

# Portfolio replication and least squares Monte Carlo with application to insurance risk management

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# Valorization

*“Nothing is more practical than a good theory.”*

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– Kurt Lewin

This addendum is dedicated to the discussion of the economic and social value added by this dissertation as well as the opportunities of transferring the knowledge presented in this thesis into practical use. According to the National Valorization Committee knowledge valorization refers to “the process of creating value from knowledge, by making knowledge suitable and/or available for social (and/or economic) use and by making knowledge suitable for translation into competitive products, services, processes and new commercial activities”. This thesis has been supported by the largest German insurer, Allianz, who is a major stakeholder and a customer of the results of this thesis. In that respect knowledge valorization has been a continuous process throughout the construction of this thesis. In the following addendum I outline the knowledge valorization of this dissertation by discussing the economic relevance of portfolio replication<sup>1</sup>, the stakeholders and target groups, the implications of the results for the industry, and the innovativeness of the research.

As the title of this thesis already suggests this work is mainly targeted at the insurance risk sector. Solvency II defines the regulatory framework for insurance supervision in the EU. It is a comprehensive concept that, among other aspects, lays down the capital requirements for the insurance industry and the quantitative rules for determining the risk capital. While the Solvency II framework offers a standard model to the calculation of risk capital requirements, particularly large insurers opt for an internal model to better represent the individual characteristics of their business. Part of the Solvency II requirement is the market-consistent valuation of the insurer’s own funds. This is a challenging task as insurance liabilities are typically not traded

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<sup>1</sup>Chapter 1 already to a large extent addresses the economic relevance and necessity to discuss portfolio replication in the context of insurance risk management, which is the topic of this dissertation. Therefore, part of the discussion in this chapter can also be found in Chapter 1.

and market values are therefore not immediately available. Moreover, many insurance products are complex in their structure and underlying dynamics and closed-form solutions to their value do not exist. Hence, numerical techniques are typically employed instead. Straightforwardly, the value of an insurance liability may simply be estimated through Monte Carlo simulation of all underlying risk factors from the risk horizon to the terminal time point, mostly the maturity time point, of the insurance policy (or a fund of pooled policies). The sample average then serves as the value estimate of the liability at the risk horizon. The problem now is, that an insurer requires the value of the liabilities under different possible economic risk factor realizations at the risk horizon in order to calculate risk capital figures as required by Solvency II. For a large insurance company, for example, it is common to consider 10000-50000 scenario realizations at the risk horizon. This means that the exercise of estimating the liability value at the risk horizon would need to be repeated 10000-50000 times. Using, for example, 1000 Monte Carlo simulations for the estimation of the liability value at each scenario realization at the risk horizon would result in a total simulation set of 10-50 million scenarios. Depending on the size and complexity of the insurance company this may quickly lead to an infeasible simulation size. The problem particularly arises for life insurance products where very long maturities such as 60 years are to be expected and minimum guarantee and profit-sharing mechanisms often additionally complicate their valuation. The problem statement has triggered the demand for alternative solutions to the valuation of (life) insurance liabilities. Commonly, approximation techniques are used in combination with Monte Carlo simulation in order to represent the valuation function of the liabilities. Given a proxy function to the value of the liabilities the estimation of the value under different economic scenarios at the risk horizon tremendously simplifies when the

proxy function is used as a substitute for the insurers liability portfolio. Among these proxy techniques, portfolio replication and Least Squares Monte Carlo (LSMC) are very popular and widely applied in the industry. Allianz, for example, employs portfolio replication for its life insurance business in order to represent market risk in its risk capital calculations (see e.g. Davidson, 2011)

Now, while both methods are already applied in insurance risk modeling, their properties have not been fully explored yet and many open questions remain. This particularly pertains to portfolio replication as a proxy method in the context of insurance liability modeling, which has so far received little attention in the academic literature compared to LSMC. LSMC is a well-known numerical technique that originates from American option pricing and has been widely discussed in the literature; see, among others, Carriere (1996), Longstaff and Schwartz (2001), Tsitsiklis and Van Roy (2001), Clement et al. (2002), Stentoft (2004), Glasserman and Yu (2004b), Egloff et al. (2007), Belomestny (2011), Gerhold (2011) and Zanger (2013). The asymptotic convergence theory for LSMC has, for example, been analyzed in Stentoft (2004), where it is shown that the LSMC estimator converges in the limit to the true value. Until now a theoretical foundation for replicating portfolios as a proxy technique has been missing in the academic literature. In that context the asymptotic convergence theory of the replicating portfolio technique has not been analyzed. Yet, understanding the asymptotic behaviour of an estimator is important when using a method. Another gap in the existing literature is a discussion on the advantages and disadvantages of the LSMC and the replicating portfolio technique and how they compare. While indications are given in the current literature (see Glasserman and Yu (2004b), Broadie and Cao (2008) and Bender and Steiner (2012)), a full-fledged discussion is presently missing. In practical applications considerations such as what quality

measures to use for the assessment of a proxy solution and what to be aware of in the set-up of the calibration scenarios are very important.

In conclusion, questions that insurance risk managers pose in choosing and setting up a proxy method in order to simplify the representation of their liabilities have not fully been answered yet by the existing literature. Apart from the internal aspiration to establish a solid internal risk model there are also external regulatory requirements. Insurers developing or extending their internal risk models within the Solvency II framework are subject to a regulatory internal model approval process, as a result of which the regulator will either accept or reject the proposed model or model changes. In order to get their internal risk models approved by the regulators, insurers have to perform a validation of their risk model. Consequently, insurers using the LSMC and/or portfolio replication technique also have to provide information regarding the validity of the method employed and why a particular method has been chosen over other alternatives.

This thesis closes the aforementioned gaps in the literature by analyzing the asymptotic properties of the replicating portfolio method, comparing the two methods under discussion and elaborating on implications in practice. In Chapters 2 and 3 it is shown that portfolio replication is a mathematically sound concept with a well-developed theoretical background. The asymptotic convergence of the replicating portfolio estimator to its true value is an important aspect with regards to the validity of the replicating portfolio method as a proxy technique for the representation of insurance liabilities. Chapter 4 discusses the differences between portfolio replication and LSMC as well as the implications of these differences for practical applications. The information provided supports the process of choosing one method over the other and stresses what to be aware of when using a method.

The results and methods presented in this thesis allow insurance risk managers, researchers and regulators to obtain a better understanding of the mathematical and asymptotic properties of the replicating portfolio technique, on the one hand, and, on the other hand, to see a clear relation to the currently more popular LSMC method. Understanding the advantages and disadvantages of these methods helps stakeholders to make better informed decisions on the choice of one method over the other. The information provided in this thesis thereby also supports insurance risk managers in explaining their model choice to the regulator. Moreover, aspects in the calibration and assessment of replicating portfolios are addressed in this thesis. For instance, in Chapter 4 the usefulness of a particular quality measure for the assessment of replicating portfolios is examined. Another example is the construction of scenarios for the calibration of the replicating portfolio. Both considerations support insurance risk managers in setting up an appropriate replicating portfolio approach.

The results of this thesis are mainly targeted at insurance risk managers and regulators in the context of Solvency II. This is because in that field proxy methods find direct application in risk capital calculations as they enable a simplified representation of insurance liabilities. However, the same idea may be leveraged to enable other interesting analysis in the insurance context. Casa and Gaffo (2013) discuss portfolio replication in the context of asset-liability management and portfolio optimization, where the replicating portfolio technique is leveraged to efficiently compare a series of different asset allocation opportunities. A careful implementation of the replicating portfolio technique is essential for the analysis to be meaningful. The results of this thesis therefore also find application in that research field. Additionally, there may be numerous other fields where the use of

proxy methods is of interest and the results of this thesis may be helpful.

As already mentioned the process of knowledge valorization outlined in this addendum is already realized by an application of the results of this thesis outside the academic world. It has been mentioned before that Allianz employs portfolio replication in its internal risk model, but also other insurers use the replicating portfolio or LSMC technique and can therefore profit from the outcomes of this thesis. This thesis also indicates various avenues for potential future research valued by the insurance industry. One option is the extension of the results of this thesis to multi-period problems. This is relevant in insurance due to dynamic lapse and surrender options, which make (life) insurance liabilities similar in its structure to American options. Furthermore, complex path-dependent insurance liabilities are more difficult to replicate and the construction of a appropriate replicating instruments is challenging. While a first step towards solving such problems is given in this thesis, much more research is required to find more general solutions. I look forward to extensions of this thesis in that direction as well as the development towards wider applicability of replicating portfolios in the insurance sector.