

# Time-consistent and market-consistent actuarial valuations

Citation for published version (APA):

Salahnejhad Ghalehjooghi, A. (2016). *Time-consistent and market-consistent actuarial valuations*. [Doctoral Thesis, Maastricht University]. Maastricht University. <https://doi.org/10.26481/dis.20161221as>

## Document status and date:

Published: 01/01/2016

## DOI:

[10.26481/dis.20161221as](https://doi.org/10.26481/dis.20161221as)

## Document Version:

Publisher's PDF, also known as Version of record

## Document license:

Unspecified

## Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

## General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.umlib.nl/taverne-license](http://www.umlib.nl/taverne-license)

## Take down policy

If you believe that this document breaches copyright please contact us at:

[repository@maastrichtuniversity.nl](mailto:repository@maastrichtuniversity.nl)

providing details and we will investigate your claim.

# Summary

The main body of this thesis discusses and introduces the methodologies to implement the “market-consistent” actuarial valuation for a hybrid liabilities consisting the underlying financial and actuarial risks. In that respect, we give a clear picture on what happens in the valuation procedure to make the pricing operator market-consistent, and how the information flow should be dealt with over the valuation period. We have provided both analytical and numerical implementations for the valuation methodology, and we have compared the results with some available methods used in industry.

The market-consistent actuarial price requires the time-consistency property to avoid the arbitrage opportunities over the valuation period. This is similar to build the tower property for a non-linear pricing operator. Time-consistent pricing operators can be created by backward iteration of one-period valuations. In Chapter 2 of this thesis, we investigate the continuous-time limits of the well-known actuarial premium principles when such backward iteration procedures are applied. This method is applied to an insurance risk process in the form of a diffusion process. We show that in the case of the diffusion process, the one-period time-consistent Variance premium principle converges to the non-linear exponential indifference price. Furthermore, we show that the Standard-Deviation and the Cost-of-Capital principle converge to the same price limit. In chapter 3, we consider the heavy-tailed nature of the insurance liabilities as a motivation to model the actuarial risk under the jump-diffusion process and get a more realistic picture of the underlying risk dynamics. We generalize the backward iteration of the one-period valuation of the insurance premium principles when the unhedgeable insurance risk can also jump by an stochastic arrival time. As a result, we no longer observe that the different premium principles converge to the same limit since each principle reflects the effect of the jump differently. In the Cost-of-Capital principle in particular, the VaR operator fails to capture the jump risk for small jump probabilities and the time-consistent price depends only on the distribution of the premium jump.

Market-Consistent valuation is a requirement by the EU insurance supervisor to value the insurance liabilities such as life and pension contracts that are not (actively) traded in the market. To obtain a market-consistent price, in Chapter 4, we combine the hedgeable financial risk with an (partially) unhedgeable actuarial risk and price the hybrid payoff using a “two-step market evaluation”. In a general setting, the valuation process comprises the no-arbitrage price of pure financial risk, the value of partially hedged actuarial risk attributable to its correlation with financial risk (if available), and finally the value of pure actuarial risk through well-known actuarial premium principles. We

implement a two-step valuation using a backward iteration method and obtain a time-consistent market-consistent (TCMC) price during the valuation period. We also provide a continuous-time limit of the TCMC price for the Variance and Standard-Deviation actuarial prices. We also provide a market-consistent version of alternative pricing methods: the Best-Estimate pricing method typically used for pension liabilities and the EIOPA's Risk-Margin method used under Solvency II to value life insurance liabilities. By comparing these prices with the TCMC price for a unit-linked contract, we show that the EIOPA Risk-Margin method acts in the correct direction to reflect part of the uncertainty attributable to the future dynamics of non-hedgeable risks, whereas Best-Estimate pricing completely ignores that uncertainty. Because the Risk-Margin method still ignores certain uncertainties, it is not fully time consistent and its gap with TCMC should not be ignored for long-dated contracts. In Chapter 5, we implement the two-step actuarial operator for a simple unit-linked contract with the equity and mortality, respectively as the underlying financial and actuarial risk drivers. We provide some insights into formulation of the EIOPA risk-margin and time-consistent price with the two-step actuarial operator under independence assumption. To perform the numerical calculations we adapt a customized version of the Least Square Monte Carlo (LSMC) method and simulate the three market-consistent prices (the two above and the best-estimate price). We show that the two-step actuarial valuation captures partial (or perfect) hedging as we observe that all three prices converge to one adjusted Best-Estimate price when the correlation between financial and actuarial risks increases.

In Chapter 6 we consider the market-consistent valuation of the participating pension contract. It is an example of the long-dated liabilities that are not fully traded and hedgeable in the market. Due to the existence of actuarial risk in their payoff, they are normally priced by non-linear premium principles. Such long-term positions make the issuer's valuation and risk management vulnerable to the dynamics of medium-term fluctuations. To reflect the effect of the medium-time evolution on the price, we require time-consistency while the premium principles are not time-consistent. The study provides a TSMC pricing method for a participating life/pension scheme with guaranteed interest rate. We consider a hybrid combination of the mortality, interest rate and equity risks in the final payoff and the profit-sharing mechanism over the life of the contract. We use the Least-Square Monte-Carlo (LSMC) method to implement the backward iteration and calculate the conditional expectations in the two-step operator. We report the TCMC price and compare it to the expected value of the discounted payoff and measure the relative risk loading and time-consistency risk premium. We also study the effect of the stochastic interest rate on the price of the contract as compared to the deterministic one.