0.05$ (with $b = 0.042$), as assumed in Chapter 6.3. With these values, the growth rates of labor force growth changes to $1 + g_L = 1.0016$, leading to a growth rate of the dependency ratio of $1 + g_{1+D} = 1.00043$. Hence, a small open economy with no influence on the interest rate will choose a higher growth rate of the active population and a higher share of time devoted to education as opposed to an economy with a flexible interest rate, which is tied to their debt. Once there is the possibility to adjust the interest rate, the economy will reduce their time devoted to education and the share of the active population. This will increase the dependency ratio.

The comparison to the model of Chapter 2 is straight forward. Whereas the steady state time share of education does not differ much from the model with exogenous labor supply of Chapter 2[24], the optimal debt to GDP ratio is much lower, namely $b_{*} = 0.038$ here, compared to $b_{*} = 0.042$ in Chapter 2. The marginal propensity to consume is slightly higher, namely 0.693 in Chapter 2 compared to 0.394 here. Hence, by introducing a debt-dependent interest rate the economy can alter their debt to influence the interest rate, which may shift the problems of an ageing population into the debt market, when the size of the active population is determined endogenously.

6.5 Changes in the Population Growth Rate

This section exogenously alters the growth rate of population to analyze the reactions of the economy if $\boldsymbol{g_N}$ falls from 0.002 to 0.001. The economy reacts to an exogenous change in the population growth rate by altering the steady state time spent in education. This in turn alters the growth rate of the active population, $\boldsymbol{g_L}$, and with it the growth rates of capital and output.

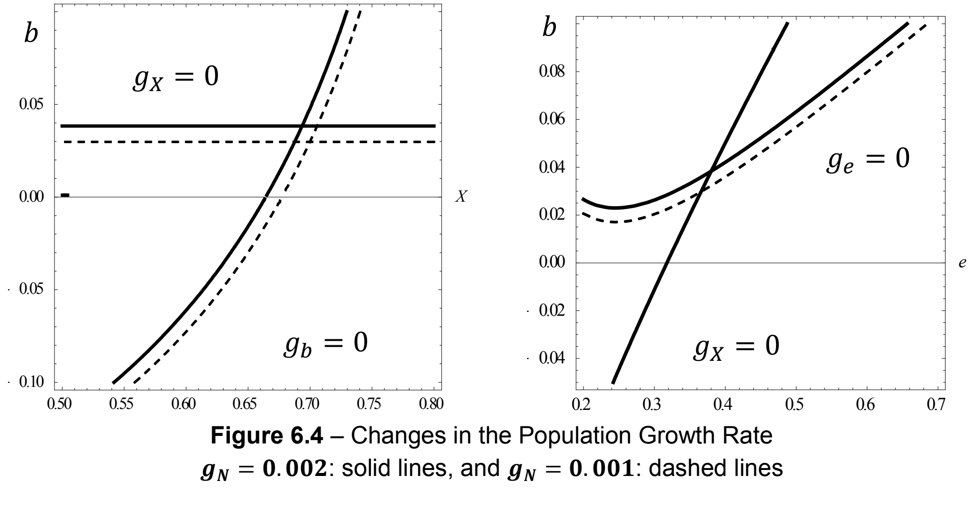

Figure 6.4 – Changes in the Population Growth Rate
$\boldsymbol{g_N = 0.002}$: solid lines, and $\boldsymbol{g_N = 0.001}$: dashed lines

[24] There is a slight difference, which is only visible in the 4th decimal place.

g_N	g_L	e_t	b_t	X_t	$r_t(1+\eta)$	$1+g_c$
0.003	0.00256	0.3953	0.0473	0.6875	0.0507	1.0298
0.002	0.00157	0.3801	0.0383	0.694	0.0496	1.0287
0.001	0.00059	0.3658	0.0297	0.6999	0.0484	1.0277
0	-0.00040	0.3523	0.0216	0.7062	0.0473	1.0267
-0.001	-0.00138	0.3396	0.0137	0.7128	0.0463	1.0257
-0.002	-0.00237	0.3276	0.0062	0.7196	0.0453	1.0248
-0.003	-0.00327	0.3213	-0.00002	0.7241	0.0444	1.0240

Table 6.2 – Changes in the Population Growth Rate

Figure 6.4 shows the movement of the $\mathbf{g_e}=\mathbf{0}, \mathbf{g_b}=\mathbf{0}$ and $\mathbf{g_X}=\mathbf{0}$ curves if $\mathbf{g_N}$ jumps from $\mathbf{0.002}$ to $\mathbf{0.001}$. The solid lines in Figure 6.4 are the same as the lines in Figure 6.3 and are drawn for a population growth rate of 0.002. The dashed lines in Figure 6.4 show the relations for a lower population growth of 0.001.

Table 6.2 shows the response of some measures of the economy to an exogenously changing growth rate of population. At first sight, when only considering the time shares in education, the results seem counter intuitive. When population growth decreases, as is the case in the recent demographic developments, the best response for education is a decrease in the time devoted to education, leaving more time in production. This is the exact opposite of what has been predicted in the previous model of Chapter 2. A similar logic applies to the other variables, such as the marginal rate of consumption, or the interest rate. But, by comparing the first two columns it becomes apparent, that the optimal response of the growth rate of the active population is always a little lower than the population growth rate. Table 6.3 shows the corresponding values of the growth rate of the dependency ratio. The growth rate of the active population does not increase proportionally with the growth rate of the total population. This means that the growth rate of the dependency ratio increases with increasing population growth. Taking this into account, the model at hand, with endogenized labor supply, does not contradict the previous model with exogenous population development, as $\boldsymbol{e_*}$ increases if the growth rate of the dependency ratio increases.

Table 6.3 shows that if $\boldsymbol{g_N}$ decreases, the optimal growth rate of the dependency ratio, which is defined as $\mathbf{1}+\boldsymbol{g_{1+D}}=\frac{\mathbf{1}+\boldsymbol{g_N}}{\mathbf{1}+\boldsymbol{g_L}}$, decreases as well.

g_N	g_L	g_{1+D}
0.003	0.00256	0.00044
0.002	0.00157	0.00043
0.001	0.00059	0.00041
0	-0.00040	0.00040
-0.001	-0.00138	0.00038
-0.002	-0.00237	0.00037
-0.003	-0.00327	0.00027
-0.004	-0.00434	0.00034
-0.005	-0.00443	-0.00058

Table 6.3 – Changes in the Population Growth Rate and the Dependency Ratio

If the growth rate of the population rate falls, the growth rate of the dependency ratio will eventually become negative. If g_N increases, it is not necessary to work longer, hence, the dependency ratio increases.

This analysis has shown that a decreasing population growth rate, as it is experienced in many developed countries nowadays, should optimally lead to a smaller growth rate of the dependency ratio. This is executed through more people in the active population as opposed to inactive. This means that there should be policies, like the possibility of a flexible retirement age, to ensure this flexibility.

6.6 Conclusion

This chapter has shown how an exogenous change of the population growth rate affects the supply of labor, the time spent in education, the marginal propensity to consume and debt in an Uzawa-Lucas growth model. It has shown that a zero growth of labor supply in times of changing population growth is not the right choice.

In this framework it is questionable whether an exogenous jump of the population growth rate, e.g. from 0.002 to 0.001, is realistic and if these results hold if the growth rate of the total population is endogenized. Chapter 7 presents some thoughts on the development of population growth and its determinants and Chapter 8 will incorporate this expression for g_N this chapter's model.

7

An Empirical Glance at Population Growth

7.1 Introduction

Chapter 6 has shown the economy’s reaction to an exogenous shock of the population growth rate in the light of an Uzawa-Lucas growth model with a Frisch-elasticity on labor supply. This was an important intermediate step to analyze the economy’s reaction to changing population dynamics. Since the population growth rate was still given exogenously, it only gives partial insights. Because of this it was only possible to analyze jumps in the population growth rate, where a slow adjustment seems more realistic. This chapter will take a glance at the empirical development of population growth. To cover a decent array of possibilities, two approaches will be chosen and compared. The first approach analyzes the influence of several socio-economic variables and health parameters on population growth which are taken from the literature. The motivation lies in the complexity of population development and the hope to be able to capture the relevant effects. Since a large amount of control variables is taken into account, this model will be called “the extensive model”. The second approach is closer to the analytical model and shows that suitably regressing the main variables, education and income, yields results with a similar fit to the extensive model. Because of the smaller number of regressors this model will be called “the reduced model”. The advantage of such a reduced model lies in its simplicity which makes it attractive for economic growth models, such as the Uzawa-Lucas growth model. This form of a reduced model which is compliable with macroeconomic growth models has not yet been analyzed. Both approaches will be estimated for 18 OECD countries, consisting of the 16 countries of Chapter 3 and, additionally, Belgium and Austria. The aim of this chapter is to increase the understanding of the dynamics of population development and to find an expression that can later be fed into the model of Chapter 6.

The chapter is structured as follows: First a theoretical background is presented with the corresponding literature in Chapter 7.2. Chapter 7.3 sets up the two models to be estimated. Chapter 7.4 summarizes the data at hand. Chapter 7.5 describes the estimation method. Chapter 7.6 reports the results for both approaches and compares the results. Chapter 7.7 concludes.

7.2 Literature and Theory

Throughout the past centuries, the evolution of population development has given rise to a discussion of whether current economic performance will be able to keep up with a changing population structure. Falling birth rates in almost all parts of the world combined with higher life expectancy raise concern whether there will be a large enough work force to support the entire population. The question of how demographic change influences economic behavior and growth covers a large body of literature in which different measures for population development are implemented into theoretical (growth) models to analyze economic development. Population development can be divided into two phenomena treated in the literature: Changes in the *size* of the population and changes in the *age structure*. Generally speaking, the growth rate of population is determined by $1 + g_N = b_t - d_t + m_t$, where b_t is the birth rate, d_t is the death rate and m_t is the net migration rate. The three measures allow for changes in the age structure and population size and are either endogenized alone or in combination. They allow for changes in the age structure and are either endogenized alone or in combination. It is important to know which “end” of the population has been manipulated as this can have the same predictions on population structure, but different effects on the population size. Lower birth rates (with constant death rates), for instance, will lead to ageing with a decreasing population size, whereas lower death rates (with constant birth rates) will lead to ageing with an increasing population size. It is hence important to always keep the purpose of the endogenization in a macroeconomic model in mind.

The majority of the literature treats the three factors separately. This is why in the following the literature estimating the birth rates and death rates is reviewed separately to be able to extract the important determinants in their development, some of them may overlap. It is the goal of this chapter to find an expression for population development in general. The role of the net migration rate is acknowledged, but migration will still be seen as exogenous. Reasons for this are the presence of political agreements and quotas. An endogenization is not suitable for this purpose. First, the literature dealing with the endogenization of the birth rates is examined in Chapter 7.2.1 and the relevant parameters are extracted. Then the same is done for the literature dealing with death rates in Chapter 7.2.2. Given the large body of literature on the topics of population development and economic growth, the review below is far from complete, but summarizes the main aspects.

7.2.1 Birth Rates

Falling birth rates and lower fertility are observable throughout all income levels[25]. Both measures can sometimes be used interchangeably; however, there is a crucial difference. Whereas the fertility rate indicates the number of children per woman in the population, the birth rate gives the number of births per citizen (or mostly per 1000). Because "birth rate" is a wider used term, the expression "birth rate" will be used whenever both terms are valid.

The existing literature distinguishes between developed and developing countries. In poor countries, an increase in the mother's education, family planning, and higher hygienic standards leads to a decrease in birth rates (Caldwell, 1979). With a higher share of women with basic education, the number of unwanted children will decrease. Murtin (2013) has shown that education has a negative effect on the birth rate in developing countries with low levels of average education. He argues that an increase in primary education decreases the birth rate. Increasing hygienic standards lead to a decreasing death rate of infants which has an immediate negative impact on the birth rate (Angeles, 2010; Barro & Becker, 1989). In richer countries falling fertility rates are associated with different reasons. Brückner and Schwandt (2013) argue that an increase in the technology level increases

[25] This is the trend throughout history starting around 1800 (Galor, 2005).

the opportunity cost of children. In this framework the time constraint of parents is taken into account. They face a trade-off between working and raising their children (Galor & Weil, 1998). With increasing education, and the associated wage increase of the parents, especially of the mother, the opportunity cost of raising children increases even further (Galor & Weil, 1993). This time-trade-off eventually results in a trade-off of “quantity” versus “quality” children (Becker, Murphy, & Tamura, 1994) and can only occur because children are seen as normal good (Brückner & Schwandt, 2013). Families with higher incomes face higher trade-offs of their time and prefer higher educated children (De La Croix & Doepke, 2003). Since children with higher education cost more time, the number of children will be reduced. This reduces fertility in the long run.

Earlier economic models examine the effect of an exogenous shock in the birth rate (Hviding & Mérette, 1998), but several recent studies endogenize birth rate choices to predict population development and the changes it causes in the economy. This endogenizing has been done via educational choices of the individual (c.f. Prettner, Bloom, and Strulik (2013)), educational choices of the parent (Alders & Broer, 2005; Liao, 2011), human capital formation of society (Ludwig, Schelkle, & Vogel, 2012) or child policies (Fanti & Gori, 2009).

From this theoretical background it can be extracted that next to education, measures for female labor force participation, early child loss (e.g. infant death rate), and income are important in the determination of population growth for the 18 OECD countries that are used for the analysis. Variables capturing these effects will be used in the analysis. A description of the variables used follows in Chapter 7.4.

7.2.2 Death Rates

Similar to falling fertility rates, falling death rates are observed in most countries, given that they are not facing times of war, epidemics or other social and political disturbances resulting in a higher than natural death rate. The two measures “mortality rate” and “life expectancy”, both used to model population development, are closely related and can sometimes be used interchangeably. It is crucial to note that they have different implications. The “mortality rate” is the number of people that die within a given year (usually denoted per 1000 individuals) and is a measurable number that can be observed every year. “Life expectancy”, on the other hand, is a calculated number that predicts the years the average individual of any given cohort is expected to live at the time of birth. This number however may change throughout the decades for the same cohort due to, for example, unforeseen technical and medical advancements, or war and warlike conflicts. R. D. Lee and Carter (1992) set up what has been described by Backus, Cooley, and Henriksen (2013) as “the leading model of mortality forecasting in the demographic literature”. They forecast mortality and life expectancy of the United States by fitting a model to the death rate matrix. This, however, is a time series analysis. While it is very useful for forecasting, it gives no insights into other determinants of falling death rates. French (2014) develops a model similar to Lee and Carter which shows the diffusion of health technology between the US and the UK. As possible explanatory variables he uses pharmaceutical expenditure, education, smoking, alcohol, health expenditure and GDP. Of these he finds only pharmaceutical expenditure and education, expressed as average years of schooling, to be significant. Cutler, Deaton, and Lleras-Muney (2006) find several reasons for the observable mortality decline, including improved nutrition, public health, urbanization, vaccination and medical treatments. They conclude that the key factors are knowledge, science and technology. Papageorgiou, Savvides, and Zachariadis (2007) find empirical evidence for the determinants of life expectancy to be health technology and its diffusion from R&D performing to non-R&D performing countries. Castelló-Climent and Domenech (2008) and Bhargava, Jamison, Lau, and Murray (2001) find evidence for the relation between life expectancy, health and human capital.

Based on the above summarized theory, several values capturing health, health expenditure and urbanization should be incorporated in the estimations. For a detailed description of the variables please refer to Chapter 7.4.

7.3 The Models

Based on the literature background of Chapter 7.2, this chapter develops the two approaches to be estimated. In the first part, Chapter 7.3.1, the extensive model is presented in which the possible determinants that were analyzed in Chapter 7.2 are included and the second part, Chapter 7.3.2, deals with the reduced model and its motivation.

7.3.1 The Extensive Model

The literature review of Chapter 7.2 shows how population development is assumed to depend on a variety of factors. These factors can be summarized in three categories: health status, health expenditure and socio economic factors. It is in the nature of this extensive model, that specific predictions about the final estimation cannot be made, as this chapter is dedicated to finding the significant factors that determine population growth. Most broadly, the estimations can be summarized as:

$$g_{N_{it}} = \beta_{0i} + \beta_{1i} * g_{Ni(t-1)} + \beta_{2i} * socio\ economic\ factors_{it} + \beta_{3i} * health\ status_{it} + \beta_{4i} health\ expenditure_{it} + \eta_i + \epsilon_{it} \tag{7.1}$$

Where β_{0i} is the unobserved time-invariant individual effect, ϵ_{it} is a disturbance term and η_i is the unobservable individual effect. $g_{N_{it}}$ is the dependent variable "population growth rate", its lag is included in the regression. $health\ status_{it}$, $health\ expenditure_{it}$ and $socio\ economic\ factors_{it}$ represent the variables of the three categories.

7.3.2 The Reduced Model

The reduced model is characterized by its limited use of variables. It will be shown, that a good estimation with just the variables of the model of Chapter 6 can compete with the estimations of the extensive model. It is common practice to include the lagged dependent variable into a dynamic macroeconomic model as the lag tends to be significant and its inclusion will thus lower the omitted variable bias. This is especially important as this model only uses very few explanatory variables. The reduced model analyses two similar estimations:

$$\log(1+g_{Nit}) = \beta_{0i} + \sum_x \beta_{1i} \log\left(1+g_{N,t-x}\right) + \sum_x \beta_{2i} \log(e_{t-x}) + \sum_x \beta_{3i} \log(e_{t-x})^2 + \sum_x \beta_{4i} \log\left(g_{y_{t-x}}\right) + \eta_i + \epsilon_{it} \quad (7.2)$$

$$\log(1+g_{Nit}) = \beta_{0i} + \sum_x \beta_{1i} \log\left(1+g_{N,t-x}\right) + \sum_x \beta_{2i} \log(e_{t-x}) + \sum_x \beta_{3i} \log(e_{t-x})^2 + \sum_x \beta_{4i} \log(Y_{t-x}) + \eta_i + \epsilon_{it} \quad (7.3)$$

Where x indicates the significant lags and β_{0i} is the unobserved time-invariant individual effect, ϵ_{it} is a disturbance term and η_i is the unobservable individual effect. g_{Nit} is again the population growth rate, e_{it} is the educational share as defined in the model and Y_{it} and $g_{y_{it}}$ are the income per capita levels and growth rates. Estimating Equations (7.2) and (7.3) in levels instead of growth rates yields no significant results.

7.4 The Data

In the analysis, the 16 OECD countries used in previous chapters are used again. They are Australia, Canada, Denmark, Spain, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Sweden, the UK and the US. Additionally, the data of Belgium and Austria are used. They were in the original data set of Chapter 3, but then cancelled out because of mismatching data. As derived in Chapter 7.3, the variables used in the extensive model can be divided into several sub categories, namely the dependent variable population growth and the explanatory categories health status, health expenditure and socio-economic factors. The reduced model only uses population growth, education, GDP per capita and its growth rate. This section will explain the variables used for each category and state their origin as well as their particularities. Additionally it will provide descriptive statistics.

The dependent variable for all regressions is the growth rate of population, denoted by g_{Nit}. It is the variable named “Population growth (annual %)” extracted from the World Development Indicators. To be closer to the models of Chapters 2 and 6, it is expressed in decimals, rather than per cent. The Netherlands, thus, has a population growth rate of 0.005, rather than 0.5% in 2010. All other variables provided in per cent of population are expressed in the same way. The variables used for the other categories are listed in Table 7.1.

Of the socio-economic variables, the variable for education, e_{it}, is the constructed variable of Chapter 3, which measures the share of the active population currently engaged in education. It is chosen because of its proximity to the model at hand. For further information on the construction and sources, please refer to Chapter 3.2. Note, that in this chapter two more countries are analyzed, namely Belgium and Austria. Both countries did not have matching data for the analysis in Chapter 3.

The other socio-economic variables are made available by the World Development Indicators: "Labor force participation rate, female (% of female population ages 15+)", $lfpf_{it}$, "Urban population (% of total)", urb_{it}, GDP per capita growth (annual %), g_{yit} and GDP per capita, Y_t. The variables g_{yit} and urb_{it} are, much like the growth rate of population, expressed in decimals rather than per cent.

Socio economic factors	Health	Health expenditure

- Education indicators
- Female labor force participation rate
- Urbanization
- Income growth or levels
	- Diseases/health threats:
	  - AIDS/HIV
	  - Cancer
	  - Diseases of the blood and blood-forming organs
	  - Diabetes mellitus
	  - Diseases of the nervous system
	  - Diseases of the circulators system
	  - Tuberculosis
	  - Cholesterol level
	- Drug consumption:
	  - Alcohol
	  - Tobacco
	- Early child mortality:
	  - Infant mortality
		- Health expenditure per capita
		- Health expenditure as % of GDP

Table 7.1 –Variables Extensive Model

The health indicators can be divided into several subcategories. Those are diseases/health threats, drug consumption and early child mortality. The category “diseases/health threats” is covered by eight variables presented below. The following variables are extracted from the OECD Health status data
set for the years 1985-2010: $AIDS_{it}$, is the variable “Deaths per 100,000 population (standardised rates) because of AIDS/HIV”, $cancer_{it}$ represents “Deaths per 100,000 population (standardised rates) because of malignant neoplasms”, the variable $blood_{it}$ stands for variable “Deaths per 100,000 population (standardized rates) because of Diseases of the blood and blood-forming organs”, $diab_{it}$is taken from “Deaths per 100,000 population (standardized rates) because of Diabetes mellitus”, $nerv_{it}$ is the “Deaths per 100,000 population (standardized rates) because of Diseases of the nervous system”, and $heart_{it}$ is the variable for “Deaths per 100,000 population (standardized rates) because of diseases of the circulatory system”. Additional to those measures, the two variables tuberculosis and cholesterol are added. Tuberculosis, tub_{it}, is measured as “Incidences of tuberculosis per 100 000” from the World development indicators. $colm_{it}$ and $colf_{it}$ are the “Mean Total Cholesterol (age-standardized estimate), Male and female” from the World Health Organization.

The section “drug consumption” of the category health covers the variables alc_{it}, “Alcohol consumption” in liters per capita of the population 15+, and $tobac_{it}$, the tobacco use in grammes per capita of the population 15+. Both are retrieved from the OECD dataset “Non-Medical Determinants of Health”.

The last section of the health category is “early child mortality”. It is represented by the measure “Mortality rate, infant”, ifd_{it} per 1000 live births, provided by the World Development Indicators.

The category “health expenditure” covers the two variables health expenditure per capita (in current US$), $hegdp_{it}$, and health expenditure, total (% of GDP), $hepc_{it}$, both taken from the world development indicators. Table 7.2 summarizes the descriptive statistics of the twenty variables mentioned above. The time frame for all variables is from 1985 to 2010.

	Mean	Maximum	Minimum	Observations
$AIDS$	1.950000	15.80000	0.000000	422
alc	10.88077	17.30000	5.800000	468
$blood$	3.050552	6.200000	0.600000	453
$cancer$	237.2219	312.4000	176.9000	453
$colf$	5.476000	6.100000	4.900000	450
$colm$	5.508222	6.100000	5.000000	450
$diab$	20.10971	44.00000	6.100000	453
e	0.365004	0.517826	0.284286	334
g_N	0.006057	0.028910	-0.004277	468
g_Y	0.019637	0.096727	-0.087067	468
$heart$	422.2477	761.1000	177.0000	453
$hegdp$	9.198619	17.08369	5.566749	288
$hepc$	3109.307	8317.486	875.7570	288
ifd	6.005342	16.50000	1.900000	468
$lfpf$	0.501190	0.712000	0.287000	468
$nerv$	23.33576	85.70000	7.300000	453
$tobac$	1978.396	3741.000	660.0000	316
tub	12.52367	70.78000	0.000000	444
$u5d$	7.200000	20.40000	2.400000	468
urb	0.761631	0.976410	0.452980	468

Table 7.2 – Descriptive Statistics

Eighteen countries are analyzed. This gives a panel of $T * N = 26 * 18$. This may change in the estimated regressions, because missing data might lead to the dropping of some cross sections. If all data were available, each variable should have a count of 468 observations. This is true for 7 of the 20 variables, as can be seen in Table 7.2. The lowest data availability is that of the category “health expenditure”. Both variables, $hepc$ and $hegdp$, have 288 observations and cover merely 61.5% of the possible observations. This may be problematic in a panel since the data is distributed unevenly. The other variables seem to be okay with a data share of at least 70% of the possible observations.

7.5 Estimation Method

Given the fact that all countries in the panel are OECD countries and that 15 of those are members of the European Union, it is likely that they may observe common shocks and, hence, are likely to exhibit cross-sectional dependence in the errors. In this case it is not wise to test the stationarity of the variables in one country without taking the others into account. For this reason, rather than the single-series unit root tests, panel unit root tests are applied here. For further explanations, please refer to Appendix 3.1. Appendix 7.1 shows strong evidence for the existence of a unit root for six out of the 20 variables and some evidence for another six. Six variables show no sign of an I(1) series, whereas two show mixed results. Phillips (1986) found that a false significant relationship between two I(1) series is likely to be found if only the levels of the I(1) series are included in a regression. He finds misleading results for conventional Wald tests if the model is estimated in levels. Phillips and Sul (2003) point out that ignored cross-sectional dependence may decrease the estimation efficiency. Pesaran (2004) developed a test for small panels. The test statistics are valid for both $T \to \infty$ and $N \to \infty$. This makes it superior to the Lagrange Multiplier test by Breusch and Pagan (1980). Results for the estimated equations are presented in Appendix 7.2.

If a linear combination of two (or more) I(1) series is stationary, the variables are said to be cointegrated (Engle & Granger, 1987). The previously found false significant relationship by Phillips (1986) is not apparent in cointegrated series. If the series are cointegrated, the Ordinary Least Square (OLS) estimation is consistent. It is, however, not wise to use a static OLS (SOLS) estimation as the estimators are asymptotically biased and asymmetric when analyzing a panel, or time series (Hamilton, 1994). A preferred estimation method needs to be able to deal with endogeneity and serial correlation in the sample that may be present in cross-section fixed effects as well as in the short run dynamics. There are several methods to overcome the issues of the SOLS estimations when analyzing single equation cointegration relationships. This chapter focuses on Fully Modified OLS (Pedroni, 2000; Phillips & Hansen, 1990).

Phillips' and Hansen's (1990) Fully Modified OLS (FMOLS) and the Dynamic OLS (DOLS) (Saikkonen, 1992; Stock & Watson, 1993) both eliminate the asymptotic bias of the SOLS. Whereas FMOLS is an estimation able to cope with the problem of serial correlation and endogeneity, DOLS takes the lag of the first differences in order to control for endogeneity (Saikkonen, 1991). The advantage of the FMOLS estimation is the absence of nuisance parameters of serial correlation patterns in the corresponding asymptotic distribution. This leads to an unbiased estimator for the standard case as well as the fixed effects model (Pedroni, 2000). The DOLS estimator takes lags and leads of the regressors such that the error term of the cointegrating equation is orthogonal to the regressors. This is based on the assumption that adding lags and leads soaks up the long run correlation between the errors (Saikkonen, 1992). The disadvantage is the loss of data corresponding with the number of lags and leads. In small samples like this one this may cause severe sample-shrinking which will make many estimations impossible. For this reason only the FMOLS estimator is applied.

7.6 Results

This chapter summarizes the results estimated in FMOLS. Chapter 7.6.1 shows the results of the extensive model and Chapter 7.6.2 deals with the results for the reduced model. Chapter 7.6.3 compares the two approaches and concludes with an analysis of the representation which will be chosen to be introduced into the Uzawa-Lucas growth model in Chapter 8.

7.6.1 Results of the Extensive Model

Equation (7.1) summarizes the type of model analyzed in this section. The variables presented in Chapter 7.4 represent the three categories health status, health expenditure and socio-economic variables. Including the variables of the category health expenditure, "health expenditure per capita" or "health expenditure per GDP", yields a near singular matrix for most of the regressions applied. In the regressions that were computable, the cross-section count was very low (5-7 countries out of 18). This is why this category had to be eliminated in the following regressions.

g_N	Model 1	Model 2	Model 3	Model 4	Model 5
$g_{N(t-1)}$	16.220 (3.191)***	1.818 (0.130)***	0.841 (0.055)***	0.766 (0.045)***	0.711 (0.036)***
e_t	46.651 (10.798)***	-0.351 (0.221)	-0.080 (0.056)		
e_t^2	-76.198 (17.275)***	0.552 (0.289)*	0.074 (0.074)	-0.027 (0.011)**	-0.040 (0.007)***
alc_t	-0.020 (0.009)*	0.001 (0.0004)**	0.0004 (0.0002)***	0.0003 (0.0002)*	0.0003 (0.0001)***
$colf_t$	0.134 (0.060)*	0.001 (0.004)	0.004 (0.002)**	0.003 (0.002)**	0.003 (0.001)**
$colm_t$	-0.131 (0.064)*	0.002 (0.006)	-0.007 (0.002)***	-0.007 (0.002)***	-0.006 (0.002)***
g_{Yt}	-0.208 (0.122)	0.019 (0.011)*	0.008 (0.004)*	0.007 (0.004)*	0.008 (0.003)**
urb_t	-0.015 (0.010)	0.0003 (0.0001)**	-0.0001 (6.73E-05)**	-0.0001 (6.29E-05)**	-0.0002 (5.18E-05)***
$nerv_t$	-0.005 (0.001)**	-0.0002 (7.29*10^-05)***	-4.31E-05 (2.22E-05)*	-7.87E-06 (1.90E-05)	
tub_t	0.003 (0.003)	-0.001 (0.0002)***	-0.0001 (8.66E-05)*	1.89E-07 (6.21E-05)	
$blood_t$	-0.048 (0.011)***	-0.001 (0.0004)**	-0.0001 (0.0002)		
$lfpf_t$	-0.032 (0.008)***	-0.001 (0.0001)***	-0.0001 (8.74E-05)		
$cancer_t$	-0.012 (0.002)***	1.31*10^-05 (4.59*10^-05)			
$diab_t$	0.031 (0.004)***	8.25*10^-05 (0.0001)			
$heart_t$	0.001 (0.0003)***	-1.46*10^-05 (1.24*10^-05)			
ifd_t	0.019 (0.010)	0.0003 (0.001)			
$AIDS_t$	0.001 (0.001)				
$tobac_t$	1.39*10^-05 (1.60*10^-05)				
Number of Observations	38	138	176	196	236
Periods	20	20	20	20	20
Cross-sections	2	8	11	13	16
Adjusted R²	-51.281	0.736	0.848	0.870	0.886
S.E. of regr.	0.027174	0.002176	0.002026	0.001896	0.001711

Table7.3 – Estimation of Equation (7.4)

Standard errors are reported in parentheses below the estimate. The significance levels are expressed as follows: no star: not significant at 10%, * significant at the 10% level ** significant at the 5% level *** significant at the 1% level

The exact representation for the first estimation is:

$$g_{Nit} = \beta_{01} + \beta_{1i}e_{it} + \beta_{2i}e_{it}^2 + \beta_{3i}AIDS_{it} + \beta_{4i}alc_{it} + \beta_{5i}blood_{it} + \beta_{6i}cancer_{it} + \beta_{7i}colf_{it} + \beta_{8i}colm_{it} + \beta_{9i}diab_{it} + \beta_{10i}g_{y,it} + \beta_{11i}heart_{it} + \beta_{12i}ifd_{it} + \beta_{13i}lfpf_{it} + \beta_{14i}nerv_{it} + \beta_{15i}tobac_{it} + \beta_{16i}tub_{it} + \beta_{17i}urb_{it} + \eta_i + \phi_t + \epsilon_{it} \quad (7.4)$$

Table 7.3 shows the estimation results in which Equation (7.4) has been estimated as Model 1. The following models are obtained by successively eliminating the most insignificant variables such that Model 5 only contains significant variables. The models are estimated in FMOLS because for most models, there was no data left after differencing for DOLS due to the small sample size.

Model 5 in Table7.3 is the only estimated model with only significant variables and is chosen as the best model to represent this relationship. The bad fit of Model 1 and the coefficient of the lagged dependent which is way above 1 can be explained by the number of included cross-sections. In Model 1 only two countries have available data for all included variables. This makes Model 1 very weak. Reducing the number of variables by eliminating the insignificant ones helps increasing the number of cross-sections. The coefficient of the lagged dependent variable, $g_{N(t-1)}$, in Model 2 is still above 1, indicating an instable system. This can also be explained by the low cross-section count of only eight countries. For this data set it is important to keep the balance between the fit of the regression and the included cross-sections. In addition, fewer observations and more regressors together make the education variable insignificant, which is at odds with the literature.

The full representation of Model 5 is:

$$g_N = c_i + 0.711 * g_{N(t-1)} - 0.040 * e^2 + 0.0003 * alc + 0.003 * colf - 0.006 * colm + 0.008 * g_Y - 0.0002 * urb \quad (7.5)$$

Equation (7.5) suggests a negative relationship between education and population growth. The constant c_i is country specific and, hence, not reported in this general representation. The influence of the other four control variables is rather low, though significant. Whereas a higher cholesterol level of men and a higher rate of urbanization decrease population growth, an increasing alcohol consumption, a higher cholesterol level of women and higher income per capita growth seem to increase population growth.

7.6.2 Results of the Reduced Model

Given the results of Chapter 7.6.1, in which four control variables were significant, but with low coefficients, this chapter will try to only include education, production (GDP per capita) and their lags to reach a comparable model. In the light of the models of Chapters 2 and 6 the reduced model is easier to handle and more accurate. In this section, Equation (7.2) and (7.3) are estimated. The extensive forms are reported in Table 7.4.

Model 6	$\log(1+g_{Nit}) = \beta_{0i} + \beta_{1i}\log\left(1+g_{Ni(t-1)}\right) + \beta_{2i}\log\left(1+g_{Ni(t-2)}\right) + \beta_{3i}\log(e_{it}) + \beta_{4i}\log\left(e_{i(t-1)}\right) + \beta_{5i}\log(e_{it})^2 + \beta_{6i}\log\left(e_{i(t-1)}\right)^2 + \beta_{7i}\log\left(g_{y_{it}}\right) + \beta_{8i}\log\left(g_{y_{i(t-1)}}\right) + \eta_i + \epsilon_{it}$
Model 7	$\log(1+g_{Nit}) = \beta_{0i} + \beta_{1i}\log\left(1+g_{Ni(t-1)}\right) + \beta_{2i}\log\left(1+g_{Ni(t-2)}\right) + \beta_{3i}\log(e_{it}) + \beta_{4i}\log\left(e_{i(t-1)}\right) + \beta_{5i}\log(e_{it})^2 + \beta_{6i}\log\left(e_{i(t-1)}\right)^2 + \beta_{7i}\log(y_{it}) + \beta_{8i}\log\left(y_{i(t-1)}\right) + \eta_i + \epsilon_{it}$

Table 7.4 – Models to be estimated

Models 6 and 7 are quite similar, as they both assume that the growth rate of population depend on education some measure of output. Given the theoretical background of Chapter 6 a log-quadratic relationship is assumed. Other forms, like log-linear, or just quadratic have been tested with insignificant results. Models 6 and 7 differ with respect to the way output is included into the estimations. Whereas Model 6 assumes a relation between the growth rate of population and several lags of level of output, Model 7 implies a relationship between the growth rate of population and the growth rate of output.

All models are estimated with two lags of the dependent variable. Including more lags is either insignificant, or reduces the cross-section count to 2-3 cross sections with a very low adjusted R^2. Table 7.5 reports the results for Model 6 and Model 7. The insignificant lags are successively eliminated.

Both approaches eventually lead to the same estimation. Model 6.2 and 7.2 are the same, with

$$\log(1+g_{Nit}) = -0.022 + 1.109\log\left(1+g_{Ni(t-1)}\right) - 0.379\log\left(1+g_{Ni(t-2)}\right) - 0.038\log(e_{it}) - 0.015\log(e_{it})^2 \tag{7.6}$$

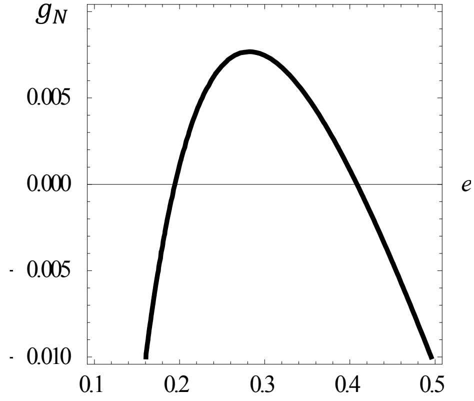

Figure 7.1 – Model 6.2 and 7.2

The constant -0.022 stems from the fixed effects coefficient. Interestingly, including neither the level of output, nor its growth rate, or their lags turns out to be significant in this panel. Solving Equation (7.5) for $\log(1+g_N)$ in steady state, where $(1+g_{Nit}) = \left(1+g_{Ni(t-1)}\right) = \left(1+g_{Ni(t-2)}\right)$, yields:

$$\log(1+g_{Nit}) = \frac{-0.022 - 0.038\log(e_{it}) - 0.015\log(e_{it})^2}{0.27} \tag{7.7}$$

To be comparable to Figure 6.2, Equation (7.7) is solved for steady state $(1+g_{Nit})$ rather than $\log(1+g_{Nit})$:

$$(1+g_{Nit}) = e_{it}^{-\frac{0.038}{0.27}} \exp\left[\frac{-0.022}{0.27} - \frac{0.015}{0.27}\log(e_{it})^2\right] \tag{7.8}$$

Equation (7.8) shows a hump-shaped relation between g_N and e_t and is displayed in Figure 7.1, which is similar again to Figure 6.2 from the calibrated theoretical model. It has a maximum at $e_t = 0.282$. Inserting the

$\log(1+g_{Nit})$	Model 6	Model 6.1	Model 6.2	Model 7	Model 7.1	Model 7.2
$\log(1+g_{Ni(t-1)})$	1.023 (0.067)***	1.129 (0.059)***	1.109 (0.059)***	1.138 (0.049)***	1.137 (0.047)***	1.109 (0.059)***
$\log(1+g_{Ni(t-2)})$	-0.261 (0.068)***	-0.373 (0.056)***	-0.379 (0.055)***	-0.347 (0.048)***	-0.379 (0.045)***	-0.379 (0.055)***
$\log(e_{it})$	-0.110 (0.058)	-0.037 (0.018)**	-0.038 (0.018)**	-0.101 (0.042)**	-0.037 (0.016)**	-0.038 (0.018)**
$\log(e_{it-1})$	0.077 (0.053)			0.061 (0.039)		
$\log(e_{it})^2$	-0.043 (0.027)	-0.015 (0.009)*	-0.015 (0.009)*	-0.039 (0.020)**	-0.015 (0.008)**	-0.015 (0.009)*
$\log(e_{it-1})^2$	0.029 (0.025)			0.022 (0.018)		
$\log(g_{y_{it}})$	-0.005 (0.006)	0.006 (0.004)				
$\log(g_{y_{it-1}})$	0.005 (0.005)					
$\log(y_t)$				0.001 (0.004)	-0.0002 (0.0007)	
$\log(y_{t-1})$				-0.001 (0.004)		
Number of Observations	235	270	270	228	270	270
Periods	21	21	21	21	21	21
Cross-sections	17	18	18	16	18	18
Adjusted R²	0.882228	0.888233	0.888014	0.878143	0.884892	0.888014
S.E. of regression	0.001714	0.001655	0.001654	0.001760	0.001677	0.001654

Table 7.5 – Estimation of Table 7.4

Standard errors are reported in parentheses below the estimate. The significance levels are expressed as follows: no star: not significant at 10%, * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level

steady state value of $e_* = 0.380$ gives a population growth of 0.0027, which is rather close to the previously inserted value of $g_N = 0.002$.

7.6.3 Comparison of the Two Approaches

The previous two sub-sections, Chapters 7.6.1 and 7.6.2, show two different approaches to give insights into the question of how population growth is composed. Whereas the extensive model of Chapter 7.6.1 deals with several socio-economic and health variables which try to explain the development of population growth, the reduced model of Chapter 7.6.2 limits its explanatory variables to just education and income, including lagged dependent variables. The two chosen estimations for the two approaches are summarized in Equations (7.5) and (7.6). The regression of the extensive model has an adjusted R^2 of 0.886 and a standard error of the regression of 0.0017 and that of the reduced model an adjusted R^2 of 0.888 and a standard error of the regression of 0.0016. This suggests a slight favoring of the reduced model and Equation (7.6). Both models show no sign of cross-sectional dependence (see Appendix 7.2). In the following chapters, the result for the reduced model is the chosen relation to be fed into the model of Chapter 6.

7.7 Conclusion

Chapter 7 has shown that estimating the population growth rate based on only education and income yields similar regression significance as including a larger number of control variables. As a result the estimated equation for the reduced model will be introduced into the theoretical model in Chapter 8. This chapter has made an effort in trying to show that only estimating the variables used in the theoretical model is a justified approach if lagged dependent variables are applied.

APPENDIX 7.1 – Unit Root Tests

This appendix shows the results of the conducted panel unit root tests. Four tests are presented: Levin, Lind and Chu; Im, Pesaran and Shin; augmented Dickey-Fuller; Phillips-Perron. For further information on the different tests, please refer to Appendix 3.1. All unit root tests presented in Table A7.1 are in levels and with no trend. It indicates strong evidence for the presence of a unit root for the variables: $cancer_{it}, hegdp_{it}, hepc_{it}, lfpf_{it}, nerv_{it}$ and $tobac_{it}$. Some evidence is present for the variables: alc_{it}, $colf_{it}$, $colm_{it}, diab_{it}$, $heart_{it}$ and urb_{it}. No evidence for the presence of a unit root can be shown for the variables: $AIDS_{it}, blood_{it}, g_{Yit}, ifd_{it}, tub_{it}$ and $u5d_{it}$. e_{it} and g_{Nit}show mixed results.

	Levin, Lin & Chu t	Im, Pesaran and Shin W-stat	ADF - Fisher Chi-square	PP - Fisher Chi-square
$AIDS$	0.0005	0.0010	0.0103	0.0525
alc	0.0592	0.2957	0.2750	0.5903
$blood$	0.0012	0.0182	0.0176	0.0347
$cancer$	1.0000	1.0000	1.0000	1.0000
$colf$	0.0000	0.6149	0.3904	0.5601
$colm$	0.0026	0.9986	0.8498	0.8226
$diab$	0.5228	0.3561	0.2190	0.0006
e	0.0008	0.4936	0.0279	0.1333
g_N	0.4855	0.0898	0.0290	0.4740
g_Y	0.0000	0.0000	0.0000	0.0000
$heart$	0.0019	0.9988	0.9842	0.9706
$hegdp$	0.9712	1.0000	1.0000	1.0000
$hepc$	0.8913	1.0000	1.0000	1.0000
ifd	0.0000	0.0000	0.0000	0.0000
$lfpf$	0.1798	0.8938	0.1375	0.2836
$nerv$	0.9999	1.0000	0.4076	0.9849
$tobac$	0.2797	0.9918	0.9824	0.7227
tub	0.0000	0.0079	0.0002	0.0153
$u5d$	0.0000	0.0000	0.0000	0.0000
urb	0.5552	1.0000	0.9988	0.0004

Table A7.1: Reported are the p-values of the different tests. Null hypothesis: common unit root

* Probabilities are computed assuming asymptotic normality

APPENDIX 7.2 – Cross-Sectional Dependence

In order to test whether there is cross-sectional dependence in the specific estimated model, Pesaran (2004) proposed a test statistic that is not likely to be distorted. Because of its stable characteristics for $T \to \infty$ and $N \to \infty$ it is valid for a wide range of panel models including homogenous and heterogeneous dynamic models. The null hypothesis is cross-sectional independence. Consider the following panel data model:

$$y_{it} = \beta_{0i} + \beta_1' x_{it} + u_{it} \tag{A7.1}$$

where x_{it} is the vector of the regressors with β_1 as the corresponding coefficients, β_{0i} is an individual-specific time-invariant fixed effects, and u_{it} is the error component that may be cross-sectionally correlated.

The null and alternative hypotheses to test for serial correlation are thus:

$$\text{H}_0\text{:}\quad \rho_{ij} = \rho_{ji} = Corr(u_{it}, u_{jt}) = 0 \qquad \forall\, t \text{ and } i \neq j \tag{A7.2}$$

$$\text{H}_1\text{:}\quad Corr(u_{it}, u_{jt}) \neq 0 \qquad \text{for some } i \neq j \tag{A7.3}$$

Pesaran (2004) has proposed the following test statistic:

$$CD = \sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right) \tag{A7.4}$$

He showed that under the null of no cross sectional dependence, $CD \xrightarrow{d} N(0{,}1)$ for $N \to \infty$ and T sufficiently large. The Pesaran CD Normal test has been applied to all models. It shows no model shows serious signs of cross sectional dependence. The results are summarized in Table A7.3.

Pesaran CD test	**Prob.**	**Pesaran CD test**	**Prob.**
Model 1	0.9793	Model 6.1	0.2317
Model 2	0.9763	Model 6.2	0.3912
Model 3	0.8498	Model 7	0.8481
Model 4	0.8768	Model 7.1	0.9566
Model 5	0.8173	Model 7.2	0.3912
Model 6	0.2139		

Table A7.3 – Pesaran's Cross-sectional dependence test for panel data
Null hypothesis: Cross-sectional independence

8

Uzawa-Lucas Model with Endogenous Population Growth and Labor Supply

8.1 Introduction

The model presented in this chapter is the final model of this dissertation. It builds on the two previous models. The model of Chapter 2 has analyzed an economy with an entirely exogenous population and labor development. The model of Chapter 6 has endogenized the choice of participation in the active population. Consequently, this chapter will introduce a model with an endogenous choice for participation in the active population together with an endogenized population development function. This is, thus, the complete endogenization of the dependency ratio. The representation of the population development function has been estimated in Chapter 7. The advantage of the chosen estimation in Chapter 7 is its simplicity which makes it possible to be introduced in an Uzawa-Lucas growth model. To my knowledge, the Uzawa-Lucas growth model has not yet been analyzed with endogenous population development and an endogenous choice of the participation in the active population. To show how exogenous shocks on the population growth rate influence the key variables, the intercept of the population development function is changed. As a policy measure the influence of increased (decreased) schooling efficiency is analyzed as well.

Chapter 8.2 introduces the population development function formally. Chapter 8.3 analyzes the steady state relations. It pays special attention to the rates of growth for the variables of the active and the total population. Chapter 8.4 analyzed two exogenous shocks. Chapter 8.4.1 deals with a not further specified shock on the population development function and Chapter 8.4.2 analyzes the impact of a change in the schooling efficiency which can be implemented as a policy measure. Chapter 8.5 compares the steady stated of this chapter's model to those of the two previous models and Chapter 8.6 concludes.

8.2 The Model

The first part of the model is similar to that of the previous models of Chapters 2 and 6. Output is formed by a Cobb-Douglas production function:

$$Y_t = A(K_t)^{1-\alpha}\big((1-e_t)h_tL_t\big)^{\alpha}\overline{h_t}^{\epsilon} \tag{8.1}$$

Where Y_t represents output, capital is given by K_t, time spent in production is $(1-e_t)$, individual human capital is h_t and labor supply is L_t. Average human capital, $\bar{h}_t^{\epsilon}$, contributes to the productivity of all factors and is modelled after Lucas (1988). The optimizing representative agent takes this externality as given. The demand for physical and human capital, provided by consumers, is determined in the firm sector in which firms maximize their profits subject to time spent in production, $(1-e_t)$, and capital, K_t.

$$\max_{(1-e_t),K_t} \pi = \mathrm{A}(K_t\,)^{1-\alpha}\big((1-e_t)h_tL_t\big)^{\alpha}\overline{h_t}^{\epsilon} - \omega_t(1-e_t)h_tL_t - r_{kt}K_t$$

$$\omega_t = \frac{\alpha Y_t}{(1-e_t)h_tL_t} \tag{8.2}$$

$$r_{kt} = (1-\alpha)\frac{Y_t}{K_t} \tag{8.3}$$

Equations (8.2) and (8.3) show the optimal wages and rental rates of capital.

Consumers are assumed to maximize their utility subject to consumption, time spent in education and labor supply. Labor supply has a Frisch elasticity, ϑ, just as in the model of Chapter 6.

$$U_t = \sum_{t=0}^{\infty} \beta^t N_t \left(\frac{c_t^{1-\sigma}}{(1-\sigma)} - \xi\frac{\left(\frac{L_t}{N_t}\right)^{1+\vartheta}}{1+\vartheta}\right)$$

Consumers face the common budget constraint.

$$N_tc_t + K_{t+1} - (1-\delta_k)K_t = \omega_t(1-e_t)h_tL_t + r_{kt}K_t + B_{t+1} - \left(1 + r\left(\frac{B_t}{Y_t}\right)\right)B_t \tag{8.4}$$

Where the RHS represents income from labor, capital and the bond market and the LHS represents the spending of consumption and investment. Human capital formation takes the well-known form as presented in Equation (8.5).

$$h_{t+1} = F\, e_t^{\gamma} h_t + (1-\delta_h) h_t \tag{8.5}$$

It depends on the time spent in education, e_t, the productivity parameter, $\gamma \leq 1$, the knowledge efficiency coefficient, F, and depreciation of human capital, δ_h. Chapter 7 has presented how population growth is formed. This expression will be used to model population growth as a constraint in this chapter. Equation (7.6) can be generalized to

$$\log(1+g_{Nit}) = \tau_0 + \tau_2 \log(e_{it}) + \tau_3 \log(e_{it})^2 + \tau_4 \log\big(1+g_{Ni(t-1)}\big) + \\ +\tau_5 \log\big(1+g_{Ni(t-2)}\big)$$

Solving for $(1+g_N)$, rather than $\log(1+g_N)$ and with $1+g_{Nt} = \frac{N_{t+1}}{N_t}$ this can be rewritten as:

$$N_{t+1} = N_t \left(\tau_1 e_t^{\tau_2} \exp^{\tau_3 \log(e_t)^2} \left(\frac{N_t}{N_{t-1}}\right)^{\tau_4} \left(\frac{N_{t-1}}{N_{t-2}}\right)^{\tau_5}\right) \tag{8.6}$$

With $\tau_1 = \exp^{\tau_0}$

Equation (8.6) represents the population development and forms the last constraint of the maximization program, in which consumers maximize their utility subject to their budget constraint, (8.4), human capital formation, (8.5), and population development (8.6):

$$\max_{c_t,e_t,L_t,B_{t+1},K_{t+1},h_{t+1},N_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(N_t \left(\frac{(c_t)^{1-\sigma}}{(1-\sigma)} - \boldsymbol{\xi} \frac{\left(\frac{L_t}{N_t}\right)^{1+\vartheta}}{1+\vartheta} \right) - \right.$$

$$\boldsymbol{\mu_t} \left[N_t c_t + K_{t+1} - (1-\delta_k)K_t - \omega_t(1-e_t)h_t L_t - r_{kt}K_t - B_{t+1} + \right.$$

$$\left. \left(1 + r\left(\frac{B_t}{Y_t}\right)\right) B_t \right] - \boldsymbol{\mu_{ht}} \left[h_{t+1} - F\, e_t^{\gamma}\, h_t \, - (1-\delta_h)h_t \right] - \boldsymbol{\mu_{Nt}} \left[N_{t+1} - \right.$$

$$\left. \left. N_t \left(\tau_1 e_t^{\tau_2} \exp^{\tau_3 \log(e_t)^2} \left(\frac{N_t}{N_{t-1}}\right)^{\tau_4} \left(\frac{N_{t-1}}{N_{t-2}}\right)^{\tau_5} \right) \right] \right)$$

The first order conditions in addition to the constraints are:

c_t: $$(c_t)^{-\sigma} = \mu_t \tag{8.7}$$

e_t: $$F\gamma e_t^{\gamma-1} h_t \mu_{ht} + \tau_1 e_t{}^{\tau 2} exp^{\tau_3 \log(e_t)^2} \left(\tau_2 + 2\tau_3 \log(e_t)\right) \left(\frac{N_t}{N_{t-1}}\right)^{\tau_4} \left(\frac{N_{t-1}}{N_{t-2}}\right)^{\tau_5} N_t \mu_{Nt} =$$

$$= \omega_t h_t L_t \mu_t \tag{8.8}$$

L_t: $$\xi \left(\frac{L_t}{N_t}\right)^{\vartheta} = \mu_t \omega_t (1-e_t) h_t \tag{8.9}$$

B_{t+1}: $$\mu_t = \beta \mu_{t+1} \left(1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right) + \frac{B_{t+1}}{Y_{t+1}} r'\left(\frac{B_{t+1}}{Y_{t+1}}\right) \right) \tag{8.10}$$

K_{t+1}: $$\mu_t = (1 - \delta_K + r_{Kt+t})\beta \mu_{t+1} \tag{8.11}$$

h_{t+1}: $$\mu_{ht} = \beta\left[\mu_{t+1}\omega_{t+1}(1-e_{t+1})L_{t+1} + \mu_{ht+1} F e_{t+1}^{\gamma} + (1-\delta_h)\mu_{ht+1}\right] \tag{8.12}$$

N_{t+1}:

$$\beta \left(\frac{\vartheta \xi \left(\frac{L_{t+1}}{N_{t+1}}\right)^{1+\vartheta}}{1+\vartheta} + \frac{c_{t+1}^{1-\sigma}}{1-\sigma} - c_{t+1}\mu_{t+1} + \right.$$

$$(1+\tau_4)\tau_1 e_{t+1}{}^{\tau 2} exp^{\tau_3 \log(e_{t+1})^2} \left(\frac{N_{t+1}}{N_t}\right)^{\tau_4} \left(\frac{N_t}{N_{t-1}}\right)^{\tau_5} \mu_{Nt+1} +$$

$$\beta(-\tau 4 + \tau 5)\tau_1 e_{t+2}{}^{\tau 2} e^{\tau 3 Log[e_{t+2}]^2} \left(\frac{N_{t+1}}{N_t}\right)^{\tau 5} \left(\frac{N_{t+2}}{N_{t+1}}\right)^{1+\tau 4} \mu_{Nt+2} -$$

$$\left. \beta^2 \tau_5 \tau_1 e_{t+3}{}^{\tau 2} e^{\tau 3 Log[e_{t+3}]^2} \left(\frac{N_{t+2}}{N_{t+1}}\right)^{\tau 5} \left(\frac{N_{t+3}}{N_{t+2}}\right)^{\tau 4} \left(\frac{N_{t+3}}{N_{t+1}}\right) \mu_{Nt+3} \right) = \mu_{Nt} \tag{8.13}$$

The following transversality conditions must hold[26]:

I. $\lim_{t\to\infty}\beta^t\mu_t K_t = 0$
II. $\lim_{t\to\infty}\beta^t\mu_{ht} h_t = 0$
III. $\lim_{t\to\infty}\beta^t\mu_{Nt} N_t = 0$

The above defines a system of 13 equations and 13 endogenous variables: $Y_t,\ K_t,\ L_t,\ h_t,\ e_t,\ N_t,\ \omega_t,\ r_{kt},\ c_t,\ B_t,\ \mu_t,\ \mu_{ht}$ and μ_{Nt}. The rates of return to bonds, physical capital and labor can be derived. They are displayed in Equations (8.14a-d).

$$\frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\mu_t}{\beta\,\mu_{t+1}} = R_{Bt+1} = R_{Kt+1} = R_{Lt+1} = R_{Ht+1} \tag{8.14a}$$

$$R_{Bt+1} = 1 + r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb}) \tag{8.14b}$$

$$R_{Kt+1} = 1 - \delta_k + (1-\alpha)\frac{Y_{t+1}}{K_{t+1}} \tag{8.14c}$$

$$R_{Lt+1} = \frac{(1+g_N)^{\vartheta}(1+g_{\omega})(1+g_{1-e})(1+g_h)}{\beta(1+g_L)^{\vartheta}} \tag{8.14d}$$

$$R_{Ht+1} = (1+g_{\omega})(1+g_L)\left[(1-e_{t+1})F\gamma e_t^{\gamma-1} + \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] + \frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}\left[\left(\tau_2 + 2\tau_3\log(e_t)\right) - \beta\left(\tau_2 + 2\tau_3\log(e_{t+1})\right)(1+g_N)(1+g_{\mu N})\ \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\,\frac{\left(Fe_{t+1}^{\gamma}+(1-\delta_h)\right)}{\left(Fe_t^{\gamma}+(1-\delta_h)\right)}\right] \tag{8.14e}$$

The first part of (8.14a) is derived from Equation (8.7). The other two equivalencies on the right hand side follow from (8.14b-c). Equation (8.14b) follows straight forwardly from Equation (8.10) with $\eta_{rb} = \frac{B_{t+1}}{Y_{t+1}}\frac{r'\left(\frac{B_{t+1}}{Y_{t+1}}\right)}{r\left(\frac{B_{t+1}}{Y_{t+1}}\right)}$ and (8.14c) is derived from Equation (8.11) where r_{kt+1} is replaced by the expression in (8.3). Equation (8.14d) is the rate of return of the active population size and is derived by taking growth rates from Equation (8.9). Equation (8.14e) is derived with Equations (8.8) and (8.12). The full derivation is presented in Appendix 8.1.

[26] For proof that the utility function has a finite integral, and hence has an interior maximum has been shown in Appendix 2.1. The same logic applies here.

In a given steady state, the interest rate is constant. Equations (8.14a) and (8.14b) then imply that $(1+g_c)$ must be constant and equal to

$$(1+g_c)=\left[\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}} \qquad (8.14a)^{I}$$

Through the first equivalence of (8.14a) this implies:

$$\left(1+g_\mu\right)=\left[\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right]^{-1} \qquad (8.14a)^{II}$$

From this follows constancy of the RHS of Equation (8.14c) from which follows constancy of $\frac{Y_{t+1}}{K_{t+1}}$. This implies equality of the growth rates of the numerator and the denominator, as has been analyzed in Chapters 2 and 6:

$$1+g_Y=\ 1+g_K=(1+g_{1-e})(1+g_L)(1+g_h)^{1+\frac{\epsilon}{\alpha}} \qquad (8.14c)^{I}$$

Equation $(8.14c)^{I}$ is the equivalent of Equations $(2.12c)^{I}$ and $(6.12c)^{I}$. It shows that the growth rates of output and capital depend on the change of time spent in production, the growth rate of human capital, and the growth rate of labor supply. Division by $(1+g_N)$ shows that the growth of income and capital per capita depend on the growth of the labor-population ratio.

The growth rate of wages is determined in the firm sector where Equation (8.2) implies:

$$1+g_\omega=\frac{1+g_Y}{(1+g_{1-e})(1+g_h)(1+g_L)} \qquad (8.2)^{I}$$

Inserting $(8.14c)^{I}$ into $(8.2)^{I}$ leads to:

$$1+g_\omega=(1+g_h)^{\frac{\epsilon}{\alpha}} \qquad (8.2)^{II}$$

If $(1+g_h)$ is assumed to be constant in steady state, then e_t must be constant. Together with the human capital formation function, Equation (8.5), this shows that $g_\omega$is constant if e_t is constant (i.e. if $g_e=0$).

The growth rate of labor can be obtained with Equation (8.14d). The expression R_{Lt+1} on the LHS of Equation (8.14d) is equal to R_{Bt+1} through Equation (8.14a). Together with Equation (8.14b) and replacing $(1+g_{\omega})$ with the expression of Equation (8.2)II in Equation (8.14d) this leads to

$$(1+g_L) = \left(1+g_N(e)\right)\left(\frac{\left(1+g_h(e)\right)^{1+\frac{\epsilon}{\alpha}}(1+g_{1-e})}{\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)}\right)^{\frac{1}{\vartheta}} \qquad (8.14\text{d})^{\text{I}}$$

Equation (8.14d)I is the same expression as (6.12e)I in Chapter 6. The essential difference is, that now not only $(1+g_h)$ depends on time spent in education, but also $(1+g_N)$, whereas the latter was exogenous in Chapter 6. The steady state growth rate of the active population depends, thus, on the interest rate, the growth rate of time spent in production and the growth rates of human capital and population, both of which depend on e_t.

The growth rates for $(1+g_Y)$ and $(1+g_K)$ which are equal, as derived in (8.14c)I, can now be analyzed. Here only $(1+g_Y)$ is shown exemplarily. All conclusions also hold for the growth rate of capital. The expression for $(1+g_L)$ of Equation (8.14d)I can be inserted into Equation (8.14c)I.

$$1+g_Y = \frac{\left(1+g_N(e)\right)\left((1+g_{1-e})(1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}}{\left(\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right)^{\frac{1}{\vartheta}}} \qquad (8.14\text{c})^{\text{II}}$$

In steady state, we can assume $(1+g_h)$ to be constant. This makes e_t constant through Equation (8.5). With e_t constant $(1+g_N)$ must be constant, through Equation (8.6). The growth rate of time spent in production cannot be different from zero in steady state, as this would eventually imply a value of $(1-e_t)$ outside the range of $[0,1]$. Figure 8.1 plots (8.14d)I and the population development function as stated below in the $g_L, g_N - e$ plane to be able to compare the steady state growth rates for the active and total population for different values of e_t. In order to do so, the population development function, Equation (8.6), must be evaluated in steady state. The growth rate of N_t is constant and, hence, $\frac{N_t}{N_{t-1}} = \frac{N_{t-1}}{N_{t-2}} = (1+g_N)$. Solving Equation (8.6) for $(1+g_N)$ then yields:

$$1 + g_N = \left[\tau_1 e_*^{\tau_2} \exp^{\tau_3 \log(e_*)^2}\right]^{\frac{1}{1-\tau_4-\tau_5}} \qquad (8.6)^{\mathrm{I}}$$

The parameters are chosen to be the same as in the previous models, namely: $\alpha = 0.6, \epsilon = 0.834, \vartheta = 3$ and $r\left(\frac{B_t}{Y_t}\right)(1 + \eta_{rb}) = 0.05$. τ_{1-5} are taken from the estimation in Chapter 7, namely Equation (7.6). They are: $\tau_1 = \exp(-0.022)$, $\tau_2 = -0.038, \tau_3 = -0.015, \tau_4 = 1.109$ and $\tau_5 = -0.379$.

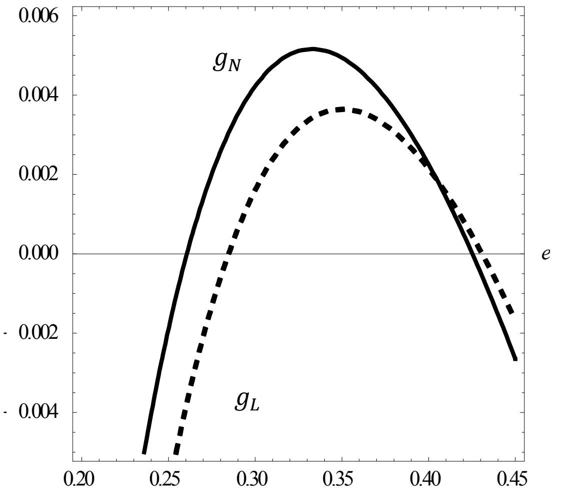

Figure 8.1 - g_N and g_L

In Figure 8.1 the solid line represents Equation $(8.6)^{\mathrm{I}}$ for the growth rate of the total population and the dashed line represents Equation $(8.14\mathrm{d})^{\mathrm{I}}$ for the growth rate of the active population. At the intersection point the dependency ratio is constant as $g_L = g_N$. To the left of the intersection point for any given e_t, population is growing more strongly than labor inputs as in the case of ageing. The vertical difference is the growth rate g_{1+D} used before. In the area where both rates are positive g_N is first increasing more strongly than g_L and then falling more strongly than g_L. To the left of the intersection point, in the area of falling growth rates, more time spent in education leads to a lower growth rate of ageing, g_{1+D}. The question now is to which value e_t tends.

8.3 Steady State Analysis

To answer the question to which value e_t tends in steady state, the dynamics in e_t, b_t and X_t are analyzed. As in the previous chapters, X_t is defined as the propensity to consume, $X_t = \frac{c_t N_t}{Y_t}$. b_t is the debt to GDP ratio, $b_t = \frac{B_t}{Y_t}$.

The dynamics in e_t can be derived from Equation (8.14e). To be able to analyze the above mentioned 3x3 dynamic system in X_t, b_t and e_t, all other time dependent variables need to be eliminated. Equation (A8.5)[III], as derived in Appendix 8.4, shows how the term $\frac{N_{t+1}\mu_{Nt}(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}$ in (8.14e) is related to the three variables X_t, b_t and e_t. This relation can be plugged into (8.14e):

$$R_{Ht+1} = (1+g_\omega)(1+g_L)\left[(1-e_{t+1})F\gamma e_t^{\gamma-1} + \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] + (1+g_N)(1-e_t)\left[\frac{\vartheta}{\beta(1+\vartheta)}\left(\frac{1-(1+g_e)e_t}{1-e_t}\left(Fe_t^{\gamma} + (1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{-\frac{1}{\vartheta}} + \frac{\sigma}{\alpha\beta(1-\sigma)}\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{\frac{1}{\sigma}}X_t\right]\frac{1}{\Gamma}\left[\left(\tau_2 + 2\tau_3 log(e_t)\right) - \beta\left(\tau_2 + 2\tau_3 log(e_{t+1})\right)(1+g_N)\left(1+g_{\mu N}\right)\ \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\ \frac{\left(Fe_{t+1}^{\gamma}+(1-\delta_h)\right)}{\left(Fe_t^{\gamma}+(1-\delta_h)\right)}\right] \quad (8.14e)^{I}$$

Where $\Gamma = \left(1/\beta - \left((1+\tau_4)\left(1+g_{\mu N}\right)(1+g_N)\ + \beta(-\tau 4+\tau 5)\left(1+g_N\right)^2\left(1+g_{\mu N}\right)^2 - \beta^2\tau_5(1+g_N)^3\left(1+g_{\mu N}\right)^3\right)\right)$

The growth rate of the wages, $(1+g_\omega)$, can be replaced by the expression in (8.2)[II]. The growth rate of labor, $(1+g_L)$ can be replaced by the expression in (8.14d)[I]. $(1+g_N)$ is replaced by its expression in (8.6)[I]. The expression for $\left(1+g_{\mu N}\right)$ is derived in Appendix 8.2 and represented by

Equation (A8.2)^{II}. All of the above expressions depend only on the three variables of the dynamic system, X_t, b_t and e_t. For better overview they are not explicitly replaced in the representations. Replacing e_{t+1} with $(1+g_e)e_t$ and R_{Ht+1} with $\left(1+r(b_{t+1})(1+\eta_{rb})\right)$ from (8.14a,b) yields:

$$\left(1+r(b_{t+1})(1+\eta_{rb})\right) = (1+g_\omega)(1+g_L)\ \Big[(1-(1+g_e)e_t)F\gamma e_t^{\gamma-1} + (1+g_e)^{1-\gamma}\left(F(1+g_e)^\gamma e_t^\gamma + (1-\delta_h)\right)\Big] + (1+g_N)(1-e_t)\left[\frac{\vartheta}{\beta(1+\vartheta)}\left(\frac{1-(1+g_e)e_t}{1-e_t}\left(Fe_t^\gamma + (1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{-\frac{1}{\vartheta}} + \frac{\sigma}{\alpha\beta(1-\sigma)}\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{\frac{1}{\sigma}}X_t\right]\frac{1}{\Gamma}\Bigg[\left(\tau_2 + 2\tau_3 log(e_t)\right) - \beta\left(\tau_2 + 2\tau_3 log\left((1+g_e)e_t\right)\right)(1+g_N)\left(1+g_{\mu N}\right)(1+g_e)^{1-\gamma}\ \frac{\left(F(1+g_e)^\gamma e_t^\gamma + (1-\delta_h)\right)}{\left(Fe_t^\gamma + (1-\delta_h)\right)}\Bigg] \qquad (8.14e)^{II}$$

Equation (8.14e)^{II} is the dynamic equation in e_t. It is the equivalent of Equations (2.12d)^{II} in Chapter 2 and Equation (6.12d)^{III} in Chapter 6. Despite the higher complexity of the expression, the major difference is the dependence on X_t. The dynamics in e_t were solely dependent on e_t and b_t in Chapters 2 and 6. In this chapter's model, they also depend on the marginal propensity to consume, X_t. The model is, thus, not separable anymore and all three variables, X_t, b_t and e_t need to be determined simultaneously.

In a next step, the dynamics of the propensity to consume are analyzed. $X_t = \frac{N_t c_t}{Y_t}$, the growth rate of X_t is thus:

$$(1+g_X) = \frac{(1+g_N)(1+g_c)}{1+g_Y} = \frac{\left[\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}+\frac{1}{\vartheta}}}{\left((1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}} \qquad (8.15)$$

In the second part of Equation (8.15) $(1+g_c)$ is replaced by the expression in Equation (8.14a)$^{\mathrm{I}}$ and the growth rates of output, $(1+g_Y)$ and population, $(1+g_N)$, are replaced by their respective expressions of Equations (8.14c)$^{\mathrm{II}}$ and (8.6)$^{\mathrm{I}}$.

In a given steady state, $g_X = 0$:

$$\left[\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}+\frac{1}{\vartheta}} = \left((1+g_h(e))^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}} \qquad (8.15)^{\mathrm{I}}$$

Equation (8.15)$^{\mathrm{I}}$ shows a generalized golden rule relation between $r(b_t)$ and $\left(1+g_h(e)\right)$ corresponding to $g_X = 0$ in the earlier models allowing for human capital externalities and imperfect capital movements.

The dynamics in $\boldsymbol{r}(\boldsymbol{b_t})$ are similar to those of Chapters 5 and 6. An estimation of the debt-dependent interest rate has been conducted in Chapter 4.3.2. The same estimation will be used for this chapter's model; for more details refer to Chapter 4.

$$r(b_t) = 0.815exp^{-0.564\log[2+b_t]^2+0.214\log[2+b_t]^3}(2+b_t)^{0.605}(1+\bar{r})^{0.566} - 1 \qquad (8.16)$$

Where $\boldsymbol{b_t} = \frac{\boldsymbol{B_t}}{\boldsymbol{Y_t}}$ is the debt to GDP ratio that determines a country's individual interest rate and $\bar{\boldsymbol{r}}$ is the world interest rate, approximated by the US interest rate, which is on average 0.05. This is included to separate worldwide influences on the individual interest rates from the debt driven influences. The elasticity of the interest rate with respect to the debt to GDP ratio, $\boldsymbol{\eta_{rb}}$, is derived from Equation (8.16) as has been done in Chapters 5 and 6 with $\boldsymbol{\eta_{rb}} = \boldsymbol{b_t}\,\frac{\boldsymbol{r}'(\boldsymbol{b_t})}{\boldsymbol{r}(\boldsymbol{b_t})}$.

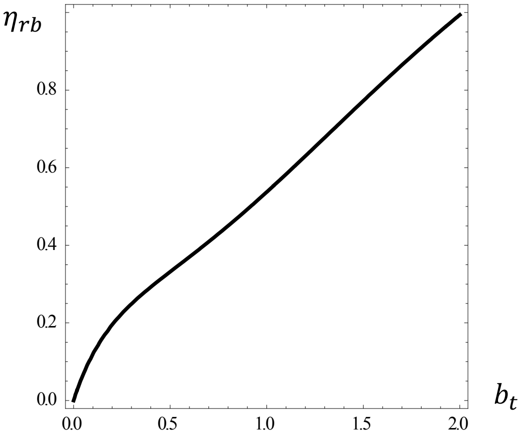

Figure 8.2 - η_{rb}

The elasticity, $\boldsymbol{\eta_{rb}}$, is increasing with a higher debt to GDP ratio, as can be seen in Figure 8.2.

$$\eta_{rb} = 20 * b_t\left(\frac{0.507exp^{-0.564\mathrm{Log}[2+b_t]^2+0.214\mathrm{Log}[2+b_t]^3}}{(2+b_t)^{0.396}} + 0.838(2+b_t)^{0.605}exp^{-0.564\mathrm{Log}[2+b_t]^2+0.214\mathrm{Log}[2+b_t]^3}\left(-\frac{1.127\mathrm{Log}[2+b_t]}{2+b_t}+\frac{0.642\mathrm{Log}[2+b_t]^2}{2+b_t}\right)\right)$$

The household sector is characterized by the budget constraint:

$$B_{t+1} = N_t c_t + K_{t+1} - (1-\delta_k)K_t - Y_t + \big(1+r(b_t)\big)B_t \qquad (8.17)$$

Equation (8.17) is the equivalent of Equation (8.4) with ω_t and r_{Kt} at their equilibrium values shown in Equations (8.2) and (8.3) and, hence, with $\omega_t(1-e_t)h_tL_t + r_{kt}K_t = Y_t$.

Because the previously determined relations hold, K_{t+1} and K_t can be replaced by $K_{t+1} = \frac{1-\alpha}{r(b_{t+1})(1+\eta_{rb})+\delta_k}\,Y_{t+1}$ from Equations (8.14a-c) with $R_{Ht+1} = R_{Bt+1} = 1 + r(b_{t+1})(1+\eta_{rb})$ for their respective periods.

$$b_{t+1} = \frac{N_t c_t}{Y_{t+1}} - \frac{\theta}{1+g_Y} + \frac{1+r(b_{t+1})(1+\eta_{rb})}{1+g_Y}\,b_t \qquad (8.17)^{\mathrm{I}}$$

Where $\theta = (1-\delta_k)\frac{1-\alpha}{r(b_{t+1})(1+\eta_{rb})+\delta_k} + 1 - (1+g_Y)\frac{1-\alpha}{r(b_{t+1})(1+\eta_{rb})+\delta_k}$ and b_t is constant in steady-state. Setting $\frac{N_t c_t}{Y_t} = X_t$ in Equation (8.17)ˡ and with $(1+g_b) = \frac{b_{t+1}}{b_t}$ it must hold that

$$(1+g_b) = \frac{1}{1+g_y}\left(\frac{1}{b_t}X_t - \frac{1+\frac{1-\alpha}{r(b_t+1)(1+\eta_{rb})+\delta_k}\big((1-\delta_k)-(1+g_y)\big)}{b_t} + 1 + r(b_t)\right) \tag{8.18}$$

In steady state, $g_b = 0$ and Equation (8.18) solved for X_t becomes

$$X_t = \Big(1+g_Y-\big(1+r(b_t)\big)\Big)b_t + \frac{(1-\alpha)(1+g_Y)}{r(b_{t+1})(1+\eta_{rb})+\delta_k}\left(\frac{(1-\delta_k)}{1+g_Y}-1\right)+1 \tag{8.18)ˡ}$$

This is the equivalent of Equation (6.15)ˡ of Chapter 6. The difference lies in the representation of $(1+g_Y)$. This chapter's growth rate of output is $(1+g_Y) = (1+g_N)\left((1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}\big(\beta(1+r(b_{t+1})(1+\eta_{rb})\,)\big)^{-\frac{1}{\vartheta}}$.

This completes the three dynamic equations for the three variables X_t, b_t and e_t. The three dynamic equations are presented in Equations (8.15), (8.18) and (8.14e)ˡˡ. These relations differ from the equivalent relations of the previous chapters with the regard that the system is no longer separable. Before, the dynamics in e_t were directly only dependent on b_t, whereas Equation (8.14e)ˡˡ shows a relation between $(1+g_e)$ and the three dynamic variables X_t, $b_t$and e_t.

In steady state all three variables are constant. This implies that their growth rates must be zero. If $g_X = g_b = g_e = 0$ is set, then the three dynamic equations (8.15), (8.18) and (8.14e)ˡˡ can be solved simultaneously. This leads to a new steady state in which $X_* = 0.691$, $b_* = 0.039$ and $e_* = 0.382$.

With these values, the optimal growth rate of the population is $1+g_N = 1.00247$, calculated with Equation (8.6)ˡ. The optimal growth rate of the active population is $1+g_L = 1.00204$, as can be calculated with Equation (8.14d)ˡ. This implies that the optimal growth rate of the dependency ratio is

Steady State Relations	***From Equation***	***Value***
$\mathbf{1}+\boldsymbol{g_h}=F\,e_*^{\gamma}+(1-\delta_h)$	(8.5)	0.01218
$\mathbf{1}+\boldsymbol{g_N}=\left[e_*^{\tau_2}\exp^{\tau_1+\tau_3\log(e_*)^2}\right]^{\frac{1}{1-\tau_4-\tau_5}}$	(8.6)[I]	0.00247
$\mathbf{1}+\boldsymbol{g_L}=\left(1+g_N(e)\right)\left(\frac{(1+g_h(e))^{1+\frac{\epsilon}{\alpha}}(1+g_{1-e})}{\beta(1+r(b_{t+1})(1+\eta_{rb}))}\right)^{\frac{1}{\vartheta}}$	(8.14d)[I]	0.00204
$\mathbf{1}+\boldsymbol{g_Y}=(1+g_N)\left((1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}\left(\beta(1+r(b_{t+1})(1+\eta_{rb}))\right)^{-\frac{1}{\vartheta}}$	(8.14c)[II]	0.03146
$\mathbf{1}+\boldsymbol{g_K}=(1+g_N)\left((1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}\left(\beta(1+r(b_{t+1})(1+\eta_{rb}))\right)^{-\frac{1}{\vartheta}}$	(8.14c)[II]	0.03146
$\mathbf{1}+\boldsymbol{g_\omega}=(1+g_h)^{\frac{\epsilon}{\alpha}}$	(8.2)[II]	0.01697
$\mathbf{1}+\boldsymbol{g_c}=\left[\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right]^{\frac{1}{\sigma}}$	(8.14a)[I]	0.02892
$\mathbf{1}+\boldsymbol{g_\mu}=\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{-1}$	(8.14a)[II]	-0.02977
$\mathbf{1}+\boldsymbol{g_{\mu_h}}=$ $(1+g_N)(1+g_h)^{\frac{\alpha+\epsilon+\epsilon\vartheta}{\alpha\vartheta}}\left[\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right]^{-\left(1+\frac{1}{\vartheta}\right)}$	(A8.4)[I]	-0.02452
$\mathbf{1}+\boldsymbol{g_{\mu N}}=$ $\left((1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}\left[\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right]^{-\left(1+\frac{1}{\vartheta}\right)}$	(A8.2)[II]	-0.0017

Table 8.1 – Steady states

Note to Table: $\alpha=0.6, \delta_h=0.03,\ F=0.055, \gamma=0.268,\ \epsilon=0.834,\ \sigma=1.06,\ \beta=0.982, \vartheta=3, \tau_1=exp(-0.022),\ \tau_2=-0.038, \tau_3=-0.015, \tau_4=1.109, \tau_5=-0.379,\ b_{t+1}=\ 0.039,$ $\eta_{rb}=\ 0.055$ and $e_*=0.38252$.

For the transversality conditions to hold, the growth rates of $\beta^t\mu_tK_t$, $\beta^t\mu_{ht}h_t$ and $\beta^t\mu_{Nt}N_t$ must be negative. With $\beta=0.982$, the growth rate of β^t is -0.018. With $g_\mu=-0.030$ and $g_K=0.031$, the growth rate of the first expression is negative. The growth rate of the second product is also negative because $g_{\mu h}=-0.012$ and $g_h=0.011$. Finally, the growth rate of the last term is negative as well because $g_{\mu N}=-0.0017$ and $g_N=0.00247$. The transversality conditions are, hence, fulfilled.

$1+g_{1+D}=1.00043$. This is the same optimal value as in Chapter 6. Further comparisons between the three models with exogenous population development (Chapter 2), endogenized labor force participation (Chapter 6) and, additionally, endogenized total population (Chapter 8) will be drawn in Chapter 8.5. All other optimal growth rates are summarized in Table 8.1.

One often mentioned root cause of ageing is a permanent shock in the population growth function. This can be modelled with an exogenous decrease of τ_1 from $\tau_1 = \exp(-0.022)$ to $\tau_1 = \exp(-0.023)$. This exogenous shock will be analyzed in Chapter 8.4, as well as an exogenous increase in the efficiency of schooling.

8.4 Exogenous Shocks

This section will analyze two exogenous shocks. First, an exogenous shock in the population development function will be analyzed. Second, a shift in the efficiency of education is analyzed.

8.4.1 Exogenous Population Shifts

The population development curve will be shifted downwards if a negative shock on the intercept τ_1 occurs, as shown in Figure 8.3. This leads to higher ageing again and is modelled by a shift from $\tau_1 = \exp(-0.022)$ to $\tau_1 = \exp(-0.023)$. Figure 8.3 shows the population development as described in Equation $(8.6)^{I}$ for the original value of $\tau_1 = \exp(-0.022)$, dashed line, and the exogenously reduced value of $\tau_1 = \exp(-0.023)$. For each value of e_t, there is less population growth.

The results of solving the system of Equations (8.15), (8.18) and $(8.14e)^{II}$ with the new value of τ_1 are presented in Table 8.2 together with the original values for better comparison.

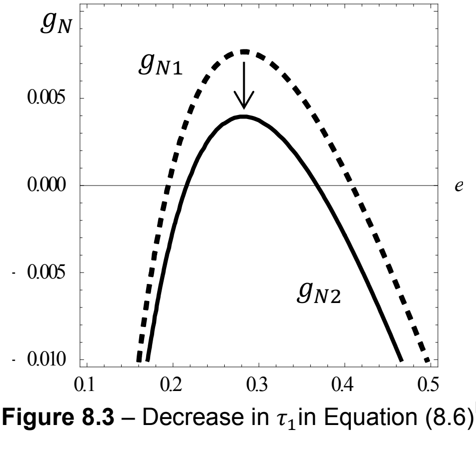

Figure 8.3 – Decrease in τ_1 in Equation $(8.6)^{I}$

	Before Shock $\tau_1 = \exp(-0.022)$	**After Shock** $\tau_1 = \exp(-0.023)$
g_{1+D}	0.00043	0.00040
g_N	0.00247	0.00073
g_L	0.00204	0.00032
e_*	0.38252	0.35855

Table 8.2 – Exogenous change of τ_1

Table 8.2 shows that an exogenous shock shifting the population development curve downwards leads to an enormous decrease in the population growth rate from $g_N = 0.00247$ in before the shock to $g_N = 0.00073$ after the shock. The only reason why this does not lead to a very high growth rate of the dependency ratio is the fact that the growth rate of the active population adjusts accordingly. It follows a very low time spent in education of only $e_t = 0.359$. In order to cope with the dramatically lower population growth rate, people need to spend more of their active time in production as opposed to education because ageing is reduced.

8.4.2 Changes in the Education System

Chapter 8.2 has shown how the economy behaves in a steady state. Throughout the last chapters the efficiency of human capital production has been kept constant. This section focuses on the consequences of a change in the efficiency of human capital production. If the educational system improves, indicated by an increase in the knowledge efficiency coefficient, F, each hour in education will be more productive in terms of human capital production. Through Equation (8.5), this will increase the growth rate of human capital if the time spent in production remains the same. The quality of schooling can be influenced by the government. An increase in F is thus used as a policy measure. This section will analyze how the endogenous growth rates of the total and the active population react and what consequences such a shift has on the growth rate of the dependency ratio. In order to do so the knowledge efficiency coefficient, F, is altered from $F = 0.055$ to $F = 0.056$, indicating an increase in the efficiency of schooling and to $F = 0.054$, indicating a decrease in the efficiency of schooling. Table 8.3 summarizes the results.

F	e_*	g_N	g_L	g_{1+D}
0.054	0.38179	0.00253	0.00212	0.00041
0.055	0.38252	0.00247	0.00204	0.00043
0.056	0.38428	0.00231	0.00184	0.00047

Table 8.3 – Changes in the Educational System

Table 8.3 shows that a decrease in the efficiency of schooling will decrease the time spent in education from $e_* = 0.38252$ for the original value of F to $e_* = 0.38428$ for a smaller F of 0.054. This will increase the population growth rate which leads to a lower growth rate of the dependency ratio. An increase in the efficiency of schooling will result in more time spent in education, from $e_* = 0.38252$ to $e_* = 0.38428$. This results in a lower population growth rate. The even lower growth rate of the active population leads to an increase in the dependency ratio.

This shows that with an increase in the efficiency of schooling more ageing can be afforded. On the other hand, a higher schooling efficiency may also be one part of the cause of the recent demographic developments as better schooling will increase the time spent in schooling which decreases the population growth rate.

8.5 Comparison of the Three Models

This chapter's model completes the stepwise endogenization of the dependency ratio in an Uzawa-Lucas growth model with international capital movements, human capital externalities and decreasing returns to schooling in human capital formation. This section will compare the three models and show how the active and inactive population and, hence, the dependency ratio behaves if more and more of the population measures are endogenized. It will also compare the different steady state time shares in education.

Table 8.4 summarizes the results for g_{1+D}, g_N, g_L and e_* for the case of a flexible interest rate. The exogenously set growth rates of the active and total population in Chapter 2 have led to an optimal share of education of $e_* = 0.380$. By implication, this was done for a stable dependency ratio and hence a growth rate of the dependency ratio of $g_{1+D} = 0$. Chapter 6 has endogenized the decision of participation in the active population. This has

	Chapter 2	Chapter 6	Chapter 8
g_{1+D}	0	0.00043	0.00043
g_N	0.002	0.00200	0.00247
g_L	0.002	0.00157	0.00204
e_*	0.380	0.38012	0.38252

Table 8.4 – Comparison of the three main models

led to a decrease in the growth rate of the active population from $g_L = 0.002$, as was set before, to $g_L = 0.00157$. Since the growth rate of the total population was still set exogenously, this has led to an increase of the growth rate of the dependency ratio from the previously implied $g_{1+D} = 0$ to a semi-flexible $g_{1+D} = 0.00043$. The model of Chapter 6 has shown a slight increase in the time spent in education from $e_* = 0.380$ to $e_* = 0.38012$. It can, thus, be concluded, that the endogenization of the choice to participate in the active population has led to a lower growth rate of the active population, compared to the exogenous case, but has increased the time spent in education. Hence, people tend to spend less time in the active population, but compensate this by a higher education and, hence, a higher productivity. This behavior increases the growth rate of the dependency ratio. For otherwise unchanged parameter values, as is the case here, this is due to the value of the Frisch parameter, ϑ.

If the development of the total population is endogenized as well and depends on the time spent in education, as has been analyzed in Chapter 7, the optimal growth rate of the population size increases from the exogenously set $g_N = 0.002$ of the previous models to $g_N = 0.00247$ in Chapter 8. The growth rate of the active population increases to $g_L = 0.00204$ as compared to both previous models. This implies an unchanged growth rate of the dependency ratio as compared to Chapter 6 at $g_{1+D} = 0.00043$. If the growth rate of the entire population is endogenized additionally to that of the active population the entire population will grow faster than previously set and the people spend more time in the active population again to be able to cope with the higher population growth. Additionally, more time will be spent in education. This increases the productivity. As explained in connection with Figure 8.1, this increase in education reduces the growth rate of the dependency ratio or prevents its increase.

8.6 Conclusion

This chapter concludes the series of models that have been analyzed in this dissertation. After the endogenization of the choice of participation in the active population in Chapter 6, this chapter has also endogenized the development of the total population, as has been estimated in Chapter 7. The results of Chapter 8 show a higher growth rate of the total population than what has been assumed exogenously of $g_N = 0.00247$ instead of $g_N = 0.002$. This leads to a growth rate of the dependency ratio of $g_{1+D} = 0.00043$, just as the analysis in Chapter 6 suggests as well. Comparing the model with entirely exogenous population dynamics, Chapter 2 to that with an entirely endogenous population development as in Chapter 8, the endogenization of the population dynamics increase the time spent in education. This suggests that with endogenous population dynamics, the best response to an ageing society is still more education.

This chapter has also analyzed the impact of a shock on the population development function and a change in the efficiency of education. Both can be seen as a cause for the recent demographic changes. It has shown that a negative shock on the population development function decreases the growth rate of the dependency ratio that can be afforded and also decreases the time spent in education. This suggests that a population growth shock is not the root cause of ageing. An increase in the efficiency of schooling on the other hand increases the time spent in education and also decreases the population growth rate, which in turn leads to an increase in the growth rate of the dependency ratio.

Given the similarity of the exogenous and endogenous values for the growth rates of the active and total population it can be said, that the exogenous approximation was very close. Nevertheless, it is still of great importance to analyze the endogenous effects as they reveal the full dynamics and dependencies, and allow for alternative adjustment strategies, which alter the solutions of the models.

APPENDIX 8.1 – Derivation of Equation (8.14e)

To derive Equation (8.14e), Equation (8.8) is solved for μ_{ht}, where $\tau_1 e_t{}^{\tau 2} exp^{\tau_3 \log(e_t)^2}\left(\tau_2 + 2\tau_3\log(e_t)\right)\left(\frac{N_t}{N_{t-1}}\right)^{\tau_4}\left(\frac{N_{t-1}}{N_{t-2}}\right)^{\tau_5} N_t$is replaced by N_{t+1} from (8.6).

$$\mu_{ht} = \frac{\omega_t h_t L_t \mu_t - \left(\tau_2 + 2\tau_3\log(e_t)\right)N_{t+1}\mu_{Nt}}{F\gamma e_t^{\gamma-1} h_t} \tag{A8.1}$$

In period $t+1$ Equation (A8.1) becomes

$$\mu_{ht+1} = \frac{\omega_{t+1} h_{t+1} L_{t+1} \mu_{t+1} - \left(\tau_2 + 2\tau_3\log(e_{t+1})\right)N_{t+2}\mu_{Nt+1}}{F\gamma e_{t+1}^{\gamma-1} h_{t+1}} \tag{A8.1$^{\text{I}}$}$$

Equations (A8.1) and (A8.1)$^{\text{I}}$ are plugged into Equation (8.12). This eliminates μ_{ht}.

$$\frac{\omega_t h_t L_t \mu_t - \left(\tau_2 + 2\tau_3\log(e_t)\right)N_{t+1}\mu_{Nt}}{F\gamma e_t^{\gamma-1} h_t} =$$
$$\beta\left[\mu_{t+1}\omega_{t+1}(1-e_{t+1})L_{t+1} + \frac{\omega_{t+1} h_{t+1} L_{t+1} \mu_{t+1} - \left(\tau_2 + 2\tau_3\log(e_{t+1})\right)N_{t+2}\mu_{Nt+1}}{F\gamma e_{t+1}^{\gamma-1} h_{t+1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] \tag{A8.2}$$

Multiplying by $F\gamma e_t^{\gamma-1} h_t$and eliminating the square brackets on the RHS yields:

$$\omega_t h_t L_t \mu_t - \left(\tau_2 + 2\tau_3\log(e_t)\right)N_{t+1}\mu_{Nt} = \beta\mu_{t+1}\omega_{t+1}(1-e_{t+1})L_{t+1}F\gamma e_t^{\gamma-1}h_t + \frac{e_t^{\gamma-1}h_t}{e_{t+1}^{\gamma-1}h_{t+1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\beta\omega_{t+1}h_{t+1}L_{t+1}\mu_{t+1} - \frac{e_t^{\gamma-1}h_t}{e_{t+1}^{\gamma-1}h_{t+1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\beta\left(\tau_2 + 2\tau_3\log(e_{t+1})\right)N_{t+2}\mu_{Nt+1} \tag{A8.2$^{\text{I}}$}$$

In the next step $\beta\mu_{t+1}\omega_{t+1}L_{t+1}h_t$ is multiplied out of the first two terms on the RHS.

$$\omega_t h_t L_t \mu_t - \left(\tau_2 + 2\tau_3\log(e_t)\right)N_{t+1}\mu_{Nt} =$$
$$\beta\mu_{t+1}\omega_{t+1}L_{t+1}h_t\left[(1-e_{t+1})F\gamma e_t^{\gamma-1} + \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] - \frac{e_t^{\gamma-1}h_t}{e_{t+1}^{\gamma-1}h_{t+1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\beta\left(\tau_2 + 2\tau_3\log(e_{t+1})\right)N_{t+2}\mu_{Nt+1} \tag{A8.2$^{\text{II}}$}$$

Solving for $\frac{\mu_t}{\beta\mu_{t+1}}$ yields:

$$\frac{\mu_t}{\beta\mu_{t+1}} = \frac{\omega_{t+1}L_{t+1}}{\omega_t L_t}\left[(1-e_{t+1})F\gamma e_t^{\gamma-1} + \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] + \frac{1}{\beta\mu_{t+1}\omega_t L_t}\left[\frac{1}{h_t}\left(\tau_2 + 2\tau_3\log(e_t)\right)N_{t+1}\mu_{Nt} - \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\beta\frac{1}{h_{t+1}}\left(\tau_2 + 2\tau_3\log(e_{t+1})\right)N_{t+2}\mu_{Nt+1}\right] \tag{A8.2$^{\text{III}}$}$$

With $\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)h_t = h_{t+1}$, Equation (8.5), $\omega_t = \frac{\alpha Y_t}{(1-e_t)h_t L_t}$ from Equation (8.2) and $\frac{\omega_{t+1}L_{t+1}}{\omega_t L_t} = (1+g_\omega)(1+g_L)$, (A8.2)$^{\text{III}}$ becomes:

$$\frac{\mu_t}{\beta\mu_{t+1}} = (1+g_\omega)(1+g_L)\left[(1-e_{t+1})F\gamma e_t^{\gamma-1} + \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] + \frac{(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}\left[\left(\tau_2 + 2\tau_3\log(e_t)\right)N_{t+1}\mu_{Nt} - \beta\left(\tau_2 + 2\tau_3\log(e_{t+1})\right)N_{t+2}\mu_{Nt+1}\frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\frac{\left(Fe_{t+1}^{\gamma}+(1-\delta_h)\right)}{\left(Fe_t^{\gamma}+(1-\delta_h)\right)}\right] \tag{A8.2$^{\text{IV}}$}$$

Finally, multiplying $N_{t+1}\mu_{Nt}$ out of the last bracket and replacing $\frac{N_{t+2}\mu_{Nt+1}}{N_{t+1}\mu_{Nt}} = (1+g_N)(1+g_{\mu N})$ yields:

$$\frac{\mu_t}{\beta\mu_{t+1}} = (1+g_\omega)(1+g_L)\left[(1-e_{t+1})F\gamma e_t^{\gamma-1} + \frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\left(Fe_{t+1}^{\gamma} + (1-\delta_h)\right)\right] + \frac{N_{t+1}\mu_{Nt}(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}\left[\left(\tau_2 + 2\tau_3\log(e_t)\right) - \beta\left(\tau_2 + 2\tau_3\log(e_{t+1})\right)(1+g_N)(1+g_{\mu N})\frac{e_t^{\gamma-1}}{e_{t+1}^{\gamma-1}}\frac{\left(Fe_{t+1}^{\gamma}+(1-\delta_h)\right)}{\left(Fe_t^{\gamma}+(1-\delta_h)\right)}\right] \tag{A8.2$^{\text{V}}$}$$

Equation (A8.2)$^{\text{V}}$ is the equivalent of Equation (8.14e).

APPENDIX 8.2 – Derivation $\left(1+g_{\mu N}\right)$ and $\left(1+g_{\mu h}\right)$

This appendix derives the steady state relations for the two co-states μ_h and μ_N. Equation (8.8) gives insights regarding the development of the co-states μ_N and μ_h. Replacing ω_t with the relation in (8.2) and dividing both sides by $\mu_{ht} h_t$ yields:

$$F\gamma e_t^{\gamma-1} + \frac{\mu_{Nt} N_t}{\mu_{ht} h_t} \tau_1 \tau_2 e_t^{\tau_2 - 1} \left(\frac{N_t}{N_{t-1}}\right)^{\tau_4} \left(\frac{N_{t-1}}{N_{t-2}}\right)^{\tau_5} = \frac{\alpha}{(1-e_t)} \frac{Y_t \mu_t}{\mu_{ht} h_t} \tag{A8.1}$$

Equation (A8.1) must hold at all times, hence both sides of the equation must grow at the same rate. With e_t and $(1+g_N)$ constant, this translates into:

$$\frac{(1+g_{\mu N})(1+g_N)}{(1+g_{\mu h})(1+g_h)} = \frac{(1+g_Y)(1+g_\mu)}{(1+g_{\mu h})(1+g_h)} \tag{A8.2}$$

And hence:

$$\left(1+g_{\mu N}\right)(1+g_N) = (1+g_Y)\left(1+g_\mu\right) \tag*{(A8.2)$^{\mathrm{I}}$}$$

This implies, that with $(1+g_h)$ constant in steady state, $(1+g_{\mu h})$ must be constant as well. Replacing $(1+g_Y)$ and $\left(1+g_\mu\right)$ on the RHS of (A8.2) with the expression in (8.14c)$^{\mathrm{II}}$ and (8.14a)$^{\mathrm{II}}$ respectively gives an expression for $\left(1+g_{\mu N}\right)$:

$$\left(1+g_{\mu N}\right) = \left((1+g_h)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}} \left[\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right]^{-\left(1+\frac{1}{\vartheta}\right)} \tag*{(A8.2)$^{\mathrm{II}}$}$$

Equation (A8.2)$^{\mathrm{I}}$ shows that $\left(1+g_{\mu N}\right)$ is constant in steady state, as $(1+g_h)$ is constant in steady state.

The steady state relation for $g_{\mu h}$ can be derived from Equation (8.12). Replacing ω_{t+1} in (8.12) with the expression in (8.2) and dividing by μ_{ht} yields:

$$1 = \alpha\beta \frac{\mu_{t+1} Y_{t+1}}{\mu_{ht} h_{t+1}} + \beta\left(1+g_{\mu h}\right)\left(F e_{t+1}^{\gamma} + (1-\delta_h)\right) \tag{A8.3}$$

We have already established above that $(1+g_{\mu h})$ is constant in steady state. Since e_{t+1} is also constant in steady state, the second term on the RHS must be constant. This leads to constancy of the first term of the RHS of Equation (A8.3). This means that numerator and denominator must grow at the same rates:

$$(1+g_{\mu h})(1+g_h)=(1+g_Y)(1+g_\mu) \tag{A8.4}$$

Replacing $(1+g_Y)$ and $(1+g_\mu)$on the RHS of (A8.4)) with the expression in (8.14c)^{II} and (8.14a)^{II} respectively gives an expression for $(1+g_{\mu h})$:

$$(1+g_{\mu h})=(1+g_N)(1+g_h)^{\frac{\alpha+\epsilon+\epsilon\vartheta}{\alpha\vartheta}}\left[\beta\left(1+r\left(\frac{B_{t+1}}{Y_{t+1}}\right)(1+\eta_{rb})\right)\right]^{-\left(1+\frac{1}{\vartheta}\right)} \tag{A8.4^{I}}$$

Since all three parts of the RHS of Equation (A8.4)^{I} are constant in steady state, it follows that $(1+g_{\mu h})$ must be constant as well.

Appendix 8.3 – Derivation μ_{Nt}

The following forms of Equation (8.6) are replaced in Equation (8.13):

$$(1+g_N) = \frac{N_{t+2}}{N_{t+1}} = \left(\tau_1 e_{t+1}^{\tau_2} \exp^{\tau_3 \log(e_{t+1})^2} \left(\frac{N_{t+1}}{N_t}\right)^{\tau_4} \left(\frac{N_t}{N_{t-1}}\right)^{\tau_5}\right)$$

$$(1+g_N) = \frac{N_{t+3}}{N_{t+2}} = \left(\tau_1 e_{t+2}^{\tau_2} \exp^{\tau_3 \log(e_{t+2})^2} \left(\frac{N_{t+2}}{N_{t+1}}\right)^{\tau_4} \left(\frac{N_{t+1}}{N_t}\right)^{\tau_5}\right)$$

$$(1+g_N) = \frac{N_{t+4}}{N_{t+3}} = \left(\tau_1 e_{t+3}^{\tau_2} \exp^{\tau_3 \log(e_{t+3})^2} \left(\frac{N_{t+3}}{N_{t+2}}\right)^{\tau_4} \left(\frac{N_{t+2}}{N_{t+1}}\right)^{\tau_5}\right)$$

This leads to

$$\beta\left(\frac{\vartheta\xi\left(\frac{L_{t+1}}{N_{t+1}}\right)^{1+\vartheta}}{1+\vartheta} + \frac{c_{t+1}^{1-\sigma}}{1-\sigma} - c_{t+1}\mu_{t+1} + (1+\tau_4)(1+g_N)\mu_{Nt+1} + \beta(-\tau 4+\tau 5)(1+g_N)(1+g_N)\mu_{Nt+2} - \beta^2\tau_5(1+g_N)^3\mu_{Nt+3}\right) = \mu_{Nt} \qquad (8.13)^{\mathrm{I}}$$

If steady state is assumed, then the growth rate of μ_{Nt} is constant and, hence, $\mu_{Nt+2} = \left(1+g_{\mu N}\right)^2 \mu_{Nt}$. Using Equation (8.7) we know that $c_{t+1} = \mu_{t+1}^{-\left(\frac{1}{\sigma}\right)}$. The second term in Equation (8.13)$^{\mathrm{I}}$, $\frac{c_{t+1}^{1-\sigma}}{1-\sigma} - c_{t+1}\mu_{t+1}$, becomes thus: $\frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}}$. Equation (8.13)$^{\mathrm{I}}$ then becomes :

$$\frac{\vartheta\xi\left(\frac{L_{t+1}}{N_{t+1}}\right)^{1+\vartheta}}{1+\vartheta} + \frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}} + (1+\tau_4)(1+g_N)\left(1+g_{\mu N}\right)\mu_{Nt} + \beta(-\tau 4+\tau 5)\left(1+g_N\right)^2\left(1+g_{\mu N}\right)^2\mu_{Nt} - \beta^2\tau_5(1+g_N)^3\left(1+g_{\mu N}\right)^3\mu_{Nt} = \frac{1}{\beta}\mu_{Nt} \qquad (8.13)^{\mathrm{II}}$$

Rearranging Equation (8.13)$^{\mathrm{II}}$ for μ_{Nt} yields:

$$\frac{\left(\frac{\vartheta\xi\left(\frac{L_{t+1}}{N_{t+1}}\right)^{1+\vartheta}}{1+\vartheta} + \frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}}\right)}{\left(1/\beta - \left((1+\tau_4)\left(1+g_{\mu N}\right)(1+g_N) + \beta(-\tau 4+\tau 5)\left(1+g_N\right)^2\left(1+g_{\mu N}\right)^2 - \beta^2\tau_5(1+g_N)^3\left(1+g_{\mu N}\right)^3\right)\right)} = \mu_{Nt}$$

$$(8.13)^{\mathrm{III}}$$

Equation (8.9) can be rearranged for $\frac{L_{t+1}}{N_{t+1}}$:

$$\frac{L_{t+1}}{N_{t+1}} = \left(\frac{1}{\xi}\mu_{t+1}\omega_{t+1}(1-e_{t+1})h_{t+1}\right)^{\frac{1}{\vartheta}} \quad (8.9)^{\mathrm{I}}$$

$\frac{L_{t+1}}{N_{t+1}}$ in Equation (8.13)$^{\mathrm{III}}$ can then be replaced by the expression in Equation (8.9)$^{\mathrm{I}}$:

$$\left[\frac{\vartheta\left(\frac{1}{\xi}\right)^{\frac{1+\vartheta}{\vartheta}}\xi\left(\mu_{t+1}\omega_{t+1}(1-e_{t+1})h_{t+1}\right)^{\frac{1+\vartheta}{\vartheta}}}{1+\vartheta} + \frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}}\right]\frac{1}{\Gamma} = \mu_{Nt} \quad (8.13)^{\mathrm{IV}}$$

Where: $\Gamma = \left(1/\beta - \left((1+\tau_4)(1+g_{\mu N})(1+g_N) + \beta(-\tau 4+\tau 5)(1+g_N)^2(1+g_{\mu N})^2 - \beta^2\tau_5(1+g_N)^3(1+g_{\mu N})^3\right)\right)$

ω_{t+1} can be replaced by the expression in (8.2): $\omega_{t+1} = \frac{\alpha Y_{t+1}}{(1-e_{t+1})h_{t+1}L_{t+1}}$

$$\left[\frac{\vartheta\left(\frac{\alpha}{\xi}\right)^{1+\frac{1}{\vartheta}}\xi}{1+\vartheta}\left(\frac{Y_{t+1}}{L_{t+1}}\mu_{t+1}\right)^{1+\frac{1}{\vartheta}} + \frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}}\right]\frac{1}{\Gamma} = \mu_{Nt} \quad (8.13)^{\mathrm{V}}$$

This is the expression used in Chapter 8.2.

Appendix 8.4 – Simplification of Equation (8.14e)

This Appendix shows the simplification of the fraction $\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}$ in front of the second term on the RHS of (8.14e). It is successively replaced in the following steps. Appendix 8.3 shows the derivation for μ_{Nt}.

$$\left[\frac{\vartheta\left(\frac{\alpha}{\xi}\right)^{1+\frac{1}{\vartheta}}\xi}{1+\vartheta}\left(\frac{Y_{t+1}}{L_{t+1}}\,\mu_{t+1}\right)^{1+\frac{1}{\vartheta}}+\frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}}\right]\frac{1}{\Gamma}=\mu_{Nt} \tag{8.13)^{V} (}$$

Where $\Gamma=\left(1/\beta-\left((1+\tau_4)(1+g_{\mu N})(1+g_N)\;+\beta(-\tau4+\tau5)(1+g_N)^2(1+g_{\mu N})^2-\beta^2\tau_5(1+g_N)^3(1+g_{\mu N})^3\right)\right)$

This can be plugged into the term $\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}$:

$$\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}=\frac{N_{t+1}(1-e_t)}{\alpha Y_t\beta\mu_{t+1}}\left[\frac{\vartheta\left(\frac{\alpha}{\xi}\right)^{1+\frac{1}{\vartheta}}\xi}{1+\vartheta}\left(\frac{Y_{t+1}}{L_{t+1}}\,\mu_{t+1}\right)^{1+\frac{1}{\vartheta}}+\frac{\sigma}{1-\sigma}\mu_{t+1}^{1-\frac{1}{\sigma}}\right]\frac{1}{\Gamma} \tag{A8.5}$$

Dissolving the brackets gives two terms:

$$\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}=\frac{\vartheta\left(\frac{\alpha}{\xi}\right)^{1+\frac{1}{\vartheta}}\xi}{\alpha\beta(1+\vartheta)}\frac{N_{t+1}(1-e_t)}{Y_t\mu_{t+1}}\left(\frac{Y_{t+1}}{L_{t+1}}\,\mu_{t+1}\right)^{1+\frac{1}{\vartheta}}\frac{1}{\Gamma}+\frac{\sigma}{\alpha\beta(1-\sigma)}\frac{(1-e_t)N_{t+1}}{Y_t}\mu_{t+1}^{-\frac{1}{\sigma}}\frac{1}{\Gamma}$$

In the second term on the RHS c_{t+1} is replaced by the expression in Equation (8.7).

$$\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t}=\frac{\vartheta\left(\frac{\alpha}{\xi}\right)^{1+\frac{1}{\vartheta}}\xi}{\alpha\beta(1+\vartheta)}\frac{N_{t+1}(1-e_t)}{Y_t\mu_{t+1}}\left(\frac{Y_{t+1}}{L_{t+1}}\,\mu_{t+1}\right)^{1+\frac{1}{\vartheta}}\frac{1}{\Gamma}+\frac{\sigma}{\alpha\beta(1-\sigma)}(1-e_t)\frac{c_{t+1}N_{t+1}}{Y_t}\frac{1}{\Gamma}$$

By definition, $\frac{c_tN_t}{Y_t}=X_t$, hence: $\frac{c_{t+1}N_{t+1}}{Y_t}=(1+g_N)(1+g_c)X_t$. This leads to:

$$\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t} = \frac{\vartheta\left(\frac{\alpha}{\xi}\right)^{1+\frac{1}{\vartheta}}\xi}{\alpha\beta(1+\vartheta)}\frac{N_{t+1}(1-e_t)}{Y_t\mu_{t+1}}\left(\frac{Y_{t+1}}{L_{t+1}}\ \mu_{t+1}\right)^{1+\frac{1}{\vartheta}}\frac{1}{\Gamma} + \frac{\sigma}{\alpha\beta(1-\sigma)}(1-e_t)(1+g_N)(1+g_c)X_t\frac{1}{\Gamma}$$

(A8.5)$^{\text{I}}$

To replace μ_{t+1} in Equation (A8.5)$^{\text{I}}$, Equation (8.9) in period t+1 is rearranged for μ_{t+1}:

$$\mu_{t+1} = \frac{\xi\left(\frac{L_{t+1}}{N_{t+1}}\right)^{\vartheta}}{\omega_{t+1}(1-e_{t+1})h_{t+1}} \tag{8.9$^{\text{II}}$}$$

ω_{t+1} is replaced by the expression in Equation (8.2).

$$\mu_{t+1} = \xi\left(\frac{L_{t+1}}{N_{t+1}}\right)^{\vartheta}\frac{L_{t+1}}{\alpha Y_{t+1}} \tag{8.9$^{\text{III}}$}$$

Equation (8.9)$^{\text{III}}$ is plugged into Equation (A8.5)$^{\text{I}}$ to eliminate μ_{t+1}.

$$\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t} = \left[\frac{\vartheta}{\beta(1+\vartheta)}(1-e_t)(1+g_Y) + \frac{\sigma}{\alpha\beta(1-\sigma)}(1-e_t)(1+g_N)(1+g_c)X_t\right]\frac{1}{\Gamma} \tag{A8.5$^{\text{II}}$}$$

The expressions $(1+g_Y)$ and $(1+g_c)$ can be replaced by their expressions in Equations (8.14c)$^{\text{II}}$ and (8.14a)$^{\text{I}}$ respectively, where $(1+g_{1-e}) = \frac{1-e_{t+1}}{1+e_t} = \frac{1-(1+g_e)e_t}{1-e_t}$ and $(1+g_h)$ is replaced by (8.5):

$$\frac{N_{t+1}\mu_{Nt}\,(1-e_t)}{\beta\mu_{t+1}\alpha Y_t} = (1+g_N)(1-e_t)\left[\frac{\vartheta}{\beta(1+\vartheta)}\frac{\left(\frac{1-(1+g_e)e_t}{1-e_t}\left(Fe_t^{\gamma}+(1-\delta_h)\right)^{1+\frac{\epsilon}{\alpha}}\right)^{1+\frac{1}{\vartheta}}}{\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{\frac{1}{\vartheta}}} + \frac{\sigma}{\alpha\beta(1-\sigma)}\left(\beta\left(1+r(b_{t+1})(1+\eta_{rb})\right)\right)^{\frac{1}{\sigma}}X_t\right]\frac{1}{\Gamma} \tag{A8.5$^{\text{III}}$}$$

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Conclusion

This dissertation addresses the question of how modern societies are able to deal with the challenges that arise with an ageing population. Given the latest developments in most OECD countries that show the continuing trend towards a strongly ageing population, it is of crucial importance to understand population dynamics on the one hand and to find ways to deal with them on the other. Whereas the answer to this question is by definition multi-dimensional, the focus of this dissertation lies on the balance between education and production. To be able to support an ageing population, the active population needs to produce more than they used to. This can be achieved in two ways, either the time spent in production is longer, which results in either longer working hours or a later retirement age, or the workers need to be more productive. It can be accomplished through a higher educated population with more individual human capital, which comes at the cost of having to spend time in education. The individuals in the economy then choose the allocation of their active time in education and production. Naturally, neither extreme is optimal. No time in education leads to a society with only low skilled workers that are even unable to read, whereas the other extreme of 100% of the time spent in education leaves no time for production. If the possibility of an entirely debt financed economy is ruled out, this leads to no consumption. Hence, there must be an intermediate optimal time allocation. It is important to note that the time spent in education is not the equivalent of the average degree. Time spent in education is a much broader concept that also includes continuous vocational training and on-the-job training. This dissertation analyzes the changes in this allocation if an economy is faced with the recent demographic developments.

To be able to model demographic developments, the age independent dependency ratio that measures the proportion of people actively involved in education or production relative to the whole population is introduced into an Uzawa-Lucas growth model with international capital movements, human capital externalities and decreasing returns to schooling time in human capital formation in Chapter 2. By calibration two steady states were found of which the one with higher education is stable. This leads to the conclusion that the optimal reaction to an exogenous increase of the growth rate of the dependency ratio is an increase in the time spent in education. It follows a stepwise endogenization of the dependency ratio, by first endogenizing the choice of participation in the active population (Chapter 6) and then endogenizing the development of the total population based on empirical grounds (Chapter 8). The endogenization of the choice of participation in the active population, as done in Chapter 6, leads to a higher share of time spent in education for the same growth rate of the population and a fixed interest rate. The growth rate of the active population decreases compared to the model with an exogenous development of the dependency ratio. This leads to less time in production and more time in education, but also a higher dependency ratio. If the interest rate is endogenized and debt-dependent a similar effect can be obtained by reducing the debt to GDP ratio. This allows for a lower interest rate and reduces the optimal time spent in education to its original value. The growth rate of the active population decreases even further. In Chapter 8, the growth rate of the population is endogenized by making it dependent on the time spent in education. This relation is hump-shaped, indicating that up until a certain point an increase in the time spent in education will increase the population growth rate, whereas it will be decreased by any further increase in the time spent in education. The analyzed countries are in the latter part. The steady state analysis shows that with a flexible interest rate the time spent in education increases slightly as compared to the previous two models. If the growth rate of the population is allowed to alter endogenously, its steady state is slightly above the previously set value, whereas the growth rate of the active population increases as well. This way, the growth rate of the dependency ratio is unchanged as compared to the model with exogenous population growth and endogenous labor force growth.

Along the way several important relations are estimated empirically. The relation between the growth rate of the dependency ratio and the time spent in education is estimated in Chapter 3 to validate the parameter choices of Chapter 2. For this analysis two new variables are constructed to fit the framework of the estimated model better. The time spent in education is constructed as a share of people in the active population that is currently engaged in education. This also includes on the job training. The second newly constructed variable is the growth rate of the dependency ratio, which is empirically constructed with the growth rate of the entire population and the growth rate of the active population as defined by the model. The analysis shows that the 16 chosen OECD countries follow a pattern similar to that of the model analyzed in Chapter 2. Additionally, to account for the fact that the analyzed countries are not in line with the originally assumed small price-taking countries, the assumption of a fixed and given interest rate is relaxed in favor of a debt-dependent interest rate. A country with higher debt has a higher risk of default and hence would be asked a higher interest rate by the creditor. The debt dependent interest rate, to later be used in the Uzawa-Lucas growth model is estimated in Chapter 4. It shows an increasing interest rate for an increasing debt to GDP ratio with diminishing returns. Chapter 7 takes an empirical glance at population development in which it compares two approaches. One takes many explanatory variables into account and the other is rather limited by only incorporating education and output, which makes it attractive for modeling purposes such as an inclusion into the Uzawa-Lucas model. Both approaches show a similar fit to the data.

Overall, the models of this dissertation have shown that when analyzing the relationship between production and education in the context of ageing the best response to the ongoing demographic changes is to increase individual human capital. This is done by a high share of the population in education. This is not meant to say that there are no other measures beyond the ones dealt with in this dissertation, like optimal retirement age, or health expenditures, to reduce ageing or deal with it.

References

Ahn, Seung C, & Schmidt, Peter. (1995). Efficient estimation of models for dynamic panel data. *Journal of econometrics, 68*(1), 5-27.

Alders, Peter, & Broer, D Peter. (2005). Ageing, fertility, and growth. *Journal of Public Economics, 89*(5), 1075-1095.

Anderson, Theodore Wilbur, & Hsiao, Cheng. (1982). Formulation and estimation of dynamic models using panel data. *Journal of econometrics, 18*(1), 47-82.

Angeles, Luis. (2010). Demographic transitions: analyzing the effects of mortality on fertility. *Journal of Population Economics, 23*(1), 99-120.

Arellano, Manuel, & Bond, Stephen. (1991). Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. *The review of economic studies, 58*(2), 277-297.

Arellano, Manuel, & Bover, Olympia. (1995). Another look at the instrumental variable estimation of error-components models. *Journal of econometrics, 68*(1), 29-51.

Backus, David, Cooley, Thomas, & Henriksen, Espen. (2013). Demography and Low Frequency Capital Flows: National Bureau of Economic Research.

Baltagi, Badi. (2008). *Econometric analysis of panel data* (Vol. 1): John Wiley & Sons.

Bardhan, Pranab K. (1967). Optimum foreign borrowing. *Essays in the Theory of Optimal Growth*, 117-128.

Barro, Robert J, & Becker, Gary S. (1989). Fertility choice in a model of economic growth. *Econometrica: journal of the Econometric Society*, 481-501.

Bazzi, Samuel, & Clemens, Michael. (2009). Blunt instruments: A cautionary note on establishing the causes of economic growth: Technical report, Center for Global Development.

Becker, Gary S, Murphy, Kevin M, & Tamura, Robert. (1994). Human capital, fertility, and economic growth *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education (3rd Edition)* (pp. 323-350): The University of Chicago Press.

Benhabib, Jess, & Perli, Roberto. (1994). Uniqueness and indeterminacy: on the dynamics of endogenous growth. *Journal of Economic Theory, 63*(1), 113-142.

Bhandari, Jagdeep S, Haque, Nadeem Ul, & Turnovsky, Stephen J. (1990). Growth, external debt, and sovereign risk in a small open economy. *IMF Staff Papers, 37*(2), 388-417.

Bhargava, Alok, Jamison, Dean T, Lau, Lawrence J, & Murray, Christopher JL. (2001). Modeling the effects of health on economic growth. *Journal of health economics, 20*(3), 423-440.

Blanchard, Olivier Jean. (1983). Debt and the current account deficit in Brazil *Financial policies and the world capital market: The problem of Latin American countries* (pp. 187-198): University of Chicago Press.

Bloom, David E, Canning, David, & Fink, Günther. (2010). Implications of population ageing for economic growth. *Oxford Review of Economic Policy, 26*(4), 583-612.

Blundell, Richard, & Bond, Stephen. (1998). Initial conditions and moment restrictions in dynamic panel data models. *Journal of econometrics, 87*(1), 115-143.

Bond, Stephen R. (2002). Dynamic panel data models: a guide to micro data methods and practice. *Portuguese economic journal, 1*(2), 141-162.

Boucekkine, Raouf, De la Croix, David, & Licandro, Omar. (2002). Vintage human capital, demographic trends, and endogenous growth. *Journal of Economic Theory, 104*(2), 340-375.

Breusch, Trevor Stanley, & Pagan, Adrian Rodney. (1980). The Lagrange multiplier test and its applications to model specification in econometrics. *The Review of Economic Studies*, 239-253.

Brückner, Markus, & Schwandt, Hannes. (2013). Income and Population Growth: Institute for the Study of Labor (IZA).

Bun, Maurice JG, & Windmeijer, Frank. (2010). The weak instrument problem of the system GMM estimator in dynamic panel data models. *The Econometrics Journal, 13*(1), 95-126.

Caldwell, John C. (1979). Education as a factor in mortality decline an examination of Nigerian data. *Population studies*, 395-413.

Castelló-Climent, Amparo, & Domenech, Rafael. (2008). Human Capital Inequality, Life Expectancy And Economic Growth*. *The Economic Journal, 118*(528), 653-677.

Cutler, David, Deaton, Angus, & Lleras-Muney, Adriana. (2006). The Determinants of Mortality. *The Journal of Economic Perspectives, 20*(3), 97-120.

Davidson, Russell, & MacKinnon, James G. (2004). *Econometric theory and methods* (Vol. 5): Oxford University Press New York.

De La Croix, David, & Doepke, Matthias. (2003). Inequality and growth: why differential fertility matters. *The American Economic Review, 93*(4), 1091-1113.

De la Croix, David, & Licandro, Omar. (1999). Life expectancy and endogenous growth. *Economics Letters, 65*(2), 255-263.

Denison, Edward F. (1962). *The sources of economic growth in the United States and the alternatives before us* (Vol. 13): Committee for Economic Development New York.

Denk, Michaela, & Weber, Michael. (2011). Avoid Filling Swiss Cheese with Whipped Cream: Imputation Techniques and Evaluation Procedures for Cross-Country Time Series. *IMF Working Papers*, 1-27.

Eaton, Jonathan, & Gersovitz, Mark. (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies*, 289-309.

Edwards, Sebastian. (1984). LDC's foreign borrowing and default risk: An empirical investigation (Vol. September, pp. 726-734): American Economic Review.

Ehrlich, Isaac, & Kim, Jinyoung. (2005). Endogenous fertility, mortality and economic growth: Can a Malthusian framework account for the conflicting historical trends in population? *Journal of Asian Economics, 16*(5), 789-806.

Engle, Robert F, & Granger, Clive WJ. (1987). Co-integration and error correction: representation, estimation, and testing. *Econometrica: journal of the Econometric Society*, 251-276.

Fanti, Luciano, & Gori, Luca. (2009). Endogenous fertility, endogenous lifetime and economic growth: the role of health and child policies: Dipartimento di Economia e Management (DEM), University of Pisa, Pisa, Italy.

Fayissa, Bichaka, & Gutema, Paulos. (2010). Dependency Ratio and the Economic Growth Puzzle in Sub-Saharan Africa. *Department of Economics and Finance Working Paper series.(Assessed on 22 December 2010 from http://ideas. repec. org/p/mts/wpaper/201010. html)*.

Foley, Duncan K. (2000). Stabilization of human population through economic increasing returns. *Economics Letters, 68*(3), 309-317.

French, Declan. (2014). International mortality modelling—An economic perspective. *Economics Letters, 122*(2), 182-186.

Frenkel, Jacob A, Razin, Assaf, & Yuen, Chi-Wa. (1996). Fiscal policies and growth in the world economy. *The MIT Press, Cambridge*.

Galasso, Vincenzo. (2008). Postponing retirement: the political effect of aging. *Journal of Public Economics, 92*(10), 2157-2169.

Galor, Oded. (2005). The demographic transition and the emergence of sustained economic growth. *Journal of the European Economic Association, 3*(2-3), 494-504.

Galor, Oded, & Weil, David N. (1993). The gender gap, fertility, and growth: National Bureau of Economic Research.

Galor, Oded, & Weil, David N. (1998). Population, Technology, and Growth: From the Malthusian Regime to the Demographic Transition and Beyond: National Bureau of Economic Research.
Gong, Gang, Greiner, Alfred, & Semmler, Willi. (2004). The Uzawa–Lucas model without scale effects: theory and empirical evidence. *Structural change and economic dynamics, 15*(4), 401-420.
Greene, William H. (2008). *Econometric analysis*: Granite Hill Publishers.
Gruescu, Sandra. (2006). *Population ageing and economic growth: education policy and family policy in a model of endogenous growth* (Vol. 1431): Springer.
Hamada, Koichi. (1969). Optimal capital accumulation by an economy facing an international capital market. *The Journal of Political Economy, 77*(4), 684-697.
Hamilton, James Douglas. (1994). *Time series analysis* (Vol. 2): Princeton university press Princeton.
Hansen, Lars Peter. (1982). Large sample properties of generalized method of moments estimators. *Econometrica: Journal of the Econometric Society*, 1029-1054.
Hviding, Ketil, & Mérette, Marcel. (1998). Macroeconomic Effects of Pension Reforms in the Context of Ageing Populations: Overlapping Generations Model Simulations for Seven OECD Countries Economics Department Working Papers No. 201. *WORKING PAPERS-ORGANISATION FOR ECONOMIC COOPERATION AND DEVELOPMENT ECONOMICS DEPARTMENT*.
Judson, Ruth A, & Owen, Ann L. (1999). Estimating dynamic panel data models: a guide for macroeconomists. *Economics letters, 65*(1), 9-15.
Kiviet, Jan F. (1995). On bias, inconsistency, and efficiency of various estimators in dynamic panel data models. *Journal of econometrics, 68*(1), 53-78.
Lane, Philip R, & Milesi-Ferretti, Gian Maria. (2007). The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004. *Journal of international Economics, 73*(2), 223-250.
Lee, Ronald. (2003). The demographic transition: three centuries of fundamental change. *The Journal of Economic Perspectives, 17*(4), 167-190.
Lee, Ronald D, & Carter, Lawrence R. (1992). Modeling and forecasting US mortality. *Journal of the American statistical association, 87*(419), 659-671.
Lee, Suk Hun. (1991). Ability and willingness to service debt as explanation for commercial and official rescheduling cases. *Journal of Banking & Finance, 15*(1), 5-27.

Lee, Suk Hun. (1993). Are the credit ratings assigned by bankers based on the willingness of LDC borrowers to repay? *Journal of Development Economics, 40*(2), 349-359.

Liao, Pei-Ju. (2011). Does demographic change matter for growth? *European Economic Review, 55*(5), 659-677.

Lucas, Robert E. (1988). On the mechanics of economic development. *Journal of monetary economics, 22*(1), 3-42.

Ludwig, Alexander, Schelkle, Thomas, & Vogel, Edgar. (2012). Demographic change, human capital and welfare. *Review of Economic Dynamics, 15*(1), 94-107.

Malthus, Thomas R. . (1826). An essay on the principle of population. *Cambridge: Cambridge University Press*.

Mankiw, N Gregory, Romer, David, & Weil, David N. (1992). A Contribution to the Empirics of Economic Growth. *The Quarterly Journal of Economics, 107*(2), 407-437.

Murtin, Fabrice. (2013). Long-term determinants of the demographic transition, 1870–2000. *Review of Economics and Statistics, 95*(2), 617-631.

Muysken, J, & Ziesemer, THW. (2013). A permanent effect of temporary immigration on economic growth. *Applied Economics, 45*(28), 4050-4059.

Muysken, J, & Ziesemer, THW. (2014). The effect of immigration on economic growth in an ageing economy. *Bulletin of Applied Economics, 1*(1), 37-65.

Nickell, Stephen. (1981). Biases in dynamic models with fixed effects. *Econometrica: Journal of the Econometric Society*, 1417-1426.

Okui, Ryo. (2009). The optimal choice of moments in dynamic panel data models. *Journal of Econometrics, 151*(1), 1-16.

Papageorgiou, Chris, Savvides, Andreas, & Zachariadis, Marios. (2007). International medical technology diffusion. *Journal of International Economics, 72*(2), 409-427.

Pedroni, Peter. (1999). Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors. *Oxford Bulletin of Economics and Statistics*

61, 653-670.

Pedroni, Peter. (2000). Fully Modified OLS for Heterogeneous Cointegrated Panels. *Advances in Econometrics, 15*, 93-130.

Pedroni, Peter. (2004). Panel cointegration: asymptotic and finite sample properties of pooled time series tests with an application to the PPP hypothesis. *Econometric theory, 20*(03), 597-625.

Pesaran, MHHM. (2004). General diagnostic tests for cross section dependence in panels. *University of Cambridge, Faculty of Economics, Cambridge Working Papers in Economics Letters, No. 0435*.
Peterman, William B. (2014). Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity: A Sensitivity Analysis: Working Paper.
Philippopoulos, Apostolis. (1991). MACROECONOMIC DYNAMICS AND OPTIMAL EXTERNAL BORROWING IN AN IMPERFECT WORLD CAPITAL MARKET*. *The Manchester School, 59*(1), 12-23.
Phillips, Peter CB. (1986). Understanding spurious regressions in econometrics. *Journal of econometrics, 33*(3), 311-340.
Phillips, Peter CB, & Hansen, Bruce E. (1990). Statistical inference in instrumental variables regression with I (1) processes. *The Review of Economic Studies, 57*(1), 99-125.
Phillips, Peter CB, & Sul, Donggyu. (2003). Dynamic panel estimation and homogeneity testing under cross section dependence*. *The Econometrics Journal, 6*(1), 217-259.
Prettner, Klaus, Bloom, David E, & Strulik, Holger. (2013). Declining fertility and economic well-being: Do education and health ride to the rescue? *Labour Economics, 22*, 70-79.
Quesada, Javier, & García-Montalvo, José. (1996). Growth and population aging: The Spanish case: Instituto Valenciano de Investigaciones Económicas, SA (Ivie).
Ricardo, D. (1817). On the Principles of Political Economy and Taxation. *Murray, London*.
Roodman, David. (2009a). How to do xtabond2: An introduction to difference and system GMM in Stata. *Stata Journal, 9*(1), 86.
Roodman, David. (2009b). A note on the theme of too many instruments*. *Oxford Bulletin of Economics and Statistics, 71*(1), 135-158.
Ruud, Paul A. (2000). Classical Econometrics: New York: Oxford University Press.
Saikkonen, Pentti. (1991). Asymptotically efficient estimation of cointegration regressions. *Econometric theory, 7*(01), 1-21.
Saikkonen, Pentti. (1992). Estimation and testing of cointegrated systems by an autoregressive approximation. *Econometric theory, 8*(01), 1-27.
Sargan, John D. (1958). The estimation of economic relationships using instrumental variables. *Econometrica: Journal of the Econometric Society*, 393-415.
Smith, Adam. (1776). An Inquiry into the Nature and Causes of the Wealth of Nations, ed. *Edwin Cannan (1776*, 282-309.
Smith, Ron P, & Fuertes, Ana-Maria. (2010). Panel time-series.

Solow, R. M. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics, 70*(1), 65-94.
Soto, Marcelo. (2009). System GMM estimation with a small sample.
Stock, James H, & Watson, Mark W. (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica: Journal of the Econometric Society*, 783-820.
Turner, John. (2007). Social security pensionable ages in OECD countries: 1949-2035. *International Social Security Review, 60*(1), 81-99.
Uzawa, Hirofumi. (1965). Optimum technical change in an aggregative model of economic growth. *International economic review, 6*(1), 18-31.
von Humboldt, Alexander. (1869). *Kosmos*: Thomas.
Windmeijer, Frank. (2005). A finite sample correction for the variance of linear efficient two-step GMM estimators. *Journal of econometrics, 126*(1), 25-51.
Wright, Robert E, Kinnunen, Jouko, Lisenkova, Katerina, & Merette, Marcel. (2014). Population Ageing and Technological Change: EcoMod.
Xie, Danyang. (1994). Divergence in economic performance: transitional dynamics with multiple equilibria. *Journal of Economic Theory, 63*(1), 97-112.
Zhang, Pengfei. (2013). A Newfound Steady State in Standard Uzawa-Lucas Model with Externality. *Available at SSRN 2262391*.
Ziesemer, Thomas. (2005). Malthus irrelevant? : Maastricht University, Maastricht Economic Research Institute on Innovation and Technology (MERIT).

Valorization Addendum

This valorization addendum discusses knowledge valorization in the context of the current dissertation and reflects the author's opinions regarding the topic "Education in Times of Population Ageing".

The main question to be asked when trying to valorize a research topic is under which circumstances academic work can be considered to be value adding. Alexander von Humboldt (1769-1859), who is considered to be one of the pioneers of academic work as we know it today, once said that all findings are just one step towards something higher (von Humboldt, 1869). In this sense, all knowledge, may it be obtained through academic research or discovered by chance, leads to a more complete picture. Drawing an ever better and a more precise picture of the world as we know it should be the highest objective for any researcher. The true value of any knowledge may not be immediately apparent, as this can only be evaluated by the way this knowledge is used to benefit, or harm society. It is not in the hands of a researcher to control, or influence how the knowledge provided by his or her research is put into practice, but he or she can give indications on how the research was intended. This is what this valorization addendum will try to achieve.

Research in the field of economics and business can be considered to be value adding, if it is of social and/or economic relevance. This dissertation aims at contributing to the question of how society can keep its consumption standards while facing a shift in population structure. The relevance of this topic has been analyzed in Chapter 1. Within the last decades the structure of the population has shifted from a pyramid form, in which there are more children and young adults than working age people and retirees, to a tree shaped figure, in which the share of children is relatively small compared to that of adults and retirees. Logically, such a shift in population structure must be followed by serious challenges when trying to support the entire economy. Fewer children lead to fewer adults who will be able to support the elderly. Adding to this effect is the growing group of retirees who no longer contribute to economic output, but rely on the economy's support. Hence, there is a need for economic research to find proper policies to counteract this vicious circle. One possible answer is given in this dissertation. In order to discuss the added value of this dissertation, its opportunities and shortcomings must be identified.

Chapter 2 sets up a model that aims at explaining the dynamics behind the interaction between population development and time spent in education. The purpose of this chapter is to introduce the base model which, to my knowledge, has not yet been presented in this way. For this model, as for most models it holds that:

> "All the theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive" – Solow (1956), p. 65

Consequently, several simplifying assumptions have been made, one of which lies in the nature of the Uzawa-Lucas model. The population is deliberately not divided into several generations to show the effect of changing generation sizes. This would complicate the analysis and would not proportionally add to the significance of the results.

This dissertation addresses the question of how modern societies are able to deal with the challenges that arise with an ageing population. Given the latest developments in most OECD countries that show the continuing trend towards a strongly ageing population, it is of crucial importance to understand population dynamics on the one hand and to find ways to deal with them on the other. Whereas the answer to this question is by definition multi-dimensional, the focus of this dissertation lies on the balance between education and production. To be able to support an ageing population, the active population needs to produce more than they used to. This can be achieved in two ways, either the time spent in production is longer, which results in either longer working hours or a later retirement age, or the workers need to be more productive. It can be accomplished through a higher educated population with more individual human capital, which comes at the cost of having to spend time in education. The individuals in the economy then choose the allocation of their active time in education and production. Naturally, neither extreme is optimal. No time in education leads to a society with only low skilled workers that are even unable to read, whereas the other extreme of 100% of the time spent in education leaves no time for production. If the possibility of an entirely debt financed economy is ruled out, this leads to no consumption. Hence, there must be an intermediate optimal time allocation. It is important to note that the time spent in education is not the equivalent of the average degree. Time spent in education is a much broader concept that also includes continuous vocational training and on-the-job training. This dissertation analyzes the changes in this allocation if an economy is faced with the recent demographic developments.

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References

Solow, R. M. (1956). A contribution to the theory of economic growth. Quarterly Journal of Economics, 70(1), 65-94.

von Humboldt, Alexander. (1869). Kosmos: Thomas.

Curriculum Vitae

Anne Edle von Gäßler was born on September 28th, 1986 in Bonn, Germany. After obtaining her Abitur in 2005, she started studying International Economic Studies at Maastricht University.

In 2009 she obtained the Bachelor of Science degree in International Economic Studies and in 2011 her Master of Science in Economic and Financial Research, both from the School of Business and Economics at Maastricht University.

During her studies she also developed an interest for teaching and obtained a tutor position in the departments of Quantitative Economics and General Economics where she taught introductory and intermediate courses in Quantitative Mathematics. Anne also took on a Research Assistant position for one year at the department for General Economics next to her master studies in which she pursued her own research project that was later part of her Master Thesis.

In 2011 Anne joined the PhD program in the department of General Economics at the Graduate School of Business and Economics at Maastricht University. During the PhD program, Anne had the opportunity to attend several conferences in which her research output was presented. One of these was the Workshop on Human Capital and Ageing at the Harvard School of Public Health, organized by David E. Bloom and Alfonso Sousa-Poza.