Matching, voting and cooperation

SUMMARY

In our daily life, we are often confronted with problems of *matching*, *voting* and *cooperation*: students have to be matched to schools, and organ transplants need to be matched to patients; presidents are elected, and laws are voted on; and at work projects are undertaken in groups. This thesis covers different topics in matching and voting theory, as well as cooperative game theory. The first chapter discusses two-sided matching problems in which workers' employment experience shapes firms' preferences. Assuming that experience assigns to each worker a set of firms he has been employed by, we first show that there are experience configurations for which no stable matching exists. We then present a sufficient condition guaranteeing existence of stable matchings for any configuration. The condition stipulates the idea that firms are size-sensitive with respect to the experience of each worker and it allows us characterize the set of stable matchings. We also provide an example showing that experience size-sensitivity does not assure the existence of a firm optimal stable matching. We contribute with the third chapter to the branch of axiomatic characterization within the field of matching theory. We assume that a resolving rule guides the selection of blocking pairs and show that two axioms (independence and top optimality) transform such a rule into a gender consistent one. Stable matchings can be reached when starting from an arbitrary individually rational matching and iteratively satisfying the pair selected by a gender consistent resolving rule.

The third chapter focuses on voting theory. We consider voting correspondences that are, besides Condorcet Consistent, immune against the two strong no show paradoxes. That is, it cannot happen that if an additional voter ranks a winning alternative on top then that alternative becomes losing, and that if an additional voter ranks a losing alternative at bottom then that alternative becomes winning. We identify the maximal voting correspondence that satisfies Condorcet Consistency and is immune against the two strong no show paradoxes. In particular, voting functions (single-valued voting correspondences) that are Condorcet Consistent and immune against the two paradoxes must select from this maximal correspondence, and we demonstrate several ways in which this can or cannot be done. We also consider a weaker version of Condorcet Consistency.

The last two chapters discuss topics of cooperation. In the fourth chapter, we introduce a new concept of farsightedness and base a new solution concept for abstract games on it. An abstract game consists of a set of states and specifies what coalitions are allowed to replace one state by another one. Agents are called farsighted if they compare the status quo to the long-term outcome following their deviation rather than to the status

they actually deviate to. What the literature has ignored so far is that if a coalition does not move out of the status quo, they might still expect another coalition to do so. Specifically, above definition of farsightedness implies that agents ignore this possibility in their reasoning. So, in fact, agents are not fully farsighted. Expectation functions assign to each state a (potentially) different state and a coalition that moves from the former to the latter, thereby creating paths between states. This endows agents with an expectation about what any potential deviation entails—namely the path of the prescribed further moves. We extend these functions by capturing coalitions' expectations about the consequences of not moving out of a state. We impose three stability and optimality axioms on extended expectation functions that reflect full farsightedness and rationality. We then show that an extended expectation function satisfies our axioms if and only if it can be associated with a non-cooperative equilibrium of the abstract game. We finally apply our solution to games in characteristic function form and matching problems.

The fifth chapter analyzes individuals' behavior if agents have to repeatedly cooperate with others. Although cooperative game theory normally abstracts away of any strategic interaction between players, we argue that in a repeated set up players would act strategically if they could. We intend to incorporate and predict such (strategic) behavior by modeling non-cooperative games based on a dynamic game with transferable utility and point-wise solution concepts for them. We model the strategic interaction between agents by allowing them to transfer some worth among periods, and, hence, indirectly affect their payoffs and the payoffs of the other players. We discuss different transfer systems, specifying what worth agents can transfer and identify Nash equilibria in the resulting non-cooperative games. Furthermore, we characterize the Nash equilibria and discuss uniqueness. We also consider particular classes of TU-games, such as market games and voting games and introduce transfer systems that seem to be suited best for these games: in market games, agents can transfer their initial endowment among periods, and in the context of voting games, we allow agents to shift their voting weights over time. Based on this idea we derive non-cooperative games and further discuss existence and uniqueness of Nash equilibria.