Making sense of uncertainty in macroeconomic model

Doctoral thesis

MAKING SENSE OF UNCERTAINTY IN MACROECONOMIC MODELS

Poramapa Poonpakdee

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in macroeconomic models

DISSERTATION

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To my parents

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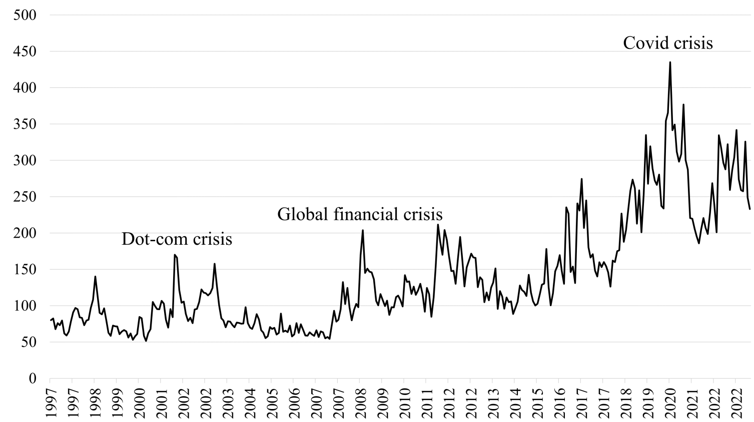


Source: Baker, Bloom, and Davis (2016)

This work is a collection of research articles studying how the economy behaves under uncertainty. Uncertainty can be broadly defined as a situation in which something is not known and historically, uncertainty always spikes during crises. Figure 1.1 shows that according to the Global Economic Policy Uncertainty Index by Baker et al. (2016), uncertainty levels rose to approximately 170 during the 2000 Dot-Com crisis, exceeded 200 during the 2008 Global Financial Crisis and peaked at 440 during the 2020 Covid Crisis. This implies a growing magnitude of uncertainty perceived by the public, increasing our exposure to vulnerable situations.

The rising uncertainty has caused conventional macroeconomic models with representative agents and rational expectations to perform less accurately at forecasting future economic conditions. This has been clearly witnessed during the Global Financial Crisis and many periods after, leaving policymakers in a difficult position. Such an issue raises scepticism on the usefulness of these conventional models in policymaking and, perhaps, economics as a whole. Accordingly, this research seeks to study the effect of uncertainty on individual expectations and to develop a theoretical macroeconomic model to replicate such an effect. We find that the model's performance in fitting and predicting output growth improves tremendously compared to its preceding benchmark. This work offers a solid foundation for further research on how uncertainty can be handled in macroeconomic models. In the subsequent paragraphs, we provide an overview of our research journey.

Starting with data

We start by looking at the relationship between macroeconomic uncertainty and people's expectations from data. A large strand of literature focuses on the expectations of professional forecasters (Clements, 2010; Dovern, 2013; Giordani & Söderlind, 2003; Glas, 2020; Glas & Hartmann, 2016; Manzan, 2011) and firms (Altig et al., 2020; Bachmann et al., 2021; Coibion et al., 2018) while the study about expectations of ordinary people like households is still limited (Baqaee, 2019; Fermand et al., 2023). **Chapter 2** contributes to this literature by providing a direct evidence on the relationship between macroeconomic uncertainty and individual expectations held by professional forecasters and households. To do this, we use four different panel survey datasets which are EU and US professional forecasters surveys, and Dutch and US household surveys. We quantify individual expectations into two dimensions: the mean expectation (first moment) and the subjective uncertainty (second moment) of income forecast distributions. Subsequently, we study these individual variables in relation to macroeconomic uncertainty, employing several macroeconomic uncertainty indices such as the Economic Policy Uncertainty index (Baker et al., 2016) and stock volatility.

Our results show that (1) macroeconomic uncertainty reduces the mean expectation of income and (2) it does not always increase subjective uncertainty. This implies that when uncertainty rises, people become more pessimistic but may be more certain or uncertain about their beliefs. Our first finding regarding pessimism is consistent with the assumptions made by most macroeconomic models. However, our second finding is both novel and at odds with many macro models which assume a monotonic positive relationship between macroeconomic uncertainty and subjective uncertainty. We observe that people, especially households, can also become more certain in periods of rising macroeconomic uncertainty. These two findings provide insight into the complexity of the relationship between macroeconomic uncertainty and individual expectations, serving as a basis for our theoretical analysis in subsequent chapters.

Connecting empirical findings to a decision-making theory

We reconcile our two empirical findings with the smooth ambiguity theory (Klibanoff et al., 2005), which we later use to study in a macroeconomic model. According to Knight (1921), uncertainty can be distinguished into risk and ambiguity, wherein risk is a situation in which the probability of the outcome is known, but the exact outcome is unknown, and ambiguity is a situation in which neither the exact outcome nor its probability is known. In macroeconomic models, risk is typically denoted as an increase in the standard deviation of the prior beliefs distribution, leading to a reduction in expected utility for a risk averse agent (Bloom, 2014; Born et al., 2018; Born & Pfeifer, 2021; Fernández-Villaverde & Guerrón-Quintana, 2020). On the other hand, ambiguity is an increase in the number of prior beliefs, leading to a lower expectation of the worst-case relative to the best-case beliefs (Ilut & Schneider, 2014, 2022). The smooth ambiguity theory allows the agent to distinguish between the effects of ambiguity and risk and to form a subjective probability of each prior belief, henceforth subjective belief. The theory suggests that an ambiguity averse agent prefers a robust expectation across different priors and holds a set of subjective beliefs that is pessimistically biased toward the worst-case scenario compared to the Bayesian belief. An increase in ambiguity will subsequently result in the subjective belief of the worst-case scenario becoming larger (Altug et al., 2020; Klibanoff et al., 2009; Marinacci, 2015). This leads to a larger negative effect of ambiguity on the agent's expectation.

Figure 1.2 presents the stylized effects of risk and ambiguity on income expectations based on the smooth ambiguity theory. For an illustrative purposes, we assume two prior beliefs: the best-case and worst-case scenarios. Risk is defined as an increase in the standard deviation of the income expectation in each scenario, and ambiguity is defined as an increase in the spread of the expected incomes between the two scenarios. $\hat{\mu}$ denotes the subjective belief of the worst-case scenario characterized by the smooth ambiguity theory[1]. $E(Y)$ is the mean of income expectation, and $\sigma(Y)$ is the standard devia-

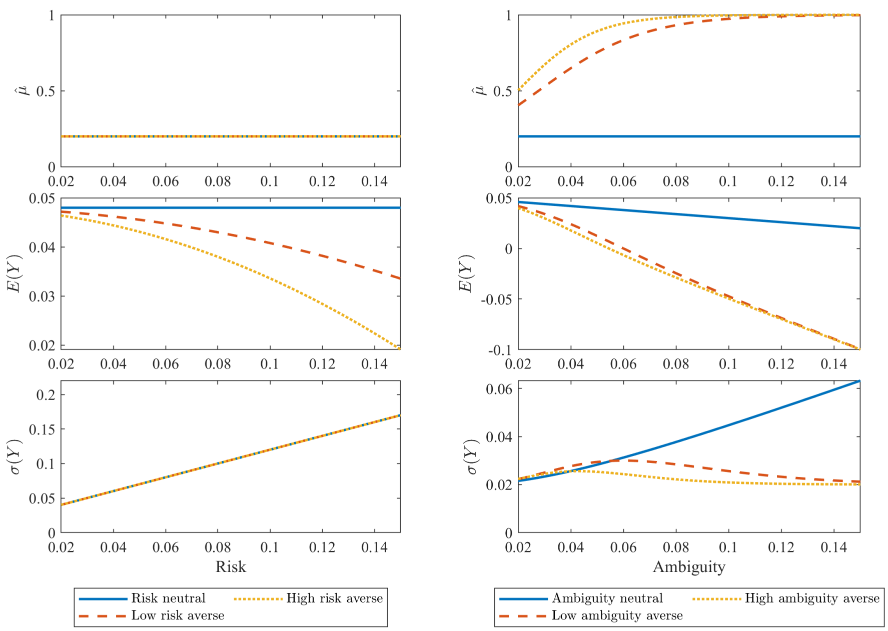
[1] Given a set of decisions $\mathbf{a}$ and Bayesian probability μ, the subjective belief of the worst-case scenario $\hat{\mu}$ is:

$$\hat{\mu} = \mu \xi^w(\mathbf{a})$$
$$\text{with } \xi^w(\mathbf{a}) = \frac{\phi'(E^w(\nu(\mathbf{a}))}{\mu\phi'(E^w(\nu(\mathbf{a})) + (1-\mu)\phi'(E^b(\nu(\mathbf{a}))}$$

where ν is a utility function whose concavity depends on risk aversion $\frac{\nu''}{\nu'} < 0$ and ϕ is a smooth ambiguity function with

Figure 1.2: Stylized effects of risk and ambiguity on income expectations

(a) Risk **(b)** Ambiguity



Note: These figures are created based on the smooth ambiguity theory by Klibanoff et al. (2005).

tion of income expectation or subjective uncertainty. Using these figures, we demonstrate that the smooth ambiguity theory can account for our first and second empirical findings when considering uncertainty in the form of ambiguity.

Panel 1.2a illustrates the impacts of risk on income expectations. According to the smooth ambiguity theory, risk does not directly affect the agent's subjective belief; thus, the subjective belief of the worst-case scenario $\hat{\mu}_t$ remains constant for all levels of risk. For a risk averse agent (dashed lines), an increase in risk leads to a decreased expectation of income, whereas for a risk neutral agent (solid lines), there is no effect. Moreover, an increase in risk will always increase subjective uncertainty. Therefore, risk can only explain the first empirical finding, not the second one.

Panel 1.2b displays the effects of ambiguity on income expectations. An ambiguity averse agent (dashed lines) increases its subjective belief of the worst-case scenario $\hat{\mu}_t$

concavity increasing with ambiguity aversion $\frac{\phi''}{\phi'} < 0$.

when confronted with greater levels of ambiguity, while this has no effect on an ambiguity neutral agent (solid lines). Ambiguity decreases income expectations because the worst-case expectations become lower relative to the best ones. This remains true for ambiguity neutral agents, who are still exposed to ambiguity despite not overreacting. More interestingly, there is a non-monotonic relationship between ambiguity and subjective uncertainty determined by ambiguity aversion. Ambiguity increases subjective uncertainty as the spread between the best-case and worst-case scenarios widens. At the same time, ambiguity averse people become more pessimistic and put a greater subjective belief in the worst-case scenario, reducing subjective uncertainty. Thus, ambiguity can explain both the first and second empirical findings.

Incorporating the theory into a model

Bridging the gap between empirical findings and theory, we develop a macroeconomic model featuring smooth ambiguity theory based on Altug et al. (2020) in **Chapter 3**. In particular, we study uncertainty in the form of ambiguity. A majority of the macroeconomic models studies uncertainty as risk, by modeling the volatility of exogenous shock to be time-varying[2]. An alternative approach is to model uncertainty as ambiguity by imposing multiple potential scenarios where the true scenario is unknown until it becomes observable (Altug et al., 2020; Backus et al., 2015; Bhandari et al., 2023; Collard et al., 2018; Gallant et al., 2019; Ilut & Schneider, 2014, 2022; Ju & Miao, 2012). Henceforth, after Chapter 3, we use uncertainty and ambiguity interchangeably, unless stated otherwise.

Chapter 3 proposes a novel approach to incorporate uncertainty in the form of ambiguity into a model. When there is uncertainty, the representative household expects the next-period economy to be in one of two scenarios: normal growth or recession. The difference in expected utilities between these two scenarios increases with the level of uncertainty, which we anchor to a macroeconomic uncertainty index. When uncertainty increases, the expected utility of the recession scenario is reduced relative to the normal growth scenario, and an ambiguity averse household puts a greater subjective belief in the recession scenario. This leads to a stronger reaction to uncertainty. With this mechanism, the negative effect of uncertainty is nonlinear, as it is more intense when the level

[2]See Fernández-Villaverde and Guerrón-Quintana (2020) for a survey of the transmission mechanisms of risk.

of uncertainty is higher. Simulations conducted using US data suggest this transmission mechanism of uncertainty can successfully reproduce the findings of Chapter 2 and has the capability to capture economic fluctuations, particularly during recession periods when uncertainty rises.

Bringing the model back to data

After exploring the theoretical transmission mechanism of uncertainty in the smooth ambiguity model, we examine whether the model can fit with the data in **Chapter 4**. The contributions of Chapter 4 include the computational methods for the business cycle models under ambiguity and the estimation result which, as far as we are aware, is the first study to estimate the level of ambiguity aversion using macroeconomic data. Closed form solutions for smooth ambiguity models are generally not available, and so they are typically solved by projection methods (Collard et al., 2018; Ju & Miao, 2012) or value function iterations (Altug et al., 2020; Jahan-Parvar & Liu, 2012).

In this research, we use a projection method with parameterized expectations algorithm and adapt the algorithm to account for the features of two scenarios and the nonlinear effect of uncertainty. We estimate the smooth ambiguity model with a nonlinear least squares approach using data from the US and major European countries. Our estimations yield many interesting insights. For instance, the out-of-sample forecast of the smooth ambiguity model is comparable to those of professional forecasters - a surprising finding since professional forecasters, on average, provide better forecasts compared to macroeconomic models (Wieland & Wolters, 2011). Moreover, the Global Financial Crisis was associated with an increase in the US representative household's aversions to both risk and ambiguity, while the Dot-com Crisis only affected risk aversion. The estimates from the models of the European countries indicate that the representative households in Italy and Spain were ambiguity averse, while those in Germany and France were close to ambiguity neutral.

This dissertation seeks to gain a deeper understanding of how uncertainty affects the economy through individual beliefs. We find that subjective beliefs and ambiguity aversion play important roles in determining an individual's response to uncertainty, which influence the effects of uncertainty on the economy as a whole. The subsequent chapters of this thesis—**Chapters 2, 3 and 4**—are devoted to the orderly development of this research topic from both empirical and theoretical perspectives. The limitations of this study and the future research agenda are discussed in **Chapter 5**, wherein its positioning within the field of economics is also addressed.

Chapter 2

Effects of macroeconomic uncertainty on expectations and subjective uncertainty: Evidence from households and professional Forecasters

Giulia Piccillo and Poramapa Poonpakdee

2.1 Introduction

Recent literature provides substantial evidence that macroeconomic uncertainty has a negative impact on economic activity (Baker et al., 2016; Bloom, 2009; Born et al., 2018; Brogaard et al., 2020; Ilut & Schneider, 2014; Jurado et al., 2015). However, few empirical studies exist that focus on the precise nature of the individual transmission mechanisms, so most macroeconomic models assume that macroeconomic uncertainty adversely affects distributions of individual expectations and then decision-making. To be specific, the common assumptions connect macroeconomic uncertainty to reduced expected utility (first moment) and to increased subjective uncertainty (second moment). These relationships lead to decisions that lower economic growth, such as an increase in precautionary saving (Fernández-Villaverde & Guerrón-Quintana, 2020), a delay of investment (Bloom, 2014) and a decrease in asset valuation (Ozoguz, 2009). The empirical evidence of the assumed relationships is however scarce. Therefore, the research question of our paper is simple, what is the relationship between macroeconomic uncertainty and individual expectations. We run regressions across four panel survey data from households and professional forecasters and provide a new empirical evidence on the relationships between macroeconomic uncertainty and individual expectations.

Our study contributes to the field of theoretical macroeconomic models by assessing the effects of macroeconomic uncertainty, which are highly dependent on the assumed mechanisms specific to the models. For instance, Born and Pfeifer (2021) demonstrate that the standard New Keynesian model with conservative parameters can only generate limited effects of macroeconomic uncertainty on the economy. In this model, increased macroeconomic uncertainty leads to higher standard deviations of total factor productivity and government spending processes. This causes a precautionary pricing behavior in firms, thus increasing price markups. However, other studies that incorporate different microfoundations and uncertainty measurements indicate that their models predict substantial effects of macroeconomic uncertainty. For instance, Ilut and Schneider (2014) use a representative business cycle model and incorporate the multiple prior theory (Gilboa & Schmeidler, 1989)[1]. They assume that macroeconomic uncertainty (which they refer to

[1] The multiple prior theory states that agents form expectations based on multiple scenarios or priors, and adopt the Maxmin criterion. If the agents are ambiguity averse, they will forecast as if they are in the worst-case scenario.

as ambiguity) decreases the expected utility of the worst-case scenario and find that it can explain a significant part of the business cycle variation.

To shed light on the link between macroeconomic uncertainty and individual expectations, we study four panel survey datasets: EU and US professional forecasters surveys (SPFs) and Dutch and US household surveys. Respondents of these surveys are asked to provide point estimates of their expected incomes or GDP growths, which form our first dependent variable. We also calculate the standard deviation from the probabilistic distributions of the respondents' forecasts, which serves as a quantitative measure of subjective uncertainty, our second dependent variable. Our results also show that macroeconomic uncertainty does not always increase and might actually decrease subjective uncertainty in households. We discuss the possibility of ambiguity aversion being able to explain this puzzle.

The outline of this paper is as follows. Section 2.2 discusses the literature on macroeconomic uncertainty, expectations and subjective uncertainty. In Section 2.3, we describe the data and methodology. Section 2.4 presents our hypotheses and Section 2.5 reports the empirical results. In Section 2.6, we discuss our results, and finally, Section 2.7 concludes.

2.2 Literature review

In this section, we summarize the empirical literature that studies macroeconomic uncertainty, our independent variable, as well as mean expectations and subjective uncertainty, our dependent variables.

2.2.1 Macroeconomic uncertainty

Given the broad definition of macroeconomic uncertainty, there is no consensus of how to measure it. Knight (1921) defines uncertainty as the agent's inability to forecast the likelihood of the events, and Knightian uncertainty is often referred to as ambiguity in the macroeconomic literature (Altug et al., 2020; Fernández-Villaverde & Guerrón-Quintana, 2020; Ilut & Schneider, 2014). Ambiguity is different from risk, and Knight defines the concept of risk as a known likelihood of the events. Although the concepts of risk and ambiguity are clearly distinct, they are often difficult to distinguish in the real world. Therefore, in this study, we refer to uncertainty as a combination of risk and ambiguity, as done in Bloom (2014). To empirically measure macroeconomic uncertainty, several indices have been developed using quantitative data and qualitative data.

Quantitative data, such as stock price volatility, forecast errors, and forecast disagreement, can be used to measure macroeconomic uncertainty. Bloom (2009) proposes to measure macroeconomic uncertainty using financial market volatility, specifically the VIX index, and shows that a rise in financial market volatility is often associated with sudden drops and subsequent rebounds in economic activities. Jurado et al. (2015) develop an uncertainty index using forecast errors assuming that macroeconomic uncertainty reduces the ability to forecast the economy. They use a broad set of economic data and large scale factor models to forecast macroeconomic variables and show that the unforecastable component can explain economic activities better than financial market volatility. Finally, the disagreement among professional forecasters is another well-known proxy for macroeconomic uncertainty as it reflects the diverse opinion amongst professional forecasters. Using a business cycle model with multiple priors preferences, Ilut and Schneider (2014) demonstrate that professional forecasters' disagreement can explain a significant part of the variation in economic growth.

A macroeconomic uncertainty index can be constructed from qualitative data such as newspapers or content in social media. The most widely used measure is the Economic Policy Uncertainty (EPU) index by Baker et al. (2016). This index is constructed by counting the frequency of the news articles containing words related to economy, policy and uncertainty. The authors demonstrate that a rise in the EPU reduces the economic activity in the US, especially in sectors sensitive to government policy. Moreover, Brogaard et al. (2020) measure the world political uncertainty by the US election cycle. They find that the returns of non-US stock markets fall when the US elections approach, indicating a high level of uncertainty.

2.2.2 Expectations

The expectation is the point estimate of the future economic condition made by an individual. According to modern theories of decision-making under uncertainty, macroeconomic uncertainty can lead to pessimistic expectations of economic prospects (Bloom, 2014; Fernández-Villaverde & Guerrón-Quintana, 2020; Gilboa & Schmeidler, 1989). To investigate how expectations are formed, researchers often utilize survey data from professional forecasters, businesses, and households, which cover a wide range of variables from inflation and GDP to individual income. In the following paragraphs, we will discuss the stylized facts of point expectations from these survey data.

Individual expectations are sensitive in an asymmetric manner to economic news. Dovern (2013) studies revisions of GDP expectations by professional forecasters and reveals that forecasters were more likely to revise their output forecasts downwards during recessions than to revise them upwards during booms. Additionally, Manzan (2011) demonstrates that US professional forecasters interpret the same information heterogeneously based on their prior beliefs. Consequently, an optimistic agent tends to attribute low weight to negative signals as they do not align with the individual prior beliefs.

Based on firms' and households' data, research has shown that asymmetric responses to signals are due to the forecasters' current beliefs and socio-economic status. Using New Zealand firm's survey, Coibion et al. (2018) find that the beliefs about current economic conditions, rather than the actual economic conditions, are positively associated with the predictions of future economic conditions. Baqaee (2019) shows that US households' inflation forecasts are more responsive to an inflation signal than a disinflation signal, demonstrating greater concern about an increase in inflation. Das et al. (2020) point out that socioeconomic status (SES), consisting of income and education, significantly influences individual macroeconomic expectations. Their analysis of the Michigan Survey of Consumers shows that low-SES respondents were generally more pessimistic than high-SES respondents; however during recessions low-SES agents perceived less negative news than high-SES agents. This suggests that socioeconomic status has an impact on mean expectation, but at a different magnitude across the boom-bust cycle.

Expectations are more accurate when the cost of information decreases. Carroll (2003) compares the forecasts of US households from the Michigan Survey of Consumers to those of US professional forecasters and finds that household expectations on the US inflation converge to those of the professional forecasters when the news increasingly reports about inflation related issues. This is because the intensity of news coverage helps reduce the cost of information, enabling households to have a better forecast that is closer to that of the professional forecasters. Lamla and Lein (2014) investigate both the intensity and content of inflation news using German consumer data. Their finding is that the news intensity improves the households' inflation forecast only when the news does not portray inflation as a negative event.

Summarizing the stylized facts from empirical studies, it appears that individual expectations respond asymmetrically to signals depending on prior beliefs and socioeconomic conditions, and are better when the cost of information is lower. These findings serve as a guide for the selection of control variables in our models.

2.2.3 Subjective uncertainty

Modern theories of decision-making under uncertainty are not conclusive on how macroeconomic uncertainty affects subjective uncertainty (Gilboa & Schmeidler, 1989; Hansen & Sargent, 2021, 2011; Klibanoff et al., 2005; Sims, 2003; Tuckett & Nikolic, 2017). At the same time, empirical studies on subjective uncertainty are few. Subjective uncertainty is often proxied by the second moment of the individual subjective forecast distribution. To obtain this distribution, survey respondents must state the probabilistic histogram of their forecasts. However, this information is costly and its quality is highly dependent on the numerical capability of respondents. For instance, Clements (2010) finds that US professional forecasters update their point estimates more frequently than their forecast distributions, suggesting that the cost of updating the forecast distribution is higher than that of the point estimate. This section highlights the relevant stylized facts of subjective uncertainty.

Subjective uncertainty depends on economic conditions and past forecasting performance. Employing the EU professional forecasters data, Glas and Hartmann (2016) discover that the subjective uncertainty of the inflation forecasts depends negatively on economic growth and positively on monetary policy surprises, but is unrelated to macroeconomic uncertainty. Altig et al. (2020) use the US firms' survey of business uncertainty and show that the sales' subjective uncertainty increases when the sale growths are volatile. In line with this finding, Bachmann et al. (2021) find that the sales' subjective uncertainty increases with the absolute sales growth rate and the forecast error in the German manufacturing firms.

Subjective uncertainty increases when the forecaster's confidence decreases or the forecasting horizon is longer. In general, professional forecasters are overconfident (Giordani & Söderlind, 2003), but this is less pronounced in the long forecasting horizon (Clements, 2014). Manzan (2021) finds that when the forecast horizon becomes longer, subjective uncertainty increases due to the fact that professional forecasters have less information for further forecast.

According to Fermand et al. (2023), socioeconomic status and individual characteristics can have an influence on subjective uncertainty, regardless of the variable being forecasted. Using the US consumer expectation survey, they document that employed people had lower subjective uncertainty than unemployed people when forecasting variables such as personal income, inflation rate, and unemployment rate. Interestingly, the authors find that even when the variables had no fundamental link, the individual's subjective uncertainty of each was positively correlated. These results suggest that there is a personal trait that influences an individual's subjective uncertainty across variables.

In summary, the empirical studies of subjective uncertainty have revealed that it fluctuates with economic conditions, decreases with improved forecasting errors and shorter forecasting horizons, and is impacted by personal traits. These findings provide useful information for selecting appropriate control variables.

2.3 Data and measurement

This paper examines the relationship between macroeconomic uncertainty, individual expectations and subjective uncertainty. We employ household surveys from the US and

Table 2.1: Correlations among the macroeconomic uncertainty indices

US indices	US EPU	VIX	Dis.	JU	EU indices	EU EPU	STOXX	Dis.	NL EPU
US EPU	1				EU EPU	1			
VIX	0.32***	1			STOXX	0.03	1		
	(0.09)					(0.11)			
Dis.	0.008	0.36***	1		Dis.	-0.14	0.48***	1	
	(0.09)	(0.09)				(0.11)	(0.10)		
JU	0.23**	0.69***	0.36***	1	NL EPU	0.34***	0.55***	0.61***	1
	(0.09)	(0.07)	(0.09)			(0.11)	(0.10)	(0.11)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, (...) is S.E. Dis. stands for SPF's disagreement.

the Netherlands, as well as surveys of professional forecasters from the US and Europe, to empirically measure these variables. In this section, we define our empirical measures and provide stylized facts about each variable.

2.3.1 Macroeconomic uncertainty

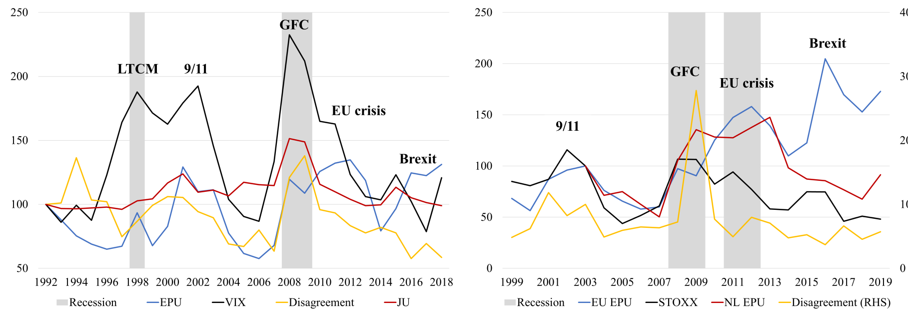
We use four macroeconomic uncertainty indices as our independent variable. The first index is the news-based component of the Economic Policy Uncertainty Index by Baker et al. (2016) (EPU), which is the most common measure used in the empirical literature on macroeconomic uncertainty. The second index is the 1-month ahead Macroeconomic Uncertainty Index by Jurado et al. (2015) (JU) which is only available for the US. Thirdly, we use stock volatility indices of the US (VIX) and Europe (STOXX). The last index is the forecast disagreement of the next-year GDP growth computed from the survey of professional forecasters in the US and Europe.

Figure 2.1 shows the indices of uncertainty over time, including the recessions and events associated with increasing uncertainty. Table 2.1 reports the correlation between the indices. Most of the indices have a significant positive correlation, supporting that they measure similar phenomena. Exceptions are the US EPU with the forecasters' disagreement, and the EU EPU with other EU indices. For example, EU EPU diverges from the STOXX and forecasters' disagreement after the Global Financial Crisis as seen in Figure 2.1b. This may be attributed to the fact that the EPU index points to policy uncertainty but STOXX represents the uncertainty in the financial markets. This is also the case for the forecast disagreement which is based on professional forecasters working in the financial sector.

Figure 2.1: Macroeconomic uncertainty indices in US and Europe

(a) US macroeconomic uncertainty

(b) EU macroeconomic uncertainty



Note: The US indices are rebased to 100 at 1992 and the EU indices are rebased to 100 at 2003. EPU stands for Economic Policy Uncertainty index (Baker et al., 2016). JU stands for 1-month ahead Macroeconomic Uncertainty Index (Jurado et al., 2015). GFC is the Global Financial Crisis and LTCM is Long Term Capital Management Fund Crisis.

2.3.2 Survey data

Before proceeding to the measurement of expectations and subjective uncertainty, we provide a brief introduction to the four survey datasets used in this study. These datasets comprise responses from professional forecasters and households. The tables with stylized statistics, as well as further details of each dataset, can be found in Appendix 2.A.

Data from the survey of professional forecasters (SPF) are collected quarterly by the respective central banks of the US and Europe. SPFs provide the next-calendar-year output predictions of individual professional forecasters. For example, in every quarter of 2010, the surveys asked for the 2011 annual GDP forecast, providing predictions of each forecaster with four different horizons. The EU SPF contains the point forecast and probabilistic distribution of the next-calendar-year real GDP growth rate, spanning from 1999Q1 to 2020Q1. The US SPF provides the point forecast of the next-calendar-year real GDP level, and the probabilistic distribution of the next-calendar-year real GDP growth rate, spanning from 1992Q1 to 2020Q1. To ensure consistency, the point forecast of the level is transformed to the point forecast of growth.

The household datasets used in this study are derived from two surveys: the survey of consumer expectations (SCE) owned by the Federal Reserve Bank of New York and the DNB Household Survey administered by Centerdata (Tilburg University, The Netherlands). The SCE provides point forecasts of the next-12-month gross personal in-

Table 2.2: Income expectations of each dataset

Dataset	Income expectations	Measurement	Timeline	Frequency
EU SPF	Next-calendar-year real GDP growth	Provided by respondents	A1999Q1-A2020Q1	Q
US SPF	Next-calendar-year real GDP growth	Transformed expected level into growth	A1992Q1-A2020Q1	Q
NL HH	Next-12-month net household income level	Calculated from probabilistic histogram	A1993-A2018	A
US HH	Next-12-month gross personal income growth	Provided by respondents	A2013M6-A2019M10	M

Note: SPF = survey of professional forecasters, HH = household, A= Annual, Q= Quarter, M= Month

come growth and its probabilistic distribution. However, its short time series–starting in 2013M6–means it does not cover recession periods. The timeline of SCE data used in this study is from 2013M6 to 2019M10. The DNB Household Survey, which began in 1993, is the longest household survey containing probabilistic distribution of the next-12-month net household income forecast but it does not provide the point forecast. We extract the point forecast from the probabilistic distribution but cannot transform a level forecast into a growth forecast due to the lack of data on household incomes in the current year. A further limitation of this survey is its annual frequency, as well as the lack of information on when households respond to the survey during the year. This means that the same year's respondents may have different economic information, yet it cannot be identified. Furthermore, the sample periods are from 1997 to 2002 and from 2008 to 2018, as the questionnaires had different structures in the years excluded.

2.3.3 Expectations

In this paper, we consider income expectations as one of our dependent variables, due to their direct connection to expected utility. A decrease in income expectation is correlated with a lower expected utility, other factors being equal. We examine two types of income expectations: the next-calendar-year real GDP growth expectation from the US and EU SPFs, and the next-12-months individual income expectation from the US and NL household surveys. In Table 2.2, we summarize the income expectations of each dataset.

Except for the Dutch household survey, three surveys provide point estimates of income expectations. For example, the US household survey says "Twelve months from now, I expect my earnings to have [increased/decreased] by ______ %" and the respondents fill their answers in the blank. The US SPF provides the forecasts of the current and the next-year GDP level. We compute the next-year expected level as a percentage of current-year expected level. We do not use the expected GDP level as our dependent variable to have consistency with that of the EU professional forecasts. It also is more common to discuss GDP growth.

The Dutch household survey asks respondents to estimate their minimum and maximum expected incomes, as well as the cumulative distribution of income. In particular, it asks what is the probability that the expected household income will be less than 20% of (Maximum expected income - Minimum expected income) + Minimum expected income? The percentage ranges from 20%, 40%, 60% and 80% (more detail in Appendix 2.A). For example, if the minimum and maximum expected income is 0 and 100k respectively. The probabilities provided are for expected income less than 20k, 40k, 60k and 80k. To calculate the mean expectation of income, we transform the cumulative distribution provided by the respondents into a probability distribution. In this example, we have probability bins of [0, 20k), [20k, 40k),...,[80k, 100k). Suppose that Y_t is the household income at t, the probability of Y_t falling into bin [20k, 40k) is $P(Y_t \in [20, 40)) = P(Y_t < 20) - P(Y_t < 40)$ and this is analogous for other bins. The mid points of each bin are 10k, 30k, 50k, 70k and 90k. Thus the expected household income of Dutch respondents is computed as follows:

$$E_{t,i}(Y_{t+1}) = \sum_{bin} P(Y_t \in bin)_{bin,t,i} \times \text{Mid Point}_{bin,t,i}$$

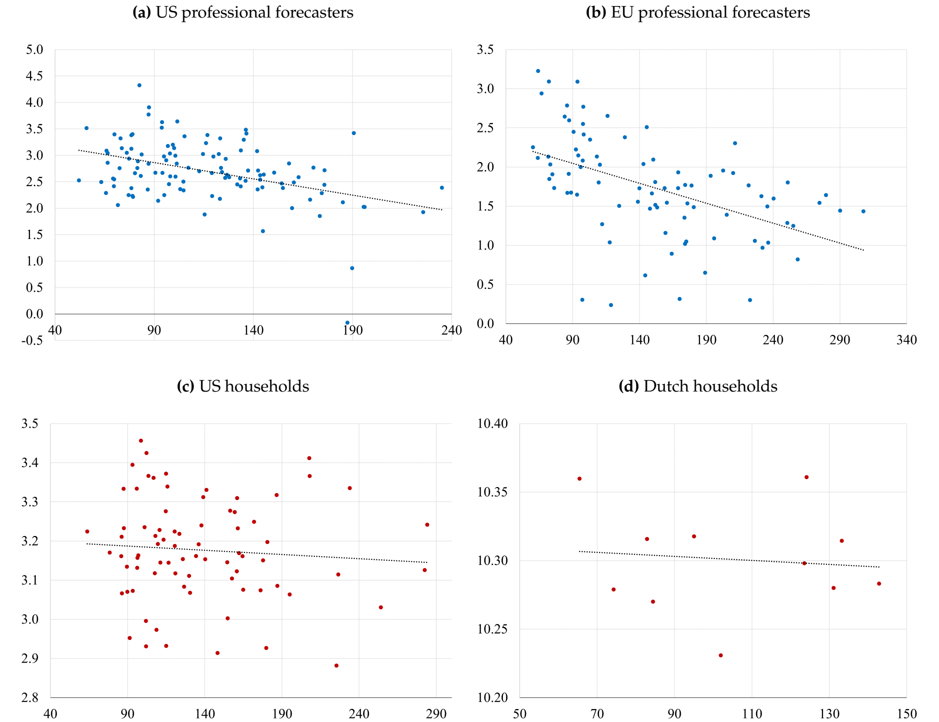
Figures 2.2 depict unconditional correlations between cross-sectional average of income expectations and EPU. A negative correlation is observable for the professional forecasters, but the relation is weak for the households.

2.3.4 Subjective uncertainty

Subjective uncertainty is the perceived uncertainty of an individual about his or her economic forecast which is usually measured as the second moment of a subjective forecast distribution (Altig et al., 2020; Enke & Graeber, 2023; Fermand et al., 2023; Giordani & Söderlind, 2003; Glas & Hartmann, 2016). Along with most literature, we use the second moment of the subjective forecast histogram as the measure of subjective uncertainty.

In our datasets, respondents are asked to provide point forecasts and probabilistic

Figure 2.2: Scatter plots between income expectations and the Economic Policy Uncertainty index



Note: The Y-axis of each graph is the cross-sectional average of individual income expectations. For the Dutch households, the level expectations are in log scale and for the other, the growth expectations are in percentage. The X-axis is the Economic Policy Uncertainty indices of US, EU and NL. Blue dots depict data from professional forecasters, while red dots represent data from households.

histograms of future income or GDP. The surveys provide ranges of forecasting bins, and ask respondents to fill in the probability that their forecast will fall within each bin. For example, the US household survey says "Suppose...that, 12 months from now, you are working in the exact same main job...Your earnings on this job, before taxes and deductions, will have increased by 0% to 1% (bin 6) with ______ percent chance." The questionnaire provides 12 bins ranging from -12% to +12%, and the respondents must fill in the blank areas. Similarly, the US and EU surveys of professional forecasters pose the same pattern of questions. These surveys implicitly ask respondents to provide a probability density of income forecasts. The Dutch household survey, however, asks questions differently, providing a cumulative probability density. Nonetheless, this difference does not affect the measurement of subjective uncertainty in our view. The second moment of the probabilistic histogram can be measured in two ways, which we explain in the following subsections.

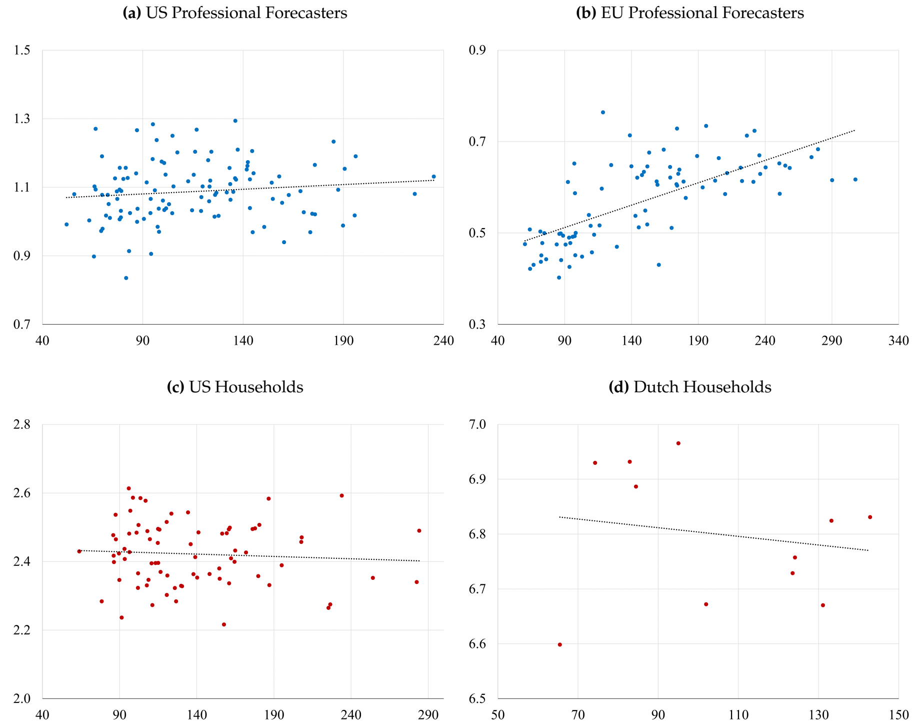
Generalized Beta distribution (GBD). The first approach is fitting a generalized Beta distribution to each individual forecaster's histogram. This method is proposed by Engelberg et al. (2009) and has been adopted by, for instance, Clements (2014) and Glas and Hartmann (2016). Fitting generalized Beta distribution yields a full analytical distribution so the researchers can study more than the second moment. The US household survey already provides the fitted estimations using the Armantier et al. (2017)'s technique which has the same concept as in Engelberg et al. (2009). We apply the same method to the US and EU surveys of professional forecasters and the Dutch household data. The quality of the fitting relies heavily on the number of forecasting bins reported by the forecasters. When less than three intervals are reported, the histogram is approximated by the triangular distribution, and when at least three bins are reported, the histogram is approximated by the beta distribution. Compared to the other datasets, more discontinuity is observed in the US household survey as half of the respondents report less than three bins.

Simple standard deviation (SSD). The second approach is a simple standard deviation. It assumes that all mass distribution is at the midpoint of the forecasting bin. Although this method does not provide a full analytical distribution, it does not suffer from the discontinuity problem in the first method. The formula of this method is as follows.

$$SSD_{t,i}(Y_{t+1}) = \sqrt{\sum_{bin} P(Y_{t+1} \in bin)_{bin,t,i} \times (\text{Mid Point}_{bin,t,i} - E_{t,i}(Y_{t+1}))^2}$$

Acknowledging the validity of subjective uncertainty being highly dependent on the statistical knowledge of respondents and the structure of the surveys, we have employed both available methods for measuring it. The correlation between these two measures is quite high (between 0.57 and 0.99) for all four surveys. Figure 2.3 shows the scatter plots between SSD subjective uncertainty and EPU. Here we do not observe a clear relationship across the four datasets. We explore this correlation in detail in Section 2.5.2.

Figure 2.3: Scatter plots between SSD subjective uncertainty and the Economic Policy Uncertainty index



Note: The Y-axis of each graph is the cross-sectional average of individual subjective uncertainty, measured in S.D. The Dutch households' subjective uncertainty is in log scale while the other are in percentage. The X-axis is the Economic Policy Uncertainty indices of US, EU and NL. Blue dots depict data from professional forecasters, while red dots represent data from households.

2.4 Hypotheses and empirical methods

In this section, we introduce two hypotheses and corresponding variables used for testing. The first hypothesis is that macroeconomic uncertainty reduces income expectations, and the second hypothesis is that macroeconomic uncertainty increases subjective uncertainty.

First hypothesis. Macroeconomic uncertainty has a negative impact on income expectations. Households and firms become pessimistic when uncertainty rises, so they spend less. As a result, economic growth declines. The theoretical models in macroeconomics have often adopted this mechanism to demonstrate that macroeconomic uncertainty adversely affects the economy (Bloom, 2009; Fernández-Villaverde & Guerrón-Quintana, 2020; Ilut & Schneider, 2014).

According to the literature survey in Section 2.2, individual expectations are persistent (Manzan, 2011) and are influenced by macroeconomic variables (Dovern, 2013), personal variables (Das et al., 2020), and good or bad news (Baqaee, 2019; Coibion et al., 2018). To identify the additional effect of macroeconomic uncertainty, we have incorporated these control variables into our model. Thus, the core equation of our first hypothesis is as follows.

$$\begin{aligned} \text{Income expectation}_{i,t} &= \alpha_1 \text{ Macroeconomic uncertainty}_t \\ +\alpha_2 \text{ Macroeconomic uncertainty}_{t-1} &+ \alpha_3 \text{ Income expectation}_{i,t-1} \\ +\alpha_4 \text{ Macroeconomic variables}_{t-1} &+ \alpha_5 \text{ Personal variables}_{i,t-1} + \epsilon_{i,t} \end{aligned} \tag{2.1}$$

t is the time period when the data is available. i is the individual index. The personal variables are only applicable in the household datasets.

Second hypothesis. Macroeconomic uncertainty increases subjective uncertainty, which reflects a decline in forecasting confidence. This concept is supported by the assumptions of many theoretical macroeconomic models (Bloom, 2014, 2009; Fernández-Villaverde & Guerrón-Quintana, 2020). For instance, Bloom (2009) contends that macroeconomic uncertainty raises the probability of default outcomes, leading to higher borrowing costs. On the other hand, Fernández-Villaverde and Guerrón-Quintana (2020) suggest that macroeconomic uncertainty increases the range of expected marginal utility of future consumption. Despite the differences in these mechanisms, they both point to a loss in forecasting confidence. A decrease in forecasting confidence can be indicated by an increase in subjective uncertainty.

The literature discussed in Section 2.2 suggests that subjective uncertainty is persistent and is influenced by both macroeconomic variables (Glas & Hartmann, 2016) and personal variables (Fermand et al., 2023). The core equation of the second hypothesis is the following.

$$\begin{aligned} \text{Subjective uncertainty}_{i,t} &= \beta_1 \text{ Macroeconomic uncertainty}_t \\ +\beta_2 \text{ Macroeconomic uncertainty}_{t-1} &+ \beta_3 \text{ Subjective uncertainty}_{i,t-1} \\ +\beta_4 \text{ Macroeconomic variables}_{t-1} &+ \beta_5 \text{ Personal variables}_{i,t-1} + \epsilon_{i,t} \end{aligned} \tag{2.2}$$

t is the time period when the data is available. i is the individual index. The personal variables are only applicable in the household datasets.

Independent and control variables. Table 2.3 summarizes the independent and control variables used in our regressions. The measurement of each variable is in Appendix 2.B. We use high and low GDP growth rates as proxies for positive and negative news. For the SPFs, we use real GDP growth since they forecast real GDP growth rates. Real GDP growth rates above 4% are considered high while those below 1% are considered low. For households, we use nominal GDP per capita growth since they forecast nominal incomes. GDP per capita growth above 4% is considered high, while those below 2% are considered low. Note that the threshold for low growth or bad news in the SPFs is when the real GDP growth rate is less than 1%, while the threshold for households is when nominal GDP per capita is less than 2%. These thresholds are derived from the top and bottom 20th percentiles of growth rates since the sample period began. For US households, we include the growth samples from 2010 to 2019, although the survey started in 2013.

To account for differences in forecast horizons, we include a quarterly fixed effect in our regressions concerning professional forecasters. This is motivated by previous research, which suggests that the forecast horizon has an influence on subjective uncertainty and income expectations (Clements, 2014; Manzan, 2021). For instance, when forecasters are making a prediction for 2011 GDP growth in the fourth quarter of 2010, they have more information than they did when making the same prediction in the first quarter of 2010.

The Dutch household data are collected throughout the year, so respondents in the same year might have different information about that year's economy. Thus, to present the best information available to the respondents, we run regressions with independent variables of the same year when the forecast was made instead of lagged data. Moreover, we will test the hypotheses with EU EPU and NL EPU in Dutch households. NL EPU would be more suitable for the Dutch dataset, however it began in 2003. As the period 2003-2007 was excluded due to inconsistencies in the questionnaire, the regression with NL EPU covers only 2008-2018 whereas the regression with EU EPU covers 1997-2002 and 2008-2018.

Empirical method. We employ panel OLS regressions that control for individual fixed effects and account for heteroskedasticity of both time and cross-section by clustering the standard errors at the individual level. Specifically, we use the STATA panel fixed effects estimator, which subtracts the mean of each variable and then runs an OLS regression. This approach enables us to control for individual fixed effects without adding individual dummies, thereby preserving degrees of freedom. Additionally, it is unlikely that individual expectations and subjective uncertainty affect macroeconomic uncertainty, suggesting that the causal relationship between dependent variables and macroeconomic uncertainty is likely to be unidirectional - from macroeconomic uncertainty to the dependent variables.

Table 2.3: Independent and control variables

Types	Variables	Note
Macroeconomic uncertainty	Economic policy index (EPU)	Available for EU, NL and the US
	Stock market volatility	VIX and STOXX
	1-month ahead macroeconomic uncertainty index (JU)	Only available for the US
	SPF's forecast disagreement of the next-year GDP growth	Available for both US and EU
Macroeconomic variables	GDP growth	
	GDP deceleration dummy	Control asymmetric response to GDP growth
	High (low) GDP growth dummy	Proxy of good (bad) news
	Recession dummy	Control economic crises
	Quarterly dummy	Control forecast horizon and only applicable in SPF
Personal variables only available for households	Good (bad) financial situation dummy	Control households' view on their financial situations
	Unemployment dummy	Control vulnerability to macroeconomic uncertainty
	College education dummy	Control literacy and socio-economic status
	Net personal income	
	Decreased net personal income dummy	Control asymmetric response to personal income
	Deficit balance sheet dummy	Control vulnerability to macroeconomic uncertainty

Note: For details, we refer to Appendix 2.B.

2.5 Results

In this section, we present the regression results for our two hypotheses, discuss the results and point out our remaining puzzles.

2.5.1 Income expectations

We explore the first hypothesis - that macroeconomic uncertainty reduces income expectations - using fixed effect OLS panel regressions. Results from surveys of professional forecasters and household surveys are reported. For each survey sample, we first present the coefficients of macroeconomic uncertainty, our variable of interest, and then the control variables to evaluate whether the results align with previous studies.

Professional forecasters

Macroeconomic uncertainty. Table 2.4 demonstrates that a rise in macroeconomic uncertainty is associated with a decrease in expected real GDP growth. This result is consistent across different macroeconomic uncertainty indices. For instance, a 1% increase in the Economic Policy uncertainty index (EPU) leads to a 0.05% and 0.54% reduction in the growth expectations of EU and US SPFs, respectively. Furthermore, the current and lagged effects of macroeconomic uncertainty have opposite signs, with the exception of the EU EPU (column 2a). This suggests that an increased growth in macroeconomic uncertainty depresses GDP forecasts. These findings lend strong support to the first hypothesis for the SPFs.

Control variables. The coefficients on the control variables are consistent with those reported in prior studies (Dovern, 2013; Manzan, 2011), and their signs remain unchanged when macroeconomic uncertainty is taken into account. The GDP expectations are found to be persistent: a 1% increase in the previous GDP forecast leads to an approximate 0.67% and 0.59% increase in the current GDP forecast for EU SPF and US SPF, respectively. Moreover, GDP growth positively influences GDP expectations, whereas the GDP deceleration dummy has a negative effect, except when Jurado et al. (2015)'s Macroeconomic Uncertainty index (JU) is employed (Column 5).

We find that the response to the news is different in the US and the EU. When examining the reaction to good and bad news, US professional forecasters only react to bad GDP growth news with the exception of JU (Column 5). Conversely, EU professional forecasters have been observed to respond only to good GDP growth news. Moreover, when there is a recession in the previous quarter, US professional forecasters tend to have higher expectations of GDP growth, while EU professional forecasters tend to have lower expectations. These optimistic forecasts held by US forecasters during recessions are in line with the findings of Bianchi et al. (2022). The different responses to news may be attributed to the fact that the two groups of professional forecasters have different priors as suggested by Manzan (2011). Since the control dummies for growth are correlated, the absolute size of these coefficients can be biased. The results only indicate that EU and US SPFs have different reactions to similar signals.

Table 2.4: Regression results of real GDP growth expectation in the surveys of professional forecasters

Expected real GDP growth$_t$	EU SPF (1999Q1-2020Q1)				US SPF (1992Q1-2020Q1)				
	(1a)	(2a)	(3a)	(4a)	(1b)	(2b)	(3b)	(4b)	(5)
Economic Policy Uncertainty index (EU, US)$_t$		-0.05*				-0.54***			
		(0.03)				(0.04)			
Economic Policy Uncertainty index (EU, US)$_{t-1}$		-0.20***				0.34***			
		(0.02)				(0.04)			
Stock market volatility (STOXX, VIX)$_t$			-0.24***				-0.60***		
			(0.02)				(0.05)		
Stock market volatility (STOXX, VIX)$_{t-1}$			0.20***				0.35***		
			(0.03)				(0.04)		
Forecast disagreement$_t$				-1.28***				-0.98***	
				(0.12)				(0.12)	
Forecast disagreement$_{t-1}$				1.50***				0.21**	
				(0.10)				(0.10)	
Macroeconomic Uncertainty index$_t$ (JU)									-3.63***
									(0.33)
Macroeconomic Uncertainty index$_{t-1}$ (JU)									2.19***
									(0.44)
Expected real GDP growth$_{t-1}$	0.68***	0.61***	0.69***	0.67***	0.60***	0.58***	0.58***	0.59***	0.57***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
GDP growth$_{t-1}$	0.03***	0.03***	0.03***	0.03**	0.09***	0.07***	0.09***	0.08***	0.12***
	(0.007)	(0.007)	(0.007)	(0.007)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
GDP deceleration D$_{t-1}$	-0.10***	-0.07***	-0.09***	-0.08***	-0.06***	-0.06***	-0.05***	-0.05**	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Above 4 % GDP growth D$_{t-1}$	0.19***	0.09***	0.19***	0.19***	-0.06**	-0.04	-0.03	-0.01	-0.11***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Below 1% GDP growth D$_{t-1}$	-0.02	0.002	0.01	0.03	-0.37***	-0.35***	-0.21***	-0.27***	-0.09
	(0.02)	(0.02)	(0.02)	(0.02)	(0.09)	(0.09)	(0.07)	(0.08)	(0.07)
Recession D$_{t-1}$	-0.16***	-0.19***	-0.17***	-0.25***	1.06***	0.89***	0.95***	1.16***	1.00***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.14)	(0.13)	(0.13)	(0.14)	(0.15)
Constant	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter D	Y	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3,917	3,917	3,917	3,917	3,431	3,431	3,431	3,431	3,431
R-squared	0.74	0.76	0.75	0.76	0.48	0.50	0.50	0.49	0.51

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, ( ) is S.E. The columns regress next year real GDP growth expectation of professional forecasters, against different macroeconomic uncertainty indices. Economic Policy Uncertainty indices (Baker et al., 2016) are from EU (2a) and US (2b). Stock market volatility indices are STOXX (3a) and VIX (3b). Forecast disagreements are from professional forecasters in EU (4a) and US (4b). Macroeconomic Uncertainty index (Jurado et al., 2015) is only available in the US (5). All independent variables ending with D are dummies.The details description of each variable is in Appendix 2.B.

Households

Macroeconomic uncertainty. Table 2.5 shows that macroeconomic uncertainty has varied effects on households' expectations of income level (NL) and income growth (US). For US households, only US EPU significantly reduces income expectations, while the other indices (VIX, forecast disagreement and JU) are insignificant. For Dutch households, EU EPU has a negative impact on their expectations, while STOXX and forecast disagreement have positive effects, and NL EPU has no effect. A robust analysis reported in Appendix 2.C.1 Table 2.14 shows that the differences in the estimated coefficients are not due to the different time frames. As discussed in Section 2.3, EPU captures uncertainty in economic policy, while STOXX and forecast disagreement capture uncertainty in financial markets. Our results suggest that households do not respond to different types of uncertainty in the same way as professional forecasters. This finding does not support the first hypothesis that macroeconomic uncertainty reduces income expectations.

Control variables. Both US and NL households' income expectations are dependent on the lagged expectations. In the US, a 1% increase in the lagged growth expectations leads to a 0.05% increase in the current expectations while in the Netherlands, a 1% increase in the lagged level expectations leads to approximately 0.10% decrease in the current expectations.

We investigate the relationships between expected income and macroeconomic variables. Including EU EPU and forecast disagreement, we find that GDP per capita growth positively impacts Dutch expectations. However, GDP per capita growth does not affect US expectations. Moreover, we observe asymmetric responses to GDP growth. When GDP per capita grows more than 4%, Dutch households tend to increase their income expectations by 0.10% - 0.28%, except when including NL EPU (column 2b). In contrast, when GDP per capita growth is less than 2%[2], Dutch households' responses are not consistent; instead, US households optimistically respond to this with an increase in their income growth expectations by 0.20% (columns 1b to 5).

The views of households on their past and future financial situations affect their

[2]The thresholds for high and low growth rates differ between SPFs and households' regressions, as the former utilizes real GDP growth while the latter employs nominal GDP per capita growth.

income expectations with expected signs. Dutch households lower their income expectations by approximately -0.20% when they were in a bad financial situation. For US households, we have data regarding their view of past and future financial situations. US households take into account the good past financial situation when considering their income expectations, but they are more influenced by their view of the future. To be specific, if US respondents believe that their future financial situations will be better (worse), their personal income expectations will increase (decrease) by 0.36% (-0.20%).

Lastly, we study the effects of personal variables, which are generally insignificant. The reason might be that we subtract the individual fixed effect that already accounts for most personal traits. This may explain households' low R-squared (0-0.23) compared to SPFs (0.5-0.7). However, we observe a significant coefficient of net personal income, which the fixed effect cannot capture. We find that a 1% increase in net personal income leads to a 0.02% increase in Dutch household income expectations, with the exception of when NL EPU and forecast disagreement are included (columns 2b and 4a).

In summary, macroeconomic uncertainty has a robust negative impact on the SPFs' expectations of real GDP growth. For households, the results are not as robust. We only find a negative impact of macroeconomic uncertainty on income expectations when the EU and US EPU indices are used.

Table 2.5: Regression results of income expectations in the household surveys

Income expectation$_t$	Dutch households					US households				
	(1997-2018)		(2008-2018)	(1999-2018)		(2013M6 - 2019M10)				
	(1a)	(2a)	(2b)	(3a)	(4a)	(1b)	(2c)	(3b)	(4b)	(5)
Economic Policy Uncertainty index (EU, NL, US)$_t$		-0.46***	0.03				-0.08**			
		(0.08)	(0.14)				(0.04)			
Economic Policy Uncertainty index (EU, NL, US)$_{t-1}$		0.01	-0.76***				0.05			
		(0.06)	(0.26)				(0.03)			
Stock market volatility (STOXX, VIX)$_t$				0.19**				-0.07		
				(0.08)				(0.06)		
Stock market volatility (STOXX, VIX)$_{t-1}$				0.21**				0.01		
				(0.09)				(0.06)		
Forecast disagreement$_t$					2.62***				0.02	
					(0.33)				(0.17)	
Forecast disagreement$_{t-1}$					-0.75***				0.03	
					(0.22)				(0.18)	
Macroeconomic Uncertainty index$_t$ (JU)										-0.07
										(1.00)
Macroeconomic Uncertainty index$_{t-1}$ (JU)										-0.30
										(1.00)
Income expectation$_{t-1}$	-0.10*	-0.12**	-0.15***	-0.10*	-0.13**	0.05***	0.05***	0.05***	0.05***	0.05***
	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
GDP per cap growth$_{t,t-1}$	0.01	0.05***	0.002	0.02	0.10***	0.02	0.02	0.02	0.02	0.02
	(0.01)	(0.05)	(0.01)	(0.01)	(0.02)	(0.05)	(0.03)	(0.03)	(0.03)	(0.03)
GDP per cap deceleration D$_{t,t-1}$	0.07***	0.15***	-0.05*	-0.01	0.05*	-0.04	-0.04	-0.03	-0.04	-0.04
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Above 4% GDP per cap growth D$_{t,t-1}$	0.21***	0.13*	0.04	0.28***	0.10**	0.02	0.02	0.02	0.02	0.02
	(0.04)	(0.04)	(0.06)	(0.05)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Below 2% GDP per cap growth D$_{t,t-1}$	-0.05	-0.05*	0.30***	-0.08**	0.01	0.17***	0.17***	0.17***	0.17***	0.16***
	(0.03)	(0.04)	(0.12)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
Recession D$_{t,t-1}$	-0.04	0.03	-0.10**	-0.03	-0.15***					
	(0.03)	(0.04)	(0.05)	(0.04)	(0.04)					
Good financial situation in the past 12 months D$_t$	-0.08	-0.56	-0.03	-0.06	-0.06	0.08***	0.08***	0.08***	0.08***	0.08***
	(0.09)	(0.08)	(0.09)	(0.08)	(0.08)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Bad financial situation in the past 12 months D$_t$	-0.20*	-0.19*	-0.17	-0.19*	-0.18	0.001	0.002	0.001	0.001	0.001
	(0.11)	(0.11)	(0.12)	(0.11)	(0.11)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Good financial situation in the next 12 months D$_t$						0.32***	0.32***	0.32***	0.32***	0.32***
						(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Bad financial situation in the next 12 months D$_t$						-0.16***	-0.16***	-0.16***	-0.16***	-0.16***
						(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Unemployed D$_t$	-0.10	-0.02	-0.02	-0.7	-0.02	-0.17	-0.17	-0.17	-0.17	-0.17
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.19)	(0.19)	(0.19)	(0.19)	(0.19)
College education D$_t$	-0.27	-0.15	-0.29	-0.21	-0.15					
	(0.24)	(0.24)	(0.18)	(0.24)	(0.24)					
Net personal income$_t$	0.03**	0.02*	0.01	0.02*	0.02					
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)					
Decreased net personal income D$_t$	0.03	0.03	0.01	0.03	0.04*					
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)					
Deficit balance sheet D$_t$	-0.04	-0.05	-0.16*	-0.05	-0.03					
	(0.10)	(0.10)	(0.09)	(0.10)	(0.10)					
Constant	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
HH income categories	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3,756	3,756	3,327	3,724	3,724	42,886	42,886	42,886	42,886	42,886
R-squared	0.00	0.006	0.02	0.00	0.01	0.23	0.23	0.23	0.23	0.23

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, ( ) is S.E. The columns regress next 12 months net household income expectation of Dutch household and next 12 months personal income expectation of US household, against different macroeconomic uncertainty indices. Economic Policy Uncertainty indices (Baker et al., 2016) are from three regions: EU (2a), Netherlands (2b) and US (2c). Stock market volatility indices are STOXX (3a) and VIX (3b). Forecast disagreements are from professional forecasters in EU (4a) and US (4b). Macroeconomic Uncertainty index (Jurado et al., 2015) is only available in the US (5). The difference in the Dutch time frame is due the availability of macroeconomic uncertainty indices. All independent variables ending with D are dummies.The detail description of each variable is in Appendix 2.B.

2.5.2 Subjective uncertainty

We explore the second hypothesis - that macroeconomic uncertainty increases subjective uncertainty - using fixed effect OLS panel regressions. Results from surveys of professional forecasters and household surveys are reported. For each survey sample, we first present the coefficients of macroeconomic uncertainty, our variable of interest, and then the control variables to evaluate whether the results align with previous studies.

Professional forecasters

Macroeconomic uncertainty. Table 2.6 reports the results of regressions that assess the effects of macroeconomic uncertainty on subjective uncertainty, based on surveys of professional forecasters. The results show that a 1% increase in EU EPU, US EPU, and VIX significantly increases subjective uncertainty among professional forecasters (columns 2a, 2b, and 3b). However, the other indices do not have significant effects. Consequently, the evidence from the surveys of professional forecasters does not convincingly support the second hypothesis.

Control variables. The subjective uncertainty of SPFs is persistent, with coefficients of lagged subjective uncertainty being approximately 0.30% for US SPF and 0.69% for EU SPF. We find that US subjective uncertainty does not significantly respond to macroeconomic variables, whereas EU subjective uncertainty does; however, the results are not robust. For example, a 1% increase in GDP growth leads to a 0.01% decrease in EU subjective uncertainty, and this drops further to around 0.02% when GDP growth is low. As the growth control dummies are interrelated, it is not possible to interpret the absolute size of individual coefficients. However, it can be inferred that the subjective uncertainty of US and EU professional forecasters have distinct responses to similar economic news, which is also the case with regards to real GDP growth expectations.

Table 2.6: Regression results of subjective uncertainty in the surveys of professional forecaster

Subjective uncertainty$_t$	EU SPF (1999Q1-2020Q1)				US SPF (1992Q1-2020Q1)				
	(1a)	(2a)	(3a)	(4a)	(1b)	(2b)	(3b)	(4b)	(5)
Economic Policy Uncertainty index (EU, US)$_t$		0.04***				0.04*			
		(0.01)				(0.02)			
Economic Policy Uncertainty index (EU, US)$_{t-1}$		0.01				-0.02			
		(0.01)				(0.02)			
Stock market volatility (STOXX, VIX)$_t$			-0.001				0.06*		
			(0.009)				(0.03)		
Stock market volatility (STOXX, VIX)$_{t-1}$			0.004				-0.02		
			(0.01)				(0.03)		
Forecast disagreement$_t$				0.05				0.04	
				(0.04)				(0.08)	
Forecast disagreement$_{t-1}$				-0.05				-0.05	
				(0.04)				(0.06)	
Macroeconomic Uncertainty index$_t$									-0.12
									(0.16)
Macroeconomic Uncertainty index$_{t-1}$									0.46*
									(0.24)
Subjective uncertainty$_{t-1}$	0.69***	0.67***	0.69***	0.69***	0.30***	0.30***	0.30***	0.30***	0.30***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
GDP growth$_{t-1}$	-0.01***	-0.01***	-0.01***	-0.01***	-0.001	0.00	-0.001	-0.002	-0.01
	(0.002)	(0.002)	(0.003)	(0.003)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
GDP deceleration D$_{t-1}$	0.01*	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.003
	(0.004)	(0.004)	(0.004)	(0.004)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Above 4 % GDP growth D$_{t-1}$	-0.00	0.02**	-0.001	-0.001	0.01	0.01	0.003	0.01	0.02
	(0.009)	(0.01)	(0.009)	(0.009)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Below 1% GDP growth D$_{t-1}$	-0.02*	-0.01*	-0.02*	-0.02**	0.05	0.05	0.02	0.04	-0.02
	(0.008)	(0.008)	(0.008)	(0.008)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
Recession D$_{t-1}$	0.02*	0.02	0.02*	0.02**	-0.02	-0.002	-0.01	-0.01	-0.09
	(0.01)	(0.01)	(0.01)	(0.009)	(0.07)	(0.07)	(0.06)	(0.07)	(0.07)
Constant	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter D	Y	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3,559	3,559	3,559	3,559	3,056	3,056	3,056	3,056	3,056
R-squared	0.76	0.76	0.76	0.76	0.44	0.44	0.43	0.43	0.43

Note: * $p < 0.1$, **$p < 0.05$, *** $p < 0.01$, ( ) is S.E. The columns regress subjective uncertainty of professional forecasters, against different macroeconomic uncertainty indices. Economic Policy Uncertainty indices (Baker et al., 2016) are from EU (2a) and US (2b). Stock market volatility indices are STOXX (3a) and VIX (3b). Forecast disagreements are from professional forecasters in EU (4a) and US (4b). Macroeconomic Uncertainty index (Jurado et al., 2015) is only available in the US (5). All independent variables ending with D are dummies.The details description of each variable is in Appendix 2.B.

Households

Macroeconomic uncertainty. Table 2.7 reports the effects of macroeconomic uncertainty on the subjective uncertainty of Dutch and US households. The results for Dutch households indicate that EU EPU decreases their subjective uncertainty, yet forecast disagreement increases it (columns 2a and 4a, respectively). These different coefficients are not due to the different time frames (see Appendix 2.C.1 Table 2.15) but rather to the different sources of uncertainty discussed previously. For US households, US EPU is found to decrease their subjective uncertainty (column 2c), while the other indices are not significant. The evidence from households thus does not support the second hypothesis, as only one significant positive relationship is observed (column 4a). The two negative relationships (columns 2a and 2c) suggest that macroeconomic uncertainty could even reduce subjective uncertainty.

Control variables. Lagged subjective uncertainty does not appear to have an effect on US households, whereas it has been found to have a small, negative influence on Dutch households. Specifically, a 1% increase in subjective uncertainty in the previous year leads to a decrease of 0.07% in subjective uncertainty in the current year.

We investigate the effects of macroeconomic variables on individual subjective uncertainty. Our findings indicate that US households' subjective uncertainty does not respond to macroeconomic variables, with the exception of a robust increase in subjective uncertainty when GDP per capita growth is below 2%. In the case of Dutch households, macroeconomic variables such as GDP per capita growth above 4% robustly decrease subjective uncertainty, whereas a deceleration of GDP per capita growth increases subjective uncertainty, except when using NL EPU and forecast disagreement (columns 2b and 4a). These results suggest that bad economic signals generally increase subjective uncertainty while good economic signals do the opposite.

The household's financial situation does not affect Dutch subjective uncertainty. In contrast, US results show that while a negative view of the financial situation does not change subjective uncertainty, a positive view of the past and future situations leads to a 0.06% and 0.10% increase respectively. This indicates that optimism leads to greater subjective uncertainty, which is not in agreement with the findings of Bachmann et al. (2021) and Altig et al. (2020) that subjective uncertainty increases with both positive and negative views.

Our results demonstrate that the effects of personal variables on subjective uncertainty are largely not significant. This may be attributed to the individual fixed effect, which captures most personal traits.

To summarize our findings, we do not observe a robust positive effect of macroeconomic uncertainty on individual subjective uncertainty. To be specific, we find a positive relationship for SPFs, using EPU indices and VIX, and for Dutch households using forecast disagreement. When we use the EU and US EPU indices, households' subjective uncertainty actually decreases. In Appendix 2.C.2, we test the robustness of this result, using the generalized-beta-distribution fitted subjective uncertainty. The positive relationship in the US SPF does not hold anymore. Therefore we conclude that there is no robust evidence supporting the second hypothesis.

Table 2.7: Regression results of subjective uncertainty in the household surveys

Subjective uncertainty$_t$	Dutch households					US households				
	(1997-2018)		(2008-2018)	(1999-2018)		(2013M6 - 2019M10)				
	(1a)	(2a)	(2b)	(3a)	(4a)	(1b)	(2c)	(3b)	(4b)	(5)
Economic Policy Uncertainty index (EU, NL, US)$_t$		-0.58**	-0.11				-0.05*			
		(0.14)	(0.25)				(0.03)			
Economic Policy Uncertainty index (EU, NL, US)$_{t-1}$		-0.23**	-0.54				-0.02			
		(0.10)	(0.39)				(0.03)			
Stock market volatility (STOXX, VIX)$_t$				0.17				-0.04		
				(0.15)				(0.05)		
Stock market volatility (STOXX, VIX)$_{t-1}$				0.37***				0.03		
				(0.14)				(0.05)		
Forecast disagreement$_t$					3.23***				0.02	
					(0.61)				(0.14)	
Forecast disagreement$_{t-1}$					-0.74*				0.02	
					(0.40)				(0.15)	
Macroeconomic Uncertainty index$_t$										-0.22
										(0.84)
Macroeconomic Uncertainty index$_{t-1}$										0.08
										(0.83)
Subjective uncertainty$_{t-1}$	-0.06*	-0.07**	-0.09***	-0.06*	-0.07**	0.005	0.005	0.005	0.005	0.005
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
GDP per cap growth$_{t,t-1}$	0.03	0.07**	-0.025	0.03	0.13***	-0.02	-0.02	-0.01	-0.02	-0.02
	(0.02)	(0.02)	(0.02)	(0.05)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
GDP per cap deceleration D$_{t,t-1}$	0.18***	0.25***	0.07	0.17***	-0.15***	-0.01	-0.01	-0.009	-0.01	-0.01
	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Above 4% GDP per cap growth D$_{t,t-1}$	-0.35***	-0.48***	-0.66***	-0.23**	-0.48***	0.01	0.02	0.01	0.01	0.01
	(0.08)	(0.07)	(0.11)	(0.09)	(0.08)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Below 2% GDP per cap growth D$_{t,t-1}$	0.03	0.02	0.30	-0.02	0.08	0.08**	0.07*	0.08*	0.08*	0.07*
	(0.06)	(0.06)	(0.19)	(0.06)	(0.08)	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)
Recession D$_{t,t-1}$	-0.09	0.07	-0.17*	-0.12	-0.22***					
	(0.07)	(0.07)	(0.09)	(0.08)	(0.08)					
Good financial situation in the past 12 months D$_t$	0.19	0.23*	0.19	0.20	0.22	0.06**	0.05**	0.05**	0.05**	0.05**
	(0.13)	(0.13)	(0.14)	(0.13)	(0.13)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Bad financial situation in the past 12 months D$_t$	-0.05	-0.03	-0.05	-0.03	-0.01	0.01	0.01	0.01	0.01	0.01
	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Good financial situation in the next 12 months D$_t$						0.11***	0.11***	0.11***	0.11***	0.11***
						(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Bad financial situation in the next 12 months D$_t$						0.002	0.003	0.002	0.002	0.002
						(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Unemployed D$_t$	-0.30**	-0.16	-0.18	-0.25*	-0.19	-0.04	-0.04	-0.04	-0.04	-0.04
	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)
College education D$_t$	-0.45	-0.24	-0.49**	-0.37	-0.30					
	(0.33)	(0.32)	(0.21)	(0.33)	(0.32)					
Net personal income$_t$	0.05*	0.04	0.03	0.05*	0.03					
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)					
Decreased net personal income D$_t$	0.02	0.02	-0.01	0.02	0.02					
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)					
Deficit balance sheet D$_t$	-0.03	-0.06	-0.21	-0.04	-0.02					
	(0.18)	(0.18)	(0.17)	(0.19)	(0.19)					
Constant	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
HH income categories	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3,703	3,703	3,279	3,671	3,671	42,475	42,475	42,475	42,475	42,475
R-squared	0.001	0.004	0.02	0.001	0.006	0.03	0.03	0.03	0.03	0.03

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, ( ) is S.E. The columns regress next 12 months net household income expectation of Dutch household and next 12 months personal income expectation of US household, against different macroeconomic uncertainty indices. Economic Policy Uncertainty indices (Baker et al., 2016) are from three regions: EU (2a), Netherlands (2b) and US (2c). Stock market volatility indices are STOXX (3a) and VIX (3b). Forecast disagreements are from professional forecasters in EU (4a) and US (4b). Macroeconomic Uncertainty index (Jurado et al., 2015) is only available in the US (5). The difference in the Dutch time frame is due the availability of macroeconomic uncertainty indices. All independent variables ending with D are dummies.The details description of each variable is in Appendix 2.B.

2.6 Discussion

In this study, we utilize four panel datasets to study the impact of macroeconomic uncertainty on individual income expectations (first moment) and subjective uncertainty (second moment). This section summarizes our main findings, discusses puzzling results and provides potential explanations.

Our results show evidence that higher macroeconomic uncertainty reduces income expectations for professional forecasters. This finding is robust across macroeconomic uncertainty indices. For households, however, this result holds significantly only for EPU. These findings are in line with the assumptions of most macroeconomic models (Born & Pfeifer, 2021; Fernández-Villaverde & Guerrón-Quintana, 2020; Lhuissier & Tripier, 2021), and decision-making theories such as multiple prior theory (Gilboa & Schmeidler, 1989) and smooth ambiguity theory (Klibanoff et al., 2005). Yet, we find that STOXX and forecast disagreement increase Dutch income expectations, implying that households respond differently to different types of uncertainty, while professional forecasters do not.

These findings regarding the first moment expectation indicate that households tend to respond differently to different types of uncertainty compared to professional forecasters. This could be due to the differences in the variables used in forecasting - GDP growth versus individual incomes - or the fundamental disparities between households and professional forecasters. Moreover, Born et al. (2018) find that macroeconomic uncertainty indices capture only a small portion of economic downturns. Our findings about heterogeneous responses of households to different uncertainty indices may explain the limited contributions of these indices to the economy.

The evidence regarding the relationship between macroeconomic uncertainty and subjective uncertainty is mixed. Positive correlations are found between the SPF's subjective uncertainty and EPU and VIX, respectively. Similarly, Dutch households exhibit a positive relationship only when using forecast disagreement. On the contrary, when EU and US EPU are used as measures of macroeconomic uncertainty, both Dutch and US households show a negative relationship. Overall, we find both significantly positive, significantly negative and insignificant effects of macroeconomic uncertainty on subjective uncertainty.

Our mixed results regarding subjective uncertainty bring up important discussions. First of all, this could point to issues with the measurement quality of subjective uncertainty. To address this, we employ two measurements of subjective uncertainty (second moment) and confirm that the positive relationship between macroeconomic uncertainty and subjective uncertainty does not robustly hold even among professional forecasters who have robust results for their first moments. Instead, we find an unexpected result of the negative relationship between EPU and the household's subjective uncertainty. The mixed result suggests that macroeconomic uncertainty has a non-monotonic effect on individual subjective uncertainty. This implies that the correlation between macroeconomic uncertainty and subjective uncertainty is complex and that the direction of the relationship varies depending on certain factors.

The concept of ambiguity aversion can be key to explaining the non-monotonic relationship between macroeconomic uncertainty and individual subjective uncertainty. In Section 2.2, we discuss that the macroeconomic uncertainty index represents both risk and ambiguity (Knightian uncertainty). Risk refers to situations where the probability of future outcomes is known, whereas ambiguity is related to a lack of knowledge about the likelihood of future outcomes. A seminal paper by Ellsberg (1961) shows that people respond differently to risk and ambiguity, tending to prefer risk. For instance, Coiculescu et al. (2019) demonstrate that risk can have positive effects on a firm's research and development, whereas ambiguity has the opposite effect. Moreover, Piccillo and Van Den Hurk (2020) show that agents who are more averse to ambiguity tend to perceive noise to be more relevant signals than they actually are[3]. Ambiguity averse agents have a preference for certainty over ambiguity, and this preference shapes their view of the world, by making them more defined even in the absence of a structured signal. Thus, the non-monotonic and diverging response to macroeconomic uncertainty of professional forecasters and households could be attributed to their differing levels of ambiguity aversion or exposure to uncertainty.

Using Klibanoff et al. (2005)'s smooth ambiguity theory, we can more precisely

[3] In their experiments, Piccillo and Van Den Hurk (2020) demonstrate that ambiguity averse participants who are exposed to salient uncertainty detect patterns in blurry pictures, even when these contain only noise. The higher the ambiguity aversion, the more illusory patterns are perceived.

explain how households' subjective uncertainty can be either increased or decreased in the presence of macroeconomic uncertainty. The households are more likely to be ambiguity averse and exposed to higher macroeconomic uncertainty than professional forecasters who have stable jobs. Following the smooth ambiguity theory, ambiguity averse households will try to avoid the spread of forecast distributions and instead prefer forecasts that are robust to uncertainty, implying that the forecast does not change drastically in different potential scenarios (Altug et al., 2020; Baliga et al., 2013; Marinacci, 2015). For example, households can have robust income forecasts when they expect their worst-case incomes to be not much different from their normal incomes. This aversion to the spread of forecast distributions can be interpreted as a preference for a low subjective uncertainty, which makes ambiguity averse households feel more at ease in uncertain conditions. Consequently, as macroeconomic uncertainty increases, the households are uncertain about their forecasts but simultaneously seek solutions to lower their subjective uncertainty. This results in a non-monotonic response to macroeconomic uncertainty.

2.7 Conclusion

This paper empirically examines the effects of macroeconomic uncertainty on income expectations and subjective uncertainty by analyzing surveys of professional forecasters in the EU and US, as well as household data from the Netherlands and the US. Our results show that macroeconomic uncertainty reduces income expectations when using the surveys of professional forecasters. However, when considering the households' income expectations, the effects of macroeconomic uncertainty vary. Additionally, we demonstrate that macroeconomic uncertainty does not necessarily increase subjective uncertainty, as is assumed by most macroeconomic models; in fact, we sometimes observe a negative relationship when considering household data. Finally, we propose that the mixed results observed in the subjective uncertainty could be attributed to ambiguity aversion, using the smooth ambiguity theory by Klibanoff et al. (2005).

As a word of caution, we point out that the four surveys have a high degree of technical variation that might affect the results. First of all, the formulations of the survey questions are different. Second, the forecasts in EU SPF and the US household survey are directly provided by the respondents while those in US SPF and the Dutch household survey are transformed or extracted. Third, the two SPFs and the US household survey provide *growth* forecasts while the Dutch household survey has *level* forecasts. Lastly, the frequency of data, time period, and numbers of observations differ across surveys. These points highlight the heterogeneity in our datasets. Therefore more research is needed to be able to generalize the outcomes.

This study reveals that the relationship between macroeconomic uncertainty, income expectation, and subjective uncertainty is not as straightforward as currently assumed by most macroeconomic models. This insight has direct consequences for our understanding of the impact of uncertainty in subjective decision-making and its consequences on the economy at large and requires further research. In the next chapter, we incorporate ambiguity aversion into a macroeconomic model and demonstrate that the model is able to replicate the empirical findings of this chapter.

Appendix

2.A Data

We test our hypotheses on four data sets, which are the survey of professional forecasters of (1) US and (2) EU, and (3) US and (4) the Netherlands' household survey. The US and EU professional forecasters' surveys are widely used in the economic literature to examine expectations of real GDP growth. The US household survey data, one of the most popular datasets available, contains useful questions regarding respondents' socioeconomic status and expectations of personal nominal income growth. This includes information such as age, gender, education, employment, household income categories, as well as respondents' views on their household financial situations. However, the US household data does not provide actual personal finance data, nor does it cover recession periods, as it only started in 2013.

The Dutch household data is employed because it encompasses the necessary elements we need. It inquires respondents to evaluate the probabilistic distributions of their household nominal income expectations, as well as providing socioeconomic status and personal financial data. This includes the amount of funds in their checking account and the income earned from jobs and financial assets. Furthermore, since its start in 1993, the survey covers major events such as the Dot-Com crisis, the Global Financial crisis, and the European Sovereign Debt crisis. This data set provides the level of income forecast, which is distinct from the growth forecasts of other surveys mentioned previously, offering further robustness to our results. To the best of our knowledge, the Dutch household survey is the longest household survey containing both subjective histogram and personal financial data. In the following sections, we describe survey questions and summary statistics of each dataset.

Table 2.8: Summary statistics of the US professional forecasters

		Mean	Std Dev.	Min	Max	N
Real GDP growth expectation (%)	overall	2.68	0.79	-1.75	7.16	4,237
	between		0.51	0.40	6.74	170
	within		0.73	-1.71	6.30	avg. 24.92
Subjective uncertainty (%)	overall	1.07	0.46	0.005	5.27	3,856
	between		0.39	0.39	2.49	169
	within		0.32	0.03	5.29	avg. 22.82

Note: Subjective uncertainty is measured in standard deviation. N of within statistics are the average number of samples of each individual.

2.A.1 Survey of professional forecasters

US professional forecasters

The forecast of the next-calendar-year real GDP growth rate is obtained by dividing the point forecast of the next-calendar-year GDP level by the point forecast of the current-calendar-year GDP level. The US SPF's series of real GDP forecasts begins in 1968Q4, but it is only from 1992 onward that the forecasting variable becomes consistent. Consequently, we use the series from 1992 onward. The survey also asks for the probability of the next year's real GDP growth falling into the interval ranging from ($-\infty$, -3%), [x%, x+0.9%] for x = -3, -2,. . . , and [6%, ∞). Prior to 2009, the lowest interval was ($-\infty$, -2%). We use this subjective histogram to measure subjective uncertainty. The results are presented in Table 2.8 which details the summary statistics of the GDP expectations and subjective uncertainty.

European professional forecasters

We use the forecast of the next-calendar-year real GDP growth. If the survey is conducted in 2010 the next-calendar-year real GDP growth is the growth rate of 2011. The real GDP series of EU professional forecasters started in 1999Q1, providing point estimates and probabilistic distributions. The interval of the distribution ranges from ($-\infty$, -1%), [x%, x+0.4%] for x = -1, -0.5, . . . , and [4%, ∞). The minimum interval covered to ($-\infty$, -6%) from 2009Q2 to 2009Q3. Table 2.9 presents summary statistics of the GDP expectations and subjective uncertainty of the EU SPF.

Table 2.9: Summary statistics of the EU professional forecasters

		Mean	Std Dev.	Min	Max	N
Real GDP growth expectation (%)	overall	1.77	0.71	-2	4.87	4,591
	between		0.44	0.7	3.1	106
	within		0.66	-1.91	3.84	avg. 43.31
Subjective uncertainty (%)	overall	0.57	0.28	0.08	2.53	4,246
	between		0.21	0.24	1.33	105
	within		0.21	-0.01	3.22	avg. 40.44

Note: Subjective uncertainty is measured in standard deviation. N of within statistics are the average number of samples of each individual.

2.A.2 Household data

US household data

The US household data comes from the survey of consumer expectations, an online survey conducted monthly since 2013 by the Federal Reserve Bank of New York. Heads of households were surveyed for up to 12 consecutive months, providing point estimates and probabilistic distributions of their next-12-months personal income growth. The distribution interval ranged from less than -12% to more than 12%. Samples outside the top and bottom fifth percentile, as well as those with invalid probabilistic distributions, were excluded.

Since 2017, the number of respondents has increased substantially, from approximately 700 to 850. These increased samples mainly comprised of people who had worked at the same place for more than one year. This change resulted in a significant decrease in both the average income growth expectation and subjective uncertainty, by 0.82% and 0.07% respectively. However, our panel regressions were not affected, as we include the individual fixed effects. Table 2.10 presents the corresponding summary statistics.

Table 2.10: Summary statistics of US households

		Mean	Std Dev.	Min	Max	N
Panel A: Gross personal income growth expectations						
Gross personal income growth expectations (%)	overall	3.17	2.70	-1	14	59,295
	between		2.56	-1	14	9,942
	within		1.61	-5.58	14.59	avg.5.96
Subjective uncertainty (%)	overall	2.42	2.42	0	26	59,295
	between		2.42	0	20.981	9,942
	within		1.35	-8.83	18.74	avg.5.96
Panel B: Demographics						
Female		47.5%				59,291
Age (Less than 40 : 40-60 : More than 60)		(36%	:48%	: 16%)		59,267
Panel C: Socioeconomic status						
Past household income (50k or less : 50k-100k : 100k or more)		(27%	:38%	: 35%)		58,850
College education		60%				59,295
Employment (Full time: Part time: Unemployed)		(97%	:2.4%	: 0.6%)		59,295

Note: Subjective uncertainty is measured in standard deviation. N of within statistics are the average number of samples of each individual.

Dutch household data

The Dutch household data comes from the DNB Household Survey administered by Centerdata (Tilburg University, The Netherlands). Questions about income expectations were added in 1995. These questions have been altered three times since then - 1997, 2003, and 2008, when they reverted back to the 1997 version. Due to the inconsistency of these questions, the answers to income expectations from the period 2003 to 2007 are excluded from our study. Thus the sample periods are 1997-2002 and 2008-2018. Additionally, we have excluded respondents who provided incorrect probabilities and those who had no income. Sources of income include work, financial investment, pension funds, unemployment benefits, and more.

The Netherlands' household survey differs from other surveys as it does not ask for point estimates of income expectations. Instead, respondents are asked to provide both the maximum and minimum expected incomes, as well as the probability of their future household income falling within a range between the two estimates. The relevant questions are:

- What do you expect to be the lowest total net yearly income your household may realize in the next 12 months?
- What do you expect to be the highest total net yearly income your household may realize in the next 12 months?

- What do you think is the probability (in percent) that the net yearly income of your household will be less than Lowest income + 20 % × (Highest income - Lowest income)? The question is also asked for the thresholds of 40%, 60%, and 80%.

If the lowest and highest expected income is 0 and 100k respectively. The probabilities provided are for expected income less than 20k, 40k, 60k and 80k. We remove respondents that do not provide valid probabilities and measure the mean expectation using the mid point of each range. In this example, the mid points are 10k, 30k, 50k, 70k and 90k. Therefore, the expected household income is computed as follows:

$$\text{Mean}_{t,i} = E_{t,i}(\text{household income}) = \sum_{range} \text{Probability}_{range,t,i} \times \text{Mid Point}_{range,t,i}$$

Table 2.11 presents the summary statistics for the income expectations, subjective uncertainty, and personal income variables. We employed the log transformation in the regression for the income expectations, subjective uncertainty, and personal income variables. The log transformation was not applied to the zero subjective uncertainty responses, as these responses indicated that the maximum and minimum expected incomes were the same.

Table 2.11: Summary statistics of NL households

		Mean	Std Dev.	Min	Max	N
Panel A: Net household income expectations						
Net HH income expectations (EUR)	overall	43,014	1.8x10^5	8.14	1.27x10^7	5,380
	between		1.17x10^5	49	4.2x10^6	2,189
	within		1.4x10^5	-4.2x10^6	8.4x10^6	avg.2.5
SSD Subjective uncertainty (EUR)	overall	2,656	25,699	0	1.63x10^6	5,380
	between		25,699	0	1.05x10^6	2,189
	within		18,363	-5.4x10^5	1.08x10^6	avg.2.5
Maximum net HH income expectation (EUR)		49,537	241,698	10	1.50x10^7	5,841
Minimum net HH income expectation (EUR)		36,878	121,526	0	9x10^6	5,841
Panel B: Demographics						
Female		40%				5,380
Age		54	14.9	21	91	5,380
Panel C: Socioeconomic status						
Head of household		74%				5,377
Past household income (43k or less : 43k-80k : 80k or more)		(25%	:43%	: 32%)		5,380
College education		51%				5,841
Employed		62%				5,377
Deficit balance sheet		5%				5,010
Annual net personal income (EUR)		31,319	21,376	-4,155	4.6x10^5	4,398
Annual gross personal income (EUR)		40,734	26,106	2	3.0x10^5	4,993

Note: N of within statistics are the average number of samples of each individual.

2.B Variable descriptions

Table 2.12: Variable descriptions for the first hypothesis

	US SPF	EU SPF	US households	Dutch households
Mean expectation	Transformation from level to growth	Point forecasts provided by respondents		Mean expectation calculated from subjective histogram (log)
GDP growth	US real GDP growth (real time)	EU real GDP growth (real time)	US nominal GDP per capita growth	Dutch nominal GDP per capita growth
GDP deceleration dummy	1 if GDP growth of the current period is less than GDP growth of the last period and 0 otherwise			
High (low) GDP growth dummy	1 if the real GDP growth rate is above 4% (below 1%)		1 if the nominal GDP per capita growth rate is above 4% (below 2%)	
Recession dummy	1 if GDP growth of current period is less than 0% and 0 otherwise			
Good (bad) financial situation dummy	NA		1 if the respondent answers that his or her family is or will be financially better (worse) off than 12 months ago and 0 otherwise	1 if the respondent answers that his or her household income is unusually high (low) household income compared to the expected income in the regular year and 0 otherwise
Unemployed dummy	NA		1 if the respondent is not working either full time or part time and 0 otherwise	
College education dummy	NA			1 if the respondent has a college education and 0 otherwise
Net personal income	NA			Actual personal income from jobs or financial assets after adjusting for taxes, rent, interest, scholarship, and so forth (log)
Decreased net personal income dummy	NA			1 if net personal income decreased from last year or 0 otherwise
Deficit balance sheet dummy	NA			1 if the respondent's balance sheet is negative or 0 otherwise

Table 2.13: Variable descriptions for the second hypothesis

	US SPF	EU SPF	US households	Dutch households
Subjective uncertainty	Standard deviation of subjective histogram			Standard deviation of subjective histogram (log)
GDP growth	US real GDP growth (real time)	EU real GDP growth (real time)	US nominal GDP per capita growth	Dutch nominal GDP per capita growth
GDP deceleration dummy	1 if GDP growth of the current period is less than GDP growth of the last period and 0 otherwise			
High (low) GDP growth dummy	1 if the real GDP growth rate is above 4% (below 1%)		1 if the nominal GDP per capita growth rate is above 4% (below 2%)	
Recession dummy	1 if GDP growth of current period is less than 0% and 0 otherwise			
Good (bad) financial situation dummy	NA		1 if the respondent answers that his or her family is or will be financially better (worse) off than 12 months ago and 0 otherwise	1 if the respondent answers that his or her household income is unusually high (low) household income compared to the expected income in the regular year and 0 otherwise
Unemployed dummy	NA		1 if the respondent is not working either full time or part time and 0 otherwise	
College education dummy	NA			1 if the respondent has a college education and 0 otherwise
Net personal income	NA			Actual personal income from jobs or financial assets after adjusting for taxes, rent, interest, scholarship, and so forth (log)
Decreased net personal income dummy	NA			1 if net personal income decreased from last year or 0 otherwise
Deficit balance sheet dummy	NA			1 if the respondent's balance sheet is negative or 0 otherwise

2.C Robustness

This section provides robustness tests of the Dutch household's first and second hypotheses results and the alternative measurement of subjective uncertainty.

2.C.1 Robustness checks of Dutch household data

We report the regression results of the Dutch households' income expectations and subjective uncertainty against the EU EPU index in two sub-periods: when the STOXX volatility index and forecast disagreement are available (1999 - 2018) and when the NL EPU index is available (2008-2018). This is to examine whether the mixed results found in Section 2.5.1 and 2.5.2 are due to the different timeframes.

First hypothesis

Table 2.14 reports the regression results of the Dutch households' income expectations against the EU EPU index in two sub-periods. We find that the timeframe does not alter the main results. From 1999 to 2018, the EU EPU index has a negative impact on households' income expectations, while the STOXX volatility index and forecast disagreement have a positive effect (Section 2.5.1 Table 2.5). From 2008 to 2018, the EU EPU index shows no effect on income expectations, similar to the outcome we observe with the NL EPU index. This suggests that the mixed effects of macroeconomic uncertainty on households' expectations may be due to the different dimensions of uncertainty captured by the uncertainty indices.

Second hypothesis

Table 2.15 reports the regression results of the Dutch households' income expectations against the EU EPU index in two sub-periods. We find that the timeframe does not alter the main results. In the sample period of 1999 to 2018, the EU EPU index has a negative impact on households' subjective uncertainty, while the forecast disagreement has a positive effect (Section 2.5.2 Table 2.7). From 2008 to 2018, the EU EPU index shows no effect on subjective uncertainty, similar to the outcome we observe with the NL EPU index. This suggests that the mixed effects of macroeconomic uncertainty on households' subjective uncertainty may be due to the different dimensions of uncertainty captured by the uncertainty indices.

Table 2.14: Dutch households' income expectations

Income expectations$_t$	(1999-2018)		(2008-2018)	
EU Economic Policy Uncertainty index $_t$		-0.46***		0.14
		(0.08)		(0.12)
EU Economic Policy Uncertainty index $_{t-1}$		0.01		0.14**
		(0.06)		(0.06)
Income expectations$_{t-1}$	-0.10*	-0.12**	-0.14**	-0.15**
	(0.05)	(0.06)	(0.06)	(0.06)
GDP per cap growth$_{t,t-1}$	0.01	0.05***	-0.02	-0.05**
	(0.01)	(0.01)	(0.01)	(0.02)
GDP per cap deceleration D$_{t,t-1}$	0.07***	0.15***	-0.01	-0.05
	(0.03)	(0.03)	(0.03)	(0.05)
Above 4% GDP per cap growth D$_{t,t-1}$	0.21***	0.13***	0.19***	0.21***
	(0.04)	(0.04)	(0.03)	(0.04)
Below 2% GDP per cap growth D$_{t,t-1}$	-0.05	-0.06*	-0.03*	-0.02
	(0.03)	(0.04)	(0.04)	(0.04)
Recession D$_{t,t-1}$	-0.04	0.03	0.07*	-0.16***
	(0.04)	(0.04)	(0.04)	(0.05)
Good financial situation in the past 12 months D$_t$	-0.08	-0.06	-0.03	-0.03
	(0.09)	(0.08)	(0.09)	(0.09)
Bad financial situation in the past 12 months D$_t$	-0.20*	-0.19*	-0.18	-0.18
	(0.11)	(0.11)	(0.12)	(0.12)
Unemployed D$_t$	-0.11	-0.03	-0.03	-0.04
	(0.07)	(0.07)	(0.07)	(0.07)
College education D$_t$	-0.27	-0.15	-0.29	-0.31
	(0.24)	(0.24)	(0.19)	(0.19)
Net personal income$_t$	0.03**	0.02*	0.01	0.01
	(0.01)	(0.01)	(0.01)	(0.01)
Decreased net personal income D$_t$	0.03	0.03	0.01	0.01
	(0.02)	(0.02)	(0.02)	(0.02)
Deficit balance sheet D$_t$	-0.04	-0.06	-0.17*	-0.16*
	(0.10)	(0.10)	(0.09)	(0.09)
Constant	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y
HH income categories	Y	Y	Y	Y
Observations	3,730	3,730	3,327	3,327
R-squared	0.00	0.006	0.04	0.04

Note: * $p < 0.1$, **$p < 0.05$, *** $p < 0.01$.The columns regress next 12 months net household income expectation of Dutch household with different timelines. All independent variables ending with D are dummies. The details description of each variable is in Appendix 2.B.

Table 2.15: Dutch Households' subjective uncertainty

Subjective uncertainty$_t$	(1999-2018)		(2008-2018)	
EU Economic Policy Uncertainty index $_t$		-0.57***		0.14
		(0.14)		(0.23)
EU Economic Policy Uncertainty index $_{t-1}$		0.23**		0.10
		(0.10)		(0.11)
Subjective uncertainty$_{t-1}$	-0.05	-0.07**	-0.09***	-0.09***
	(0.03)	(0.03)	(0.03)	(0.03)
GDP per cap growth$_t$	0.02	0.09***	-0.03	-0.07
	(0.02)	(0.02)	(0.02)	(0.04)
GDP per cap deceleration D$_t$	0.18***	0.25***	0.02	-0.03
	(0.04)	(0.05)	(0.05)	(0.09)
Above 4% GDP per cap growth D$_t$	-0.36***	-0.48***	-0.52***	-0.50***
	(0.08)	(0.08)	(0.08)	(0.08)
Below 2% GDP per cap growth D$_t$	0.02	0.02	0.02	0.03
	(0.06)	(0.06)	(0.06)	(0.06)
Recession D$_t$	-0.10	0.07	-0.18***	-0.25**
	(0.07)	(0.07)	(0.07)	(0.10)
Good financial situation in the past 12 months D$_t$	0.19	0.22*	0.19	0.19
	(0.13)	(0.13)	(0.14)	(0.14)
Bad financial situation in the past 12 months D$_t$	-0.05	-0.03	-0.05	-0.06
	(0.15)	(0.15)	(0.15)	(0.15)
Unemployed D$_t$	-0.30**	-0.16	-0.18	-0.19
	(0.13)	(0.13)	(0.13)	(0.13)
College education D$_t$	-0.45	-0.25	-0.49**	-0.49**
	(0.33)	(0.32)	(0.21)	(0.21)
Net personal income$_t$	0.05*	0.03	0.03	0.03
	(0.03)	(0.03)	(0.03)	(0.03)
Decreased net personal income D$_t$	0.01	0.02	-0.01	-0.01
	(0.04)	(0.04)	(0.04)	(0.04)
Deficit balance sheet D$_t$	-0.03	-0.06	-0.22	-0.21
	(0.19)	(0.19)	(0.17)	(0.17)
Constant	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y
HH income categories	Y	Y	Y	Y
Observations	3,677	3,677	3,279	3,2379
R-squared	0.001	0.004	0.02	0.02

Note: * $p < 0.1$, **$p < 0.05$, *** $p < 0.01$. The columns regress the subjective uncertainty of Dutch households with different timelines. All independent variables ending with D are dummies. The details description of each variable is in Appendix 2.B.

2.C.2 Generalized Beta distribution for the second hypothesis

In this section, we present the results of subjective uncertainty using the generalized Beta distribution (GBD) to examine whether the findings in Section 2.5.2 are the same across different measures.

Professional forecasters

Table 2.16 reports the regression results of the SPFs' GBD subjective uncertainty. In the EU SPF data, we find that the positive effect still holds with the EU EPU index, while it does not hold anymore in the US SPF data.

Households

Table 2.17 shows the regression results for the GBD subjective Uncertainty of households. We find that the results are consistent with those from Section 2.5.2. For Dutch households, the negative and positive coefficients remain significant for the EU EPU index and forecast disagreement, respectively. Regarding US households, the negative coefficient of the US EPU index is found to be more significant, while the negative coefficient of the VIX index also becomes statistically significant.

Table 2.16: SPF subjective uncertainty

Subjective uncertainty$_t$	EU SPF (1999Q1-2020Q1)				US SPF (1992Q1-2020Q1)				
	(1a)	(2a)	(3a)	(4a)	(1b)	(2b)	(3b)	(4b)	(5)
Economic Policy Uncertainty index (EU, US)$_t$		0.03**				-0.02			
		(0.01)				(0.02)			
Economic Policy Uncertainty index (EU, US)$_{t-1}$		0.004				-0.009			
		(0.01)				(0.02)			
Stock market volatility (STOXX, VIX)$_t$			-0.001				-0.01		
			(0.007)				(0.02)		
Stock market volatility (STOXX, VIX)$_{t-1}$			0.008				0.03*		
			(0.01)				(0.02)		
Forecast disagreement$_t$				0.04				-0.03	
				(0.04)				(0.04)	
Forecast disagreement$_{t-1}$				-0.06				0.01	
				(0.04)				(0.04)	
Macroeconomic Uncertainty index$_t$									-0.35***
									(0.11)
Macroeconomic Uncertainty index$_{t-1}$									0.41**
									(0.17)
Subjective uncertainty$_{t-1}$	0.74***	0.72***	0.74***	0.74***	0.45***	0.45***	0.45***	0.45***	0.45***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
GDP growth$_{t-1}$	-0.01***	-0.01***	-0.005**	-0.006*	0.004	-0.006	-0.004	-0.004	-0.004
	(0.001)	(0.002)	(0.002)	(0.003)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
GDP deceleration D$_{t-1}$	0.004	0.00	0.004	0.004	0.005	0.004	0.003	0.005	0.004
	(0.003)	(0.003)	(0.003)	(0.003)	(0.006)	(0.006)	(0.006)	(0.006)	(0.007)
Above 4 % GDP growth D$_{t-1}$	-0.01	0.01	-0.01	-0.01	0.003	0.003	0.00	0.004	0.005
	(0.009)	(0.009)	(0.009)	(0.009)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Below 1% GDP growth D$_{t-1}$	-0.01	-0.01	-0.1	-0.01	-0.02	-0.04	-0.02	-0.008	-0.03
	(0.008)	(0.008)	(0.008)	(0.008)	(0.02)	(0.03)	(0.02)	(0.03)	(0.03)
Recession D$_{t-1}$	0.02**	0.02**	0.02***	0.03***	0.04	0.02	0.02	0.04	-0.007
	(0.009)	(0.009)	(0.009)	(0.008)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Constant	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter D	Y	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3,058	3,058	3,058	3,058	2,405	2,405	2,405	2,405	2,405
R-squared	0.79	0.79	0.79	0.79	0.55	0.55	0.55	0.55	0.55

Note: * $p < 0.1$, **$p < 0.05$, *** $p < 0.01$. The columns regress subjective uncertainty of professional forecasters, against different macro uncertainty indices. Economic Policy Uncertainty indices (Baker et al., 2016) are from EU (2a) and US (2b). Stock market volatility indices are STOXX (3a) and VIX (3b). Forecast disagreements are from professional forecasters in EU (4a) and US (4b). Macroeconomic Uncertainty index (Jurado et al., 2015) is only available in the US (5). All independent variables ending with D are dummies. The details description of each variable is in Appendix 2.B.

Table 2.17: Households' subjective uncertainty

Subjective uncertainty$_t$	Dutch households					US households				
	(1997-2018)		(2008-2018)	(1999-2018)		(2013M6 - 2019M10)				
	(1a)	(2a)	(2b)	(3a)	(4a)	(1b)	(2c)	(3b)	(4b)	(5)
Economic Policy Uncertainty index (EU, NL, US)$_t$		-0.44***	-0.18				-0.08***			
		(0.15)	(0.28)				(0.02)			
Economic Policy Uncertainty index (EU, NL, US)$_{t-1}$		-0.28**	-0.19				-0.01			
		(0.11)	(0.43)				(0.02)			
Stock market volatility (STOXX, VIX)$_t$				0.18				-0.07**		
				(0.15)				(0.04)		
Stock market volatility (STOXX, VIX)$_{t-1}$				0.31**				0.04		
				(0.15)				(0.04)		
Forecast disagreement$_t$					2.31***				0.09	
					(0.67)				(0.11)	
Forecast disagreement$_{t-1}$					-0.33				0.06	
					(0.44)				(0.12)	
Macroeconomic Uncertainty index$_t$										-0.88
										(0.66)
Macroeconomic Uncertainty index$_{t-1}$										0.64
										(0.66)
Subjective uncertainty$_{t-1}$	-0.07**	-0.09**	-0.11***	-0.08**	-0.08**	0.05***	0.05***	0.05***	0.05***	0.05***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
GDP per cap growth$_{t,t-1}$	0.02	0.08***	-0.02	0.02	0.09***	-0.001	-0.004	0.004	-0.006	0.00
	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
GDP per cap deceleration D$_{t,t-1}$	0.13***	0.17***	-0.01	0.03	0.12**	0.00	-0.00	0.005	-0.006	0.002
	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Above 4% GDP per cap growth D$_{t,t-1}$	-0.24***	-0.35***	-0.48***	-0.14	-0.33***	0.02	0.02	0.02	0.02	0.02
	(0.08)	(0.08)	(0.12)	(0.10)	(0.09)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Below 2% GDP per cap growth D$_{t,t-1}$	-0.01	0.01	0.11	-0.05	0.01	0.06*	0.06	0.07*	0.06	0.06
	(0.06)	(0.06)	(0.21)	(0.07)	(0.09)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)
Recession D$_{t,t-1}$	-0.05	0.11	-0.09	-0.12	-0.22***					
	(0.07)	(0.08)	(0.10)	(0.09)	(0.09)					
Good financial situation in the past 12 months D$_t$	0.15	0.19	0.18	0.17	0.18	0.07***	0.06***	0.07***	0.07***	0.07***
	(0.15)	(0.15)	(0.15)	(0.15)	(0.14)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Bad financial situation in the past 12 months D$_t$	-0.08	-0.05	-0.06	-0.06	-0.05	0.03	0.03	0.03	0.03	0.03
	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Good financial situation in the next 12 months D$_t$						0.08***	0.08***	0.08***	0.08***	0.08***
						(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Bad financial situation in the next 12 months D$_t$						-0.009	-0.007	-0.009	-0.009	-0.009
						(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Unemployed D$_t$	-0.29**	-0.16	-0.14	-0.24*	-0.20	-0.15	-0.14	-0.14	-0.14	0.14
	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
College education D$_t$	-0.55*	-0.39	-0.57***	-0.49	-0.45					
	(0.32)	(0.32)	(0.22)	(0.31)	(0.31)					
Net personal income$_t$	0.05	0.04	0.03	0.05	0.04					
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)					
Decreased net personal income D$_t$	0.01	0.01	-0.03	0.01	0.01					
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)					
Deficit balance sheet D$_t$	0.08	0.05	-0.11	0.06	0.08					
	(0.19)	(0.19)	(0.17)	(0.20)	(0.20)					
Constant	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
HH income categories	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3,470	3,470	3,069	3,439	3,439	45,081	45,081	45,081	45,081	45,081
R-squared	0.002	0.001	0.04	0.003	0.001	0.38	0.37	0.38	0.38	0.38

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The columns regress next 12 months net household income expectation of Dutch household and next 12 months personal income expectation of US household, against different macro uncertainty indices. Economic Policy Uncertainty indices (Baker et al., 2016) are from three regions: EU (2a), Netherlands (2b) and US (2c). Stock market volatility indices are STOXX (3a) and VIX (3b). Forecast disagreements are from professional forecasters in EU (4a) and US (4b). Macroeconomic Uncertainty index (Jurado et al., 2015) is only available in the US (5). The difference in the Dutch time frame is due the availability of macro uncertainty indices. All independent variables ending with D are dummies. The details description of each variable is in Appendix 2.B.

Chapter 3

Ambiguous business cycles, recessions and uncertainty

Giulia Piccillo and Poramapa Poonpakdee

3.1 Introduction

Many empirical studies suggest that uncertainty has adverse effects on the economy through the expectations or beliefs in future events (Baker et al., 2016; Bloom, 2014; Born et al., 2018; Jurado et al., 2015). However, most macroeconomic models fail to accurately capture the magnitude of the negative uncertainty effect, especially in crises (Born & Pfeifer, 2021; Ng & Wright, 2013; Wieland & Wolters, 2011). This can be attributed, in part, to the complex relationship between uncertainty and people's beliefs. As examined in Chapter 2, the assumed relationship between macroeconomic uncertainty[1] and people's beliefs in most models is sometimes inconsistent with the empirical findings. In addition, we argue that ambiguity aversion might explain this complex relationship. In this chapter, we replicate the effect of macroeconomic uncertainty on individual beliefs in a macroeconomic model that accounts for ambiguity aversion. With this microfoundation, we then study the effects of macroeconomic uncertainty on the economy, particularly in crises.

The exact relationship between macroeconomic uncertainty and people's beliefs is still largely a matter for discussion. As a starting point for the analysis, we focus on three empirical stylized facts that describe the impact of macroeconomic uncertainty on the first and second moments of subjective beliefs as well as on the magnitude of this impact over time. First, Chapter 2 and other empirical studies show that macroeconomic uncertainty makes people more pessimistic. This effect is more evident in households, which are typically more pessimistic than professional forecasters. For instance, Bhandari et al. (2023) shows that the US households are pessimistic compared to the US professional forecasters, and this pessimistic distortion (belief wedge) increases with macroeconomic uncertainty. The second stylized fact is that the effect of macroeconomic uncertainty on subjective uncertainty is not monotonic. A high subjective uncertainty indicates that an individual has a wide probability distribution around her fixed point belief, and she is therefore less confident about her fixed point estimate. In Chapter 2, we find that Economic Policy Uncertainty has a different impact on the subjective uncertainty of professional forecasters and households. When it increases, the professional forecasters' subjec-

[1] Macroeconomic uncertainty is measured by indices such as the Economic Policy Uncertainty index (Baker et al., 2016), the 1-month macroeconomic uncertainty index (Jurado et al., 2015), and implied volatility indices.

tive uncertainty increases while households' subjective uncertainty decreases. Finally, the empirical evidence suggests that the effect of macroeconomic uncertainty on the economy is nonlinear and it is disproportionally stronger during periods of high macroeconomic uncertainty (Jackson et al., 2020; Ng & Wright, 2013). Using a Markov-switching VAR model, Lhuissier and Tripier (2021) shows that the adverse effect of uncertainty on the US output is four times greater in a "distress" regime[2].

In this paper, we develop a macroeconomic model to replicate the mentioned three stylized facts and investigate if it can replicate the negative effects of uncertainty on the economy, especially during recessions. The model is based on the smooth ambiguity model from Altug et al. (2020), which accounts for uncertainty in the form of ambiguity and household's attitude toward ambiguity (ambiguity aversion). The term ambiguity refers to Knightian uncertainty in which probabilities of future outcomes are unknown, as opposed to risk when the probabilities are known (Knight, 1921). We depart from Altug et al. (2020) in two ways. First, our household is uncertain whether the economy will be in a recession or a period of normal growth, whereas, in their model, the two scenarios are high and low persistent technological processes. Second, uncertainty or ambiguity in Altug et al. (2020)'s model increases the variance of the prior belief of each scenario. Instead, we propose that uncertainty increases the difference in expected utilities between the two scenarios and this difference is anchored to an empirical macroeconomic uncertainty index.

Using simulations, we demonstrate that our smooth ambiguity model can replicate the three stylized facts regarding the relationships between macroeconomic uncertainty and people's beliefs, and ambiguity aversion is an important factor determining these correlations. Moreover, simulations with the US data show that the model with an appropriate level of ambiguity aversion can capture the large output drops in recessions, even with the out-of-sample exercise. Lastly, the smooth ambiguity model has the potential to be a forecasting tool as its performance in predicting output growth rate is comparable to that of professional forecasters.

The rest of this paper proceeds as follows. In Section 3.2, we document how

[2]The authors define this regime as a major disturbance in financial markets, imbalances in macroeconomic conditions, and increased uncertainty.

macroeconomic models have incorporated uncertainty. Then, Section 3.3 introduces our key assumption and presents the supporting evidence for the assumption. In Section 3.4, we describe our model and discuss the implications of uncertainty in the model. Section 3.5 demonstrates that the smooth ambiguity model can replicate the three stylized facts using the simulations. Then, we simulate output growth using the real data from the US and discuss its performance in Section 3.6. Finally we conclude in Section 3.7.

3.2 Modeling uncertainty

In this section, we provide a survey of the macroeconomic literature that incorporates uncertainty in the business cycle model. This literature is vast and uses various - not always transferable - definitions of the concept of uncertainty. In order to avoid any confusion in the jargon, we organize the literature according to the specific definition of uncertainty used. Knight (1921) defines the components of uncertainty by distinguishing between risk, where the likelihood of an event is known, and ambiguity, where the likelihood is unknown. Ambiguity is often referred to as Knightian uncertainty. To show the difference between these two modeling approaches, we first study models of risk, and then models of ambiguity.

Risk. A large strand of literature studies uncertainty as a time-varying volatility assuming that the data generating process is known but is changing over time. According to the distinction above, and in line with Fernández-Villaverde and Guerrón-Quintana (2020), uncertainty in this case is more similar to the modern concept of risk. In this vein, Born and Pfeifer (2021) studies the effects of uncertainty through markup channels, in which uncertainties are measured by time-varying volatility of the TPF and government spending processes. They find that a two S.D. uncertainty shock can generate only a 0.0035% decrease in output unless employing more extreme and less common parameters such as a risk aversion of 20[3]. Fernández-Villaverde and Guerrón-Quintana (2020) introduces uncertainty in TFP, financial frictions and preference processes. In their estimation, this time-varying volatility explains a significant part of economic fluctuations; for example, financial frictions uncertainty can account for 63% of output volatility. Lhuissier and Trip-

[3] According to standard models such as Slobodyan and Wouters (2012), the standard value of risk aversion is around 2.

ier (2021) creates a Markov-switching model with two economic regimes: tranquil and distress periods, in which uncertainty is the volatility of TFP. In their estimation, the monitoring cost in the distress period is higher than the tranquil period, so the risk premium is higher in periods of distress, which derails investment. This mechanism amplifies the negative effect of uncertainty in the distress period by a factor of four.

Ambiguity. Another way to model uncertainty is to impose multiple potential scenarios where the true scenario is unknown until time t, when it becomes observable. In this version of uncertainty, before the true scenario is known, the likelihood of the events is unknown, so it is closer to ambiguity in Knight (1921)'s framework. In the business cycle model, ambiguity is usually interpreted as a range of scenarios with different data generating processes. The agent knows data generating process of each scenario but does not know which one is true. To model the agent's behavior, a specific preference toward ambiguity is necessary. In general, ambiguity aversion implies that the agent is worse-off when exposed to uncertainty. For example, in multiple priors preferences (Gilboa & Schmeidler, 1989), and robust preferences (Hansen & Sargent, 2011), the ambiguity averse agents behave as if they are in the worst-case scenario. Ilut and Schneider (2014) adopts multiple priors preferences in which uncertainty or ambiguity is defined as a decrease in the worst-case expected utility which is proxied by professional forecasters' disagreement. Uncertainty in their model accounts for 70% of output volatility. Using robust preferences, Bhandari et al. (2023) derives household's belief wedges of inflation and unemployment,[4] which increase during uncertain periods. The belief wedges make the household's worst-case belief more pessimistic, and, with this mechanism, the model can match the volatility of output, inflation, and unemployment. Although agents in these two models are ambiguity averse, their attitude toward ambiguity is not adjustable and cannot be distinguished from the attitude toward risk.

Smooth ambiguity preferences (Klibanoff et al., 2005) differentiate between risk and ambiguity aversion. Only extremely ambiguity averse agents will always adopt the worst-case scenario (Klibanoff et al., 2005; Marinacci, 2015). Altug et al. (2020) use smooth ambiguity preferences with two scenarios: high and low persistent technological progresses, where the true scenario is unknown. In their model, the agent is ambiguity

[4]Bhandari et al. (2023) measures the belief wedge as a difference between expectations of consumers and professional forecasters.

averse and learns about the probability of the true scenario, using Bayes' rule. Due to ambiguity aversion, the agent always puts more weight on the low-utility scenario than Bayes' rule. The authors label this behavior a pessimistic belief distortion because the agent's belief is more pessimistic than the Bayesian belief. A higher uncertainty means a larger variance of a Bayesian prior, and the simulations show that uncertainty increases the volatility of the economy but the magnitude is small.

Our model is an extension of the smooth ambiguity model by Altug et al. (2020). We choose the smooth ambiguity model for three reasons. First, it nests the properties of multiple priors preferences and robust preferences as special cases and distinguishes between the attitudes toward ambiguity and risk (Ju & Miao, 2012; Marinacci, 2015). In this way, we will focus on the result of changes to ambiguity. Second, the properties of the smooth ambiguity model are in line with a growing micro finance literature (Guidolin & Liu, 2016; Nowzohour & Stracca, 2020; Pulford, 2009). For example, uncertainty impacts the economy through pessimistic beliefs, and the magnitude of pessimism is conditional on individual ambiguity attitudes. Finally, the smooth ambiguity model can be tested in a macro setting using available variables from empirical data.

3.3 Uncertainty and expected utilities

This section discusses our key assumption about the relationship between uncertainty and expected utilities. We begin by providing intuitions to motivate the assumption, followed by empirical evidence to support it.

We depart from the model of Altug et al. (2020) in two ways. First, the two scenarios in our paper are a normal economic growth scenario versus an economic recession scenario (and not a low versus high persistence of technological growth). Second, we anchor the ratio of expected utilities of the two scenarios using empirical data. We propose that the ratio of expected utilities is time varying. There are periods when expected utilities in the good and bad scenarios are relatively similar, and periods when a deep crisis is feared, meaning that the ratio of the expected utilities in the two possible outcomes is more relevant[5]. To pin down this ratio, we use an empirical macroeconomic uncertainty

[5] This ratio is not the same as the agent's probability belief in the likelihood that one scenario will be realized over the other.

index. The remainder of this section discusses our key assumption in some detail and shows the empirical grounds on which it stands.

Uncertainty affects the dynamics of the model when two conditions are satisfied. First, the household believes that the economy could at least potentially fall into a recession. If the household has a belief that with 100% probability there will be no recession, this condition is not satisfied. Second, the household expects that the utilities of the recession and normal growth scenarios are different. Let μ_t be a Bayesian belief of the recession probability, $E_t(V_{t+1}^R)$ be the expected utility at time t for the economy to be in recession at time $t+1$, and $E_t(V_{t+1}^{NR})$ be the expected utility when the economy is in the period of normal growth at $t+1$. V indicates the utility, and superscripts R and NR indicate recession and normal growth scenarios, respectively. Therefore, uncertainty is relevant when:

$$\mu_t > 0 \text{ and } E_t(V_{t+1}^{NR}) > E_t(V_{t+1}^R) \tag{3.1}$$

We assume that uncertainty affects the expected utilities in the two scenarios asymmetrically and illustrate this through a practical example. Currently Stefanie has a permanent position in a large firm, and she might not be severely affected if there is a recession. Therefore, her expected utility in the recession scenario is close to the normal growth scenario. However, twenty years ago, Stefanie had an entry-level job in a start-up, so she would have been severely affected in case of a recession. Thus, in the past Stefanie's expected utility in the recession scenario was much lower than in the normal growth scenario. In this example, the young Stefanie is exposed to higher uncertainty of recession than the current Stefanie. It implies that uncertainty increases the spread of the expected utilities between the two scenarios.

Because in a representative agent model the expected utilities of Stefanie are the average utilities in the whole economy, we anchor the ratio of $E_t(V_{t+1}^R)$ in relation to $E_t(V_{t+1}^{NR})$ to the empirical series of macroeconomic uncertainty:

$$M_t = \frac{E_t(V_{t+1}^{NR})}{E_t(V_{t+1}^R)} \tag{3.2}$$

A description of the roles of the two concepts in the dynamics is given in Section 3.4.2.

where M_t is a time-series of macroeconomic uncertainty.

3.3.1 Empirical evidence

In this section, we use empirical evidence to motivate the assumption to anchor the ratio of expected utilities to a macroeconomic uncertainty measure. We use GDP growth expectations from the US professional forecasters as a proxy for expected utilities. Ceteris paribus, we assume that:

$$\frac{E_t(V_{t+1}^{NR})}{E_t(V_{t+1}^{R})} \propto \frac{E_t^j(Y_{t+1}^{NR})}{E_t^j(Y_{t+1}^{R})} \tag{3.3}$$

where $E_t^j(Y_{t+1}^{NR})$ is forecaster j's next-year GDP growth expectation if GDP growth will be positive, and $E_t^j(Y_{t+1}^{R})$ is forecaster j's next-year GDP growth expectation if GDP growth will be negative (in a recession).

The survey of US professional forecasters provides an individual subjective histogram of next-year GDP growth expectations. Here each forecaster fills in his or her subjective probabilities that GDP growth will be within a given bin. In this survey, the bin ranges from ($-\infty$, -3%), [x%, x+0.9%] for $x \in \{-3, -2, \ldots, 5\}$, and [6%, ∞) [6]. We use this subjective histogram to calculate the expected GDP growth in each scenario.

$$E_t^j(Y_{t+1}^{NR}) = \frac{\sum_i P_t^j([\underline{x}_i, \bar{x}_i], \underline{x}_i \geq 0) \times Y_{i,t+1}}{\sum_i P^j(Y_{i,t+1} \geq 0)} \tag{3.4}$$

$$E_t^j(Y_{t+1}^{R}) = \frac{\sum_i P_t^j([\underline{x}_i, \bar{x}_i], \bar{x}_i < 0) \times Y_{i,t+1}}{\sum_i P^j(Y_{i,t+1} < 0)} \tag{3.5}$$

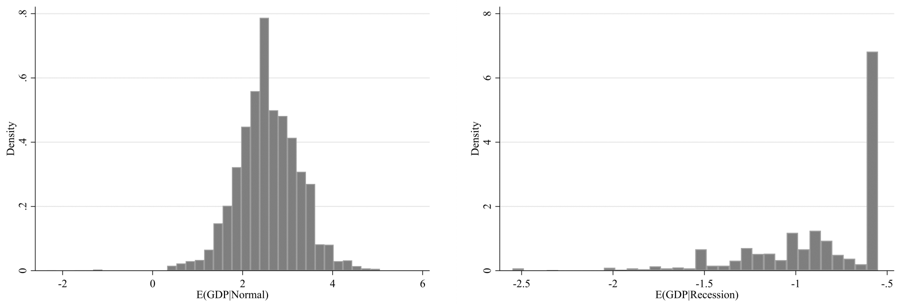
where i is the bin index, $\underline{x}_i$ is the lower bound of bin i, and $\bar{x}_i$ is the upper bound of bin i, $Y_{i,t+1}$ is the mid point of bin i, and P_t^j is the probability of forecaster j of each bin.

Figure 3.1 shows histograms of $E_t^j(Y_{t+1}^{NR})$ and $E_t^j(Y_{t+1}^{R})$ from 1992Q1 to 2020Q1. In most of the quarters, the forecasters believe with a 100% probability that there will be no recession next year. In these cases, the difference between two scenarios is zero and uncertainty is not relevant according to our model. When the forecasters believe that the economy can possibly be in a recession, the average expected GDP growth in the recession scenario is -0.87%, and the average GDP growth forecast in the normal scenario is 2.37%.

[6] During 1985 - 2019, our sample period, the size of the bin was changed once at the beginning of the Global Financial crisis.

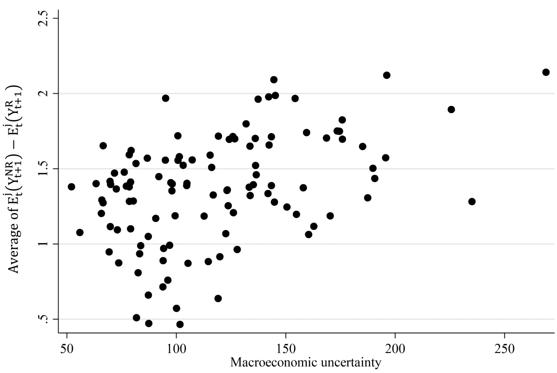
Figure 3.1: Histograms of GDP growth expectations in the normal and recession scenarios

(a) $E_t^j(Y_{t+1}^{NR})$ **(b)** $E_t^j(Y_{t+1}^{R})$



Because both scenarios are now conceived as possible, uncertainty becomes relevant in the model. The Y axis of Figure 3.2 shows the cross-sectional average of the difference between $E_t^j(Y_{t+1}^{NR})$ and $E_t^j(Y_{t+1}^{R})$[7]. This is plotted against macroeconomic uncertainty on the X axis, measured by the US Economic Policy Uncertainty index, and the positive relationship is visible to the naked eye. Table 3.1 shows the result of a panel regression of the difference against the US Economic Policy Uncertainty Index. When macroeconomic uncertainty increases by 1% the difference between the two expected GDP growth rates increases by 0.18%. Hence the difference in expected GDP growth rates is proportional (positively correlated) with macroeconomic uncertainty.

Figure 3.2: Difference in GDP growth expectations and macroeconomic uncertainty



Note: Y-axis is the average difference between point estimates of GDP growth forecasts in the normal and recession scenarios. X-axis is the US Economic Policy Uncertainty index (Baker et al., 2016).

[7] We use the difference rather than the ratio because the expectations are growth rates. The difference between two expected growth rates reflects the deviation between two expected utilities, better than the ratio. Suppose that the expected growth rates of the two scenarios are -5% and 5% when uncertainty is high. In this case the ratio is -1 and the difference is 10. When uncertainty is low the expected growth rates are -1% and 1%. The ratio is still -1 but the difference is 2, implying a decreased uncertainty. Thus the difference is more appropriate.

Table 3.1: Effect of uncertainty on the difference between expected GDP growth rate

	Difference between the expected GDP growth rates
Macroeconomic uncertainty growth$_t$	0.18***
	(0.06)
GDP growth$_t$	-0.03**
	(0.08)
Difference$_{t-1}$	0.46***
	(0.06)
Constant	Y
Quarter FE	Y
Individual FE	Y
Observations	3,259
R-squared	0.57

Note: * $p < 0.1$, **$p < 0.05$, *** $p< 0.01$. Macroeconomic uncertainty is the US Economic Policy Uncertainty index by Baker et al. (2016). The dependent variable is the difference between point estimates of GDP growth forecasts for normal and recession scenarios. The GDP forecasts are from the survey of US professional forecasters. The model is a fixed-effect regression that controls for heteroskedasticity.

3.4 Model

This section describes the representative-agent model with smooth ambiguity preferences based on Altug et al. (2020). Our model differs from theirs in two main ways. First, the two scenarios in Altug et al. (2020) are the periods of high and low persistent technological process, whereas ours are the periods of normal growth and recession. Second, in Altug et al. (2020), higher uncertainty or ambiguity is measured by a larger variance of the Bayesian prior. Here, we use a macroeconomic uncertainty index M_t to proxy for the level of macroeconomic uncertainty and anchor it to the spread of expected utilities between the two scenarios. A summary of Altug et al. (2020) is provided in Appendix 3.A.

To precisely study the role of smooth ambiguity preferences, we employ a real business cycle model with a simple setup. According to Cogley and Nason (1995), real business cycle models have three main transmission mechanisms of shocks: capital accumulation, intertemporal substitution, and various types of adjustment costs. We focus on the intertemporal substitution channel since it is closely related to agents' expectations and smooth ambiguity preferences. Our model features two representative agents: a household and a firm. The setup of these agents is close to the original model in Altug et al. (2020) and those in Greenwood et al. (1988) and Christiano and Eichenbaum (1992). We assume that three market clearing conditions are satisfied for each period. First, the good market is cleared in such a way that production is equal to consumption plus savings. Second, savings are equal to investment under an equilibrium rental rate, meaning that the household's savings are transformed into an investment in capital without any friction. Lastly, the firm's labor demand is equal to the household's labor supply under an equilibrium wage.

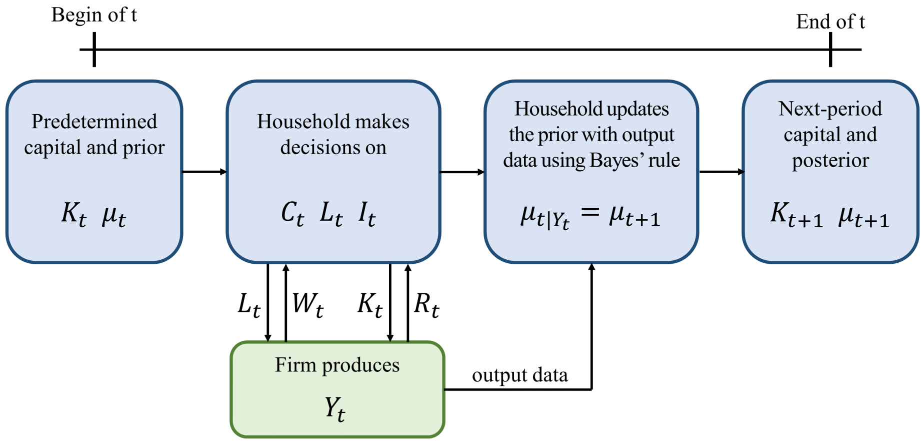
In our economy, the household has smooth ambiguity preferences, is a Bayesian learner and is uncertain whether there will be a recession in the next period. Smooth ambiguity preferences imply that the household tries to *smooth* out its expected utilities across the two scenarios. This is conceptually analogous to consumption smoothing in the sense that the consumption is smoothed out over time. When consumption smoothing is perfect, the expected consumption remains constant over time; comparably, when ambiguity smoothing is perfect, the expected utility is the same for both recession and normal growth scenarios. In contrast to the household, the firm is not directly subject to uncertainty. We assume it makes optimizing decisions based on the currently observable information. Uncertainty indirectly affects the firm only through the household's decisions.

Figure 3.3 illustrates the timeline of decision-makings and Bayesian updating. At each period, the household chooses how much to consume, work and invest given a predetermined capital and its Bayesian prior of recession (i.e. the probability of recession). The household provides labor and capital to the firm. The firm provides wage and a rental fee on capital to the household and produces output. The profit from the production will be transferred to the household. The household uses the observed output data to update the Bayesian prior of the recession, which will be used in the next period.

3.4.1 Household

This section describes the household's maximization problem and shows how we incorporate uncertainty into the model. The household forms the expected utilities of the two scenarios: recession and a normal growth period. The expected utilities will be evaluated with the following smooth ambiguity function: $\phi(E_t(V_{t+1})) = \frac{[E_t(V_{t+1})]^{1-\gamma}}{1-\gamma}$, where $\gamma \geq 0$ is the ambiguity aversion parameter and $E_t(V_{t+1})$ is the expected utility of period

Figure 3.3: Decision-making and Bayesian updating



where K_t is capital, C_t is consumption, L_t is labor, I_t is investment, Y_t is output, W_t is the labor wage, R_t is the rental fee on capital, μ_t is the Bayesian prior of recession, μ_{t+1} is the Bayesian posterior of recession.

$t+1$. The concavity of the function ϕ captures the reaction to ambiguity, which can be interpreted as aversion to mean-preserving spreads. When the spread of expected utilities increases, the mean expected utility decreases, implying that the ambiguity averse household are better off when the spread between expected utilities of the two scenarios is smaller. The combination of expected utilities, ambiguity aversion, and Bayesian beliefs plays an important role in the household's decision-making process. The household's objective function is the following:

$$\begin{aligned} \max_{C_t, L_t, I_t} V(C_t, L_t) &= \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu} \\ &+ \beta\phi^{-1}\left[\Big(\mu_t\phi(E_t(V(C_{t+1}^R, L_{t+1}^R))) + (1-\mu_t)\phi(E_t(V(C_{t+1}^{NR}, L_{t+1}^{NR})))\Big)\right] \\ &\text{subject to } C_t + I_t = W_t L_t + R_t K_t + \Pi_t \end{aligned} \tag{3.6}$$

where $\phi(E_t(V_{t+1})) = \frac{[E_t(V_{t+1})]^{1-\gamma}}{1-\gamma}$ is the smooth ambiguity function, $\gamma \geq 0$ is ambiguity aversion, C_t is consumption, I_t is investment, L_t is labor, K_t is capital, R_t is the rental price of capital, W_t is the wage rate, Π_t is the firm's profit distributed to the household, β is the discount factor and μ_t is the Bayesian belief of the recession at t+1. $\sigma > 0$ is risk aversion, $\nu > 0$ is the disutility of labor. To reduce the number of abbreviations, L_t is defined for both labor supply and demand, and I_t is defined for both savings and investment since the market clearing conditions are satisfied in each period.

The Lagrangian equation is:

$$\max_{C_t,L_t,I_t} \Bigg\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu}$$
$$+ \beta\phi^{-1}\left[\Big(\mu_t\phi(E_t(V(C_{t+1}^R, L_{t+1}^R))) + (1-\mu_t)\phi(E_t(V(C_{t+1}^{NR}, L_{t+1}^{NR})))\Big)\right]$$
$$- \Lambda_t\left(C_t + I_t - W_tL_t - R_tK_t - \Pi_t\right)\Bigg\}$$

The first order optimality conditions for C_t and L_t are:

$$\Lambda_t = C_t^{-\sigma} \tag{3.7}$$
$$\Lambda_t = \frac{L_t^{\nu}}{W_t} \tag{3.8}$$

Conditions 3.7 and 3.8 imply that the substitution rate between consumption and labor is proportional to the wage rate. The intertemporal substitution between current and future consumption can be determined by the marginal utility of consumption and the marginal expected utility of investment. The choice of consumption naturally implies the choice of investment, as they are subject to the same budget constraint. Nonetheless, we look at the first order condition of investment I_t separately to depict the effect of uncertainty via the marginal expected utility of investment. This will facilitate clear communication when discussing the effect of uncertainty in the subsequent sections. The first order optimality condition for I_t is:

$$\Lambda_t = \beta\Upsilon_t\left(\mu_t\xi_t^R\frac{\partial E_t(V_{t+1}^R)}{\partial I_t} + (1-\mu_t)\xi_t^{NR}\frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t}\right) \tag{3.9}$$
$$\text{where } \Upsilon_t = \frac{\mu_tE_t(V_{t+1}^R)^{-\gamma} + (1-\mu_t)E_t(V_{t+1}^{NR})^{-\gamma}}{\left(\mu_tE_t(V_{t+1}^R)^{1-\gamma} + (1-\mu_t)E_t(V_{t+1}^{NR})^{1-\gamma}\right)^{\frac{-\gamma}{1-\gamma}}} \tag{3.10}$$
$$\xi_t^k = \frac{E_t(V_{t+1}^k)^{-\gamma}}{\mu_tE_t(V_{t+1}^R)^{-\gamma} + (1-\mu_t)E_t(V_{t+1}^{NR})^{-\gamma}} \tag{3.11}$$
$$k \in \{R, NR\}$$

Condition 3.9 is the smooth ambiguity Euler equation which contains the marginal expected utilities of investment $\frac{\partial E_t(V_{t+1}^k)}{\partial I_t}$ for the two scenarios, the Bayesian beliefs μ_t, the scaling factor Υ_t and belief distortions ξ_t^k. The weights attached to the recession scenario and the normal growth scenario are $\mu_t\xi_t^R$ and $(1-\mu_t)\xi_t^{NR}$ respectively. We call these weights subjective beliefs because they consist of a nonbehavioral part, Bayesian beliefs, and a behavioral part, belief distortions. We write Υ_t and ξ_t^k in the forms of Equations 3.10 and 3.11 because we formulate ξ_t^k as a Radon-Nikodym derivative that effectively distorts from the Bayesian belief to the subjective belief. Marinacci (2015) and Klibanoff et al. (2009) define the Radon-Nikodym derivative of the smooth ambiguity function as $\frac{\phi'(E_t(V_{t+1}))}{E_{\mu_t}(\phi'(E_t(V_{t+1})))}$, which we use for ξ_t^k. As ξ_t^k is defined, Υ_t naturally follows, and does not affect the subjective beliefs related to the two scenarios. According to Marinacci (2015), the maximization under the expected utility with smooth ambiguity preferences under Bayesian beliefs is equivalent to the expected utility without smooth ambiguity preferences under subjective beliefs.

If the household is ambiguity neutral $\gamma = 0$, ξ_t^k will equal one so the household's subjective belief is the Bayesian belief. If the household is ambiguity averse $\gamma > 0$, ξ_t^R is greater than ξ_t^{NR} because $E_t(V_{t+1}^R)$ is smaller than $E_t(V_{t+1}^{NR})$. Thus, the ambiguity averse household's subjective belief is biased toward the recession scenario compared to the Bayesian belief. Since the recession scenario has a lower expected utility, we refer to this weighting scheme as pessimistic belief distortions, following Altug et al. (2020), Collard et al. (2018) and Ju and Miao (2012).

According to Section 3.3, we assume that the ratio between expected utilities of normal and recession scenarios can be approximated by a macroeconomic uncertainty index, $M_t = \frac{E_t(V_{t+1}^{NR})}{E_t(V_{t+1}^{R})}$. We substitute $E_t(V_{t+1}^R) = \frac{E_t(V_{t+1}^{NR})}{M_t}$ into the Euler equation (Eq.

3.9) and solve the partial derivatives. We obtain:[8]

$$\Lambda_t = \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta))\Upsilon_t \left(\frac{\mu_t \xi_t^R}{M_t} + (1-\mu_t)\xi_t^{NR} \right) \tag{3.12}$$

$$\text{where } \Upsilon_t = \frac{\mu_t M_t^{\gamma} + (1-\mu_t)}{\left(\mu_t M_t^{\gamma-1} + (1-\mu_t)\right)^{\frac{-\gamma}{1-\gamma}}}$$

$$\xi_t^R = \frac{M_t^{\gamma}}{\mu_t M_t^{\gamma} + (1-\mu_t)}$$

$$\xi_t^{NR} = \frac{1}{\mu_t M_t^{\gamma} + (1-\mu_t)}$$

Λ_{t+1}^{NR} is the marginal utility of consumption in the normal scenario.

R_{t+1}^{NR} is the rental price of capital in the normal scenario.

Now, the belief distortions ξ_t^k and a scaling factor Υ_t become a function of macroeconomic uncertainty M_t, the Bayesian belief μ_t, and ambiguity aversion γ. The next section discusses how these variables affect belief distortions.

3.4.2 Belief distortions

This section discusses the dynamics of the belief distortions ξ_t^k. In order to illustrate this analytically, we divide our analyses into three cases. First we discuss the benchmark case when $M_t = 1$ or $\mu_t = 0$ and therefore uncertainty is not relevant. Then we compare it with the cases when uncertainty is relevant in the model $M_t > 1$ and $\mu_t > 0$ by analyzing the ambiguity neutral and ambiguity averse cases.

Benchmark. In this benchmark economy, the household behaves as if it will be in a normal growth period with certainty, so the Euler equation is reduced to one normal growth scenario as follows:

$$\Lambda_t = \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta)) \tag{3.13}$$

Ambiguity neutral. When there is uncertainty $M_t > 1$ and $\mu_t > 0$, the household will take the recession scenario into account. If the household is ambiguity neutral $\gamma = 0$, uncertainty will have an impact through the average expectation of the household (since

[8] We assume that the second-order effect of capital on uncertainty is very small and can be ignored. The derivation is in Appendix 3.B.

a recession is also taken into account) but there will be no belief distortion $\xi_t^k = 1$ and no scaling factor $\Upsilon_t = 1$. Thus, the household is purely Bayesian, and the ambiguity neutral Euler equation is:

$$\begin{aligned}\Lambda_t &= \beta\left(\mu_t \frac{\partial E_t(V_{t+1}^R)}{\partial I_t} + (1-\mu_t)\frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t}\right) \\ &= \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta))\left(\frac{\mu_t}{M_t} + (1-\mu_t)\right) \end{aligned} \tag{3.14}$$

The ambiguity neutral Euler equation is the linear combination of the marginal expected utilities of investment, weighted by Bayesian beliefs. Once we disentangle these expectations to compare them to the benchmark model, we obtain Equation 3.14 where $\frac{\mu_t}{M_t} + (1-\mu_t)$ is a ratio of the expected marginal utilities to the benchmark model. Since $M_t > 1$ and $\mu_t > 0$, the ratio is smaller than one. Thus the marginal expected utility of investment in the benchmark model is greater than that in the ambiguity neutral model. This implies that the household's expectation becomes lower when uncertainty exists although it is ambiguity neutral.

Ambiguity averse. If the household is ambiguity averse $\gamma > 0$, the belief distortions will be different from 1 and the scaling factor will be greater than 1. Thus, the ambiguity averse Euler equation is:

$$\begin{aligned}\Lambda_t &= \beta\Upsilon_t\left(\mu_t\xi_t^R \frac{\partial E_t(V_{t+1}^R)}{\partial I_t} + (1-\mu_t)\xi_t^{NR}\frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t}\right) \\ &= \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta))\Upsilon_t\left(\frac{\mu_t\xi_t^R}{M_t} + (1-\mu_t)\xi_t^{NR}\right) \end{aligned} \tag{3.15}$$

In the ambiguity averse Euler equation, the marginal expected utilities of investment are weighted by the Bayesian beliefs, the belief distortions and the scaling factor.
$\Upsilon_t\left(\frac{\mu_t\xi_t^R}{M_t} + (1-\mu_t)\xi_t^{NR}\right)$ indicates a ratio of the expected marginal utility of the ambiguity averse model to the benchmark model. Since $\xi_t^R > \xi_t^{NR}$, the ambiguity averse household is always biased toward the recession scenario compared to the Bayesian belief, regardless of the scaling factor. The marginal expected utility of investment in the ambiguity averse model is less than that in the ambiguity neutral model and the benchmark model. This means that the investment in future capital becomes less attractive for the ambiguity averse household than for the ambiguity neutral household.

Table 3.2: Dynamics of belief distortions and its effect on the expected utility

When the variable increases	ξ_t^R	ξ_t^{NR}	Υ_t	$\Upsilon_t\left(\frac{\mu_t\xi_t^R}{M_t}+(1-\mu_t)\xi_t^{NR}\right)$
Ambiguity aversion (γ)				
$\gamma = 0$ (ambiguity neutral)	1	1	1	$\frac{\mu_t}{M_t}+(1-\mu_t)$
$0<\gamma<1$	↑	↓	↑	↓
$\gamma>1$	↑	↓	↓	↓
$\gamma\to\infty$	$\frac{1}{\mu_t}$	0	1	$\frac{1}{M_t}$
Bayesian belief (μ_t)				
$\mu_t = 0$ (no uncertainty)	M_t^γ	1	1	1
$0<\mu_t<1$	↓fast	↓slow	↑then↓	↓
$\mu_t=1$	1	$\mathrm{M}_t^{-\gamma}$	1	$\frac{1}{M_t}$
Macroeconomic uncertainty (M_t)				
$M_t = 1$ (no uncertainty)	1	1	1	1
$M_t>1$	↑	↓	↑	↓
$M\to\infty$	$\frac{1}{\mu_t}$	0	a constant	0

Note: ↓: decrease, ↑: increase

As we can see, in addition to its direct effect on expected utilities, macroeconomic uncertainty indirectly impacts the decision-making process through the scaling factor and the belief distortions. Both factors then affect the ratio to the benchmark model. Table 3.2 summarizes how the belief distortions and the scaling factor respond to ambiguity aversion γ, Bayesian belief μ_t, and macroeconomic uncertainty M_t. The downward arrow (upward arrow) means decrease (increase) when these three variables increase. The last column shows the ratio to the benchmark model. As the ratio in this column decreases, the marginal expected utility of investment becomes smaller compared to the benchmark.

According to Table 3.2, we can draw three implications. First, ambiguity aversion γ increases pessimistic belief distortions. When ambiguity aversion increases, the belief distortions are more biased toward the recession scenario as ξ_t^R increases while ξ_t^{NR} decreases. Υ_t increases until $\gamma = 1$ and then decreases. As a result, the total weight on the recession scenario increases more than the total weight on the normal growth scenario. Therefore, the marginal expected utility of investment decreases. If the household is extremely ambiguity averse ($\gamma\to\infty$), the belief distortion toward the recession will be $\frac{1}{\mu_t}$ such that the total weight of recession is one and the total weight of normal growth is zero. Then the household will become a Maxmin optimizer and acts as if it will be in a recession. This implication is in line with Altug et al. (2020), Marinacci (2015), and Ju and Miao (2012).

Second, the Bayesian beliefs μ_t have a hedging effect against the belief distortions. When μ_t increases, ξ_t^R decreases faster than ξ_t^{NR} does, implying that the belief distortion toward recession is smaller when the Bayesian belief of recession is larger. This can be interpreted as the ambiguity averse household avoiding the extreme expectation to minimize the loss when the situation turns out unexpected. Baliga et al. (2013) show that the hedging effect can cause the polarization of beliefs when there is ambiguous information and heterogeneous agents. When the information is ambiguous, the ambiguity averse agents prefer not to extremely deviate from their Bayesian priors to hedge against the forecast error loss. If the agents hold heterogeneous prior beliefs their posterior beliefs will polarize toward their prior beliefs.

Lastly, macroeconomic uncertainty M_t increases pessimistic belief distortions. When macroeconomic uncertainty increases, the belief distortions are more biased toward the recession scenario as ξ_t^R increases while ξ_t^{NR} decreases. Moreover, when macroeconomic uncertainty increases the scaling factor rises, which further amplifies the deviating effects of ξ_t^R and ξ_t^{NR}. As a result, the ambiguity averse household puts more weight on the recession scenario so the ratio decreases. This negative effect of macroeconomic uncertainty on average expected utilities is in line with the findings in Chapter 2 that macroeconomic uncertainty reduces output growth expectations.

To summarize, ambiguity aversion γ and macroeconomic uncertainty M_t increase the pessimistic belief distortions while Bayesian beliefs of recession μ_t have a hedging effect against the belief distortions. The pessimistic belief distortions lead to a lower average marginal expected utility of investment, which makes investment into future capital less attractive.

3.4.3 Firm and market clearing conditions

In this economy, we define the firm as simple as possible. There is one representative firm that produces one good. Our firm is not directly affected by uncertainty since its decisions are based on current information. The firm maximizes profits as follows:

$$\max_{K_t, L_t} \Pi_t = Y_t - W_t L_t - R_t K_t \tag{3.16}$$

$$\text{where } Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$$

$$K_t = (1-\delta)K_{t-1} + I_t$$

$$Z_t = \exp(a_t)$$

$$a_t = (1-\rho)\bar{a} + \rho a_{t-1} + \sigma_a \epsilon_t^a; \epsilon_t^a \sim \mathcal{N}(0,1)$$

Π_t is profit, Y_t is output, K_t is capital, L_t is labor, I_t is investment, W_t is the wage rate, and R_t is the rental price of capital. α is the capital share in production and δ is the depreciation rate of capital. Finally, Z_t is the total productivity factor (TFP) which is developing as an AR(1) process, around mean $\bar{a}$. The first order optimality conditions of the firm are:

$$W_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha} \tag{3.17}$$

$$R_t = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{3.18}$$

Until now, two market clearing conditions have been imposed: labor supply being equal to demand and savings being equal to investment. Thus, only good market clearing conditions have to be specified:

$$Y_t = C_t + I_t \tag{3.19}$$

3.5 Replication of the three stylized facts

This section uses the subjective belief of recession derived in Section 3.4 to show how our model can possibly replicate the three stylized facts listed in the introduction. Note that the subjective belief of the normal growth scenario is one minus the subjective belief of recession. Hence, the discussion of the recession scenario also covers the normal growth scenario. The following subsection briefly presents the solution approach of the model and how we do simulations. Next, we demonstrate how our model can replicate the three stylized facts using mathematical analyses and model simulations.

3.5.1 Solution approach and simulation setup

We use the parameterized expectations algorithm (PEA) to solve the smooth ambiguity model. We cannot solve the model using a standard linearization method for two reasons. First, the effect of ambiguity aversion is captured by the concavity of the smooth ambiguity function in the Euler equation 3.15 so the linearization will make this effect disappear. Second, the household forms the expectations based on two scenarios so the number of variables at t is smaller than the number of variables at $t+1$. To be precise, the variables at t are $\{\Lambda_t, C_t, I_t, L_t, K_t, Y_t, W_t, R_t, Z_t, a_t, M_t, \mu_t\}$ while the variables at $t+1$ also include $\{\Lambda^{NR}_{t+1}, R^{NR}_{t+1}\}$. PEA is a projection method to approximate the household's conditional expectations in the Euler equation by a parametric function. Using PEA, we first solve the household's conditional expectations in the benchmark model (no uncertainty). The expectation's rule in the benchmark model is similar to that of the normal growth scenario because in both cases macroeconomic uncertainty is irrelevant for the household. Given the expectation's rule in the benchmark model, we know how the household forms their expectations of Λ^{NR}_{t+1} and R^{NR}_{t+1}. Therefore, we can solve for the household's conditional expectations in the smooth ambiguity model which contains two scenarios. The details about the solution method can be found in Appendix 3.C.

Next, we discuss the simulation set up. For the purpose of the simulation, we assume macroeconomic uncertainty to follow a stationary AR(1) process:

$$M_{t+1} = c + \rho^M M_t + \sigma_M \epsilon^M_{t+1}$$

$$\text{where c is constant and } \epsilon^M_t \sim \mathcal{N}(0,1)$$

We fit the parameters of this stochastic process of macroeconomic uncertainty to the US Economic Policy Uncertainty index[9]. For the posterior updating of the recession proba-

[9] We log transform the EPU index and divide the index by its minimum value so that the index starts from one. Then we fit the transformed index with an AR(1) model. We use the fitted standard deviation divided by four to simulate macroeconomic uncertainty, so that the volatility of macroeconomic uncertainty does not dominate the volatility of technological progress. Finally, we rebase the simulated macroeconomic uncertainty so that it is greater than or equal to one.

bility μ_t, we use Bayes' rule:

$$\mu_{t+1} = \frac{\mu^{prior}_{t+1|t} L(g^R)}{\mu^{prior}_{t+1|t} L(\bar{g}^R) + (1 - \mu^{prior}_{t+1|t}) L(\bar{g}^{NR})} \tag{3.20}$$

$$L(\bar{g}^k) = \frac{\exp(-0.5(E_t(g_{t+1}) - \bar{g}^k)^2/(\kappa\sigma_g)^2)}{\sqrt{2\pi}\kappa\sigma_g} \tag{3.21}$$

$$E_t(g_{t+1}) = \rho_{sm} \log\left(\frac{y_t}{y_{t-1}}\right) + (1 - \rho_{sm}) E_{t-1}(g_t) \tag{3.22}$$

$$\mu^{prior}_{t+1|t} = \rho_{sm}\mu_t + (1 - \rho_{sm})\mu^{prior} \tag{3.23}$$

where $k \in \{R, NR\}$, y_t is output at t with a zero growth steady state, $\mu^{prior}_{t+1|t}$ is a prior of the recession probability at $t+1$ given information at t, μ^{prior} is a constant prior of the recession probability, κ is a multiplier of the standard deviation, ρ_{sm} is a smoothing parameter, and $L(.)$ is a standard likelihood.

The means ($\bar{g}^{NR}, \bar{g}^R$) and the standard deviations (σ_g) of output growth are obtained during the simulations. $\bar{g}^{NR}$ is calculated when the simulated growth rates are greater than zero, and vice versa. For example, if the ambiguity aversion is set to zero, $\bar{g}^{NR}$, $\bar{g}^R$ and σ_g are 0.68%, -0.69% and 0.89% respectively. If the ambiguity aversion is set to 10, $\bar{g}^{NR}$, $\bar{g}^R$ and σ_g are 0.68%, -0.72% and 0.93% respectively.

When solving a model with a high ambiguity aversion, we find that PEA sometimes does not converge. To ensure the convergence of PEA, we make the Bayesian updating process sufficiently smooth. In order to do this, we increase the standard deviation with a multiplier κ. The increased standard deviation during the learning process is not new in the literature. Kurz et al. (2013) show that the Bayesian learning process can exhibit a higher variance than the empirical distribution due to the learning feedback from the data [10]. Moreover, we include a smooth parameter ρ_{sm} to make the expectation lag dependent and to prevent the prior from becoming one or zero. This is necessary because the Bayesian learning process stops updating when the prior is either one or zero. We arbitrarily choose the Bayesian prior μ^{prior} to be 0.1[11] and run the simulations with $\kappa = \{1, 1.5, 2, 2.5, 3, 3.5, 4\}$ and $\rho_{sm} = \{0.2, 0.4, 0.6, 0.8\}$. We present the results from $\rho_{sm} = 0.4$ because it is the most robust value in the sense that the PEA converges for all

[10] Kurz et al. (2013) state that the increased variance is unrestricted in most literature.
[11] We tried running simulations other values of μ^{prior} and the qualitative result does not change.

$\kappa > 1$ and any values of γ from 0 to 20. We set $\kappa = 1.5$ because it is the smallest value that makes PEA converge until $\gamma = 20$. The sensitivity analysis of the PEA convergence is presented in Appendix 3.D.1. In Section 3.5, we run the simulations of T = 2500 periods with $T_{\text{begin}} = 500$ to ensure that the initial guesses do not affect the solution. Finally, the structural and technological progress parameters are standard. Table 3.3 summarizes our parameters.

Table 3.3: Parameters for simulations

Parameter	Description	Value
Structural parameters		
β	discount factor	0.99
σ	risk aversion	2
ν	curvature in labor disutility	1
δ	capital depreciation rate	0.025
α	capital share	0.3
Total factor productivity		
$\bar{a}$	long-run TFP growth rate	0.02
ρ	persistence of technology growth	0.9
σ_a	volatility of technology	0.032
Macroeconomic uncertainty		
c	constant	0.34
ρ^M	persistence of macro uncertainty	0.71
σ_M	volatility of macro uncertainty	0.02
Bayesian updating		
κ	multiplier of S.D.	1.5
ρ_μ	smoothing parameter	0.4
μ^{prior}	prior	0.1

3.5.2 First stylized fact

Macroeconomic uncertainty makes people more pessimistic (see our Chapter 2, Bhandari et al. (2023) or Bianchi et al. (2022)). We relate pessimism with the household's subjective belief. An increase in the subjective belief of recession means that the household believes the economy will be more likely to be in recession. In the smooth ambiguity model, we assume that the occurrence of the next-period recession follows a Bernoulli distribution where the outcome is either one or zero. We define the household's subjective belief as the first moment of the Bernoulli distribution or the probability of the recession according to the household as follows:

$$\text{Subjective belief}_t = \mu_t \xi_t^R \tag{3.24}$$
$$\text{where } \xi_t^R = \frac{M_t^\gamma}{\mu_t M_t^\gamma + (1-\mu_t)}$$

We take the first derivative of the subjective belief with respect to macroeconomic uncertainty to mathematically show the effect of macroeconomic uncertainty:

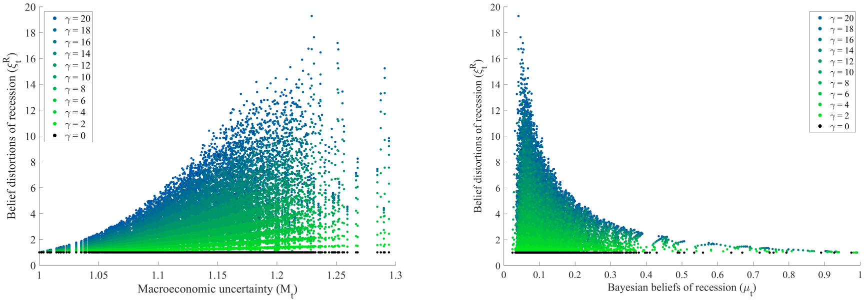
$$\frac{\partial \mu_t \xi_t^R}{\partial M_t} = (1-\mu_t \xi_t^R)\mu_t \xi_t^R \frac{\gamma}{M_t} \tag{3.25}$$

Since subjective belief $\mu_t \xi_t^R$ is always between zero and one, $\frac{\partial \mu_t \xi_t^R}{\partial M_t}$ is always greater than or equal to zero. Therefore, macroeconomic uncertainty positively impacts the subjective belief of recession. This is in line with what discussed in Section 3.4.2. When macroeconomic uncertainty increases, the belief distortion toward the recession ξ_t^R increases, so the subjective belief of recession $\mu_t \xi_t^R$ increases.

We use the simulations described in Section 3.5.1 to demonstrate that macroeconomic uncertainty increases pessimistic belief distortions and it positively correlates with the subjective belief of recession. All simulations have the same set of structural parameters, the technological process, and macroeconomic uncertainty but they have different levels of ambiguity aversion γ. In each simulation, we simulate 2500 observations but only use the last 2000 observations to exclude the effect of the initial guess. Figure 3.4 shows the simulated belief distortions of recession ξ_t^R against macroeconomic uncertainty M_t and Bayesian beliefs of recession μ_t. Black dots represent belief distortions in the ambiguity neutral model $\gamma = 0$. Green dots represent belief distortions in the low ambiguity averse models and blue dots represent belief distortions in the high ambiguity averse models.

As seen in Figure 3.4a, the belief distortions of the ambiguity neutral household is always one, implying that the household does not have belief distortions and is purely Bayesian. For the ambiguity averse households, the belief distortions of recession increase with macroeconomic uncertainty and these effects become stronger as the household is more ambiguity averse. However, the high belief distortions are mostly bounded by the low Bayesian beliefs of recession (Figure 3.4b). This implies that Bayesian beliefs have

Figure 3.4: Belief distortions of recession ξ_t^R



(a) Macroeconomic uncertainty M_t

(b) Bayesian beliefs of recession μ_t

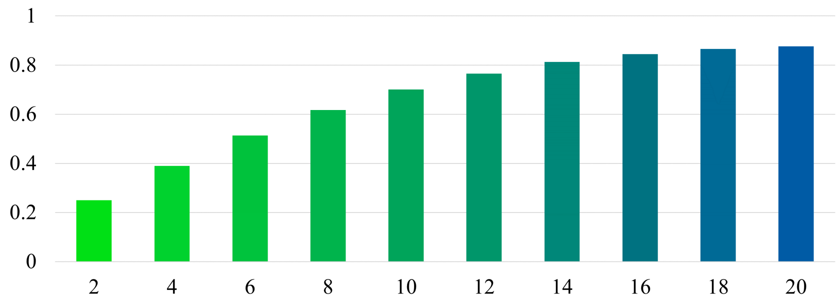
hedging effects against the belief distortions of recession as discussed in Section 3.4.2. In other words, the ambiguity averse household refrains from making extreme expectations that are further away from the Bayesian beliefs. Note that macroeconomic uncertainty is exogenous and Bayesian beliefs are predetermined, so these two variables' concurrent effects on belief distortions are independent.

To measure the effect of macroeconomic uncertainty on subjective beliefs, we transform M_t and $\mu_t \xi_t^R$ into log scale and calculate correlations between the two variables. Figure 3.5 reports the correlations of the models with different levels of ambiguity aversion (x-axis). Each correlation covers 2000 simulated data points and has a p-value less than 1%. The effects of macroeconomic uncertainty on the subjective belief of recession are positive and they become larger as ambiguity aversion increases. For example, if the level of ambiguity aversion is 2, the correlation between macroeconomic uncertainty and subjective belief is 0.25. If the level of ambiguity aversion is 20, the correlation could increase to 0.88. Therefore, macroeconomic uncertainty makes ambiguity averse households more pessimistic as they believe that the next-period recession probability is higher when macroeconomic uncertainty rises.

3.5.3 Second stylized fact

Macroeconomic uncertainty can have both positive and negative effects on subjective uncertainty (see our Chapter 2 or Glas (2020)). Intuitively, this fact implies that the households can be more or less uncertain about their subjective beliefs when facing a higher

Figure 3.5: Correlations between macroeconomic uncertainty and subjective beliefs of recession



Note: We calculate the correlations between macroeconomic uncertainty and subjective beliefs of recession on a log scale. The black means p-value is less than 1%. The x-axis is the parameter of ambiguity aversion γ.

macroeconomic uncertainty. The household's subjective uncertainty is defined as the second moment of the subjective beliefs, indicating how confident the household is about its first-moment belief $\mu_t \xi_t^R$ (Altig et al., 2020; Fermand et al., 2023). Assuming that the subjective belief follows Bernoulli distribution, the second moment of the subjective belief is as follows:

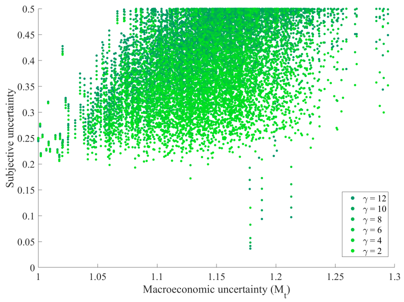
$$\text{Subjective uncertainty}_t = \sqrt{\mu_t \xi_t^R \times (1 - \mu_t \xi_t^R)} \qquad (3.26)$$
$$\text{where } \xi_t^R = \frac{M_t^\gamma}{\mu_t M_t^\gamma + (1 - \mu_t)}$$

The derivative of subjective uncertainty with respect to macroeconomic uncertainty is as follows:

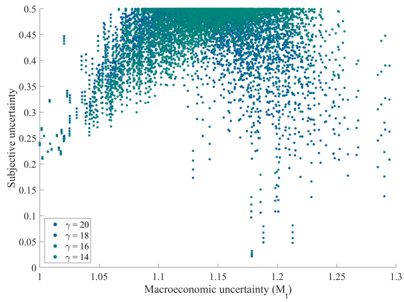
$$\frac{\partial \sqrt{\mu_t \xi_t^R (1 - \mu_t \xi_t^R)}}{\partial M_t} = \sqrt{\mu_t \xi_t^R (1 - \mu_t \xi_t^R)} \frac{1 - 2\mu_t \xi_t^R}{2} \frac{\gamma}{M_t} \qquad (3.27)$$

The sign of this derivative depends on the sign of $\frac{1-2\mu_t \xi_t^R}{2}$, since the other terms are positive. If $\mu_t \xi_t^R$ is less than 0.5, the derivative is greater than zero or vice versa. This implies that the effect of macroeconomic uncertainty on subjective uncertainty is positive when subjective belief is between 0 and 0.5. When subjective belief is between 0.5 and 1, the effect of macroeconomic uncertainty becomes negative. On top of that, macroeconomic uncertainty increases the subjective belief of recession and ambiguity aversion strengthens this effect as discussed in Section 3.5.2. Therefore, the relationship between macroeconomic uncertainty and subjective uncertainty depends on the level of ambiguity aversion.

Figure 3.6: Macroeconomic uncertainty and subjective uncertainty



(a) Low ambiguity aversion

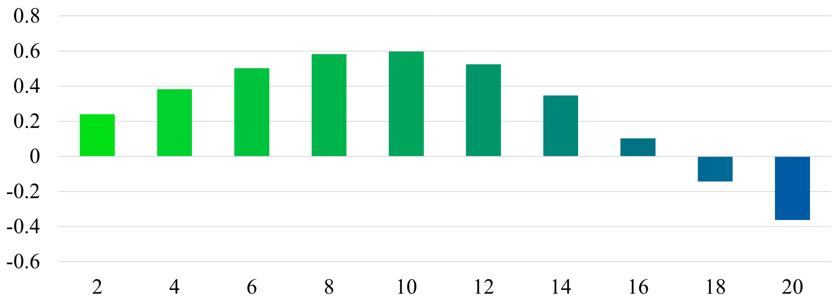


(b) High ambiguity aversion

Using the same simulations in Section 3.5.2, we show the scatter plots between macroeconomic uncertainty and subjective uncertainty in Figure 3.6. Subjective uncertainty in the low ambiguity aversion models generally increases with macroeconomic uncertainty (Figure 3.6a). However, in the high ambiguity aversion models, subjective uncertainty decreases when macroeconomic uncertainty is sufficiently high (Figure 3.6b). We do not show the ambiguity neutral model because its subjective belief does not depend on the current level of macroeconomic uncertainty as discussed in Section 3.5.2. Thus the ambiguity neutral subjective uncertainty is not related to macroeconomic uncertainty.

To show the average effect of macroeconomic uncertainty on subjective uncertainty, we report the correlations between macroeconomic uncertainty and subjective uncertainty in Figure 3.7. The x-axis indicates the parameters of ambiguity aversion and the correlations are calculated using a log scale of data points in Figure 3.6. We can see that subjective uncertainty has mixed responses to macroeconomic uncertainty depending on the levels of ambiguity aversion. When ambiguity aversion is low, macroeconomic uncertainty increases subjective uncertainty but, when ambiguity aversion is sufficiently high, it decreases subjective uncertainty. A decreased subjective uncertainty means that the household has a stronger belief toward one scenario. This switch in the correlations is due to the large pessimistic belief distortions in the high ambiguity averse economy. When macroeconomic uncertainty increases, the pessimistic belief distortions spike such that the subjective belief of recession becomes greater than 50% and subjective uncertainty decreases. Therefore, facing the same macroeconomic uncertainty shock, the low ambiguity averse households become more uncertain, while the high ambiguity averse

Figure 3.7: Correlations between macroeconomic uncertainty and subjective uncertainty



Note: We calculate the correlations between macroeconomic uncertainty and subjective uncertainty on a log scale. The black bar means p-value is less than 1%. The x-axis is the parameter of ambiguity aversion (γ).

households become less uncertain about their subjective beliefs.

3.5.4 Third stylized fact

The empirical evidence from the previous literature suggests that the effect of macroeconomic uncertainty on the economy is nonlinear and is stronger when macroeconomic uncertainty is higher (Jackson et al., 2020; Lhuissier & Tripier, 2021; Ng & Wright, 2013). To analyze this issue with our model, recall that the smooth ambiguity Euler equation is:

$$\Lambda_t = \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta))\Upsilon_t \left(\frac{\mu_t \xi_t^R}{M_t} + (1 - \mu_t)\xi_t^{NR} \right) \tag{3.28}$$

where $\Upsilon_t \left(\frac{\mu_t \xi_t^R}{M_t} + (1 - \mu_t)\xi_t^{NR} \right)$ is the ratio of the marginal expected utility of investment to the benchmark model (henceforth, we call it the ratio).

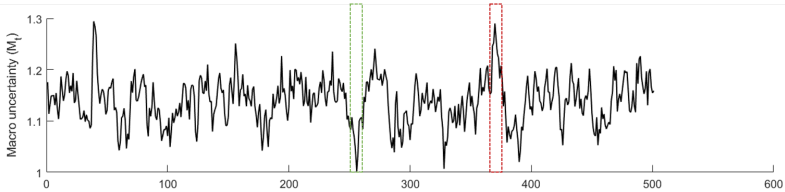
Equation 3.28 shows that macroeconomic uncertainty enters the model non-linearly. It affects the household's average expected utility directly through an increase in the spread of expected utilities between the two scenarios and indirectly through subjective beliefs. When macroeconomic uncertainty increases, the recession scenario's expected utility is relatively lower than the normal growth scenario. Moreover, the subjective belief of recession rises due to the increased macroeconomic uncertainty. As a result, the expected utility of the recession scenario, while decreasing, becomes more relevant to the household's average expected utility. This mechanism creates a nonlinear effect of macroeconomic uncertainty on the economy. However, when the Bayesian belief of recession is close to zero, the ratio to the benchmark model converges to one regardless of the levels of macroeconomic uncertainty and ambiguity aversion (see Table 3.2). This implies that the household's pessimism is tightly bounded when its Bayesian belief describes the recession as very unlikely. Therefore, the nonlinear effect of macroeconomic uncertainty on the economy is bounded by Bayesian beliefs.

Figure 3.8 illustrates the time series of the ratios to the benchmark model, Bayesian beliefs and macroeconomic uncertainty. For visualization, we pick a time frame with 500 observations to show in these figures. By the simulation setup, macroeconomic uncertainty is the same for all smooth ambiguity models (Figure 3.8a). Bayesian beliefs of recession are not clearly distinguishable across the levels of ambiguity aversions except for some periods (Figure 3.8b) but the ratios to the benchmark model differ across models (Figure 3.8c). The ratio of the ambiguity neutral model (black line) stays above 0.9 for the whole period. The downward deviation of the ambiguity neutral model comes from the Bayesian beliefs and the increased spread of the expected utilities due to macroeconomic uncertainty. The ratios of the ambiguity averse models are always smaller than that of the ambiguity neutral model because pessimistic belief distortions amplify the downward deviations.

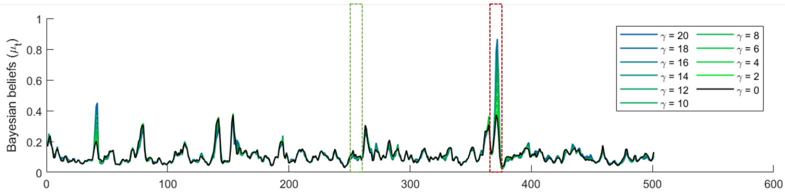
The green and red boxes in Figure 3.8 indicate the periods when macroeconomic uncertainty is at its lowest (one) and its highest respectively (1.3). When macroeconomic uncertainty is one, the ratios of all models equal one, implying that the smooth ambiguity model converges to the benchmark model. When macroeconomic uncertainty is substantially high, the Bayesian beliefs of recession spike but the magnitudes differ across the levels of ambiguity aversion. For example, the Bayesian belief of the ambiguity neutral model is 37.5% while that of the ambiguity averse $\gamma = 20$ model increases to 86.7%. As a result, the ratios of the ambiguity averse model can go down to approximately 0.8, implying 20% downward deviation from the benchmark model. This highlights that Bayesian beliefs and ambiguity aversion strengthen the nonlinear effect of macroeconomic uncertainty during extreme situations like economic crises.

Figure 3.8: Ratios of the smooth ambiguity models to the benchmark model

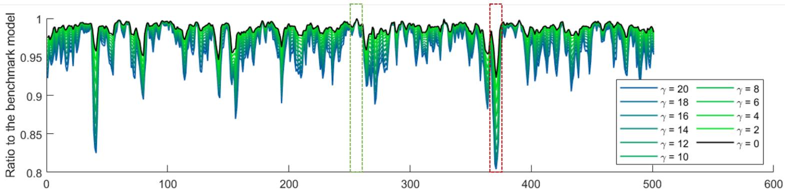
(a) Macroeconomic uncertainty (M_t)



(b) Bayesian beliefs of recession (μ_t)



(c) Ratios to the benchmark model



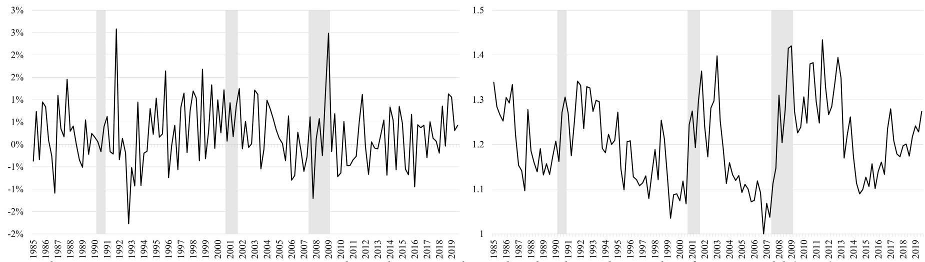
3.6 Simulation with the US data

At the introduction of this chapter, we argue that macroeconomic models struggle to capture the severity of economic crises, possibly because their microfoundations do not adequately reflect the relationship between uncertainty and people's beliefs as observed in the data. Section 3.5 demonstrates that our smooth ambiguity model can replicate these relationships and that ambiguity aversion is an important factor determining them. In this section, we investigate whether our model captures the negative output growth during recessions, and to what extent ambiguity aversion may be influencing its performance.

To evaluate the model's performance when fitting output growth data, we use the simulation of US output growth across the benchmark, ambiguity neutral and ambiguity averse models. The utilization-adjusted TFP by Fernald (2014) is used as a measure for a_t (Figure 3.9a). It is a quarterly TFP series for the US business sector, adjusted for labor

Figure 3.9: US data from Q1 1985 to Q4 2019

(a) Utilization-adjusted technological growth (a_t) **(b)** Macroeconomic uncertainty (M_t)



Note: The grey areas are economic recessions. The utilization-adjusted technological growth is from Fernald (2014). Macroeconomic uncertainty is the Economic Policy Uncertainty index (Baker et al., 2016) in log scale and we rebase the index to start from one.

and capital utilization. This series fits well with our model because it contains only technological change, that is barely affected by the business cycle. Secondly, we use the US Economics Policy Uncertainty index by Baker et al. (2016) as M_t. The index is divided by its minimum value, so it is always bigger than or equal to one (Figure 3.9b). The highest macroeconomic uncertainty was in Q3 2011 when the US reached its fiscal cliff, and the lowest one was in Q4 2006 before the Global Financial crisis. The US simulations have 141 periods (1985Q1-2019Q4) and we use samples in 1985 and 1986 as burn-in to avoid the effect of an initial guess. All structural parameters are the same as those in Table 3.3.

We show the model performance both in and out of sample. We solve the model with the PEA method using the in-sample period from Q1 1985 to Q4 2006 (more details in Appendix 3.C). Then, we use the result from the in-sample simulation to simulate output growth for the whole sample from Q1 1985 to Q4 2019. The in-sample period (Q1 1985 - Q4 2006) covers two economic crises in 1990 and 2001 and the out-of-sample period (Q1 2007 - Q4 2019) includes the great recession. Moreover, we compare the simulations using model-based Bayesian beliefs and the recession probability from the US professional forecasters assuming that they are Bayesian on average. This comparison serves as an exercise for model's applicability. As mentioned in Section 3.5.1, the PEA convergence highly depends on the Bayesian updating process. If we can circumvent this technical limitation by using an external source such as survey data to proxy μ_t, the model's practicality can be significantly enhanced.

As a word of caution, our model does not contain any friction which normally is an important factor to explain the fluctuations in the business cycle (Smets & Wouters,

Table 3.4: Simulation performance of the smooth ambiguity model with model-implied Bayesian beliefs

	Ambiguity aversion (γ)						
	BM	0	4	8	12	16	20
Correlations with actual growth	0.10	0.42	0.52	0.53	0.57	0.59	0.59
Root mean square errors							
Overall	0.59	0.54	0.73	1.49	1.52	1.39	1.29
Normal periods	0.42	0.57	0.72	1.37	1.36	1.25	1.18
Recessions	1.37	1.03	0.78	2.10	2.27	2.07	1.84

Note: The table presents the performance of out-of-sample simulations. The first row shows the correlations between simulated growth and actual growth of the US quarterly GDP, and the other rows show the root mean square errors. All values are in percentage points. The simulated growth rates are 4-quarter moving averages of the original simulations. BM means benchmark model.

2007). Therefore, we use the 4-quarter moving average of the original simulated series as mechanical friction. All results and graphs presented in the following sections are obtained from the moving average series unless stated otherwise.

3.6.1 Simulation using the model-based Bayesian beliefs

This section discusses the performance of the US GDP growth simulations and compares the model-based Bayesian belief to the SPF's recession probability. For the Bayesian updating, we use the formula in Section 3.5.1 with the following parameters: $\kappa = 2, \rho_\mu = 0.5, \mu^{prior} = 16\%, \bar{g}^R = -0.45\%, \bar{g}^{NR} = 0.36\%$, and $\sigma_g^k = 0.56\%$ where $k = \{R, NR\}$[12]. μ^{prior} is the average recession probability by the US professional forecasters. $\bar{g}^R$, $\bar{g}^{NR}$, and σ_g^k are obtained from the demeaned US quarterly GDP growth during 1985Q1 and 2019Q4. We subtract the mean of actual GDP growth rates because the model assumes zero growth. The average quarterly GDP growth rate from Q1 1985 to Q4 2019 is 0.42%.

We divide our result discussion into three main parts: the correlations with actual data, the root mean square errors (RMSE) during the recession and normal growth periods[13], and the model-based Bayesian beliefs. Table 3.4 reports the correlations with actual growth data and RMSE of the difference between simulations and actual data. Figure 3.10 presents the 4-quarter moving average of the simulated GDP growth rates and the demeaned actual GDP growth rates. Finally, Figure 3.11 depicts the model-based Bayesian beliefs of recessions and the professional forecasters' recession probability.

[12] The sensitivity analysis is in Appendix 3.D.2

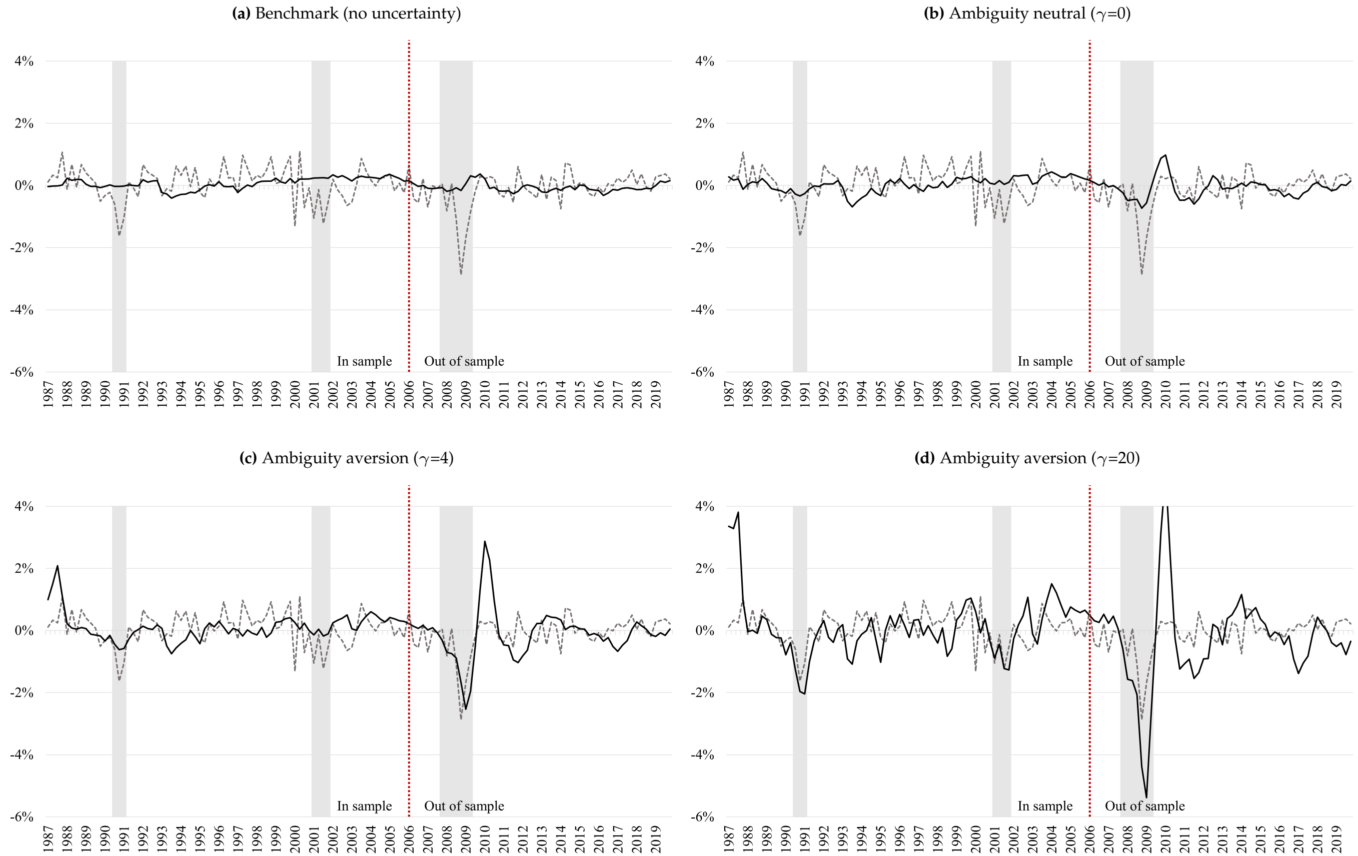
[13] The correlation measures how closely the data points align with a linear relationship between model-implied and actual output growth rates. Root mean square error assesses the average size of the differences between model-implied and actual output growth rates. They capture different aspects of the data fitness.

Correlations. According to Table 3.4, the benchmark simulation (without uncertainty, as defined in Section 3.4.2) barely matches with the actual GDP growth as their correlation is 0.10. In the smooth ambiguity models, the correlations improve to 0.42 - 0.59. This indicates that the technological process, which is the only shock in the benchmark model, contributes very little to cyclical fluctuations. The improved correlation in the smooth ambiguity model is due to macroeconomic uncertainty, ambiguity aversion and Bayesian beliefs.

Root mean square errors. According to Table 3.4, the overall root mean square errors of smooth ambiguity models are ranging from 0.54% - 1.52%. In normal growth periods, the higher parameters of ambiguity aversion generate higher volatility, so the model performance deteriorates. During recessions, the ambiguity averse model with $\gamma = 4$ performs best since it has the lowest RMSE of recessions at 0.78%. Looking at the out-of-sample simulations in Figure 3.10, the smooth ambiguity model with $\gamma = 4$ is able to mimic the depth of the recession from the 2008 Global Financial crisis significantly better than the benchmark model and other smooth ambiguity models. The ambiguity neutral model generates an insufficient depth while the ambiguity aversion $\gamma = 20$ model generates too much depth. These results show a trade-off of RMSEs between the normal growth periods and recessions and suggest that the ambiguity aversion between 0 and 4 could be an optimal value.

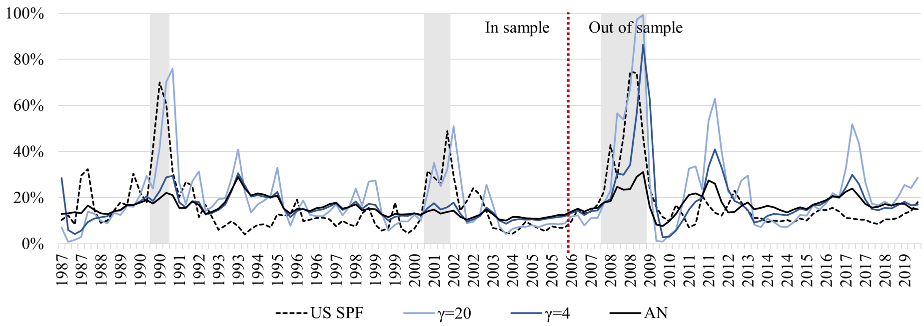
Model-based Bayesian belief. To assess whether the model-based Bayesian belief of recession is sensible, we compare it with the average next-quarter recession probability of professional forecasters as measured in the survey of US professional forecasters in Figure 3.11. During the recession periods, the model-based Bayesian beliefs (solid lines) increase similarly to the SPF's recession probability, but the magnitudes differ across the levels of ambiguity aversion. For example, during the Global Financial crisis (Q2 2009), the SPF's recession probability went up to 74%. Bayesian belief in the ambiguity neutral model is 31.06% while those in the ambiguity averse $\gamma = 4$ and 20 models are 86.40% and 99.31%, respectively. During normal growth periods, the SPF's recession probability mostly stays below 20%, while the model-based Bayesian beliefs spike several times, especially in the ambiguity averse $\gamma = 20$ model. This also corresponds with the volatile output growth in those models.

Figure 3.10: US quarterly output growth simulations



Note: The black solid line is the output growth simulated by the smooth ambiguity model. The simulations are 4-quarter moving averages. The in-sample period is from Q1 1985 to Q4 2006, and the out-of-sample period is from Q1 2007 to Q4 2019. The dashed grey line is the demeaned actual output growth data. The grey areas are economic recessions.

Figure 3.11: Model-based Bayesian beliefs and the SPF's recession probability



Note: The solid lines are the model-based Bayesian beliefs of recession generated from the smooth ambiguity model with different levels of ambiguity aversion (γ). The dashed line is the US professional forecasters' recession probability or the anxious index. AN means the ambiguity neutral model ($\gamma = 0$). The left side of the dashed red line is the in-sample period from Q1 1985 to Q4 2006 and the right side of the dashed red line is the out-of-sample period from Q1 2007 to Q4 2019. The grey areas are economic recessions.

To summarize, we demonstrate that the smooth ambiguity model fits the data better than the benchmark model (no uncertainty) and generates sufficient negative growth during recessions. To be specific, the ambiguity averse model can capture the decline in GDP during the crisis better than the ambiguity neutral model because pessimistic belief distortions negatively impact output growth. When output growth decreases, the next-period Bayesian beliefs become higher because the household uses declined output growth to update the Bayesian posterior. The higher Bayesian belief further reduces output growth in the next period. Moreover, our model-based Bayesian beliefs increase during crises, similar to the SPF's recession probability. However, in normal growth periods, the model-based Bayesian beliefs are more volatile than the SPF's. In Appendix 3.D.2, we present the correlations between model-based and actual output growth rates across different smoothing parameters of Bayesian belief updating for robustness checks. The correlations range from 0.34 to 0.52 in the ambiguity neutral models, depending on the Bayesian updating parameters. The correlations in the ambiguity averse models are higher on average ranging between 0.42 and 0.58.

Table 3.5: Simulation performance of the smooth ambiguity model with SPF-implied Bayesian beliefs

			Ambiguity aversion (γ)					
	SPF	BM	0	4	8	12	16	20
Correlations with actual growth	0.72	0.10	0.63	0.64	0.65	0.65	0.65	0.64
Root mean square errors								
Overall	0.46	0.59	0.55	0.67	0.82	0.93	0.98	0.98
Normal periods	0.38	0.42	0.58	0.70	0.82	0.90	0.93	0.93
Recessions	0.83	1.37	0.35	0.49	0.84	1.11	1.23	1.24

Note: The table presents the performance of out-of-sample simulations with the SPF's recession probability. The first row shows the correlations between simulated growth and actual growth of US quarterly GDP, and the other rows show the root mean square errors. All value are in percentage points. The simulated growth rates are 4-quarter moving averages of the original simulations except for SPF. SPF means the forecasts of professional forecasters. BM means the benchmark model.

3.6.2 Simulation using the professional forecasters' recession probability

As shown in the previous section, the model-implied Bayesian belief of recession moves in line with the SPF's recession probability, particularly in the recession periods. In this section, we use the SPF's recession probability as a proxy for μ_t. The purpose of this simulation is to experiment with the case when the household forms the Bayesian belief the same as the recession probability implied by SPF. For this exercise, the benchmark model is not a suitable reference point to assess the performance of the smooth ambiguity performance because it is not designed to incorporate the SPF's recession probability. Thus, in addition to the benchmark model, we use the current-quarter GDP growth forecasts by the US professional forecasters[14] as a reference point for assessing the model's performance. The real GDP forecast of US professional forecasters has been available since 1992.

Our results are discussed in two main parts: the correlations with actual data and the root mean square errors during the recession and normal growth periods. Table 3.5 reports the correlations with the actual growth data and root mean square errors. Figure 3.12 presents the professional forecasters' output growth forecasts and the 4-quarter moving average of the simulated GDP growth rates compared to the demeaned actual GDP growth rates.

Correlations. The correlation of the ambiguity neutral model is 0.63 and the correlations of the ambiguity averse models range between 0.64 and 0.65. Compared to the

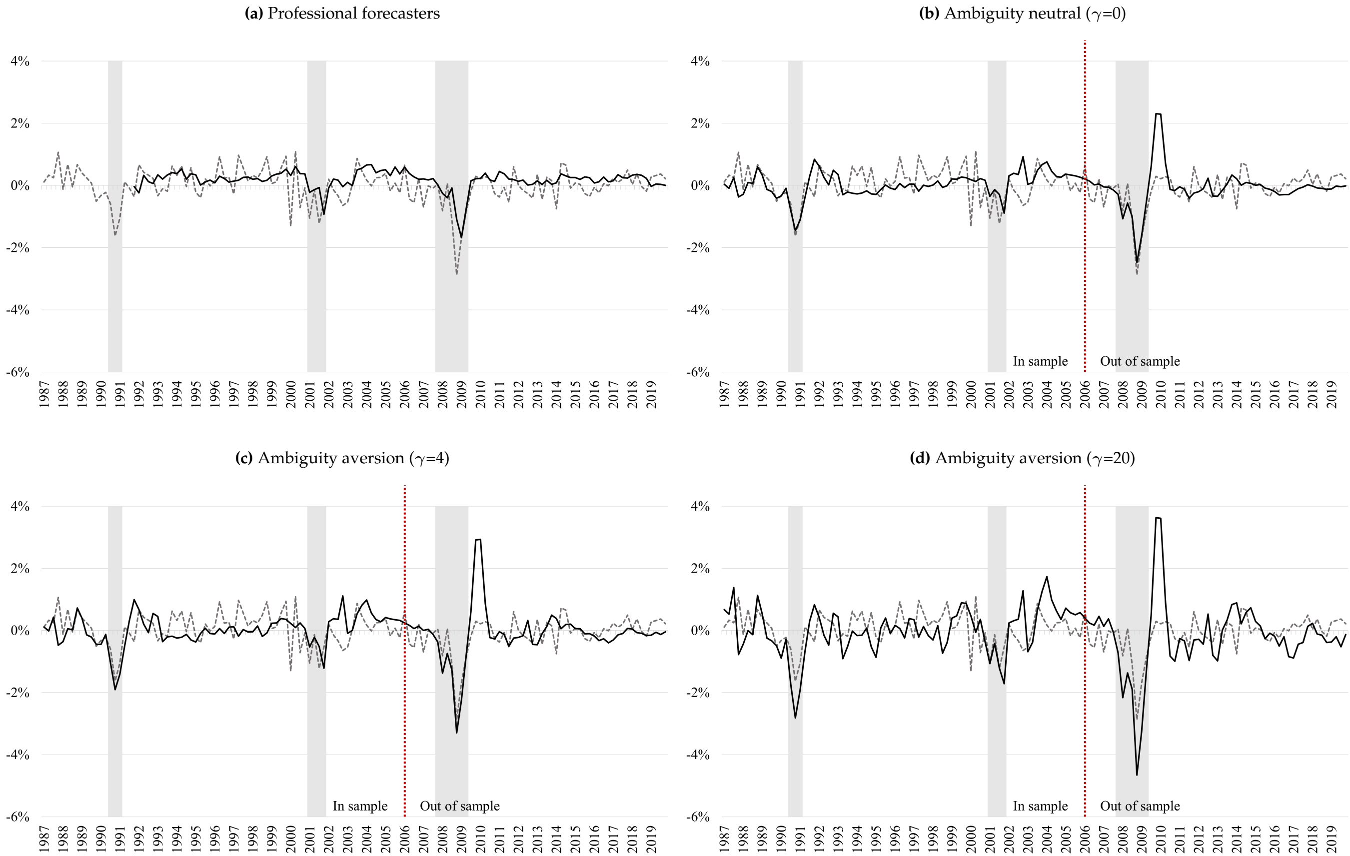
[14] To measure the current-quarter GDP growth forecasts from the SPF, we calculate the log difference between the average GDP level forecast for the current quarter and the actual GDP level of the previous quarter that was available to the forecasters when making the forecasts. This method is also used in the Federal Reserve of Philadelphia's report of SPF. We subtract the SFPs' forecast with the mean GDP growth (0.42%) to fit the forecast with the zero-growth model.

models using model-based Bayesian beliefs in Section 3.6.1, the correlations of the smooth ambiguity models have noticeably increased. However, they still perform worse than the SPF's forecast that correlates 0.72 with the actual GDP growth.

Root mean square errors. Looking at the out-of-sample RMSE, the SPF's forecasts do better than our models except for the recession periods. If the parameter of ambiguity aversion is less than 8, the recession's RMSEs for the ambiguity averse model are lower than those for the SPF's forecast. As seen in Figure 3.12b and 3.12c, the smooth ambiguity model is able to generate the depth of the 2008 recession better than the SPF's forecast. For example, at the deepest point of GFC (Q4 2008), the actual output growth was -2.88%. The SPF's forecast was -1.09%, the ambiguity neutral model gives -2.47%, and the ambiguity averse $\gamma = 4$ model predicts -3.30%. This result is in favor of the smooth ambiguity model. Most model-based forecasts failed to capture the Great Recession and its turning point because of the fixed parameters and a mean-reverting property. The output forecasts by professional forecasters, on average, tend to perform better than economic models because they adjust to the news faster than the models (Ng & Wright, 2013; Wieland & Wolters, 2011). Since our smooth ambiguity models have the SPF's recession probability, they can utilize the information capacity of professional forecasters.

To summarize, the smooth ambiguity models with the SPF-implied Bayesian belief perform better than the smooth ambiguity models with model-based Bayesian beliefs. Moreover, when the level of ambiguity aversion is less than eight, the smooth ambiguity model with the SPF's probability predicts the GFC more accurately than the SPF's forecasts. This shows that the smooth ambiguity model has the potential as a forecasting tool. However, it is important to bear in mind the possible bias in these findings. The SPF's recession probability might have taken over the role of ambiguity aversion in explaining economic fluctuation. Using the model-based Bayesian beliefs, the model's correlations with the GDP growth data can improve from 0.42 to 0.59, from the ambiguity neutral to the ambiguity averse models (see Table 3.4). On the contrary, with the SPF-implied Bayesian belief, the model's correlations improve at most from 0.63 to 0.65 (see Table 3.5). This implies that the ambiguity aversion of the models with the SPF's recession probability plays a minor role in replicating the US output growth.

Figure 3.12: US output growth forecasts by professional forecasters and the model simulations



Note: Panel (a) shows the professional forecasters' output growth forecasts in the solid black line, while in the other panels, the black solid line is the output growth simulated by the smooth ambiguity model. The simulations are 4-quarter moving averages. The in-sample period is from Q1 1985 to Q4 2006, and the out-of-sample period is from Q1 2007 to Q4 2019. The dashed grey line is the demeaned actual output growth data. The grey areas are economic recessions.

3.7 Conclusion

In this paper, we study the effects of uncertainty on the economy using a business cycle model with smooth ambiguity preferences, based on Altug et al. (2020). We contribute to the existing literature by modeling uncertainty in the form of ambiguity in a macroeconomic model. Specifically, we use the macroeconomic uncertainty index to anchor uncertainty in our model, which equals the ratio between the expected utilities of normal growth and recession scenarios. With this assumption, we study the transmission mechanism of macroeconomic uncertainty and its effects on the household's beliefs and the economy.

We find that the smooth ambiguity model can replicate three empirical stylized facts: the households' pessimistic beliefs, the mixed responses of subjective uncertainty, and the nonlinear effects of macroeconomic uncertainty. Ambiguity aversion plays an important role in shaping the effect of uncertainty on a representative household's beliefs. Using data from the US, we demonstrate in a simple simulation setting that the smooth ambiguity model with relevant ambiguity aversion can capture large declines in output during economic recessions better than the benchmark model and provide forecasts comparable to professional forecasters' predictions. The improved performance of smooth ambiguity models implies that it is necessary to differentiate between risk and ambiguity in macroeconomic models and suggests that ambiguity is likely to be more relevant to households, particularly in crises.

Finally, the simulations demonstrate that the macroeconomic uncertainty indices and recession probabilities computed from the survey of professional forecasters have the potential to forecast output growth. In the next chapter, we evaluate our model with real-world data by estimating it with data from the US and major European countries.

Appendix

3.A Summary of Altug et al. (2020)'s model

Altug et al. (2020) present a social planner maximization model in which the agent holds the ambiguity smoothing preferences of Klibanoff et al. (2005). The following sections explain their model.

3.A.1 Source of uncertainty and belief

The growth of total production factor (TPF) consists of 2 components, a long-run one and a temporary one. To the agent or social planner, the long-run component ($\bar{g}$) is known but the temporary component (x_t) is ambiguous.

The data generating processes of TFP growth, temporary component and TFP are defined as following:

$$g_{A,t+1} = \bar{g} + x_{t+1} + \sigma_A \epsilon_{A,t+1}$$

$$x_{t+1} = \rho x_t + \sigma_x \epsilon_{x,t+1}$$

$$A_{t+1} = A_t \exp(g_{A,t+1})$$

Social planner tries to forecast the temporary component. She or he knows that, at any time, the temporary component is either in a high persistent or a low persistent stage. Therefore, the agent has two forecasts which are:

- $\hat{x}_{k,t}$: the temporary TFP component for state k (high/low persistence), following the Kalman filter
- η: the belief of probability that the economy is in a low persistent stage following the Bayesian rule

3.A.2 Production technology and social planner's problem

The production function is:

$$y_t = k_t^a (A_t n_t)^{1-a}$$
$$k_{t+1} = (1-\delta)k_t + i_t$$

The ambiguity smoothing social planner has the following indirect value function:

$$\hat{J}(\hat{k}_t, \mu_t) = \max_{\hat{c}_t, n_t, \hat{i}_t} \left\{ \frac{(\hat{c}_t^{\nu} l_t^{1-\nu})^{1-\gamma}}{1-\gamma} + \beta \left[E_{\mu_t} \left(E_{x_t} \left[\hat{J}(\hat{k}_{t+1}, \mu_{t+1}) \exp(\gamma(1-\nu) g_{A,t+1}) \right] \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \right\}$$

subject to

$$\hat{c}_t + \hat{i}_t \leq \hat{k}_t^a n_t^{1-a}$$
$$\exp(g_{A,t+1})\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \hat{i}_t$$
$$l_t + n_t \leq 1$$
$$\hat{i}_t \geq 0$$
$$\mu_t = (\hat{x}_{l,t}, \hat{x}_{h,t}, \eta_t)$$
$$\hat{x}_{k,t} \sim \text{ Kalman filter}$$
$$\eta_t \sim \text{ Bayesian updating}$$

3.B Derivation for Section 3.4.1

Here, we show the derivation of the Euler equation with uncertainty. First, we substitute $E_t(V_{t+1}^R) = \frac{E_t(V_{t+1}^{NR})}{M_t}$ into the Euler equation. We have:

$$
\begin{aligned}
\Lambda_t &= \beta \Upsilon_t \left(\mu_t \xi_t^R \frac{\partial \frac{E_t(V_{t+1}^{NR})}{M_t}}{\partial K_{t+1}} + (1-\mu_t)\xi_t^{NR} \frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t} \right) \\
&= \beta \Upsilon_t \left(\mu_t \xi_t^R \left(\frac{1}{M_t} \frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t} + E_t(V_{t+1}^{NR}) \frac{\partial M_t^{-1}}{\partial I_t} \right) + (1-\mu_t)\xi_t^{NR} \frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t} \right) \\
&= \beta \Upsilon_t \left(\frac{\mu_t \xi_t^R}{M_t} E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta)) + (1-\mu_t)\xi_t^{NR} E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta)) \right) \\
&\because \frac{\partial M_t^{-1}}{\partial I_t} \approx 0 \\
&= \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta)) \Upsilon_t \left(\frac{\mu_t \xi_t^R}{M_t} + (1-\mu_t)\xi_t^{NR} \right)
\end{aligned}
$$

where $\Upsilon_t = \dfrac{\mu_t M_t^{\gamma} + (1-\mu_t)}{\left(\mu_t M_t^{\gamma-1} + (1-\mu_t)\right)^{\frac{-\gamma}{1-\gamma}}}$

$$\xi_t^R = \frac{M_t^{\gamma}}{\mu_t M_t^{\gamma} + (1-\mu_t)}$$

$$\xi_t^{NR} = \frac{1}{\mu_t M_t^{\gamma} + (1-\mu_t)}$$

Λ_{t+1}^{NR} is the marginal utility of consumption in the normal scenario.

R_{t+1}^{NR} is the rental price of capital in the normal scenario.

Investment I_t could have a very small or zero second-order effect on the current uncertainty M_t. Therefore, we assume that $\frac{\partial M_t^{-1}}{\partial I_t} \approx 0$.

3.C Solution method

This section describes the computational method and the parameterized expectations algorithm (PEA) that we use to solve our model. In Chapter 4, we provide a more detailed description of the PEA. The PEA technique and programming code are adapted from Collard (2015) and we add the moving bound technique by Maliar and Maliar (2003) to reduce the possibility that the algorithm will explode. To get a zero-growth steady state, we define $y_t \equiv \frac{Y_t}{Z_t}, k_{t+1} \equiv \frac{K_{t+1}}{Z_t}, \pi_t \equiv \frac{\Pi_t}{Z_t}, c_t \equiv \frac{C_t}{Z_t}, i_t \equiv \frac{I_t}{Z_t}, w_t \equiv \frac{W_t}{Z_t}, \lambda_t \equiv \frac{\Lambda_t}{Z_t^{-\sigma}}$, and $z_t \equiv \frac{Z_t}{\exp(\bar{a})}$. R_t and L_t are already stationary. We obtain a system of equations as presented in Box 3.C.1

We use the parameterized expectations algorithm to solve our model. Basically, the method is to approximate the Euler equation with a parametric function. We first solve the benchmark model without uncertainty, then the smooth ambiguity model with uncertainty. To solve the benchmark model, we define the parametric function as follow:

$$P^{NR}(\lambda_{t-1}^{NR}, k_t^{NR}, z_t; \theta^{NR}) = \theta^{NR,\lambda}\lambda_{t-1}^{NR} + \theta^{NR,k}k_t^{NR} + \theta^{NR,z}z_t + u_t^{NR} \text{ where } E(u_t^{NR}) = 0$$

At the end of this iteration, we obtain the parametric function of the expectation in the normal growth scenario θ^{NR}, which we use to solve the smooth ambiguity model. The parametric function for the smooth ambiguity model is:

$$P^{SA}(\lambda_{t-1}, k_t, z_t, M_t, \mu_t; \theta, \theta^{NR}) = \theta^{\lambda}\lambda_{t-1} + \theta^{k}k_t + \theta^{z}z_t + \theta^{M}M_t + \theta^{\mu}\mu_t + \theta^{M\mu}M_t\mu_t + u_t$$

where $\theta = \{\theta^{\lambda}, \theta^{k}, \theta^{z}, \theta^{M}, \theta^{\mu}, \theta^{M\mu}\}, E(u_t) = 0$

We assume that the technological process is same for both scenarios since the technology is developed by the firm who is not exposed to uncertainty. This implies that TFP is exogenous to the business cycle and is the same in the benchmark model. Box 3.C.2 describes the parameterized expectations algorithm to solve the model.

Box 3.C.1: Equilibrium conditions

Household:

$$\lambda_t = \beta E_t \left[\lambda_{t+1}^{NR}\left(\frac{z_{t+1}}{z_t}\right)^{-\sigma}(R_{t+1}^{NR}+1-\delta)\right]\Upsilon_t\left[\mu_t\xi_t^R\frac{1}{M_t}+(1-\mu_t)\xi_t^{NR}\right]$$

$$\lambda_t = c_t^{-\sigma}$$

$$\lambda_t = \frac{\exp((1-\sigma)\bar{a})L_t^{\nu}}{z_t^{1-\sigma}w_t}$$

Firm:

$$k_{t+1} = (1-\delta)k_t\frac{z_{t-1}}{z_t}+i_t$$

$$y_t = \exp(\alpha\bar{a})z_{t-1}^{\alpha}k_t^{\alpha}L_t^{1-\alpha}$$

$$w_t = (1-\alpha)\frac{y_t}{L_t}$$

$$R_t = \alpha\frac{y_t}{k_t}\frac{z_t}{z_{t-1}}$$

$$z_t = \exp(a_t-\bar{a})$$

Good market clearing:

$$y_t = c_t + i_t$$

Exogenouse processes:

$$a_t = (1-\rho_a)\bar{a}+\rho_a a_{t-1}+e_t^a;\, e_t^a \sim \mathcal{N}(0,\sigma_a^2)$$

$$M_t = (1-\rho_M)\bar{M}+\rho_M M_{t-1}+e_t^M;\, e_t^M \sim \mathcal{N}(0,\sigma_M^2)$$

Bayes' rule is:

$$\mu_{t+1} = \frac{\mu_{t+1|t}^{prior}L(g^R)}{\mu_{t+1|t}^{prior}L(\bar{g}^R)+(1-\mu_{t+1|t}^{prior})L(\bar{g}^{NR})}$$

$$L(\bar{g}^k) = \frac{\exp(-0.5(E_t(g_{t+1})-\bar{g}^k)^2/(\kappa\sigma_g)^2)}{\sqrt{2\pi}\kappa\sigma_g}$$

$$E_t(g_{t+1}) = \rho_{sm}\log\left(\frac{y_t}{y_{t-1}}\right)+(1-\rho_{sm})E_{t-1}(g_t)$$

$$\mu_{t+1|t}^{prior} = \rho_{sm}\mu_t+(1-\rho_{sm})\mu^{prior}$$

where $k \in \{R, NR\}$, y_t is output at t with a zero growth steady state, $\mu_{t+1|t}^{prior}$ is a prior of the recession probability at $t+1$ given information at t, μ^{prior} is a constant prior of the recession probability, κ is a multiplier of the standard deviation, ρ_{sm} is a smoothing parameter, and $L(.)$ is a standard likelihood. Capital market clearing (savings equal to investment) and labor market clearing (labor supply equal to labor demand) are satisfied.

Box 3.C.2: Parameterized Expectations algorithm

PEA for θ^{NR}

1. Set an initial guess for $\theta^{NR} = \{1, 0, 0\}$ and $S = \{a_t\}_{t=1}^{T}$ is generated from an exogenous process. Consequently, $\{z_t\}_{t=1}^{T}$ is given.
2. At iteration i and for the given θ_i^{NR}, generate $\{\lambda_t^{NR}\}_{t=1}^{T}$ using $\lambda_t^{NR} = P^{NR}(\lambda_{t-1}^{NR}, k_t^{NR}, z_t; \theta_i^{NR})$, and $\{c_t^{NR}, i_t^{NR}, w_t^{NR}, R_t^{NR}, L_t^{NR}, y_t^{NR}, k_{t+1}^{NR}\}_{t=1}^{T}$ using the equilibrium conditions except the Euler equation
3. Let $X(\theta_i^{NR}) = \{\lambda_t^{NR}, c_t^{NR}, i_t^{NR}, w_t^{NR}, R_t^{NR}, L_t^{NR}, y_t^{NR}, k_{t+1}^{NR}; \theta_i^{NR}\}_{t=1}^{T}$ and for given upper and lower bounds, $\bar{X}_i$ and $\underline{X}_i$, set $X(\theta_i^{NR}) = \bar{X}_i$ for any element in $X(\theta_i^{NR}) > \bar{X}_i$ and set $X(\theta_i^{NR}) = \underline{X}_i$ for any element in $X_i < \underline{X}_i$
4. Generate $\{\hat{\lambda}_t^{NR}\}_{t=1}^{T-1}$ using $\hat{\lambda}_t^{NR} = \beta\left(\lambda_{t+1}^{NR}\left(\frac{z_{t+1}}{z_t}\right)^{-\sigma}(R_{t+1}^{NR} + 1 - \delta)\right)$
5. Obtain $\hat{\theta}_{i+1}^{NR}$ by regressing $\{\hat{\lambda}_t^{NR}\}_{t=1}^{T}$ against $\{\lambda_{t-1}^{NR}, k_t^{NR}, z_t\}_{t=1}^{T}$ such that:
$$\hat{\lambda}_t^{NR} = \theta^{NR,\lambda}\lambda_{t-1}^{NR} + \theta^{NR,k}k_t + \theta^{NR,z}z_t + u_t^{NR} \text{ where } E(u_t^{NR}) = 0$$
6. Update $\theta_{i+1}^{NR} = \omega\hat{\theta}_{i+1}^{NR} + (1-\omega)\theta_i^{NR}; \omega = 0.25$ and if any variable hits the bounds in step 3, expand the bounds for the next iteration according to the following formula:
$$\bar{X}_{i+1} = X_s^{NR}(1 + \Delta_i)$$
$$\underline{X}_{i+1} = X_s^{NR}(1 - \Delta_i)$$
where $\Delta_i = 0.05 + 0.01i$, $X_s^{NR} =$ steady state values of variables in X
7. Go back to step 2 and iterate until $|\theta_i^{NR} - \theta_{i-1}^{NR}| < 10^{-6}$ and no variable hits the bounds

---

PEA for θ

1. Set an initial guess for θ_i and $S = \{a_t, M_t\}_{t=1}^{T}$ is generated from an exogenous process.
2. At iteration i, for the given θ_i, generate $\{\lambda_t\}_{t=1}^{T}$ using $\lambda_t = P^{SA}(\lambda_{t-1}, k_t, z_t, M_t, \mu_t; \theta_i)$, and $\{c_t, i_t, w_t, R_t, L_t, y_t, \xi_t^R, \xi_t^{NR}, \Upsilon_t, k_{t+1}, \mu_{t+1}\}_{t=1}^{T}$ using the equilibrium conditions except the Euler equation
3. Let $X(\theta_i) = \{\lambda_t, c_t, i_t, w_t, R_t, L_t, y_t, \xi_t^R, \xi_t^{NR}, \Upsilon_t, k_{t+1}; \theta_i\}_{t=1}^{T}$ and for given upper and lower bounds, $\bar{X}_i$ and $\underline{X}_i$, set $X(\theta_i) = \bar{X}_i$ for any element in $X(\theta_i) > \bar{X}_i$ and set $X(\theta_i) = \underline{X}_i$ for any element in $X_i < \underline{X}_i$
4. Given the θ^{NR} obtained from the previous PEA, generate $\{\lambda_t^{NR}, R_t^{NR}\}_{t=1}^{T}$ using $P^{NR}(\lambda_{t-1}^{NR}, k_t, z_t; \theta^{NR})$. We use k_t instead of k_t^{NR} because the household form an expectation given that capital is predetermined. Thus for the smooth ambiguity household, k_t^{NR} is not a predetermined variable but rather the expected capital in the normal growth scenario.
5. Generate $\{\hat{\lambda}_t\}_{t=1}^{T-1}$ where $\hat{\lambda}_t = \beta\left[\lambda_{t+1}^{NR}\left(\frac{z_{t+1}}{z_t}\right)^{-\sigma}(R_{t+1}^{NR} + 1 - \delta)\right]\Upsilon_t\left[\mu_t\xi_t^R\frac{1}{M_t} + (1-\mu_t)\xi_t^{NR}\right]$
6. Obtain $\hat{\theta}_{i+1}$ by regressing $\{\hat{\lambda}_t\}_{t=1}^{T}$ against $\{\lambda_{t-1}, k_t, z_t, M_t, \mu_t\}_{t=T_{\text{begin}}}^{T-1}$ such that:
$$\hat{\lambda}_t = \theta^\lambda\lambda_{t-1} + \theta^k k_t + \theta^z z_t + \theta^M M_t + \theta^\mu\mu_t + \theta^{M\mu}M_t\mu_t + u_t \text{ where } E(u_t) = 0$$
7. Update $\theta_{i+1} = \omega\hat{\theta}_{i+1} + (1-\omega)\theta_i; \omega = 0.25$ and if any variable hits the bounds in step 3, expand the bounds for the next iteration according to the following formula:
$$\bar{X}_{i+1} = X_s(1 + \Delta_i)$$
$$\underline{X}_{i+1} = X_s(1 - \Delta_i)$$
where $\Delta_i = 0.05 + 0.01i$, $X_s =$ steady state values of variables in X
8. Go back to step 2 and iterate until $|\theta_i - \theta_{i-1}| < 10^{-6}$ and no variable hits the bounds

Note: We limit the maximum number of iterations to 5000. In Section 3.5, we run the simulations of T = 2500 periods with $T_{\text{begin}} = 500$ to ensure that the initial guesses do not affect the solution. For the US simulation in Section 3.6, T = 141 periods (1985Q1-2019Q4) and we set $T_{\text{begin}} = 9$, meaning that we use 1985 and 1986 as burn-in samples.

3.D Robustness

This section reports the sensitivity analysis of the smooth parameters (κ and ρ_{sm}) in the Bayesian updating. Recall, Bayes' rule is:

$$\begin{aligned}
\mu_{t+1} &= \frac{\mu_{t+1|t}^{prior} L(g^R)}{\mu_{t+1|t}^{prior} L(\bar{g}^R) + (1 - \mu_{t+1|t}^{prior}) L(\bar{g}^{NR})} \\
L(\bar{g}^k) &= \frac{\exp(-0.5(E_t(g_{t+1}) - \bar{g}^k)^2/(\kappa\sigma_g)^2)}{\sqrt{2\pi}\kappa\sigma_g} \\
E_t(g_{t+1}) &= \rho_{sm} \log\left(\frac{y_t}{y_{t-1}}\right) + (1 - \rho_{sm}) E_{t-1}(g_t) \\
\mu_{t+1|t}^{prior} &= \rho_{sm}\mu_t + (1 - \rho_{sm})\mu^{prior}
\end{aligned}$$

where $k \in \{R, NR\}$, y_t is output at t with a zero growth steady state, $\mu_{t+1|t}^{prior}$ is a prior of the recession probability at $t+1$ given information at t, μ^{prior} is a constant prior of the recession probability, κ is a multiplier of the standard deviation, ρ_{sm} is a smoothing parameter, and $L(.)$ is a standard likelihood.

First, we discuss the limitation of PEA convergence. Second, we show the US simulation's performance, particularly the correlations between simulated output growths and actual output growths.

3.D.1 Parameterized expectations algorithm convergence

We run the simulations with ambiguity aversion $\gamma = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ to see the highest value that PEA can converge. Table 3.6 reports the highest level of ambiguity aversion. If the value is 20, it means that the PEA converges for all γ from 0 to 20. NA means the model does not converge at all. The higher κ or the lower ρ_{sm} mean that the Bayesian updating is smoother. We find that $\rho_{sm} = 0.4$ is the most robust smoothing parameter since the PEA converges for all $\kappa > 1$.

Table 3.6: the highest level of ambiguity aversion that PEA converges

κ	ρ_{sm} 0.2	0.4	0.6	0.8
1	8	4	NA	NA
1.5	14	20	4	0
2	20	20	NA	NA
2.5	20	20	0	20
3	20	20	4	20
3.5	0	20	10	8
4	20	20	20	16

3.D.2 US simulation's performance

We run the simulations using the US data with ambiguity aversion $\gamma = \{0, 4, 20\}$ to show the out-of-sample performances across different levels of smooth parameters. Table 3.7 reports the correlations between the 4-quarter-moving-average simulated output growth and actual output growth. The higher κ or the lower ρ_{sm} mean that the Bayesian updating is smoother.

We observed that the correlations vary when the parameters of the Bayesian updating process are changed; however, the extent of this variation was limited. The correlations for the ambiguity neutral models vary from approximately 0.34 to 0.52, those for the ambiguity averse $\gamma = 4$ models range from 0.42 to 0.57, and the correlations for the ambiguity averse $\gamma = 20$ models range from 0.49 to 0.58.

Table 3.7: Correlations between simulated and actual output growth

	$\gamma = 0$				$\gamma = 4$				$\gamma = 20$			
$\kappa\|\rho_{sm}$	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
1	0.52	0.45	0.37	0.36	0.57	0.45	0.39	0.36	0.56	0.45	0.41	0.45
1.5	0.39	0.49	0.47	0.45	0.51	0.46	0.51	0.45	0.56	0.58	0.49	0.46
2	0.36	0.39	0.45	0.51	0.46	0.52	0.49	0.46	0.55	0.58	0.58	0.55
2.5	0.35	0.36	0.39	0.43	0.44	0.47	0.51	0.52	0.53	0.56	0.57	0.56
3	0.35	0.35	0.37	0.39	0.43	0.44	0.47	0.51	0.51	0.55	0.56	0.55
3.5	0.35	0.35	0.36	0.37	0.42	0.43	0.45	0.48	0.50	0.54	0.56	0.55
4	0.35	0.34	0.35	0.36	0.42	0.42	0.43	0.45	0.49	0.52	0.55	0.54

Note: The table presents the performance of out-of-sample simulations. The first row shows the correlations between simulated growth and actual growth of the US quarterly GDP. All values are in percentage points. The simulated growth rates are 4-quarter moving averages of the original simulations.

Chapter 4

Ambiguous business cycles: an empirical assessment

Poramapa Poonpakdee

4.1 Introduction

Research has revealed that the effect of uncertainty on the economy is nonlinear, with increasingly severe impacts as uncertainty levels rise (Bianchi et al., 2018; Bloom, 2014; Jackson et al., 2020; Lhuissier & Tripier, 2021; Ng & Wright, 2013). This has posed a challenge to most standard business cycle models[1], which rely on first-order approximations that are typically accurate in periods of low uncertainty (Christiano et al., 2018; Wieland & Wolters, 2011). In Chapter 3, we demonstrate that the smooth ambiguity model based on Altug et al. (2020) can replicate the relationships between macroeconomic uncertainty and people's beliefs and has the potential to capture economic fluctuations. In this chapter, we estimate the smooth ambiguity model using data from the United States and large European countries, including Germany, Italy, France and Spain. We study how uncertainty affects these countries and assess the model performance in data fitting.

Literature on macroeconomic models under uncertainty mainly revolves around three issues: measurement of uncertainty, transmission channels, and solution approaches. Our paper contributes to the literature on these three topics.

First, uncertainty in macroeconomic models involves the concepts of risk and ambiguity. Risk, when the likelihood of outcome is known, is measured as time-varying volatility (Fernández-Villaverde & Guerrón-Quintana, 2020; Lhuissier & Tripier, 2021), while ambiguity often relates to multiple scenarios where the agent does not know the true data generating process (Altug et al., 2020; Bhandari et al., 2023; Born et al., 2018; Ilut & Schneider, 2014). However, only a limited number of macro models connect these two concepts of uncertainty to observable uncertainty. For example, Ilut and Schneider (2014) use forecast disagreements among professional forecasters as a measure for ambiguity in their business cycle model. In our paper, we define uncertainty using the concept of ambiguity. A representative household is exposed to uncertainty and believes the next-period economy may fall into either a recession or a period of normal economic growth. We anchor the ratio of the expected utilities between these two scenarios using a macroeconomic uncertainty index to connect theoretical uncertainty to empirical uncertainty.

[1] We refer to the model with representative agents with rational expectations.

Second, the transmission channels of uncertainty have been extensively studied in the literature, such as financial friction (Chatterjee & Milani, 2020; Christiano et al., 2018; Fernández-Villaverde & Guerrón-Quintana, 2020; Lhuissier & Tripier, 2021), price and wage mark-up (Born & Pfeifer, 2021), investment adjustment cost (Bloom, 2009), and agents' expectations (Altug et al., 2020; Bhandari et al., 2023; Ilut & Schneider, 2014). For instance, Bhandari et al. (2023) investigate the belief distortions of a household with robustness preferences. In uncertain periods, the household tends to focus more on minimizing distortions in its expectations rather than maximizing its utility, ultimately reducing economic activity. Our paper focuses on the expectations channel, as uncertainty has been shown to induce biases through this channel. We incorporate the Klibanoff et al. (2005)'s smooth ambiguity preferences into a simple business cycle model based on Altug et al. (2020). An increase in macroeconomic uncertainty leads to an ambiguity averse household being more concerned about the recession scenario while its expectation of the recession scenario worsens.

Finally, as an alternative to loglinear solutions, various approaches have been utilized to account for the nonlinear effects of uncertainty, for instance, higher-order perturbations (Born & Pfeifer, 2021; Fernández-Villaverde & Guerrón-Quintana, 2020) and nonlinear or markov-switching VARs (Bianchi et al., 2018; Jackson et al., 2020; Lhuissier & Tripier, 2021). The smooth ambiguity models, for which there is no closed-form solution, are generally solved by projection methods (Collard et al., 2018; Gallant et al., 2019; Ju & Miao, 2012) or value function iterations (Altug et al., 2020; Jahan-Parvar & Liu, 2012). To solve the model, we apply a parameterized expectation algorithm. This projection method preserves the nonlinearity in the transmission mechanism and determines the expectations of the two scenarios without needing to impose an additional decision rule for each scenario. Notably, while smooth ambiguity models have been used to fit financial asset returns (Gallant et al., 2019), their application to estimate macroeconomic variables remains scarce. We estimate the smooth ambiguity model to minimize the distance between the model-generated and actual output growth rates. To the best of our knowledge, this is the first study to measure the level of ambiguity aversion using macroeconomic data.

Our model's estimation uses three empirical time series as inputs: the Economic policy uncertainty index (Baker et al., 2016), the recession probability computed from the survey of professional forecasters, and the utilization-adjusted technological process (Comin et al., 2020; Fernald, 2014). The result shows that accounting for uncertainty and ambiguity aversion significantly improves the model performance in fitting output growth rates compared to the benchmark model (no uncertainty). This result holds for the US and the European countries. The model's out-of-sample forecasts of US output growth are comparable to those of US professional forecasters, and it performs even better during recessions. Additionally, we find that the Dot-com crisis contributed to an increase in risk aversion but had no impact on ambiguity aversion. The Global Financial crisis increased both aversions to risk and ambiguity structurally.

This paper is organized as follows: Sections 4.2 and 4.3 present the transmission channel of uncertainty in the smooth ambiguity model and the model's steady state, respectively. The methodology to evaluate the model is described in Section 4.4. The main results are presented and discussed in Section 4.5, while Section 4.6 presents robustness tests with alternative estimators. Finally, Section 4.7 summarizes the paper.

4.2 Uncertainty in the smooth ambiguity model

This section discusses the transmission channel of uncertainty in our model. To begin, we define uncertainty according to Knight (1921), which describes it as a situation where the likelihood of future outcomes is unknown. Risk, on the other hand, is used to refer to situations with known likelihoods. Macroeconomic literature frequently refers to Knightian uncertainty as ambiguity (Altug et al., 2020; Collard et al., 2018; Fernández-Villaverde & Guerrón-Quintana, 2020; Ilut & Schneider, 2014; Ju & Miao, 2012). In our paper, the relevance of (Knightian) uncertainty is contingent upon two conditions. First, the household must perceive that the economy could enter either a normal growth period or a recession. If the household is certain that the economy will not enter the recession scenario, then uncertainty will not impact their decisions. Secondly, the household expects different utilities from the two scenarios. If the household is indifferent between the two scenarios, uncertainty does not matter. We summarize these two necessary conditions for uncertainty to be relevant as follows:

$$\mu_t > 0 \text{ and } E_t(V_{t+1}^{NR}) > E_t(V_{t+1}^{R}) \tag{4.1}$$

where μ_t is a Bayesian belief of the recession probability, $E_t(V_{t+1}^{R})$ is the expected utility at time t for the economy to be in recession at time $t+1$, and $E_t(V_{t+1}^{NR})$ is the expected utility when the economy is in the period of normal growth at $t+1$. V indicates the utility and superscripts R and NR are recession and normal growth scenarios (no recession), respectively.

To connect theoretical uncertainty to empirical uncertainty, we assume that the ratio of $E_t(V_{t+1}^{R})$ in relation to $E_t(V_{t+1}^{NR})$ equals the empirical series of macroeconomic uncertainty M_t. Since $E_t(V_{t+1}^{NR})$ is greater than $E_t(V_{t+1}^{R})$, M_t is larger than one.

$$M_t = \frac{E_t(V_{t+1}^{NR})}{E_t(V_{t+1}^{R})} \text{ where } M_t > 1 \tag{4.2}$$

Throughout this paper, the term 'uncertainty' will refer to the two conditions in Equation 4.1 and the term 'macroeconomic uncertainty' will be specific to Equation 4.2.

4.2.1 Transmission channels of macroeconomic uncertainty

This section describes how macroeconomic uncertainty is transmitted through expected utilities and agent's beliefs. We begin by introducing the household's optimality conditions and incorporating macroeconomic uncertainty into the model. We then discuss the theoretical impacts of macroeconomic uncertainty.

The household is Bayesian and has smooth ambiguity preferences. The household maximizes its value function V_t with respect to consumption, labor, and investment[2]

[2] L_t is defined for both labor supply and demand, and I_t is defined for both savings and investment since the market clearing conditions are satisfied in each period. Investment I_t determines the next-period capital K_{t+1}. We assume a standard process of capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$ where δ is the depreciation rate.

thus the objective function is:

$$
\begin{aligned}
&\max_{C_t, L_t, I_t} V(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu} \\
&+ \beta\phi^{-1}\left[\Big(\mu_t\phi(E_t(V(C_{t+1}^R, L_{t+1}^R))) + (1-\mu_t)\phi(E_t(V(C_{t+1}^{NR}, L_{t+1}^{NR})))\Big)\right] \qquad (4.3) \\
&\text{subject to } C_t + I_t = W_t L_t + R_t K_t + \Pi_t
\end{aligned}
$$

where $\phi(E_t(V_{t+1})) = \frac{[E_t(V_{t+1})]^{1-\gamma}}{1-\gamma}$ is the smooth ambiguity function, $\gamma \geq 0$ is ambiguity aversion[3], C_t is consumption, I_t is investment, L_t is labor, K_t is capital, R_t is the rental price of capital, W_t is the wage rate, Π_t is the firm's profit distributed to the household, β is the discount factor, $\sigma > 0$ is risk aversion, and $\nu > 0$ is the disutility of labor. Lastly, μ_t is the Bayesian belief of recession at $t+1$. The optimality conditions for C_t, L_t and I_t can be summarized as follows:

$$
W_t = L_t^\nu C_t^\sigma \qquad (4.4)
$$

$$
\Lambda_t = \beta\Upsilon_t\left(\mu_t\xi_t^R\frac{\partial E_t(V_{t+1}^R)}{\partial I_t} + (1-\mu_t)\xi_t^{NR}\frac{\partial E_t(V_{t+1}^{NR})}{\partial I_t}\right) \qquad (4.5)
$$

$$
\begin{aligned}
\text{where } \Lambda_t &= C_t^{-\sigma} \\
\Upsilon_t &= \frac{\mu_t E_t(V_{t+1}^R)^{-\gamma} + (1-\mu_t)E_t(V_{t+1}^{NR})^{-\gamma}}{\left(\mu_t E_t(V_{t+1}^R)^{1-\gamma} + (1-\mu_t)E_t(V_{t+1}^{NR})^{1-\gamma}\right)^{\frac{-\gamma}{1-\gamma}}} \\
\xi_t^k &= \frac{E_t(V_{t+1}^k)^{-\gamma}}{\mu_t E_t(V_{t+1}^R)^{-\gamma} + (1-\mu_t)E_t(V_{t+1}^{NR})^{-\gamma}} \\
k &\in \{R, NR\}
\end{aligned}
$$

Equation 4.4 implies that the substitution rate between consumption and labor is proportional to the wage rate. Equation 4.5 is the smooth ambiguity Euler equation which equates a Lagrange multiplier Λ_t to the marginal expected utilities of investment $\frac{\partial E_t(V_{t+1}^k)}{\partial I_t}$ for the two scenarios weighted by the Bayesian beliefs μ_t, the scaling factor Υ_t and belief distortions ξ_t^k. According to the assumption in Equation 4.2, we substitute $E_t(V_{t+1}^R) = \frac{E_t(V_{t+1}^{NR})}{M_t}$ into Equation 4.5 and solve for the partial derivatives. The Euler equation be-

[3] Section 3.4.1 provides details about the smooth ambiguity function.

comes:[4]

$$\Lambda_t = \beta E_t(\Lambda_{t+1}^{NR}(R_{t+1}^{NR} + 1 - \delta))\Upsilon_t \left(\frac{\mu_t \xi_t^R}{M_t} + (1-\mu_t)\xi_t^{NR} \right) \tag{4.6}$$

$$\Upsilon_t = \frac{\mu_t M_t^{\gamma} + (1-\mu_t)}{\left(\mu_t M_t^{\gamma-1} + (1-\mu_t)\right)^{\frac{-\gamma}{1-\gamma}}} \tag{4.7}$$

$$\xi_t^R = \frac{M_t^{\gamma}}{\mu_t M_t^{\gamma} + (1-\mu_t)} \tag{4.8}$$

$$\xi_t^{NR} = \frac{1}{\mu_t M_t^{\gamma} + (1-\mu_t)} \tag{4.9}$$

where Λ_{t+1}^{NR} : the marginal utility of consumption in the normal scenario.

R_{t+1}^{NR} : the rental price of capital in the normal scenario.

For simplicity, we define macroeconomic uncertainty as a simple stationary process. We assume the Bayesian updating of μ_t to be a function B of information $\mathcal{I}_t$ and prior belief μ_t^{prior}.

$$M_t = (1-\rho_M)\bar{M} + \rho_M M_{t-1} + e_t^M; e_t^M \sim \mathcal{N}(0, \sigma_M^2) \tag{4.10}$$

$$\mu_t = B(\mu_t^{prior}, \mathcal{I}_t) \tag{4.11}$$

Equation 4.6 shows that macroeconomic uncertainty has been incorporated into the model through the spread of expected utilities between the two scenarios, a scaling factor Υ_t, and belief distortions ξ_t^k. As noted by Altug et al. (2020), the scaling factor does not affect the first order condition since it attaches to both scenarios equally; however, belief distortions do, since they distort the household's beliefs away from the Bayesian beliefs. If ξ_t^k is one, there is no distortion, meaning the household's beliefs are the same as Bayesian beliefs. We define the combination of Bayesian beliefs and belief distortions as subjective beliefs. Therefore, $\mu_t\xi_t^R$ and $(1-\mu_t)\xi_t^{NR}$ are the subjective beliefs of recessions and normal growth periods, respectively, and their sum is one.

The dynamics of belief distortions depend on macroeconomic uncertainty M_t, ambiguity aversion γ, and Bayesian belief of recession μ_t. All else being equal, an increase in macroeconomic uncertainty or ambiguity aversion leads to an increase (decrease) in belief distortion towards the recession (normal growth) scenario. This behavior is referred

[4]The full derivation of Eq. 4.6 is described in Chapter 3 Appendix 3.B.

to as pessimistic belief distortions (Altug et al., 2020; Ju & Miao, 2012; Marinacci, 2015). However, when the Bayesian belief of recession is higher, the belief distortion towards the recession (normal growth) scenario is lower (greater). This indicates that the household avoids forming an extreme subjective belief toward one particular scenario when it is aware that the Bayesian belief is heavily oriented towards that scenario[5].

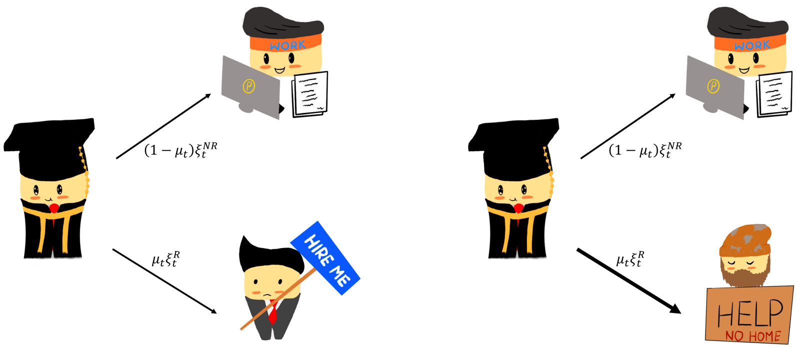
Next, we discuss the transmission channels of uncertainty in our model. If $\mu_t > 0$ and $M_t > 1$, uncertainty affects the household's decision. Macroeconomic uncertainty M_t affects the economy non-linearly through expected utilities and subjective beliefs. First, it directly decreases the average expected utility of investment by widening the spread of expected utilities between the two scenarios. Second, if the household is ambiguity averse, macroeconomic uncertainty increases subjective beliefs of recession due to pessimistic belief distortions. Thus, the effect of macroeconomic uncertainty is stronger when it is higher. This nonlinear effect of macroeconomic uncertainty has been extensively documented in the macro literature such as Jackson et al. (2020), Ng and Wright (2013) and Lhuissier and Tripier (2021).

Figure 4.1 illustrates a real-life example of the effects of macroeconomic uncertainty. Consider a representative Ph.D. candidate who is ambiguity averse. When macroeconomic uncertainty is low, the Ph.D. candidate expects to secure an academic position in the normal growth scenario but anticipates not finding any job in the recession scenario (Figure 4.1a). When macroeconomic uncertainty is high, this Ph.D. candidate expects to become homeless in the recession scenario (Figure 4.1b). In this case, the recession scenario's expected utility is significantly lower than the normal growth scenario compared to the case of low macroeconomic uncertainty. Since the Ph.D. is ambiguity averse, his subjective uncertainty increases with macroeconomic uncertainty, meaning that he is more concerned about the recession scenario. This leads to a pronounced effect of macroeconomic uncertainty.

Finally we describe the firm in this economy. There is one representative firm that produces one good. The firm is not directly exposed to uncertainty and maximizes profits

[5] In Chapter 3 Section 3.4.2, we examine the dynamics of belief distortions in greater detail.

Figure 4.1: An example of macroeconomic uncertainty's effects on a Ph.D. candidate



(a) Low macroeconomic uncertainty **(b)** High macroeconomic uncertainty

as follows:

$$\max_{K_t,L_t} \Pi_t = Y_t - W_t L_t - R_t K_t$$
$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$$
$$K_t = (1-\delta)K_{t-1} + I_t$$
$$Z_t = \exp(a_t)$$
$$a_t = (1-\rho)\bar{a} + \rho a_{t-1} + \sigma_a \epsilon_t^a \text{ where } \epsilon_t^a \sim \mathcal{N}(0,1)$$

Π_t is profit, Y_t is output, K_t is capital, L_t is labor, I_t is investment, W_t is the wage rate, and R_t is the rental price of capital. α is the capital share in production and δ is the depreciation rate of capital. Finally, Z_t is the total productivity factor (TFP) which is developing as an AR(1) process, around mean $\bar{a}$. The first order optimality conditions of the firm are:

$$W_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha} \tag{4.12}$$
$$R_t = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{4.13}$$

4.3 Steady state

In this section, we discuss the steady state of the smooth ambiguity model. Determining the steady state is not trivial, given the decision-making under uncertainty is based on two scenarios. We will first review the concepts of steady states in business cycle models that involve multiple scenarios and then explain the steady state of our model.

4.3.1 Steady state in the literature

This section summarizes three types of business cycle models with multiple scenarios or regimes and their respective steady states. These studies do not explicitly or conceptually define their steady state, although it is assumed to exist when solving the model. Therefore, we focus on whether the steady state contains multiple scenarios and under which conditions the steady state model will be in one or multiple scenarios.

Altug et al. (2020)'s smooth ambiguity model has one steady state scenario due to the agent learning the true data generating process (DGP) through Bayes' rule. Their model has two scenarios regarding the technological process: high persistence and low volatility, or low persistence and high volatility, with one of these scenarios being the true DGP. The agent does not know which TFP process is the trueth but eventually learns it. As the agent learns the true process, the two-scenario model converges into a one-true-scenario model, where ambiguity aversion and Bayesian beliefs no longer matter. Note that Altug et al. (2020) do not explicitly discuss the model steady state but it is shown in their appendix that the Bayesian probability converges to either one or zero depending on which DGP is set up as true.

Second, under the multiple priors preferences (maxmin), the steady state model has one scenario: the worst case (Bianchi et al., 2018; Ilut & Schneider, 2014). For example, in the Ilut and Schneider (2014)'s model, the agent is uncertain about future technological growth and behaves as if it is always in the worst-case scenario. Technological growth consists of past growth, innovations (pure shocks) and ambiguous deterministic sequences, and it is impossible to learn this deterministic sequence from the data. Thus, the agent always expects it to be the worst, and the model assumes that a higher level of ambiguity leads to a lower expectation. From the agent's perspective, the steady state is entailed in the worst-case scenario where the ambiguous component is different from zero. However, when solving the model, the authors use the econometrician's perspective based on the true DGP, assuming that the ambiguous component is zero in the steady state. Thus, there is a distinction in the steady states between the worst-case believer and the econometrician. Ilut and Schneider (2014) explain that this discrepancy positively associates with the level of ambiguity which is proxied by professional forecasters' disagreement. In other words, the difference in expectations between the worst-case believer and the econometrician is the level of ambiguity. Furthermore, they assume that the average level of ambiguity, i.e. the steady state ambiguity, is smaller than the volatility of technology growth, restricting the deviation of the worst-case believer from the econometrician. In short, the multiple priors agent does not know the true DGP, but the steady state model has one scenario since the agent always decides as if it is in the worst-case scenario.

Finally, in regime switching models, the agent considers all regimes at the steady state, taking the average of the regime dependent variables (Benigno et al., 2020; Bianchi et al., 2018; Lhuissier & Tripier, 2021; Liu et al., 2011). For example, Lhuissier and Tripier (2021) study the effects of uncertainty in two regimes: "distress"[6] and "tranquil" and use the ergodic means of the two regimes weighted by the estimated regime switching probabilities at the steady state. In this way, the agent's decisions are contingent upon the probability of each scenario, allowing them to consider multiple scenarios in the steady state.

In summary, the steady states of business cycle models with multiple scenarios can largely be categorized into three cases. First, if the agent finally learns the true DGP (Altug et al., 2020), then the steady state model is anchored by the true scenario. Second, regardless of the true DGP, if the agent only considers one particular scenario such as the worst-case scenario (Bianchi et al., 2018; Ilut & Schneider, 2014), then the steady state is subject to that scenario. Finally, if these two conditions are not applicable as in the regime switching models, the values of steady state are the average of multiple scenarios weighted by the probability of each scenario. Using these concepts, we discuss our model's steady state in the next section.

[6]The distress regime includes periods such as the 9/11 attacks, the dot-com bubble, and the global financial crisis.

4.3.2 Steady states in our smooth ambiguity model

In this section, we define our model's steady state, which we refer to as an *ambiguous steady state*. Subsequently, we demonstrate analytically that the ambiguous steady state can capture the features of a conventional one-scenario steady state (benchmark steady state) and a worst-case steady state.

To develop an idea of the steady state, we consider the three cases derived from the literature review in Section 4.3.1. First, does the household learn the true scenario or regime at a steady state? This condition is not applicable in our smooth ambiguity model as we do not impose a single true scenario structurally. At each point in time, either normal growth or recession scenarios can be true thus the household does not learn a single true scenario at the steady state. Consequently, its expectations are not restricted to one scenario. Second, does the household always behave as if the economy will fall into only one scenario? The answer is no. According to the smooth ambiguity preference, the household's expectations consider both scenarios, and so does its behaviour. However, there is an exception when the household is extremely ambiguity averse. In this extreme case, the household always expects the worst-case scenario (Klibanoff et al., 2005; Marinacci, 2015), which is the recession scenario in our model. Finally, since the household does not meet either of these two cases, our smooth ambiguity model's steady state should take both scenarios into account, like regime switching models. However, the regime-switching model uses an empirical or objective probability, while in our model the probability is the subjective belief, a combination of Bayesian beliefs and belief distortions. Therefore, our steady state is conceptually different from that in the regime switching model, so we define it as an ambiguous steady state wherein the household's expectations account for both scenarios, weighted by subjective beliefs.

To summarize, we do not structurally apply a single true scenario in the model, and our household always considers two scenarios based on smooth ambiguity preferences. Therefore, the household's previous experience with these scenarios can still influence their steady state expectations, even in the absence of shock. Consequently, these expectations, subject to normal growth and recession scenarios, influence the economy in an ambiguous steady state.

Next, we analyze the ambiguous steady state and show that it contains the properties of benchmark and worst-case steady states. The ambiguous steady state is mainly determined by ambiguity aversion γ and the long run values of Bayesian belief μ_s and macroeconomic uncertainty M_s. These values also determine the household's subjective beliefs, that is, how much weight they put on each scenario, as well as the expected utility in the recession scenario relative to the normal growth scenario. The three values are incorporated into our model through Euler's equation as a deviation from the benchmark model (see Section 3.4.2). Hence we discuss the ambiguous steady state as a deviation from the benchmark steady state, focusing on the Euler equation and the marginal expected utility of investment, denoted as Λ_s.

We describe the benchmark steady state as the steady state of a benchmark model, where there is no uncertainty (i.e., $M_t = 1$ or $\mu_t = 0$ for all t). We denote the variable in the benchmark steady state as X_s^{NR}, which is similar to the steady state in the standard one-scenario business cycle model.

Ambiguous steady state. The ambiguous steady state differs from the benchmark steady state if $\mu_s \in (0,1)$ and $M_s > 1$. $\mu_s \in (0,1)$ implies the household believes the economy could fall into one of the two scenarios thus its expectations take both scenarios into account. $M_s > 1$ implies that the household's expected utility in the recession scenario is less than the normal growth scenario. Consequently, the marginal expected utility of investment of the ambiguous steady state Λ_s is lower than that of the benchmark steady state Λ_s^{NR} as shown by the steady state Euler's equation (Eq. 4.14).

$$\Lambda_s = \beta\Lambda_s^{NR}(R_s^{NR} + 1 - \delta)\Upsilon_s \left[\mu_s\xi_s^R\frac{1}{M_s} + (1-\mu_s)\xi_s^{NR}\right] \tag{4.14}$$

$$\text{where } \xi_s^R = \frac{M_s^\gamma}{\mu_s M_s^\gamma + (1-\mu_s)}$$

$$\Lambda_s < \Lambda_s^{NR} \text{ since } \mu_s \in (0,1), M_s > 1$$

If M_s, μ_s, or γ increases, while all else remains equal, then Λ_s will decrease. As the marginal expected utility of investment Λ_s declines, the expected return from the investment will decrease, thereby prompting a reduction in investment. This can cause a decrease in capital and output at a steady state. Consequently, an economy that has experienced uncertainty may have a lower steady state output than the benchmark economy. In the following paragraphs, we will discuss two specific cases of ambiguous steady states: the benchmark and the worst case.

Benchmark steady state. The ambiguous steady state is the same as the benchmark steady state if $\mu_s = 0$ or $M_s = 1$[7]. In this situation, the household is certain that there will be no recession once the economy is in a steady state or makes decisions such that their utility in a recession is no different from that of a normal growth scenario. In both cases, uncertainty is irrelevant to the household at the steady state. Thus, the marginal expected utility of investment Λ_s is the same as that in the benchmark model Λ_s^{NR} as seen in Equation 4.15. This implies that the household's decisions at the steady state are robust to uncertainty.

$$\begin{aligned} \Lambda_s &= \beta \Lambda_s^{NR}(R_s^{NR} + 1 - \delta) \\ \Lambda_s &= \Lambda_s^{NR} \end{aligned} \tag{4.15}$$

Worst-case steady state. The ambiguous steady state will be the same as the worst-case steady state if $\mu_s = 1$ or $\gamma \to \infty$. In this case, the subjective belief of recession converges to one. Recall, the subjective belief of recession is:

$$\mu_s \xi_s^R = \mu_s \frac{M_s^\gamma}{\mu_s M_s^\gamma + (1 - \mu_s)} \tag{4.16}$$

If $\mu_s = 1$, $\xi_s^R = 1$ such that $\mu_s \xi_s^R = 1$. If $\gamma \to \infty$, $\xi_s^R \to \frac{1}{\mu_s}$ such that $\mu_s \xi_s^R \to 1$. This implies that the household's expectations only consider the recession scenario at the steady state. Thus, the deviation of the worst-case steady state from the benchmark steady state fully depends on the long run macroeconomic uncertainty M_s as shown in Equation 4.17.

$$\begin{aligned} \Lambda_s &= \beta \Lambda_s^{NR}(R_s^{NR} + 1 - \delta) \frac{1}{M_s} \\ \Lambda_s &< \Lambda_s^{NR} \text{ since } M_s > 1 \end{aligned} \tag{4.17}$$

In conclusion, the ambiguous steady state takes into account the average of the

[7] If $M_t = 1$ ($\mu_t = 0$) for all t, then it follows that $M_s = 1$ ($\mu_s = 0$); however, the reverse does not necessarily hold.

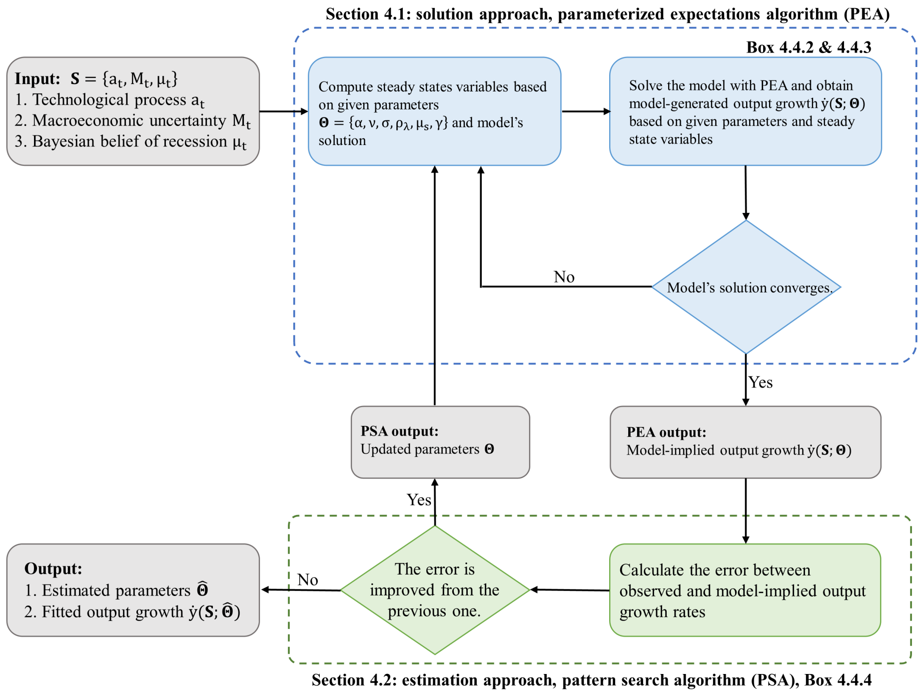
expected utilities between two scenarios, which are determined by the household's subjective beliefs. Depending on these beliefs, the ambiguous steady state can converge to either the benchmark or the worst-case steady state, or remain in between the two. It is important to note that the ambiguous steady state differs from the risky steady state, as the latter takes into account risk, which affects the economy at a second order (Coeurdacier et al., 2011), whereas ambiguity has a first order effect.

4.4 Solution and estimation approaches

We provide in-depth discussion of our solution and estimation methods, which are one of the main contributions of this chapter. The numerical algorithms used to solve and estimate our model are the parameterized expectation algorithm (PEA) and the pattern search algorithm (PSA) respectively. We first describe an overall solution and estimation approaches depicted in Figure 4.2. Then we discuss the solution and estimation methods separately. At the end of this section, we describe the data and setup used for the estimation.

Figure 4.2 summarizes how we implement the parameterized expectation algorithm and pattern search algorithm. We use empirical time series to pin down three variables: a_t, M_t and μ_t, which serve as inputs for the model. The PEA is used to solve the model and to generate model-implied output growth, which are then used as an input for the PSA. The PSA then searches for a set of parameters that minimizes the difference between the model-implied and observed output growth. This procedure provides an estimated set of parameters and corresponding fitted output growth.

Figure 4.2: Solution and estimation flowcharts



4.4.1 Solution approach

The solution used in this paper is an iterative process. It involves PEA to solve the model with the given parameters and initial values of the steady state. The obtained solution is then used to update the steady state values. Subsequently, we utilize this new steady state to again solve the model with PEA. We repeat these steps until the PEA solution reaches the desired level of accuracy. This section describes the parameterized expectation algorithm we use to solve the model, followed by an explanation of how we solve the steady states.

Parameterized expectations algorithm

In this section, we explain why we use PEA to solve to model and describe the algorithm. The standard linearization method is not suitable to solve the model for two reasons. First, the first-order approximation eliminates the concave property of the right

Box 4.4.1: Equilibrium conditions

Household:

$$\lambda_t = \beta E_t \left[\lambda_{t+1}^{NR} \left(\frac{z_{t+1}}{z_t} \right)^{-\sigma} (R_{t+1}^{NR} + 1 - \delta) \right] \Upsilon_t \left[\mu_t \xi_t^R \frac{1}{M_t} + (1 - \mu_t) \xi_t^{NR} \right]$$

$$\lambda_t = c_t^{-\sigma}$$

$$\lambda_t = \frac{\exp((1-\sigma)\bar{a}) L_t^{\nu}}{z_t^{1-\sigma} w_t}$$

Firm:

$$k_{t+1} = (1-\delta) k_t \frac{z_{t-1}}{z_t} + i_t$$

$$y_t = \exp(\alpha \bar{a}) z_{t-1}^{\alpha} k_t^{\alpha} L_t^{1-\alpha}$$

$$w_t = (1-\alpha) \frac{y_t}{L_t}$$

$$R_t = \alpha \frac{y_t}{k_t} \frac{z_t}{z_{t-1}}$$

$$z_t = \exp(a_t - \bar{a})$$

Good market clearing:

$$y_t = c_t + i_t$$

Note: Market clearing conditions also include savings equivalent to the investment and labor supply equivalent to labor demand. a_t, M_t and μ_t are observed from data.

side of the smooth ambiguity Euler equation, which then also eliminates the pessimistic belief distortions, the core mechanism of the model. Bhandari et al. (2023) proposed a perturbation technique to solve a business cycle model with robust preferences, which has similarly disappearing belief distortions with linearization. However, their approach is not used here due to the fact that we have another challenge. The second reason is that we have an underdetermined system with 9 equilibrium equations and 11 variables: $\{\Lambda_t, C_t, I_t, L_t, K_t, Y_t, W_t, R_t, Z_t\}$ and $\{\Lambda_{t+1}^{NR}, R_{t+1}^{NR}\}$. PEA can address these two issues.

To solve the model, we first transform the system of equations into the zero-growth steady state by normalizing the variables: $y_t = \frac{Y_t}{Z_t}$, $i_t = \frac{I_t}{Z_t}$, $k_{t+1} = \frac{K_{t+1}}{Z_t}$, $c_t = \frac{C_t}{Z_t}$, $i_t = \frac{I_t}{Z_t}$, $w_t = \frac{W_t}{Z_t}$, $\lambda_t = \frac{\Lambda_t}{Z_t^{-\sigma}}$, and $z_t = \frac{Z_t}{\exp(\bar{a})}$, where R_t and L_t are assumed to be stationary and $\bar{a}$ is a steady state technological growth. a_t, M_t and μ_t are observed from the data. See Box 4.4.1 for a list of transformed equilibrium conditions. Then we use the parameterized expectation algorithm to approximate the household's conditional expectations with a parametric function. This function includes an interaction component, which captures the concavity of the household's Euler equation. According to Barañano et al. (2002), this approach reproduces the effect of the utility function's curvature more accurately than a log linear approach. We can also determine $\{\lambda_{t+1}^{NR}, R_{t+1}^{NR}\}$ by solving for the household's expectations conditional on the normal growth scenario. We then use this solution to solve the household's expectations with respect to both scenarios. Notably, the PEA solution for the normal growth scenario is identical to that of the benchmark model, which assumes the household makes its decisions with no uncertainty. Next, we explain PEA step-by-step.

The parameterized expectations algorithm is based on the intuition that the household makes decisions that are consistent with its expectations. PEA learns the decision rule in each iteration and finds a solution that is in accordance with the household's expectations. We assume that the household's conditional expectation is based on a mixture of one-period lagged expectations, predetermined variables and observable variables, with ρ_λ representing the weight on the lagged expectation. The lagged expectation acts as a friction in the model since we have not imposed any structural friction in our model[8].

Define a smooth ambiguity parametric function P^{SA} as a combination of observable variables (z_t, M_t, μ_t) and predetermined variables (k_t)[9]. We include the interaction term $M_t\mu_t$ in the P^{SA} function to capture the nonlinear effect of macroeconomic uncertainty.

$$P^{SA}(k_t, z_t, M_t, \mu_t; \theta) = \theta^c + \theta^k k_t + \theta^z z_t + \theta^M M_t + \theta^\mu \mu_t + \theta^{M\mu} M_t \mu_t + u_t \tag{4.18}$$
$$\text{where } \theta = \{\theta^c, \theta^k, \theta^z, \theta^M, \theta^\mu, \theta^{M\mu}\}, E(u_t) = 0$$

The parameterized Euler equation can be written as a combination of lagged expectation λ_{t-1} and the parametric function P^{SA}:

$$\begin{aligned}\lambda_t &= \beta E_t \left[\lambda_{t+1}^{NR} \left(\frac{z_{t+1}}{z_t}\right)^{-\sigma} (R_{t+1}^{NR} + 1 - \delta)\right] \Upsilon_t \left[\mu_t \xi_t^R \frac{1}{M_t} + (1-\mu_t)\xi_t^{NR}\right] \\ &= \rho_\lambda \lambda_{t-1} + (1-\rho_\lambda) E_t(P^{SA}(k_t, z_t, M_t, \mu_t; \theta)) \end{aligned} \tag{4.19}$$

Before solving the model, we still have to address the underdetermined system to pin down $\{\lambda_t^{NR}, R_t^{NR}\}$. To do so, we solve the benchmark model to find a parametric function that represents how the household forms expectations in periods of no uncer-

[8] Smets and Wouters (2007) show that frictions help improve the model's performance in fitting data.
[9] An observable variable is obtained from data. A predetermined variable is fixed at t given all information observed at $t-1$.

tainty, i.e., the normal growth scenario. The parametric function of the benchmark model P^{NR} includes the technological process z_t as an observable variable and capital k_t as a predetermined variable:

$$P^{NR}(k_t, z_t; \theta^{NR}) = \theta^{NR,c} + \theta^{NR,k} k_t^{NR} + \theta^{NR,z} z_t + u_t^{NR} \tag{4.20}$$
$$\text{where } \theta^{NR} = \{\theta^{NR,c}, \theta^{NR,k}, \theta^{NR,z}\}, E(u_t^{NR}) = 0$$

The parameterized Euler equation of the benchmark model or the normal growth scenario can be written as a combination of lagged expectation λ_{t-1}^{NR} and the parametric function P^{NR}:

$$\begin{aligned}\lambda_t^{NR} &= \beta E_t \left[\lambda_{t+1}^{NR} \left(\frac{z_{t+1}}{z_t} \right)^{-\sigma} (R_{t+1}^{NR} + 1 - \delta) \right] \\ &= \rho_\lambda \lambda_{t-1}^{NR} + (1 - \rho_\lambda) E_t(P^{NR}(k_t, z_t; \theta^{NR}))\end{aligned} \tag{4.21}$$

We use the algorithm in Box 4.4.2 to solve for θ^{NR} in P^{NR} of the expectation in the normal growth scenario and the algorithm in Box 4.4.3 to solve for θ in P^{SA} of the smooth ambiguity model. The parameterized expectations algorithm is adapted from Collard (2015) and incorporates the moving bound technique of Maliar and Maliar (2003) to avoid explosive solutions. Before proceeding to the boxes, two points must be noted. First, the initial value of each variable is set to be its steady state value, which we explain how to calculate in the subsequent section. Second, the technological process in the benchmark and smooth ambiguity models is the same. The works of Fernald (2014) and Basu et al. (2006) suggest that the pure technological process is exogenous to the firm and household's decision-making regarding the utilization of capital and labor. Consequently, it is reasonable to assume that the technological process is independent of Bayesian beliefs and macroeconomic uncertainty, implying that the technology does not associate with (cyclical) uncertainty. We therefore can assume that the technological process is the same in both the benchmark and smooth ambiguity models.

Box 4.4.2: Parameterized expectations algorithm to solve for θ^{NR}

1. Set an initial guess for $\theta^{NR} = \{1, 0, 0\}$ and $S = \{a_t\}_{t=1}^{T}$ is observed from data.
2. At iteration i and for the given θ_i^{NR}, generate (1) $\{\lambda_t^{NR}\}_{t=1}^{T}$ using $\lambda_t^{NR} = \rho_\lambda \lambda_{t-1}^{NR} + (1 - \rho_\lambda) P^{NR}(k_t^{NR}, z_t; \theta_i^{NR})$ where ρ_λ is the weight on the lagged expectation and $z_t = \exp(a_t)$, and (2) $\{c_t^{NR}, i_t^{NR}, w_t^{NR}, R_t^{NR}, L_t^{NR}, y_t^{NR}, k_{t+1}^{NR}\}_{t=1}^{T}$ using the equilibrium conditions except the Euler equation
3. Let $X(\theta_i^{NR}) = \{\lambda_t^{NR}, c_t^{NR}, i_t^{NR}, w_t^{NR}, R_t^{NR}, L_t^{NR}, y_t^{NR}, k_{t+1}^{NR}; \theta_i^{NR}\}_{t=1}^{T}$ and for given upper and lower bounds, $\bar{X}_i$ and $\underline{X}_i$,
   - Set $X(\theta_i^{NR}) = \bar{X}_i$ if any element in $X(\theta_i^{NR}) > \bar{X}_i$ and
   - Set $X(\theta_i^{NR}) = \underline{X}_i$ if any element in $X_i(\theta_i^{NR}) < \underline{X}_i$
4. Generate $\{\hat{\lambda}_t^{NR}\}_{t=1}^{T-1}$ using $\hat{\lambda}_t^{NR} = \beta \left(\lambda_{t+1}^{NR} \left(\frac{z_{t+1}}{z_t} \right)^{-\sigma} (R_{t+1}^{NR} + 1 - \delta) \right)$
5. Obtain $\hat{\theta}_{i+1}^{NR}$ by regressing $\{ \frac{\hat{\lambda}_t^{NR} - \rho_\lambda \lambda_{t-1}^{NR}}{1-\rho_\lambda} \}_{t=1}^{T}$ against $\{1, k_t^{NR}, z_t\}_{t=1}^{T}$ such that:
$$\frac{\hat{\lambda}_t^{NR} - \rho_\lambda \lambda_{t-1}^{NR}}{1 - \rho_\lambda} = \theta^{NR,c} + \theta^{NR,k} k_t + \theta^{NR,z} z_t + u_t^{NR} \text{ where } E(u_t^{NR}) = 0$$
6. Update $\theta_{i+1}^{NR} = \omega \hat{\theta}_{i+1}^{NR} + (1 - \omega)\theta_i^{NR}; \omega = 0.5$ and if any variable hits the bounds in step 3, expand the bounds for the next iteration according to the following formula:
$$\bar{X}_{i+1} = X_s^{NR}(1 + \Delta_i)$$
$$\underline{X}_{i+1} = X_s^{NR}(1 - \Delta_i)$$
where $\Delta_i = 0.05 + 0.01i$, $X_s^{NR} =$ steady state values of variables in X
7. Go back to step 2 and iterate until $\left| \frac{\theta_i^{NR} - \theta_{i-1}^{NR}}{\theta_{i-1}^{NR}} \right| < 10^{-6}$ and no variable hits the bounds

Box 4.4.3: Parameterized expectations algorithm to solve for θ

1. Set an initial guess for $\theta = \{1, 0, 0, 0, 0, 0\}$ and $S = \{a_t, M_t, \mu_t\}_{t=1}^{T}$ is observed from data.
2. At iteration i, for the given θ_i, generate (1) $\{\lambda_t\}_{t=1}^{T}$ using $\lambda_t = \rho_\lambda \lambda_{t-1} + (1 - \rho_\lambda) P^{SA}(k_t, z_t, M_t, \mu_t; \theta_i)$ where ρ_λ is the weight on the lagged expectation and $z_t = \exp(a_t)$, and (2) $\{c_t, i_t, w_t, R_t, L_t, y_t, \xi_t^R, \xi_t^{NR}, \Upsilon_t, k_{t+1}\}_{t=1}^{T}$ using the equilibrium conditions except for the Euler equation
3. Let $X(\theta_i) = \{\lambda_t, c_t, i_t, w_t, R_t, L_t, y_t, \xi_t^R, \xi_t^{NR}, \Upsilon_t, k_{t+1}, \mu_{t+1}; \theta_i\}_{t=1}^{T}$ and for given upper and lower bounds, $\bar{X}_i$ and $\underline{X}_i$,
   - Set $X(\theta_i) = \bar{X}_i$ if any element in $X(\theta_i) > \bar{X}_i$ and
   - Set $X(\theta_i) = \underline{X}_i$ if any element in $X_i(\theta_i) < \underline{X}_i$
4. Given the θ^{NR} obtained from the PEA in the benchmark model, generate $\{\lambda_t^{NR}, R_t^{NR}\}_{t=1}^{T}$ using $P^{NR}(k_t, z_t; \theta^{NR})$. We use k_t instead of k_t^{NR} because the household forms an expectation given that capital is predetermined. Thus for the smooth ambiguity household, k_t^{NR} is not a predetermined variable but rather the expected capital in the normal growth scenario.
5. Generate $\{\hat{\lambda}_t\}_{t=1}^{T-1}$ where $\hat{\lambda}_t = \beta \left[\lambda_{t+1}^{NR} \left(\frac{z_{t+1}}{z_t} \right)^{-\sigma} (R_{t+1}^{NR} + 1 - \delta) \right] \Upsilon_t \left[\mu_t \xi_t^R \frac{1}{M_t} + (1 - \mu_t)\xi_t^{NR} \right]$
6. Obtain $\hat{\theta}_{i+1}$ by regressing $\{ \frac{\hat{\lambda}_t - \rho_\lambda \lambda_{t-1}}{1-\rho_\lambda} \}_{t=1}^{T}$ against $\{1, k_t, z_t, M_t, \mu_t, M_t\mu_t\}_{t=T_{\text{begin}}}^{T-1}$ such that:
$$\frac{\hat{\lambda}_t - \rho_\lambda \lambda_{t-1}}{1 - \rho_\lambda} = \theta^c + \theta^k k_t + \theta^z z_t + \theta^M M_t + \theta^\mu \mu_t + \theta^{M\mu} M_t \mu_t + u_t \text{ where } E(u_t) = 0$$
7. Update $\theta_{i+1} = \omega \hat{\theta}_{i+1} + (1 - \omega)\theta_i; \omega = 0.5$ and if any variable hits the bounds in step 3, expand the bounds for the next iteration according to the following formula:
$$\bar{X}_{i+1} = X_s(1 + \Delta_i)$$
$$\underline{X}_{i+1} = X_s(1 - \Delta_i)$$
where $\Delta_i = 0.05 + 0.01i$, $X_s =$ steady state values of variables in X
8. Go back to step 2 and iterate until $\left| \frac{\theta_i - \theta_{i-1}}{\theta_{i-1}} \right| < 10^{-6}$ and no variable hits the bounds

Steady states

The benchmark and ambiguous steady states are determined. We first calculate 9 steady state values of the benchmark model $\{\lambda_s^{NR}, c_s^{NR}, i_s^{NR}, L_s^{NR}, k_s^{NR}, y_s^{NR}, w_s^{NR}, R_s^{NR}, z_s^{NR}\}$. To do this, we solve the following system of equations[10]:

$$R_s^{NR} = \frac{1}{\beta} + \delta - 1 \tag{4.22}$$

$$\lambda_s^{NR} = \frac{\exp((1-\sigma)\bar{a})(L^{NR})_s^{\nu}}{w_s} \tag{4.23}$$

$$\lambda_s^{NR} = (c_s^{NR})^{-\sigma} \tag{4.24}$$

$$y_s^{NR} = \exp(\alpha\bar{a})(k_s^{NR})^{\alpha}(L_s^{NR})^{1-\alpha} \tag{4.25}$$

$$w_s^{NR} = (1-\alpha)\frac{y_s^{NR}}{L_s^{NR}} \tag{4.26}$$

$$R_s^{NR} = \alpha\frac{y_s^{NR}}{k_s^{NR}} \tag{4.27}$$

$$i_s^{NR} = \delta k_s^{NR} \tag{4.28}$$

$$y_s^{NR} = c_s^{NR} + i_s^{NR} \tag{4.29}$$

$$z_s^{NR} = 1 \tag{4.30}$$

For the smooth ambiguity model, there are 11 steady state values $\{\lambda_s, c_s, i_s, L_s, k_s, y_s, w_s, R_s, z_s, M_s, \mu_s\}$ that satisfy the original and parameterized Euler equations, as shown in Equations 4.31a and 4.31b respectively as well as equilibrium conditions which are analogous to those of Equations 4.23 to 4.30.

$$\lambda_s = \beta\left[\lambda_s^{NR}(R_s^{NR} + 1 - \delta)\right]\Upsilon_s\left[\mu_s\xi_s^R\frac{1}{M_s} + (1-\mu_s)\xi_s^{NR}\right] \tag{4.31a}$$

$$\lambda_s = \rho_\lambda\lambda_s + (1-\rho_\lambda)(\theta^c + \theta^k k_s + \theta^z z_s + \theta^M M_s + \theta^\mu \mu_s + \theta^{M\mu} M_s\mu_s) \tag{4.31b}$$

$$\text{where } \Upsilon_s = \frac{\mu_s M_s^{\gamma} + (1-\mu_s)}{\left(\mu_s M_s^{\gamma-1} + (1-\mu_s)\right)^{\frac{-\gamma}{1-\gamma}}}$$

$$\xi_s^R = \frac{M_s^{\gamma}}{\mu_s M_s^{\gamma} + (1-\mu_s)}$$

$$\xi_s^{NR} = \frac{1}{\mu_s M_s^{\gamma} + (1-\mu_s)}$$

λ_s^{NR}, R_s^{NR} : the steady state values in the benchmark model

[10] We can use the parameterized Euler equation 4.21 instead of the original Euler equation 4.23 to calculate the steady state values of the benchmark model. The values are not significantly different.

In the PEA, we set the $t = 0$ value of each variable to its steady state value and solve the model to obtain θ. As an initial step, we use 9 steady state values of the benchmark model and calculate the steady state Bayesian belief μ_s and M_s from external sources, such as empirical data. Once we obtain θ, we compute the steady state of the smooth ambiguity model again. Note that, we have 10 steady state equations to calculate the 11 steady state variables. Thus, we must pin down either M_s or μ_s using external sources. In this paper, we estimate μ_s from data while calculating M_s from the equations.

The difference between benchmark and ambiguous steady states is in their respective Euler equations. For example, in the benchmark steady state, the rental rate R_s^{NR} is directly determined by the discount rate β and the depreciation rate δ. However, in the case of an ambiguous steady state, the expected marginal utility of investment λ_s is reduced due to the presence of macroeconomic uncertainty M_s and Bayesian belief μ_s, leading to a decrease in investment and capital. Consequently, this results in an increase in the rental rate at the steady state.

4.4.2 Estimation approach

In this section, we describe the estimation method. Our estimation is a nonlinear least squares method (NLS) which minimizes the distance between the model-implied and empirical output growth rates. We define the model-implied output growth as $\dot{y}(S_t;\Omega) = \log\left(\frac{y(S_t;\Omega)}{y(S_{t-1};\Omega)}\right)$, where S_t is a set of observable variables at time t and $\Omega = \{\alpha, \nu, \sigma, \rho_\lambda, \mu_s, \gamma\}$ be a set of parameters to be estimated. The model is fitted with $\dot{y}_t^{\text{obs}}$, the observed output growth rate. We use a pattern search algorithm to find the set of estimated parameters $\hat{\Omega}$ that minimizes the root mean square errors (RMSE) between these two variables:

$$\text{RMSE}(\hat{\Omega}) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\dot{y}(S_t;\hat{\Omega}) - \dot{y}_t^{\text{obs}}\right)^2}$$

Additionally, we employ the delta method to compute the asymptotic standard errors of the estimated parameters. Suppose that Ω is the set of true parameters,

$$\begin{aligned}
\dot{y}(S_t;\hat{\Omega}) &\approx \dot{y}(S_t;\Omega) + \nabla\dot{y}(S_t;\Omega)(\hat{\Omega}-\Omega) \\
\hat{\Omega}-\Omega &\approx (\dot{y}(S_t;\hat{\Omega}) - \dot{y}(S_t;\Omega)) \left(\nabla\dot{y}(S_t;\hat{\Omega})\right)^{-1} \\
\hat{\Omega}-\Omega &\longrightarrow_d \mathcal{N}(0,\hat{\sigma}^2 F'F)
\end{aligned}$$

where $\hat{\sigma} = RMSE(\hat{\Omega})$, $F = \left(\nabla\dot{y}(S_t;\hat{\Omega})\right)^{-1}$ and $\nabla\dot{y}(S_t;\hat{\Omega})$ is a gradient matrix with respect to $\hat{\Omega}$[11].

The NLS approach, which is also used in Carroll et al. (2019), has the advantage of allowing us to use the observable data as the target which the estimated model seeks to fit. Thus, any failure to match the target can be used to study the limitations of the model and derive a useful economic explanation. In contrast, maximum likelihood estimation used in most macroeconomic models relies on linearization around a steady state in combination with Bayesian estimation to maximize data density. Since the true likelihood is unknown, it is difficult to interpret the gap between the estimated model and the target. Other estimation targets are methods of moments (Duffie & Singleton, 1993; Kim & Ruge-Murcia, 2009) and indirect inference (Guerron-Quintana et al., 2017; Theodoridis, 2011). The methods of moment estimator seeks to minimize the gap between model-implied and data-implied moments, while indirect inference focuses on fitting the impulse response computed from the data. For the robustness check, we provide the results of these estimators in Section 4.6.

In the context of the nonlinear least squares method, we briefly discuss the econometric issues associated with estimating DSGE models: weak identification, stochastic singularity, and small-sample distortion as pointed out by Ruge-Murcia (2007). NLS generally suffers from weak identification less than other estimators since it does not require the selection of moments or likelihood to estimate the model. Furthermore, stochastic singularity is relevant to linearized DSGE models (Ruge-Murcia, 2007) thus it does not apply in our estimation as we do not linearize the model. Lastly, small-sample distortion

[11] We compute the gradient matrix numerically using $\nabla\dot{y}(S_t;\Omega) = \frac{\dot{y}(S_t;\hat{\Omega}+h)-\dot{y}(S_t;\hat{\Omega}-h)}{2h}$ where $h = \max\{10^{-7},\hat{\Omega}10^{-7}\}$

leads to a discrepancy between the asymptotic standard errors and the actual variability of the estimated parameters, which affects the significance of the estimated parameters. Ruge-Murcia (2007) demonstrates that all three estimations methods (maximum likelihood, methods of moments, indirect inference) suffer from small-sample distortion to varying degrees. In particular, maximum likelihood estimation tends to produce asymptotic standard errors that are larger than the actual standard errors for all estimated parameters. This is expected to be the case for NLS as well, given the similarity in asymptotic properties between NLS and ML under the Gaussian distribution.

We use the pattern search algorithm[12] to search for the set of parameters that minimize the distance between the model-implied and observed output growth rate. To illustrate how the pattern search algorithm works, we provide an example with two parameters: $\Omega = \{\gamma, \sigma\}$ in Box 4.4.4.

Box 4.4.4: Pattern search algorithm - an example of two parameters

Let S_t be observed variables.

1. Set an initial guess of the parameter to $\Omega_0 = \{\gamma_0, \sigma_0\}$ and the initial updating interval to $m_1 = 0.2$. The upper bound is set to $\bar{\Omega} = \{\bar{\gamma}, \bar{\sigma}\}$ and the lower bound is set to $\underline{\Omega} = \{\underline{\gamma}, \underline{\sigma}\}$. Rescale the parameters using $x(\Omega_0) = \frac{\Omega_0 - \underline{\Omega}}{\bar{\Omega} - \underline{\Omega}}$ such that $0 \leq x(\Omega_0) \leq 1$.
2. At iteration i, compute the four pairs of updated parameters $x(\Omega_i)$ as following:
$$\begin{pmatrix} \mathrm{x}(\Omega_{i-1}) \\ \mathrm{x}(\Omega_{i-1}) \\ \mathrm{x}(\Omega_{i-1}) \\ \mathrm{x}(\Omega_{i-1}) \end{pmatrix} + \begin{pmatrix} \mathrm{m}_i & 0 \\ 0 & \mathrm{m}_i \\ \text{-}\mathrm{m}_i & 0 \\ 0 & \text{-}\mathrm{m}_i \end{pmatrix} = \begin{pmatrix} \mathrm{x}_1(\Omega_i) \\ \mathrm{x}_2(\Omega_i) \\ \mathrm{x}_3(\Omega_i) \\ \mathrm{x}_4(\Omega_i) \end{pmatrix}$$
If any element is less than 0, we set it to 0 or if any element is more than 1, we set it to 1.
3. For each pair, revert the $x(\Omega_i)$ parameters back to their original scales using $\Omega_i = x(\Omega_i)(\bar{\Omega} - \underline{\Omega}) + \underline{\Omega}$ and solve the model with PEA. Then, compute model-implied output growth $\dot{y}(S_t; \Omega_i)$ rate and RMSE(Ω_i) using:
$$\dot{y}(S_t; \Omega_i) = \log y(S_t; \Omega_i) - \log y(S_{t-1}; \Omega_i)$$
$$\text{RMSE}(\Omega_i) = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \left(\dot{y}(S_t; \Omega) - \dot{y}_t^{\text{obs}}\right)^2}$$
4. If any $x(\Omega_i)$ pair yields RMSE that is lower than or equals the RMSE(Ω_{i-1}):
   - Set the new parameter $x(\Omega_i)$ to the pair that generates the lowest RMSE
   - Expand the updating interval by setting $m_{i+1} = m_i \times 2$

   If no $x(\Omega_i)$ pair yields RMSE that is lower than RMSE(Ω_{i-1}):
   - Set the new parameter $x(\Omega_i)$ to $x(\Omega_{i-1})$
   - Shrink the updating interval by setting $m_{i+1} = m_i \times 0.5$
5. Go back to Step 2 and iterate until $|x(\Omega_i) - x(\Omega_{i-1})| < 10^{-6}$ or $m_i < 10^{-6}$
6. After the patternsearch algorithm, we run Nelder-Mead simplex algorithm[a] used in Carroll et al. (2019), to ensure that we have obtained the local minimum.
7. Set the solution $\hat{\Omega}$ as a new initial value of the parameter and iterate until the parameter values converge, with a tolerance of 10^{-6}.

[a] This is *fminsearch* function in Matlab. See Lagarias et al. (1998) for detail.

[12] This is *patternsearch* function in *MATLAB*.

Data

We use three empirical time series as the model inputs. The TFP growth a_t is utilization-adjusted technological growth of US and Europe from Fernald (2014) and Comin et al. (2020) respectively. These TFP series are measured by a comparable methodology (Fernald, 2014) and are suitable for our model since it is assumed to measure 'pure technology' and is thus exogenous to the business cycle. Macroeconomic uncertainty M_t is the Economics Policy Uncertainty index of respective countries from Baker et al. (2016), log-scaled to reduce volatility and divided by its minimum such that it is greater than one. Bayesian belief of recession μ_t is the recession probabilities computed from the survey of professional forecasters (SPF). We use it as a proxy for the Bayesian belief of recession, assuming that professional forecasters are Bayesian on average. The recession probability of the US is for the next quarter while that of the European countries is for the next calendar year. As individual country recession probabilities in Europe are limited, the EU recession probability is used to estimate all European models.

The estimation is conducted over the period of 1985Q1 to 2019Q4 for the US and 1999Q1 to 2018Q4 for Europe, with the exception of Spain, whose Economic Policy Uncertainty index began in 2001Q1. A list of data sources used is provided in Appendix 4.A.

Table 4.1 reports the correlations between the empirical time series. Macroeconomic uncertainty and SPF-implied Bayesian belief of recession are positively correlated except for Spain. The correlation is higher in the US than in the European countries, most likely due to the fact that the SPF-implied Bayesian belief of the US is country-specific while that in Europe is not. Moreover, technological progress is found to be uncorrelated with macroeconomic uncertainty and recession probabilities, indicating that total factor productivity is independent of uncertainty. Two exceptions are Germany and Spain, where total factor productivity is positively correlated with the SPF-implied Bayesian belief. This correlation is driven by a single data point in 2009Q2 when SPF-implied Bayesian belief of recession was 37% and total factor productivity growth in Germany and Spain spiked to 3.40% and 1.48%, respectively. Excluding this data point, the correlation becomes insignificant.

Table 4.1: Correlations of the empirical time series used in estimations

	US		DE		IT		FR		ES	
	M_t	μ_t	M_t	μ_t	M_t	μ_t	M_t	μ_t	M_t	μ_t
M_t	1		1		1		1		1	
μ_t	0.4756***	1	0.2510**	1	0.2476**	1	0.2601**	1	-0.0138	1
a_t	0.0153	0.1394	0.1186	0.2963***	-0.0624	-0.0250	-0.0906	-0.1029	0.0650	0.2056*

Note: US: United States, DE: Germany, IT: Italy, FR: France, ES: Spain. ***: p-value $<$ 1%. ** $<$5%, * $<$10%

Estimation setup

We estimate six parameters: $\Omega = \{\alpha, \nu, \sigma, \rho_\lambda, \mu_s, \gamma\}$. The initial parameter values and bounds are summarized in Table 4.2. The initial value of capital share is set to $\alpha = 0.3$. We tried estimating the capital share in the European countries but the estimated values are unreasonably low. We think that this could be due to the fact that the recession probability of each European country is unavailable individually. Therefore, we fix the parameter of capital share to the average capital income share to provide more information to the European models. We compute the capital share income using 1 - average labor share income and get: DE=0.38, IT=0.40, FR=0.38, and ES=0.39. The initial value of risk aversion is set to the standard value of $\sigma = 2$. The labor disutility parameter is set to $\nu = 1.5$ based on estimates of the Frisch elasticity [13], which is between 0 to 0.5 in micro level data and 2 to 4 in macro level data (Peterman, 2016). The initial value of ρ_λ is arbitrarily set to 0.5, and the initial value for the steady state Bayesian belief is set to the average of the recession probability from the survey of professional forecasters.

There is inconclusive evidence regarding the magnitude of ambiguity aversion. Altug et al. (2020) and Backus et al. (2015) use an ambiguity aversion value ranging from 5 to 50 in their quantitative exercises. Collard et al. (2018) calibrate an ambiguity aversion parameter between 6.65 and 17.75 to fit with a risk-free rate. Gallant et al. (2019) estimate a coefficient of ambiguity aversion to be between 6.96 and 23.37 in their consumption-based asset pricing models. As such, we experiment with four initial values of ambiguity aversion, namely 0, 5, 10, and 20, and report the results that have the lowest root mean squares errors.

The remaining three parameters are fixed commonly used in the literature (Fernández-

[13]Since the labor supply is not a main focus of this paper, we keep the parameter related to labor simple. Parameter ν in our model is close to the reciprocal of Frisch elasticity, although the formulation of Frisch elasticity in macroeconomic models is varied.

Table 4.2: Initial parameters for estimations

Parameter	Description	Initial value	Bound [lower, upper]
α	capital share	0.3	[0,1]
ν	labor disutility	1.5	[0,20]
σ	risk aversion	2	[0,20]
ρ_λ	weight on the lagged expectations	0.5	[0,1]
μ_s	SS Bayesian belief of recession	average of SPF	[0,1]
γ	ambiguity aversion	0,5,10,20	[0,40]

Note: For the estimations in European countries, we fix α to the average capital share income of respective countries: DE=0.38306, IT=0.40488, FR=0.38256, and ES=0.39256. SS means steady state.

Villaverde & Guerrón-Quintana, 2020; Slobodyan & Wouters, 2012; Smets & Wouters, 2007): the quarterly depreciation rate is $\delta = 0.025$, the discount factor $\beta = 0.99$, and the average TFP growth $\bar{a}$ is the average value of the empirical TFP growth.

4.5 Empirical results

In this section, we present the estimation results of the smooth ambiguity model. We first discuss the estimations of US output growth, which includes the analysis of estimated parameters, model's performance, out-of-sample forecasts, and the time variations of attitudes toward risk and ambiguity. Then, we present the cross country's estimations of ambiguity averse models in selected European countries: Germany, Italy, Spain, and France.

4.5.1 US estimation results

This section compares the estimation of US across three models: the benchmark model (BM), the ambiguity neutral model (AN), and the ambiguity averse model (AA). The BM allows only the total factor productivity shocks and ignores uncertainty. The AN incorporates two shocks from TFP and macroeconomic uncertainty, while assuming that the household is ambiguity neutral ($\gamma = 0$). The AA allows for the same shocks as the AN, and the household is allowed to be ambiguity averse ($\gamma \geq 0$); the parameter of ambiguity aversion is estimated in the AA.

Table 4.3 presents the estimated parameters, the values of steady state and the

Table 4.3: Estimation results of the US output growth

Estimated parameter	Description	BM	AN	AA
α	capital share	0.34	0.29	0.34
		(0.08)	(0.06)	(0.12)
ν	labor disutility	7.14	4.006	6.44
		(4.85)	(0.84)	(3.43)
σ	risk aversion	0.42	0.49	0.43
		(0.41)	(0.17)	(0.11)
ρ_λ	weight of the lagged expectations	0.86	0.54	0.08
		(0.06)	(0.22)	(0.59)
μ_s	SS Bayesian belief of recession		0.0007	1.00
			(1.18)	(0.54)
γ	ambiguity aversion			3.35
				(2.28)
Steady state	Description	BM	AN	AA
i_s/y_s	share of investment in output	0.24	0.20	0.22
c_s/l_s	ratio of consumption to labor	2.49	1.85	2.38
M_s	macroeconomic uncertainty		1.00	1.00
$\mu_s\xi_s^R$	subjective belief of recession		0.0007	1.00
RMSE	Periods	BM	AN	AA
	all periods	0.53%	0.42%	0.41%
	recession periods	1.15%	0.42%	0.41%
	normal growth periods	0.43%	0.42%	0.41%

Note: All models were estimated using the parameterized expectations algorithm and pattern search algorithm described in Section 4.4. BM stands for Benchmark model, AN is Ambiguity neutral model where γ is fixed to 0. AA is the ambiguity averse model where γ is estimated. SS means steady state and RMSE stands for root mean square error. The standard error of the estimated parameter is in (...).

root mean square errors. The asymptotic standard error (ASE) is shown in the parentheses. A small standard error implies that a change in the parameter around its estimated values leads to a large increase in the RMSE, indicating that the objective function is highly convex around the estimated value. As we pointed out in Section 4.4.1, the ASE of nonlinear least squares could be subject to small-sample distortions, so it should be interpreted with caution. Our estimation analysis is divided into four parts: estimated parameters, steady states, model fit, and out-of-sample forecast.

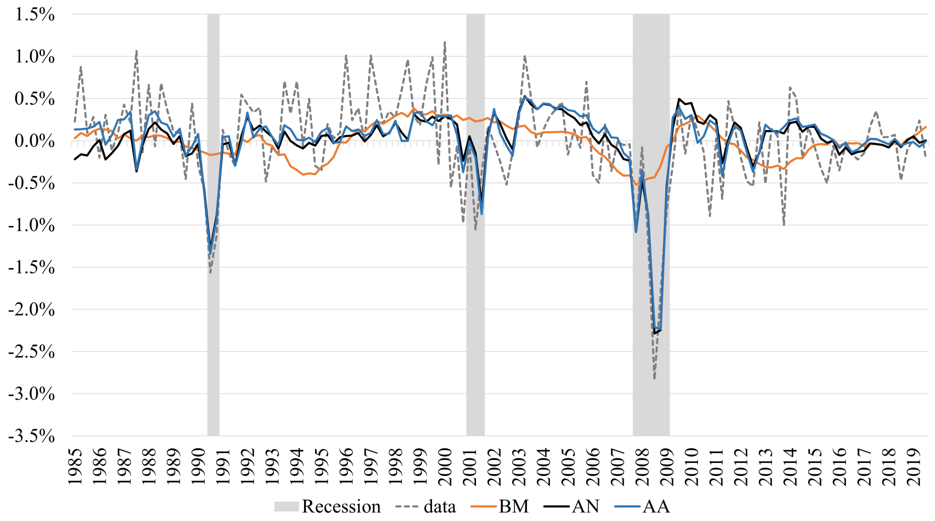
Parameters. The estimated capital share ranges between 0.29 to 0.34 which is close to the capital share income in the US (0.36 - 0.41). However, the estimated risk aversion is between 0.42 to 0.49, significantly lower than the standard values (1 - 2) found in the literature on the business cycle model. This discrepancy could be due to two reasons. First, we minimize the RMSEs, as opposed to maximizing the likelihood in other studies (Fernández-Villaverde & Guerrón-Quintana, 2020; Slobodyan & Wouters, 2012; Smets &

Wouters, 2007). In Section 4.6.2, we show that the maximum likelihood estimation with the Bayesian technique obtains a risk aversion value closer to the standard value. Second, an intuitive explanation is that the inclusion of ambiguity aversion may affect the estimated degree of risk aversion. Comparing smooth ambiguity models with ambiguity aversion against those without ambiguity aversion[14], Gallant et al. (2019) report the estimated risk aversion to be lower when ambiguity aversion is present. This finding implies that ambiguity aversion may partly take the role of risk aversion in determining households' responses to uncertainty. However, the second reason does not explain a low level of risk aversion in the benchmark model (BM). Thus, we believe that the difference in the risk aversion estimates between our paper and a standard value (1-2) is mainly due to the solution and estimation technique used. However, there is no definitive evidence or consensus on the best method, so we also present results from other techniques in Section 4.6.

When comparing the estimated parameters across three models, we find that uncertainty and ambiguity aversion have a substantial impact on the weight of lagged expectations ρ_λ and labor disutility ν. These parameters primarily measure friction in the economy. In the benchmark model, the parameter for the weight of lagged expectations is 0.86, whereas it is 0.54 in the ambiguity neutral model (AN). This implies that the expectations of these households are mainly driven by past information, resulting in large frictions in the economies. Conversely, ρ_λ in the ambiguity averse model (AA) is only 0.08, not significantly different from 0. It implies that the ambiguity averse household mainly uses current information to form its expectations and that the economic fluctuations in the ambiguity averse model are generated endogenously, in contrast to being autoregressive. However, this does not mean that the ambiguity averse model has less friction than other models. The estimated labor disutility parameter in the ambiguity averse model is larger than that in the ambiguity neutral model. A greater labor disutility parameter indicates heightened sensitivity to changes in labor supply, which leads to less volatile labor supply and, consequently, more friction in the economy. As illustrated in Figure 4.3, the fitted output growth in the benchmark model (orange line) is significantly smoother than that of the smooth ambiguity models, due to a large ρ_λ and ν.

[14] A smooth ambiguity model without ambiguity aversion is equivalent to an Epstein and Zin model

Figure 4.3: Fitted time series of the US quarterly real output growth

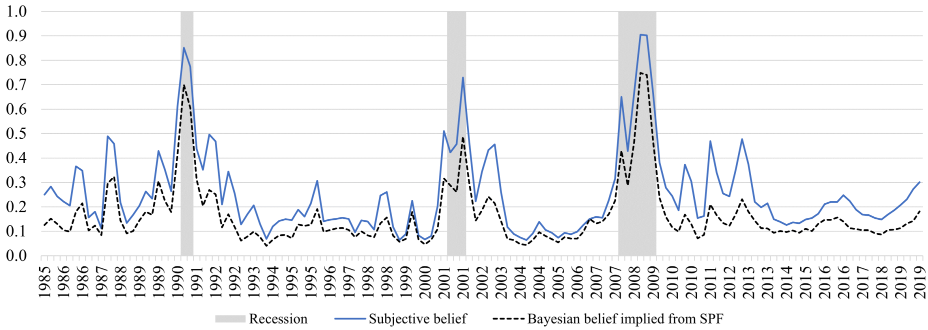


Note: The results of Table 4.3 are illustrated through the solid lines, which represent the fitted real GDP growth. The dashed line represents the actual quarterly output growth minus the mean. The models are as follows: BM - Benchmark model; AN - Ambiguity Neutral model; AA - Ambiguity Averse model.

The ambiguity aversion of the US representative household is estimated to be 3.35, indicating that the household has a pessimistic belief distortion towards a recession scenario $\xi_t^R \geq 1$. This results in a subjective belief of recession probability that is higher than the SPF-implied Bayesian belief. Figure 4.4 compares the Bayesian beliefs computed from the SPF-implied recession probability and the model-implied subjective beliefs of recession. To calculate the subjective belief, we use the formula in Equation 4.6 given that ambiguity aversion $\gamma = 3.3481$. For instance, in the fourth quarter of 2008, the SPF-implied Bayesian belief of recession μ_t was 75% and macroeconomic uncertainty M_t was 1.42, resulting in a model-implied subjective belief $\mu_t \xi_t^R$ of 90.45%. This can be interpreted that the household believes there is a 75% chance of recession occurring in the next quarter, however, due to ambiguity aversion, it behaves as if the probability is 90.45%. The asymptotic standard error of ambiguity aversion is 2.28, implying that the level of ambiguity aversion is not significantly different from zero. This is in line with the small difference of RMSEs between AN and AA models.

Steady state. Surprisingly, all smooth ambiguity models have one-scenario steady states but these have different implications. The ambiguity neutral model has a steady

Figure 4.4: SPF-implied Bayesian belief and model-implied subjective belief of recessions



Note: The Bayesian belief of recession (dashed line) is proxied by the next-quarter recession probability of the US professional forecasters. The model-implied subjective belief of recession is calculated with ambiguity aversion $\gamma = 3.3481$.

state Bayesian belief of 0.07%, and a steady state macroeconomic uncertainty of 1.00. This indicates that the household believes there is only a 0.07% chance of recession. Furthermore, this small probability does not have any effect, as the household is almost indifferent between the two scenarios ($M_s = 1.00$). In contrast, the ambiguity averse model has a worst-case steady state where Bayesian belief and macroeconomic uncertainty are 1. This implies that the household's expectation of a steady state is anchored to the worst-case scenario, the recession scenario. However, the household is indifferent between the normal growth and recession scenarios ($M_s = 1.00$) thus macroeconomic uncertainty does not impact the steady state decision. These results suggest that the US representative household makes decisions such that its expected utility is robust to uncertainty at the steady state[15].

The asymptotic standard errors of steady state Bayesian belief are large for both AN and AA models, indicating that RMSE is insensitive to changes in the estimated μ_s. The steady state Bayesian belief of recession in the ambiguity averse model is only weakly significant at a 10% confidence level. This implies that the household's steady state belief does not have a significant effect on economic fluctuation outside of the steady state.

Uncertainty and ambiguity aversion can have indirect effects on the steady state through other parameters. For instance, uncertainty can reduce the expected return from investment, thus discouraging the household from investing. As Table 4.3 shows, the

[15]We check whether the one-scenario steady states are the outcome of the estimation setup. Instead of estimating μ_s and computing M_s, we estimate M_s and calculate μ_s. The results of the one-scenario steady state still hold with $\mu_s = 0$ and $M_s > 1$.

steady state share of investment in output in the benchmark model (0.24) is larger than that in the smooth ambiguity models (0.20-0.22). Additionally, ambiguity aversion can lead the household to prioritize its current utility (consuming more and working less). Consequently, the ratio of consumption to labor in the ambiguity averse model is 2.38, which is higher than the ratio of 1.85 observed in the ambiguity neutral model.

Model fit. The smooth ambiguity models clearly outperform the benchmark model in terms of data fitting. The RMSEs of the smooth ambiguity models are markedly lower than the BM, particularly in recession periods. Moreover, the RMSE of the ambiguity averse model is marginally better than that of the ambiguity neutral model in both recession and normal growth periods. These results suggest that adding uncertainty helps significantly improve data fitting for the US. This is further highlighted in Figure 4.3, which clearly demonstrates the distinction between the benchmark model (orange line) and smooth ambiguity models (black and blue lines).

Table 4.4: Out-of-sample forecast performance

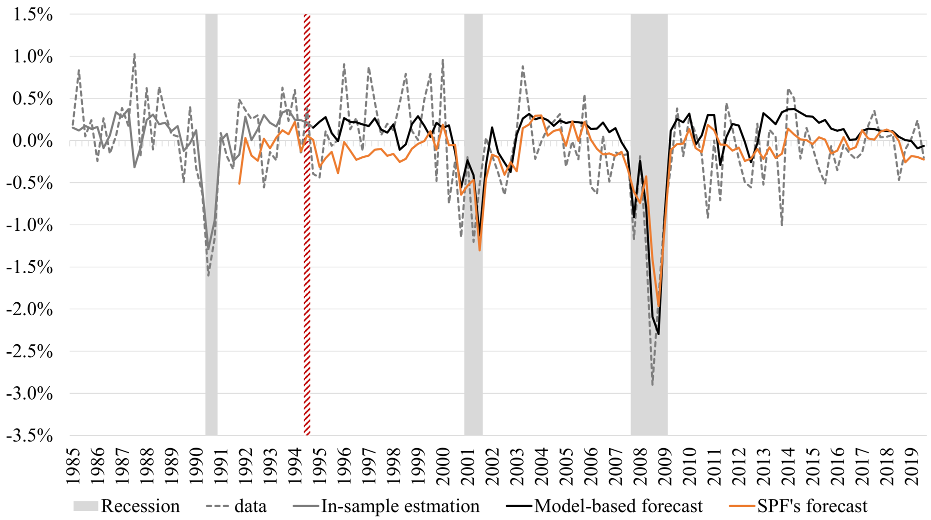
RMSEs	SPF	Model
All periods	0.44%	0.44%
Recession periods	0.68%	0.49%
Normal growth periods	0.40%	0.43%

Out-of-sample forecast. The good fit of our model is reflected in the relatively accurate out-of-sample forecast. Table 4.4 reports the RMSEs and Figure 4.5 depicts the out-of-sample forecasts of US output growth, generated from our model and from the survey of US professional forecasters[16]. To forecast the output growth at time t, we estimate the model up until time $t-1$ and use the Economic Policy Uncertainty Index and the SPF-implied recession probability at time t to simulate the output growth at time t. We excluded the Fernald (2014)'s technological progress data from the predictions as this time series is constructed ex-post the release of GDP. We use the first 10 years (1985Q1 -1994Q4) as a calibrating period and recursively estimate the model to forecast each quarter from 1995Q1 until 2019Q4.

Our model-based forecast was comparable to the SPF's forecast, with an overall

[16]To measure the quarterly GDP growth forecasts from the Survey of Professional Forecasters (SPF), we calculate the log difference between the average GDP level forecast for the current quarter and the actual GDP level of the previous quarter that was available to the forecasters when making the forecasts. This method is also employed in the Federal Reserve Bank of Philadelphia's report on the SPF. Subtracting the SFPs' forecast from the average GDP growth rate allows us to fit the forecast to the zero-growth model. The US professional forecasters' real GDP forecasts have been available since 1992.

Figure 4.5: Out-of-sample forecast of the US quarterly real output growth



Note: The forecast period is 1995Q1 - 2019Q2 as indicated by the dashed vertical red line. We estimate the model until time $t-1$ and forecast the output growth at t utilizing the US Economic policy uncertainty index and the Survey of US Professional Forecasters' recession probability at time t.

RMSE of 0.44% for the SPF's forecast and 0.44% for the model-based forecast as reported in Table 4.4. During the periods of normal growth, the SPF's forecast slightly outperformed our model (0.40% vs 0.43%). Interestingly, the greatest discrepancy was observed in the forecasts during recessions, with the SPF's RMSE being 0.68% and our model-based forecast performing better at 0.49%. This result is surprising, as most model-based forecasts have been unable to accurately capture the Great Recession and its turning point due to fixed parameters and a mean-reverting property. On average, US professional forecasters tend to outperform economic models as they are able to adjust to the new information faster (Ng & Wright, 2013; Wieland & Wolters, 2011). However, it is important to note that the out-of-sample forecasts of the smooth ambiguity model are based on revised GDP data, whereas the SPF used real-time GDP, and therefore may be subject to potential biases.

4.5.2 Time variation of the attitudes toward risk and ambiguity

In this section, we explore the time variation of risk and ambiguity aversion as measured by recursive estimations used to do the out-of-sample forecasts. This result highlights the relationship of the two major economic crises, the Dot-com crisis and the Global Financial crisis (GFC), with households' attitudes towards risk and ambiguity.

Figure 4.6 demonstrates the time variation of ambiguity and risk aversion from 1995Q1 to 2019Q4. Prior to the Dot-com crisis and following its conclusion, the level of ambiguity aversion remained at zero, indicating that household was ambiguity neutral. However, risk aversion decreased from 0.34 to 0.27 prior to the Dot-Com crisis, which is consistent with the risk-taking behavior and low risk premium that contributed to the financial market bubble of that time.

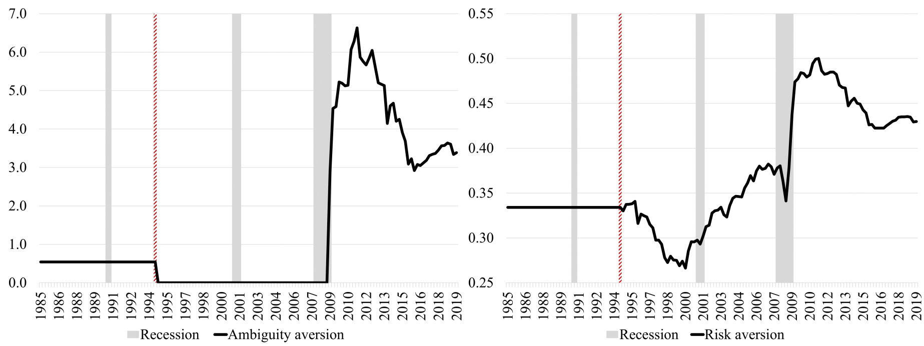
When the bubble burst in 2000Q2, risk aversion began to increase and continued rising even after the Dot-com crisis concluded while ambiguity aversion remained zero throughout these periods. An increase in risk aversion before the GFC might appear counterintuitive, considering the risk-taking activities such as subprime lending. Bekaert et al. (2013) find that monetary policy accommodation reduces risk aversion, resulting in a decrease in risk aversion during the period from 2002 to 2005 and a subsequent rise between 2006 and 2007. However, they show that the overall level of risk aversion in these periods was relatively low.

After the GFC, ambiguity aversion experienced a sharp increase, peaking at 6.63 in 2010Q3. Subsequently, it decreased and stabilized around 3 since 2016Q4. Risk aversion displays a similar movement but it briefly decreased in the middle of GFC. Afterward, it stabilized around 0.43, almost twice the level of pre Dot-com crisis.

Our results emphasize the different relationship between crises and the representative household's aversion to both ambiguity and risk. The dot-com crisis seems to contribute to the increased risk aversion, but had no effect on ambiguity aversion. In contrast, the Global Financial Crisis associates with a structural rise in both parameters. The increase in risk aversion indicates that the marginal utility of consumption decreases, which in turn reduces consumption. The increase in ambiguity aversion results in a lower expected marginal utility of investment, discouraging investment. This could be one of the reasons why the US economic recovery from the GFC is slower when compared to the Dot-com crisis.

Figure 4.6: Dynamics of ambiguity aversion and risk aversion in the US

(a) Ambiguity aversion **(b)** Risk aversion



4.5.3 European estimation results

In this section, we investigate the estimations of European countries, namely Germany (DE), Italy (IT), France (FR), and Spain (ES). These four countries were selected due to the availability of TFP data. The results presented in Table 4.5 are the ambiguity averse models with the lowest RMSEs from all initial values of ambiguity aversion indicated in Table 4.2. We report the full results including the benchmark and ambiguity neutral models in Appendix 4.D[17]. Table 4.5 reports the results of the ambiguity averse models including estimated parameters, steady state values of selected variables and the change of RMSE from the benchmark model. We divide our discussion into three parts: estimated parameters, steady state and model fitness.

[17] The out-of-sample estimations and the time-varying ambiguity aversion and risk aversion cannot be provided. This is due to the technical limitations of having a small sample size in each European country from 1999Q4 to 2018Q4, making the PEA solution unsolvable in many cases.

Table 4.5: Estimation results of European countries

Estimated parameter	Description	DE	IT	FR	ES
ν	labor disutility	2.36	0.94	2.97	3.00
		(0.10)	(0.38)	(0.56)	(0.21)
σ	risk aversion	0.08	0.48	0.58	0.80
		(0.02)	(0.19)	(0.20)	(0.27)
ρ_λ	weight of the lagged expectations	0.00	0.57	0.46	0.79
		(0.21)	(0.12)	(0.11)	(0.08)
μ_s	SS Bayesian belief of recession	0.03	0.00	1.00	0.00
		(0.20)	(0.36)	(258.91)	(0.006)
γ	ambiguity aversion	0.00	3.55	0.00	12.86
		(2.57)	(2.53)	(1.42)	(2.55)
Steady state	Description	DE	IT	FR	ES
M_s	macroeconomic uncertainty	1.00	1.00	1.00	1.38
$\mu_s \xi_s^R$	subjective belief of recession	0.03	0.00	1.00	0.00
Change of RMSE relative to BM		DE	IT	FR	ES
	all periods	-0.15	-0.07	-0.01	-0.17
	recession periods	-0.48	-0.16	0.02	-0.49
	normal growth periods	0.04	-0.03	-0.03	0.02

Note: DE: Germany, IT: Italy, FR: France, ES: Spain. All models are estimated with the methods described in Section 4.4. The reported results are those with the lowest root mean square errors among the different initial values of ambiguity aversion. SS means steady state and RMSE stands for root mean square error. Change of RMSE relative to BM is calculated by the RMSE of ambiguity aversion model minus the RMSE of benchmark model. The definition of recessions is when there are negative GDP growth for two consecutive quarters. See the list of recession dates in Appendix 4.A. The standard error of the estimated parameter is in (...).

Parameters. The parameters of labor disutility range from 0.94 (Italy) to 3.00 (Spain), while the parameters of risk aversion vary across countries, from 0.08 (Germany) to 0.80 (Spain). This finding is inconsistent with Albonico et al. (2019), who estimate that these countries tend to have similar labor disutility and risk aversion. The discrepancy may have been caused by estimation methods and the model setup, in particular, Albonico et al. (2019)'s models do not include ambiguity aversion. Gallant et al. (2019) compare smooth ambiguity models with ambiguity aversion to those without ambiguity aversion (Epstein and Zin model). They show that the level of estimated risk aversion reduces when there is ambiguity aversion. The weight of lagged expectations also exhibits considerable variation, with Germany having the lowest weight of 0.00, and the other countries having higher weights (0.49 to 0.80). This result is in line with Bräuning and van der Cruijsen (2019) that, in comparison to other European countries, German households attach a lower weight to previous information when forming inflation expectations.

Our estimation suggests that France and Germany are ambiguity neutral, whereas Italy and Spain are ambiguity averse. Specifically, Spain has an ambiguity aversion of 12.86, whereas Italy's ambiguity aversion is 3.55, which is not significantly different from zero. The level of ambiguity aversion has implications for the subjective belief of recession. For example, in 2012Q4, the SPF-implied Bayesian belief of recession was 29.02% across all countries, whereas the subjective belief of French and German representative households was also 29.02% since they are ambiguity neutral. Italy's subjective belief of recession was 46.21%, and that of Spain was 96.25% due to pessimistic belief distortions. These high subjective beliefs of recession reduce the marginal expected utility of investments, resulting in a decrease in investment. This is consistent with the findings of Albonico et al. (2019), who suggest that the slow recovery from crises in Italy and Spain is largely due to low investment.

Steady state. All countries have one scenario steady state except for Germany. The steady state Bayesian belief in Germany is 0.03, meaning the German representative household expects a recession to occur with a 3% chance if any shock hits the steady state. Despite this, macroeconomic uncertainty is relatively low at 1.00 compared to the average macroeconomic uncertainty of 1.25, thus the decisions of the household in the steady state are unlikely to be impacted by uncertainty. France has a worst-case steady state in which the Bayesian belief of recession and macroeconomic uncertainty is equal to 1. This implies that the representative household in France is certain that a shock hitting the steady state will result in recession and, consequently, they make decisions such that the expected utility of recession is the same as that of a period of normal growth. Finally, Italy and Spain have a benchmark steady state, where their Bayesian beliefs of recession are zero, indicating that they are certain that the recession will not occur once the economy reaches the steady state. However, the steady state macroeconomic uncertainty of the two countries is different, at 1.00 and 1.38 respectively. This difference in uncertainty implies to a worse expectation of recession among the Spanish representative household, compared to that of the Italian representative household but does not have an effect on the steady state.

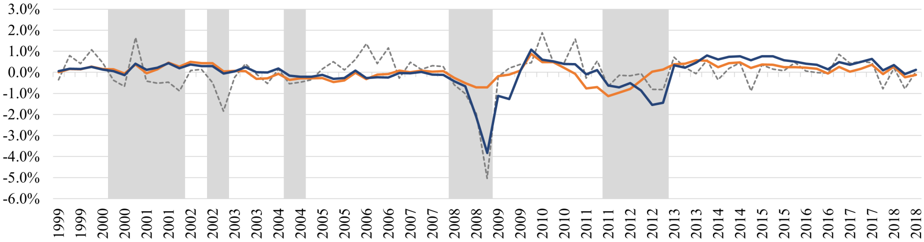
Similar to the US estimation, the asymptotic standard errors of steady state Bayesian belief are large for all European countries. In particular, the estimated μ_s of Germany and France are not statistically distinguishable from zero, indicating that their steady states are not significantly different from the benchmark steady state.

Model fitness. The fitness of the smooth ambiguity model for all countries has improved from the benchmark model, decreasing by -0.01 to -0.17. Figure 4.7 illustrates the actual output growth rate and the fitted output growth rate from the benchmark (BM) and ambiguity aversion (AA) models. With the exception of France, the decrease in RMSEs is mainly attributable to the more accurate predictions during the recession periods. The reduction in recession RMSEs is between -0.16 to -0.49 in comparison to the benchmark model. In the case of France, the improvement from the benchmark model is quite marginal (-0.01). This could be explained by the fact that the technological process in France is highly cyclical due to the variation in the 35-hour workweek policy, as noted by Comin et al. (2020). Thus, the benchmark model that includes only TFP shock already performs well in France. Moreover, a caution should be taken when interpreting the results for Spain, as the data fitness of BM in Spain is poor (see Figure 4.7d) and may cause bias in the results.

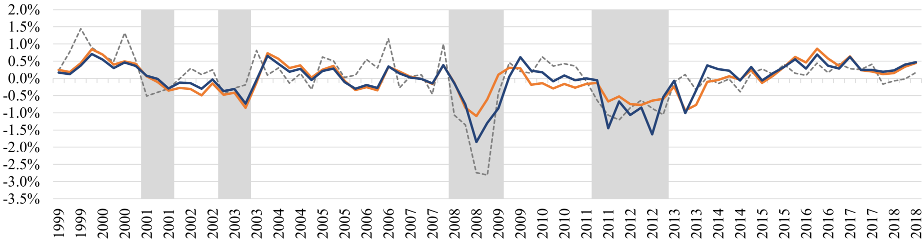
In conclusion, our empirical assessments indicate that the smooth ambiguity model outperforms the benchmark model in data fitness, particularly during recession periods. We observed this to be true for both the US and large European countries. This suggests that ambiguity aversion amplifies the impact of uncertainty on the economy and pessimistic belief distortions can be a significant transmission channel of macroeconomic uncertainty. In the case of the US representative household, the Global Financial Crisis seemed to result in a structural rise in ambiguity aversion, whereas the Dot-com crisis, a less severe one, did not have a similar impact. This is consistent with the findings for European countries, where those that experienced more severe crises, such as Italy and Spain, appear to have higher levels of ambiguity aversion.

Figure 4.7: Estimation of the quarterly real output growth in the European countries

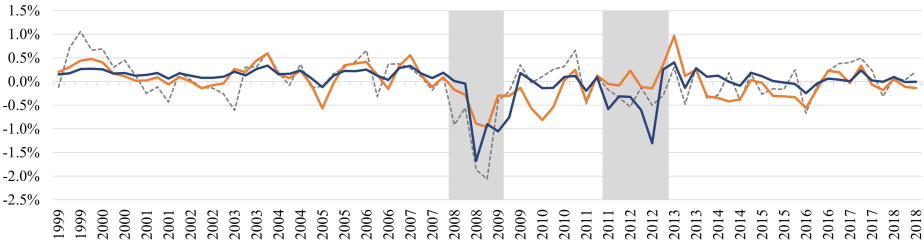
(a) Germany (DE)



(b) Italy (IT)



(c) France (FR)



(d) Spain (ES)

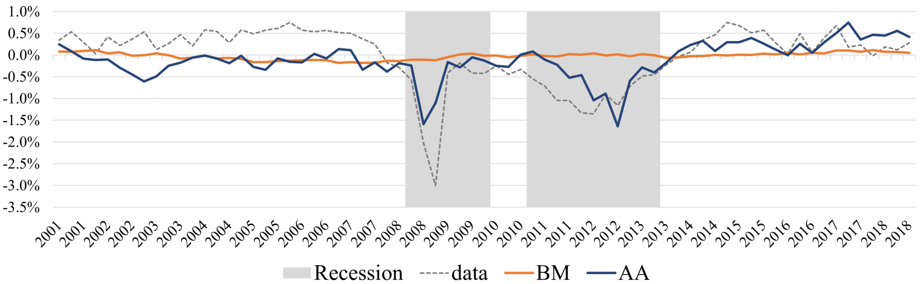


Table 4.6: Estimation results with alternative estimators

Parameter	Description	NLS	ML	SMM	II
α	capital share	0.34	0.34	0.41	0.50
ν	labor disutility	6.44	5.37	2.51	1.71
σ	risk aversion	0.43	0.43	0.61	0.67
ρ_λ	weight of the lagged expectations	0.08	0.49	0.23	0.00
μ_s	SS Bayesian belief of recession	1.00	1.00	0.98	1.00
γ	ambiguity aversion	3.35	11.35	3.04	0.00
Steady state	Description	NLS	ML	SMM	II
M_s	macroeconomic uncertainty	1.00	1.00	1.00	1.00
$\mu_s\xi_s^R$	subjective belief of recession	1.00	1.00	0.98	1.00
RMSE	Periods	NLS	ML	SMM	II
	all periods	0.41%	0.48%	0.52%	0.64%
	recession periods	0.41%	0.59%	0.86%	1.35%
	normal growth periods	0.41%	0.46%	0.48%	0.53%

Note: All models were estimated using the parameterized expectations algorithm and pattern search algorithm described in Section 4.4. SS means steady state and RMSE stands for root mean square error.

4.6 Robustness

The purpose of robustness tests is to show the extent to which our main results are affected by the estimation method. We first compare the estimations of nonlinear models using nonlinear least squares (our main result), maximum likelihood, simulated methods of moments, and indirect inference. We then report the results of the linearized model using the standard maximum likelihood with Bayesian techniques commonly employed in macroeconomic models.

4.6.1 Nonlinear models with different estimators

In this section, we compare the results of ambiguity averse models estimated with maximum likelihood (ML), simulated methods of moments (SMM), and indirect inference (II), to those estimated with nonlinear least squares (NLS) of the US. In particular, NLS aims to reduce the distance between a model's generated and observed output growths. ML seeks to maximize the sum of the log likelihood so that the observed output growths are most likely within the model. SMM attempts to minimize the weighted distance between the selected moments implied from the model and observed data. Lastly, II focuses on fitting a model's impulse response to observed data. The details of each estimator are described in Appendix 4.B.

Table 4.6 presents the results of fitting US output growth from these estimators where the first column is the same as the AA result in Section 4.5.1. We do not report the standard errors as they are not comparable across estimators due to different computational methods. Estimates of capital share, risk aversion, and SS Bayesian belief of recession are found to be similar across estimators. Capital share is estimated to be between 0.34 and 0.50, while risk aversion is estimated to be between 0.43 and 0.67. The steady state Bayesian belief is estimated to be close to one, indicating that there is one-scenario steady state. However, other parameters vary greatly across estimators, for example, the levels of ambiguity aversion range from 0 to 11.35. This discrepancy across estimators indicates that different objective functions can yield different results, despite all estimators being asymptotically consistent.

The RMSEs of maximum likelihood, simulated methods of moments, and indirect inference are 0.48%, 0.52%, and 0.64%, respectively, higher than that of NLS. The ML and NLS estimations are theoretically supposed to yield the same results under normality assumptions but the output growth rate is not normally distributed (Fagiolo et al., 2008), resulting in different outcomes. The high RMSEs of SSM and II are mainly attributed to the recession periods as the recession RMSEs are more than twice than those of the NLS estimation [18].

In conclusion, the estimates of capital share, risk aversion, and SS Bayesian belief of recession are similar but other parameters vary greatly across estimators. This finding is consistent with that in Ruge-Murcia (2007), wherein it is stated that different estimators yield different estimation results, though to a lesser degree than in our case. This has important implications for generalizing our estimation results and those of other studies. In this paper, we choose to present the NLS esitmation as our main result since it is capable of fitting the data more accurately than the other estimators. Future research should investigate which estimator best approximates a business cycle model.

[18] The results for SMM and II are highly sensitive to the choice of moments and Vector Autoregression models used for estimation. For the SMM estimator, we fit the model to four moments: mean, variance, skewness and correlation with the lagged component. The II estimation is fitted with an AR(1) process. Moreover, an identity weighting matrix is employed for a simple computational process. Ruge-Murcia (2012, 2020) demonstrates that the identity weight results in a larger asymptotic standard error than the optimal weight. Further explanation of these issues is beyond the scope of this paper.

Table 4.7: Estimation results with the maximum likelihood and Bayesian technique

Parameter	Description	BM	AN	AA
α	capital share	0.11	0.13	0.14
		(0.03)	(0.03)	(0.03)
ν	labor disutility	2.08	3.29	7.18
		(0.72)	(0.52)	(0.93)
σ	risk aversion	1.32	1.37	1.52
		(0.22)	(0.21)	(0.26)
ρ_λ	weight of lagged expectations	0.21	0.23	0.74
		(0.22)	(0.11)	(0.06)
μ_s	SS Bayesian belief of recession		0.18	0.16
			(0.02)	(0.02)
γ	ambiguity aversion			17.77
				(1.66)
Steady state	Description	BM	AN	AA
M_s	macroeconomic uncertainty		1.00	1.00
$\mu_s \xi_s^R$	Subjective belief of recession		0.18	0.17
Log likelihood		1010.99	1324.79	1338.31
RMSE	Periods	BM	AN	AA
	all periods	0.60%	0.56%	0.48%
	recession periods	1.34%	1.17%	0.74%
	normal growth periods	0.45%	0.46%	0.45%

Note: BM is benchmark model, AN is ambiguity neutral model ($\gamma = 0$). AA is ambiguity averse model where the prior of ambiguity aversion is uniform distribution with a range from 0 to 40. SS means steady state and RMSE stands for root mean square error. The standard deviation is stated in (...). Macroeconomic uncertainty and subjective belief of recession are not estimated but implied from the model thus the standard deviation is not available. Log likelihood of the model is measured by the modified harmonic mean method.

4.6.2 Linearized models with maximum likelihood and Bayesian technique

This section compares our main results to those from standard maximum likelihood estimation used by most macroeconomic models[19]. This method employs Bayesian techniques to run the maximum likelihood estimation, which requires the linearization around the steady state of the model and the assumption of prior distributions for each parameter to be estimated. The mean priors are similar to the initial values of estimated parameters as showed in Section 4.4.2. Further details of the estimation are provided in Appendix 4.C. For simplicity, we refer to this estimator as Linear-ML. Table 4.7 reports the posterior mean, standard deviation, log likelihood and RMSEs of the Linear-ML estimation.

The estimated parameters of Linear-ML differ from those obtained by the NLS estimator, though the model performance is consistent across both estimators. The log like-

[19] We solve the model in *Matlab* and estimate the model in *Dynare*.

lihood of the ambiguity neutral and ambiguity averse models are 1324.79 and 1338.31, respectively, performing better than the benchmark model with a log likelihood of 1010.99. This result is in line with the main finding in Section 4.5.1. The posterior means are closer to their priors, such as the posterior risk aversion ranges between 1.32 and 1.51 (prior of 2) and the steady state Bayesian beliefs are 18% and 16% (prior of 16.18%). Note that, despite these Bayesian beliefs, the steady-state decision remains robust to uncertainty due to a low macroeconomic uncertainty at 1.00.

We find that linearization reduces the transmission of uncertainty, requiring a higher level of ambiguity aversion to compensate. The Linear-ML posterior of ambiguity aversion is 17.77, significantly higher than 11.35 of the ML estimator in the nonlinear ambiguity averse model. Table 4.7 further reveals that the recession RMSE is only slightly improved from the BM to AN models (1.34% to 1.17%) but is markedly reduced from AN to AA models (1.17% to 0.74%). This is inconsistent with our main estimation of nonlinear models in Section 4.5.1 which shows a great improvement from BM to AN. Thus, we can infer that linearization largely diminishes the effects of macroeconomic uncertainty on the spread of the expected utility between the two scenarios, which is the only transmission mechanism in the ambiguity neutral model. However, the effect due to pessimistic belief distortions caused by ambiguity aversion is still preserved, albeit to a lesser degree and at a cost of a high level of ambiguity aversion.

In conclusion, we find that the main results in Section 4.5.1 are qualitatively consistent when the standard maximum likelihood method is employed. The smooth ambiguity models fit the data better than the benchmark model in terms of log likelihood. The estimated parameters between NLS and Linear-ML estimators, however, varied. Linear-ML results are more closely aligned with the standard results, likely due to their priors. As argued in Section 4.6.1, this has important implications for generalizing our estimation results and those of other related studies. Accordingly, future research should investigate which estimator best approximates a business cycle model.

4.7 Conclusion

We bring the smooth ambiguity model developed in Chapter 3 to the data and empirically examine the extent to which uncertainty and ambiguity aversion contribute to economic fluctuations. To do so, we estimate a smooth ambiguity model in which a representative household believes that the economy could enter one of the two scenarios: recession or normal growth. Our model captures the transmission of uncertainty through the increased spread of expected utilities between the two scenarios and the pessimistic belief distortions of the representative household. We solve the model using parameterized expectations algorithm, which preserves the nonlinear effect of uncertainty and allows us to pin down the household's expectation conditional on each scenario.

The estimation suggests that the smooth ambiguity model outperforms the benchmark model in terms of data fitness for the United States and major European countries. With a relevant level of ambiguity aversion, the model is able to capture the large output drop during recession periods. Our out-of-sample forecast for the US further supports this notion, implying that ambiguity aversion and pessimistic belief distortions could be important determinants of the severity of the crisis. Moreover, for the US representative household, the Global Financial Crisis led to a structural increase in ambiguity aversion, whereas it remained unchanged throughout the Dot-com crisis. This may explain why the recovery from the GFC was slower than the Dot-com crisis.

This study has been one of the first attempts to evaluate the smooth ambiguity model using macroeconomic data. It contributes to the literature of business cycle by mapping theoretical uncertainty to empirical uncertainty, utilizing the parameterized expectations algorithm to solve the model, and estimating ambiguity aversion across countries. Despite this, there are two main limitations that must be acknowledged. Firstly, the absence of recession probability data of individual countries in Europe can potentially bias the results of the respective samples. Secondly, we measure the Bayesian belief of recession by the recession probability from the survey of professional forecasters, assuming that the professional forecasters are Bayesian and ambiguity neutral on average. However, Manzan (2021) finds that the professional forecasters occasionally violate the Bayesian learning rule. This indicates that professional forecasters may not be ambiguity neutral, which could lead to an underestimation of ambiguity aversion according to our model. Finally, the scope of this study was limited by the model's simplicity, meaning that considerable more work will need to be done to examine important topics such as uncertainty in the supply side, the incorporation of price dynamics or fiscal and monetary policies.

Appendix

4.A Data sources

4.A.1 US data

- **Real quarterly output growth**: U.S. Bureau of Economic Analysis, retrieved from FRED, https://fred.stlouisfed.org/series/GDPC1; 21 January 2023.
- **US Economic Policy Uncertainty index**: Baker et al. (2016) retrieved from https://www.policyuncertainty.com/us_monthly.html, 21 January 2023.
- **Recession probability**: Survey of Professional forecasters, Federal Reserve Bank of Philadelphia, retrieved from https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/anxious-index, 21 January 2023.
- **Utilization-adjusted technological process**: Fernald (2014) retrieved from https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp, 21 January 2023.
- **Recession dates**: NBER's US Business Cycle Expansions and Contractions

4.A.2 Europe data

- **Real quarterly output growth**: Eurostat, retrieved from FRED, https://fred.stlouisfed.org/categories/32291, 15 June 2022.
- **Economic Policy Uncertainty index** Baker et al. (2016) retrieved from https://www.policyuncertainty.com/europe_monthly.html, 13 June 2022.
- **Recession probability**: We compute the probabilities of next-year GDP growth to be less than zero from probabilistic distributions provided by individual forecasters and use the cross sectional average for our estimation. Survey of Professional forecasters, European Central Bank is retrieved from https://www.ecb.europa.eu/stats/ecb_surveys/survey_of_professional_forecasters/html/all_data.en.html, 13 June 2022.

- **Utilization-adjusted technological process**: Comin et al. (2020) retrieved from https://tomgschmitz.wordpress.com/about, 3 August 2022.
- **Recession dates**: In order to identify recession dates, we assess whether there has been negative growth in two consecutive quarters. If there is a one quarter gap between two recession episodes, then we define that quarter as a period of recession. Specifically, for the sovereign debt crisis, we use the common interval across all countries from 2011Q4 to 2013Q1.
  - Germany: 2000Q1 - 2002Q1, 2002Q4 - 2003Q1, 2004Q3 - 2004Q4, 2008Q2 - 2009Q1, 2011Q4 - 2013Q1
  - Italy: 2001Q1 - 2001Q4, 2003Q1 - 2003Q3, 2008Q2 - 2009Q2, 2011Q4 - 2013Q1
  - France: 2008Q2 - 2009Q2, 2011Q4 - 2013Q1
  - Spain: 2008Q3 - 2010Q1, 2011Q1-2011Q3, 2011Q4-2013Q3
- **Capital share income**: We compute the capital share income as 1 - labor share of GDP. The labor share of GDP data is published by United Nations Sustainable Development Goals - United Nations retrieved from Our World in Data https://ourworldindata.org/grapher/labor-share-of-gdp, 31 January 2023.

4.B Alternative estimators

This section describes three alternative estimators, maximum likelihood (ML), simulated methods of moments (SMM), and indirect inference (II), which were used in Section 4.6.1. The nonlinear least squares (NLS) estimator is the main estimator, and its results are presented in Section 4.5. Each estimator has a different objective function but uses the same pattern search algorithm described in Section 4.4.2. NLS minimizes the distance between model-generated and observed output growth. ML maximizes the sum of the log likelihood such that the observed output growth is the most probable in the model. SMM minimizes the weighted distance of selected moments, which are implied from the model and observed data. Lastly, II fits the impulse response of the model to the observed data. Let Ω be a set of parameter, $\dot{\mathbf{y}}(\Omega)$ be the model-implied output growth and $\dot{\mathbf{y}}^{\text{obs}}$ be the observed output growth. Table 4.8 summarizes the objective functions of all estimators.

Table 4.8: Objective function of each estimator

Estimator	Objective function
Nonlinear least squares (NLS)	$\min(\dot{\mathbf{y}}(\Omega) - \dot{\mathbf{y}}^{\text{obs}})'(\dot{\mathbf{y}}(\Omega) - \dot{\mathbf{y}}^{\text{obs}})$ where $\dot{\mathbf{y}}$: output growth column matrix
Maximum likelihood (ML)	$\max \sum_t \log L(\dot{y}_t^{\text{obs}}; \Omega)$ where L: normal density function The normal density function is obtained from *Matlab* code: fitdist($\dot{\mathbf{y}}(\Omega)$, 'Normal')
Simulated methods of moments (SMM)	$\min(\mathbf{m}(\Omega) - \mathbf{m}^{\text{obs}})\mathbf{W}(\mathbf{m}(\Omega) - \mathbf{m}^{\text{obs}})'$ where a row matrix $\mathbf{m} = [\dot{y}_t, \dot{y}_t^2, \dot{y}_t^3, \dot{y}_t\dot{y}_{t-1}]$, $\mathbf{W}$: identity weighting matrix
Indirect inference (II)	$\min(\mathbf{p}(\Omega) - \mathbf{p}^{\text{obs}})'\mathbf{W}(\mathbf{p}(\Omega) - \mathbf{p}^{\text{obs}})$ where a row matrix $\mathbf{p}$ contains parameters of the AR(1) process AR(1) process is estimated from *Matlab* code: estimate(varm(4,1), [$\dot{\mathbf{y}}(\Omega)$,$\mathbf{M}$, $\boldsymbol{\mu}$, $\mathbf{z}$]) $\mathbf{W}$: identity weighting matrix

4.C Bayesian estimation method

This section describes the Bayesian estimation method. To estimate the model, we substitute the original Euler equation (Eq. 4.6) with the parameterized Euler equation (Eq. 4.19) in the equilibrium conditions and linearize the model around steady states. Table 4.9 shows the equilibrium conditions in original and linear forms. The structural parameters were estimated using Bayesian estimation and Monte Carlo Markov Chain (MCMC). A sample of 20,000 draws was created and the first 10,000 draws were used as burn-in. We used the prior variance as the MCMC jumping covariance defining the transition probability function to the next draw. A step size was chosen such that the acceptance rate is between 0.2 and 0.4. Given the structural parameters in each draw, the PEA coefficients (θ) and steady states were computed using the methods described in Section 4.4.2.

Table 4.9: Equilibrium conditions

Original model	Linearized model
$\lambda_t = \rho_\lambda \lambda_{t-1} + (1-\rho_\lambda)(\theta^c + \theta^k k_t + \theta^z z_t + \theta^M M_t + \theta^\mu \mu_t + \theta^{M\mu}(M_t \mu_t))$	$\lambda_s \tilde{\lambda}_t = \rho_\lambda \lambda_s \tilde{\lambda}_{t-1} + (1-\rho_\lambda)(\theta^k k_s \tilde{k}_t + \theta^z \tilde{z}_t + \theta^M M_s \tilde{M}_t + \theta^\mu \mu_s \tilde{\mu}_t + \theta^{M\mu} M_s \mu_s (\tilde{M}_t + \tilde{\mu}_t))$
$\lambda_t = c_t^{-\sigma}$	$\tilde{\lambda}_t = -\sigma \tilde{c}_t$
$\lambda_t = \frac{\exp((1-\sigma)\bar{a}) L_t^\nu}{z_t^{1-\sigma} w_t}$	$\tilde{\lambda}_t = \nu \tilde{L}_t - (1-\sigma)\tilde{z}_t - \tilde{w}_t$
$k_{t+1} = (1-\delta) k_t \frac{z_{t-1}}{z_t} + i_t$	$k_s \tilde{k}_{t+1} = (1-\delta)(k_s \tilde{k}_t + \tilde{z}_{t-1} - \tilde{z}_t) + i_s \tilde{i}_t$
$y_t = \exp(\alpha \bar{a}) z_{t-1}^\alpha k_t^\alpha L_t^{1-\alpha}$	$\tilde{y}_t = \alpha \tilde{z}_{t-1} + \alpha \tilde{k}_t + (1-\alpha)\tilde{L}_t$
$w_t = (1-\alpha)\frac{y_t}{L_t}$	$\tilde{w}_t = \tilde{y}_t - \tilde{L}_t$
$R_t = \alpha \frac{y_t}{k_t} \frac{z_t}{z_{t-1}}$	$\tilde{R}_t = \tilde{y}_t - \tilde{k}_t + \tilde{z}_t - \tilde{z}_{t-1}$
$y_t = c_t + i_t$	$y_s \tilde{y}_t = c_s \tilde{c}_t + i_s \tilde{i}_t$

Note: $\tilde{x}_t = \frac{x_t - x_s}{x_s}$ where x_s is the steady state.

We do not directly estimate the Bayesian process but use the next-quarter recession probability from the survey of US professional forecasters as a observed data for μ_t. For simplicity, we assume that the Bayesian belief follows an AR(1) process:

$$\tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \sigma_\mu \epsilon_t^\mu$$

The technological process and macroeconomic uncertainty are also obtained from the data. We assume that they follow AR(1) processes:

$$\tilde{z}_t = \rho_a \tilde{z}_{t-1} + \sigma_a \epsilon_t^z$$
$$\tilde{M}_t = \rho_M \tilde{M}_{t-1} + \sigma_M \epsilon_t^M$$

In total, we use four time-series data for the estimation: the quarterly GDP per capita (GDP), the utilization-adjusted technological progress by Fernald (2014) (TFP), the US Economic Policy Uncertainty index by Baker et al. (2016) (EPU) [20], and the next-quarter recession probability from the US professional forecasters (SPF). The last three data are

[20]For the US Economic Policy Uncertainty index, we take the log scale to reduce the volatility and divide the index its minimum value, so it is always bigger than or equal to one.

the same as those used in Section 4.4.2. The measurement equations are:

$$d \log GDP_t = \tilde{y}_t - \tilde{y}_{t-1} + ME_y$$

$$d \log TFP_t = \tilde{z}_t - \tilde{z}_{t-1}$$

$$EPU_t = M_s \tilde{M}_t + M_s$$

$$SPF_t = \mu_s \tilde{\mu}_t + \mu_s$$

where $d \log$ is the log difference and ME_y is measurement error of output. $d \log GDP_t$ and $d \log TFP_t$ are demeaned and the model is estimated during the sample period of 1985Q1 - 2019Q4. The estimation of the benchmark model has y_t and z_t as observables. The ambiguity neutral and ambiguity averse models use all four observables. The Bayesian priors are summarized in Table 4.10.

Table 4.10: Priors of main parameters

Parameter	Description	Type	Mean	S.D.
Parameters related to ambiguity				
γ	ambiguity aversion	U	between 0 and 40	
ρ_λ	weight on lagged expectation	U	between 0 and 1	
μ_s	steady-state Bayesian belief of recession	U	between 0 and 1	
Other structural parameters				
σ	risk aversion	IG	2	0.5
ν	labor disutility	IG	1.5	0.5
α	capital share	B	0.3	0.01
Bayesian beliefs parameters				
ρ_μ	persistence of Bayesian belief	B	0.7	0.1
σ_μ	volatility of Bayesian beliefs	IG	0.2	0.001
Macro uncertainty parameters				
ρ_M	persistence of macro uncertainty	B	0.7	0.01
σ_M	volatility of macro uncertainty	IG	0.05	0.01
Technological progress parameters				
ρ_a	persistence of technology growth	B	0.95	0.01
σ_a	volatility of technology	IG	0.008	0.001
Measurement error				
ME_y	measurement error of output	IG	0.006	0.0001

Note: B: Beta distribution, IG: Inverse gamma distribution. U:Uniform distribution

4.D Full estimation results

4.D.1 Estimation results in European countries

Table 4.11: Germany

Parameter	Description	BM	AN	AA
α	capital share	0.38	0.38	0.38
ν	labor disutility	2.42	2.34	2.36
		(0.0003)	(0.71)	(0.10)
σ	risk aversion	0.03	0.08	0.08
		(0.01)	(0.004)	(0.02)
ρ_λ	weight of the lagged expectations	0.71	0.00	0.00
		(0.10)	(0.18)	(0.21)
μ_s	Bayesian belief of recession		0.03	0.03
			(0.19)	(0.20)
γ	ambiguity aversion			0.00
				(2.57)
M_s	macroeconomic uncertainty		1.00	1.00
RMSE	Periods	BM	AN	AA
	all periods	0.82%	0.66%	0.66%
	recession periods	1.26%	0.77%	0.77%
	normal growth periods	0.58%	0.62%	0.62%

Table 4.12: Italy

Parameter	Description	BM	AN	AA
α	capital share	0.40	0.40	0.40
ν	labor disutility	1.20	0.79	0.94
		(0.006)	(0.05)	(0.28)
σ	risk aversion	0.41	0.50	0.48
		(0.18)	(0.14)	(0.19)
ρ_λ	weight of the lagged expectations	0.56	0.59	0.57
		(0.12)	(0.08)	(0.12)
μ_s	Bayesian belief of recession		0.07	0.00
			(69.75)	(0.36)
γ	ambiguity aversion			3.55
				(3.53)
M_s	macroeconomic uncertainty		1.02	1.00
RMSE	Periods	BM	AN	AA
	all periods	0.54%	0.48%	0.47%
	recession periods	0.78%	0.63%	0.62%
	normal growth periods	0.45%	0.42%	0.42%

Note: All models were estimated using the parameterized expectations algorithm and pattern search algorithm described in Section 4.4. BM stands for Benchmark model, AN is Ambiguity neutral model where γ is fixed to 0. AA is the ambiguity averse model where γ is estimated.

Table 4.13: France

Parameter	Description	BM	AN	AA
α	capital share	0.38	0.38	0.38
ν	labor disutility	1.36	2.97	2.97
		(0.0002)	(0.28)	(0.56)
σ	risk aversion	0.66	0.58	0.58
		(0.20)	(0.10)	(0.20)
ρ_λ	weight of the lagged expectations	0.54	0.46	0.46
		(0.10)	(0.10)	(0.11)
μ_s	Bayesian belief of recession		1.00	1.00
			(101.29)	(258.91)
γ	ambiguity aversion			0.00
				(1.42)
M_s	macroeconomic uncertainty		1.00	1.00
RMSE	Periods	BM	AN	AA
	all periods	0.38%	0.37%	0.37%
	recession periods	0.60%	0.63%	0.63%
	normal growth periods	0.33%	0.30%	0.30%

Table 4.14: Spain

Parameter	Description	BM	AN	AA
α	capital share	0.39	0.39	0.39
ν	labor disutility	20.00	1.82	3.00
		(35.31)	(0.84)	(0.21)
σ	risk aversion	4.56	1.03	0.80
		(16.13)	(0.46)	(0.27)
ρ_λ	weight of the lagged expectations	0.00	0.81	0.79
		(0.09)	(0.11)	(0.08)
μ_s	Bayesian belief of recession		0.00	0.00
			(0.02)	(0.006)
γ	ambiguity aversion			12.86
				(2.55)
M_s	macroeconomic uncertainty		2.88	1.38
RMSE	Periods	BM	AN	AA
	all periods	0.69%	0.63%	0.52%
	recession periods	1.12%	0.94%	0.63%
	normal growth periods	0.45%	0.49%	0.48%

Note: All models were estimated using the parameterized expectations algorithm and pattern search algorithm described in Section 4.4. BM stands for Benchmark model, AN is Ambiguity neutral model where γ is fixed to 0. AA is the ambiguity averse model where γ is estimated.

4.D.2 Maximum likelihood estimation of the US

Table 4.15: Posterior estimations

Structural parameter	Description	BM	AN	AA
α	capital share	0.11	0.13	0.14
		(0.03)	(0.03)	(0.03)
ν	labor disutility	2.08	3.29	7.18
		(0.72)	(0.52)	(0.93)
σ	risk aversion	1.32	1.37	1.52
		(0.22)	(0.21)	(0.26)
ρ_λ	weight of lagged expectations	0.21	0.23	0.74
		(0.22)	(0.11)	(0.06)
γ	ambiguity aversion			17.77
				(1.66)
μ_s	Bayesian belief of recession		0.18	0.16
			(0.02)	(0.02)
Other parameter	Description	BM	AN	AA
ρ_a	persistence of technological process	0.95	0.95	0.96
		(0.22)	(0.02)	(0.02)
ρ_M	persistence of macroeconomic uncertainty		0.93	0.92
			(0.02)	(0.02)
ρ_μ	persistence of Bayesian belief		0.76	0.77
			(0.04)	(0.04)
σ_a	volatility of technological process	0.007	0.007	0.007
		(0.0004)	(0.0004)	(0.0004)
σ_M	volatility of macroeconomic uncertainty		0.07	0.07
			(0.004)	(0.004)
σ_μ	volatility of Bayesian belief		0.43	0.41
			(0.05)	(0.05)
ME_y	volatility of output measurement error	0.006	0.006	0.005
		(0.0003)	(0.0003)	(0.0003)
Log likelihood		1010.99	1324.79	1338.31

Note: BM is benchmark model, AN is ambiguity neutral model ($\gamma = 0$). AA is ambiguity averse model where the prior of ambiguity aversion is uniform distribution with a range from 0 to 40. The standard deviation is stated in (...). The log likelihood of the model is measured by the modified harmonic mean method.

Chapter 5

Conclusion

This research aims to gain a deeper understanding of how uncertainty affects the economy through individual beliefs from both empirical and theoretical points of view. The motivation for this topic comes from the fact that macroeconomic uncertainty has been shown to reduce economic output empirically. However, macroeconomic models under uncertainty (primarily in the form of risk) still struggle to reflect this effect, particularly during recessions. Our research suggests that individuals have different reactions to uncertainty in the forms of risk and ambiguity. Incorporating ambiguity in the macroeconomic model can improve the model's performance, especially during crises. The following paragraphs detail our findings, the main limitations, and propose future work. Additionally, we provide an overview of future research in macroeconomic models and our position relative thereto.

When examining the survey datasets in Chapter 2, we find that the effect of macroeconomic uncertainty on individual beliefs is not as straightforward as most macroeconomic models assume. Especially the relationship between macroeconomic uncertainty and subjective uncertainty is not always positive, implying that people do not necessarily become more uncertain in the face of macroeconomic uncertainty. To reconcile this puzzling result, we consider the distinction between risk and ambiguity purposed by Knight (1921), which suggests that people do not perceive uncertainty solely as risk, but that ambiguity also plays an important role in their expectations.

In Chapter 3, we study a theoretical macroeconomic model that incorporates uncertainty in the form of ambiguity, aiming to replicate the stylized facts documented in Chapter 2 and other existing empirical studies. Our model is based on the smooth ambiguity model by Altug et al. (2020), in which the representative agent differentiates between risk and ambiguity. We extend this model by introducing a macroeconomic uncertainty index as a model variable, allowing us to trace the effects of uncertainty with greater precision. The effect of macroeconomic uncertainty is amplified by ambiguity aversion as the household becomes more pessimistic and more certain about its pessimistic beliefs when uncertainty rises. We solve the model using a parameterized expectations algorithm and estimate the model with a nonlinear least squares estimator as described in Chapter 4. To the best of our knowledge, this is the first attempt to quantify the level of ambiguity aversion through macroeconomic data. Our estimations suggest that ambiguity aversion is associated with the severity of economic crises and that a higher ambiguity aversion appears to delay the economic recovery from the crisis.

We point out the main limitations and propose future work on our research. First, we highlight that ambiguity and risk impact people's expectations differently, with ambiguity potentially being more important than risk during recessions. However, the empirical data we employed as a proxy for uncertainty does not accurately distinguish between risk and ambiguity. A natural progress is to develop and utilize a more precise measurement of uncertainty in ambiguity and risk. For example, Izhakian (2020) proposes a method to measure ambiguity from the data, and Coiculescu et al. (2019) empirically shows that the firm's investment in innovation decreases with ambiguity but increases with risk measured from financial data.

Second, our macroeconomic model focuses on the demand-side uncertainty and pessimistic belief distortions caused by ambiguity aversion as the main mechanism in the model. We have not explored other transmission channels of uncertainty such as price-markups, financial intermediaries and supply-side uncertainty. For instance, our model would not be able to fully account for the Corona crisis which was characterized by both demand- and supply-side uncertainty shocks. Further research may take our method to incorporate macroeconomic uncertainty and expand the smooth ambiguity model to examine other pertinent topics.

In the broader picture, the Global Financial Crisis of 2008 highlighted the inadequacies of traditional macroeconomic models during periods of heightened uncertainty. This has prompted many innovations in the effort to address this issue, of which our research is a part. One strand of macroeconomic models, such as those previously mentioned in this thesis and ours, focuses on uncertainty shock as a main determinant of economic fluctuation. These models assume that agents have rational expectations, wherein the decision is consistent with their expectations and uncertainty influences their decisions through the expectations on future events (Altug et al., 2020; Fernández-Villaverde & Guerrón-Quintana, 2020; Ilut & Schneider, 2022). As a deviation from rational expectations, research has also explored adaptive learning models, wherein decisions are not consistently adherent to expectations whose parameters are updated based on past data. This approach has been found to perform better than the rational expectation model without uncertainty (Milani, 2012; Slobodyan & Wouters, 2012). Recently, research has been gaining increasing interest in heterogeneous agent or agent-based models, which emphasize the interactions between different agents as a main driver of economic fluctuations. These models simulate the heuristic behavior of individual agents to capture the aggregate economic fluctuations (Poledna et al., 2023). All these approaches demonstrate both advantages and disadvantages and provide valuable insight into the behavior of the economy in uncertain times.

It is undeniable that at this stage, no single model or theory can satisfactorily explain the economic fluctuations given the complex relationship between uncertainty and individual beliefs. To that end, this dissertation therefore tries to provide a comprehensive analysis of the effects of uncertainty on people's beliefs and the economy through both empirical and theoretical approaches. We argue that it is essential to differentiate between ambiguity and risk in order to gain a more profound understanding of uncertainty. According to the smooth ambiguity model, those who are ambiguity averse become more pessimistic in the presence of increasing uncertainty or ambiguity, and sometimes they can become more certain about their pessimistic beliefs. This behavior further amplifies the effects of uncertainty during times of recession. Our study reconciles the micro effect of uncertainty on people's expectations with the macro effect on the economy. By utilizing this mechanism, the model's performance is notably improved, especially during recession periods.

To conclude this four-year research journey, we have highlighted the importance of connecting the uncertainty study in macroeconomic literature to the decision-making theory in microeconomic literature. We demonstrate that empirical studies can provide a useful basis for bridging the two strands of literature. Gaining a deeper understanding of how uncertainty impacts individual expectations can give us a comprehensive perspective of the economy's response to uncertainty. This eventually helps us construct a better macroeconomic model for studying uncertainty. Going forward, economic research should strive to integrate features of both macro and micro literature to gain further insight into how the economy works.

Bibliography

Albonico, A., Calés, L., Cardani, R., Croitorov, O., Ferroni, F., Giovannini, M., Hohberger, S., Pataracchia, B., Pericoli, F. M., Raciborski, R., Ratto, M., Roeger, W., & Vogel, L. (2019). Comparing post-crisis dynamics across euro area countries with the global multi-country model. Economic Modelling. https://doi.org/10.1016/j.econmod.2019.04.016

Altig, D., Barrero, J. M., Bloom, N., Davis, S. J., Meyer, B., & Parker, N. (2020). Surveying business uncertainty. Journal of Econometrics. https://doi.org/10.1016/j.jeconom.2020.03.021

Altug, S., Collard, F., Çakmaklı, C., Mukerji, S., & Özsöylev, H. (2020). Ambiguous business cycles: A quantitative assessment. Review of Economic Dynamics. https://doi.org/10.1016/j.red.2020.04.005

Armantier, O., Topa, G., van der Klaauw, W., & Zafar, B. (2017). An overview of the survey of consumer expectations. Economic Policy Review.

Bachmann, R., Carstensen, K., Lautenbacher, S., & Schneider, M. (2021). Uncertainty and change: Survey evidence of firms' subjective beliefs. CESifo Working Paper Series. https://doi.org/10.3386/w29430

Backus, D., Ferriere, A., & Zin, S. (2015). Risk and ambiguity in models of business cycles. Journal of Monetary Economics. https://doi.org/10.1016/j.jmoneco.2014.12.005

Baker, S. R., Bloom, N., & Davis, S. J. (2016). Measuring economic policy uncertainty. Quarterly Journal of Economics. https://doi.org/10.1093/qje/qjw024

Baliga, S., Hanany, E., & Klibanoff, P. (2013). Polarization and ambiguity. American Economic Review. https://doi.org/10.1257/aer.103.7.3071

Baqaee, D. R. (2019). Asymmetric inflation expectations, downward rigidity of wages, and asymmetric business cycles. Journal of Monetary Economics. https://doi.org/10.1016/j.jmoneco.2019.04.014

Barañano, I., Iza, A., & Vázquez, J. (2002). A comparison between the log-linear and the parameterized expectations methods. Spanish Economic Review. https://doi.org/10.1007/s101080200039

Basu, S., Fernald, J. G., & Kimball, M. S. (2006). Are technology improvements contractionary? American Economic Review. https://doi.org/10.1257/aer.96.5.1418

Bekaert, G., Hoerova, M., & Lo Duca, M. (2013). Risk, uncertainty and monetary policy. Journal of Monetary Economics. https://doi.org/10.1016/j.jmoneco.2013.06.003

Benigno, G., Foerster, A., Otrok, C., & Rebucci, A. (2020, April). Estimating macroeconomic models of financial crises: An endogenous regime-switching approach. https://doi.org/10.3386/w26935

Bhandari, A., Borovicka, J., & Ho, P. (2023). Survey data and subjective beliefs in business cycle models. FRB Richmond Working Paper.

Bianchi, F., Ilut, C. L., & Schneider, M. (2018). Uncertainty shocks, asset supply and pricing over the business cycle. Review of Economic Studies. https://doi.org/10.1093/restud/rdx035

Bianchi, F., Ludvigson, S. C., & Ma, S. (2022). Belief distortions and macroeconomic fluctuations. American Economic Review. https://doi.org/10.1257/aer.20201713

Bloom, N. (2014). Fluctuations in uncertainty. Journal of Economic Perspectives. https://doi.org/10.1257/jep.28.2.153

Bloom, N. (2009). The impact of uncertainty shocks. Econometrica. https://doi.org/10.3982/ECTA6248

Born, B., Breuer, S., & Elstner, S. (2018). Uncertainty and the great recession. Oxford Bulletin of Economics and Statistics. https://doi.org/10.1111/obes.12229

Born, B., & Pfeifer, J. (2021). Uncertainty-driven business cycles: Assessing the markup channel. Quantitative Economics. https://doi.org/10.3982/QE1297

Brogaard, J., Dai, L., Ngo, P. T. H., & Zhang, B. (2020). Global political uncertainty and asset prices. Review of Financial Studies. https://doi.org/10.1093/rfs/hhz087

Bräuning, C., & van der Cruijsen, C. (2019). Learning dynamics in the formation of european inflation expectations. Journal of Forecasting. https://doi.org/10.1002/for.2553

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. Quarterly Journal of Economics. https://doi.org/10.1162/00335530360535207

Carroll, C. D., Slacalek, J., & Sommer, M. (2019). Dissecting saving dynamics: Measuring wealth, precautionary, and credit effects. NBER Working Papers. https://doi.org/10.3386/w26131

Chatterjee, P., & Milani, F. (2020). Perceived uncertainty shocks, excess optimism-pessimism, and learning in the business cycle. Journal of Economic Behavior and Organization. https://doi.org/10.1016/j.jebo.2020.09.007

Christiano, L. J., & Eichenbaum, M. (1992). Liquidity effects and the monetary transmission mechanism. American Economic Review.

Christiano, L. J., Eichenbaum, M. S., & Trabandt, M. (2018). On DSGE models. Journal of Economic Perspectives. https://doi.org/10.1257/jep.32.3.113

Clements, M. (2014). Forecast uncertainty—ex ante and ex post: U.s. inflation and output growth. Journal of Business and Economic Statistics. https://doi.org/10.1080/07350015.2013.859618

Clements, M. P. (2010). Explanations of the inconsistencies in survey respondents' forecasts. European Economic Review. https://doi.org/10.1016/j.euroecorev.2009.10.003

Coeurdacier, N., Rey, H., & Winant, P. (2011). The risky steady state. American Economic Review. https://doi.org/10.1257/aer.101.3.398

Cogley, T., & Nason, J. M. (1995). Output dynamics in real-business-cycle models. American Economic Review.

Coibion, O., Gorodnichenko, Y., & Kumar, S. (2018). How do firms form their expectations? new survey evidence. American Economic Review. https://doi.org/10.1257/aer.20151299

Coiculescu, G., Izhakian, Y., & Ravid, S. A. (2019). Innovation under ambiguity and risk. Journal of Financial and Quantitative Analysis. https://doi.org/10.2139/ssrn.3428896

Collard, F. (2015, October). Parameterized expectations algorithm. Retrieved June 29, 2021, from http://fabcol.free.fr/notes.html

Collard, F., Mukerji, S., Sheppard, K., & Tallon, J.-M. (2018). Ambiguity and the historical equity premium. Quantitative Economics. https://doi.org/10.3982/QE708

Comin, D. A., Gonzalez, J. Q., Schmitz, T. G., & Trigari, A. (2020). A new measure of utilization-adjusted TFP growth for europe and the united states. NBER Working Papers. https://doi.org/10.3386/w28008

Das, S., Kuhnen, C. M., & Nagel, S. (2020). Socioeconomic status and macroeconomic expectations. Review of Financial Studies. https://doi.org/10.1093/rfs/hhz041

Dovern, J. (2013). When are GDP forecasts updated? evidence from a large international panel. Economics Letters. https://doi.org/10.1016/j.econlet.2013.06.007

Duffie, D., & Singleton, K. J. (1993). Simulated moments estimation of markov models of asset prices. Econometrica. https://doi.org/10.2307/2951768

Ellsberg, D. (1961). Risk, ambiguity, and the savage axioms. Quarterly Journal of Economics. https://doi.org/10.2307/1884324

Engelberg, J., Manski, C. F., & Williams, J. (2009). Comparing the point predictions and subjective probability distributions of professional forecasters. Journal of Business and Economic Statistics. https://doi.org/10.1198/jbes.2009.0003

Enke, B., & Graeber, T. (2023). Cognitive uncertainty. Quarterly Journal of Economics.

Fagiolo, G., Napoletano, M., & Roventini, A. (2008). Are output growth-rate distributions fat-tailed? some evidence from OECD countries. Journal of Applied Econometrics. https://doi.org/10.1002/jae.1003

Fermand, E., Kuhnen, C. M., Li, G., & Ben-David, I. (2023, March 14). Extrapolative uncertainty and household economic behavior. https://doi.org/10.2139/ssrn.3293399

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. FRBSF Working Paper.

Fernández-Villaverde, J., & Guerrón-Quintana, P. A. (2020). Uncertainty shocks and business cycle research. Review of Economic Dynamics. https://doi.org/10.1016/j.red.2020.06.005

Gallant, A. R., R Jahan-Parvar, M., & Liu, H. (2019). Does smooth ambiguity matter for asset pricing? Review of Financial Studies. https://doi.org/10.1093/rfs/hhy118

Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. Journal of Mathematical Economics. https://doi.org/10.1016/0304-4068(89)90018-9

Giordani, P., & Söderlind, P. (2003). Inflation forecast uncertainty. European Economic Review. https://doi.org/10.1016/S0014-2921(02)00236-2

Glas, A. (2020). Five dimensions of the uncertainty–disagreement linkage. International Journal of Forecasting. https://doi.org/10.1016/j.ijforecast.2019.07.010

Glas, A., & Hartmann, M. (2016). Inflation uncertainty, disagreement and monetary policy: Evidence from the ECB survey of professional forecasters. Journal of Empirical Finance. https://doi.org/10.1016/j.jempfin.2016.05.001

Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. American Economic Review.

Guerron-Quintana, P., Inoue, A., & Kilian, L. (2017). Impulse response matching estimators for DSGE models. Journal of Econometrics. https://doi.org/10.1016/j.jeconom.2016.09.009

Guidolin, M., & Liu, H. (2016). Ambiguity aversion and underdiversification. Journal of Financial and Quantitative Analysis. https://doi.org/10.1017/S0022109016000466

Hansen, L. P., & Sargent, T. J. (2021). Macroeconomic uncertainty prices when beliefs are tenuous. Journal of Econometrics. https://doi.org/10.1016/j.jeconom.2019.11.010

Hansen, L. P., & Sargent, T. J. (2011, November 28). Robustness. Princeton University Press.

Ilut, C. L., & Schneider, M. (2014). Ambiguous business cycles. American Economic Review. https://doi.org/10.1257/aer.104.8.2368

Ilut, C. L., & Schneider, M. (2022). Modeling uncertainty as ambiguity: A review. NBER Working Papers. https://doi.org/10.3386/w29915

Izhakian, Y. (2020). A theoretical foundation of ambiguity measurement. Journal of Economic Theory. https://doi.org/10.1016/j.jet.2020.105001

Jackson, L. E., Kliesen, K. L., & Owyang, M. T. (2020). The nonlinear effects of uncertainty shocks. Studies in Nonlinear Dynamics and Econometrics. https://doi.org/10.1515/snde-2019-0024

Jahan-Parvar, M., & Liu, H. (2012). Ambiguity aversion and asset prices in production economies. Review of Financial Studies. https://doi.org/10.2139/ssrn.2012740

Ju, N., & Miao, J. (2012). Ambiguity, learning, and asset returns. Econometrica. https://doi.org/10.3982/ECTA7618

Jurado, K., Ludvigson, S. C., & Ng, S. (2015). Measuring uncertainty. American Economic Review. https://doi.org/10.1257/aer.20131193

Kim, J., & Ruge-Murcia, F. J. (2009). How much inflation is necessary to grease the wheels? Journal of Monetary Economics. https://doi.org/10.1016/j.jmoneco.2009.03.004

Klibanoff, P., Marinacci, M., & Mukerji, S. (2005). A smooth model of decision making under ambiguity. Econometrica. https://doi.org/10.1111/j.1468-0262.2005.00640.x

Klibanoff, P., Marinacci, M., & Mukerji, S. (2009). Recursive smooth ambiguity preferences. Journal of Economic Theory. https://doi.org/10.1016/j.jet.2008.10.007

Knight, F. H. (1921). Risk, uncertainty and profit. Hart, Schaffner, Marx.

Kurz, M., Piccillo, G., & Wu, H. (2013). Modeling diverse expectations in an aggregated new keynesian model. Journal of Economic Dynamics and Control. https://doi.org/10.1016/j.jedc.2013.01.016

Lagarias, J. C., Reeds, J. A., Wright, M. H., & Wright, P. E. (1998). Convergence properties of the nelder–mead simplex method in low dimensions. SIAM Journal on Optimization. https://doi.org/10.1137/S1052623496303470

Lamla, M. J., & Lein, S. M. (2014). The role of media for consumers' inflation expectation formation. Journal of Economic Behavior and Organization. https://doi.org/10.1016/j.jebo.2014.05.004

Lhuissier, S., & Tripier, F. (2021). Regime-dependent effects of uncertainty shocks: A structural interpretation. Quantitative Economics. https://doi.org/10.3982/QE1298

Liu, Z., Waggoner, D. F., & Zha, T. (2011). Sources of macroeconomic fluctuations: A regime-switching DSGE approach. Quantitative Economics. https://doi.org/10.3982/QE71

Maliar, L., & Maliar, S. (2003). Parameterized expectations algorithm and the moving bounds. Journal of Business and Economic Statistics. https://doi.org/10.1198/073500102288618793

Manzan, S. (2021). Are professional forecasters bayesian? Journal of Economic Dynamics and Control. https://doi.org/10.1016/j.jedc.2020.104045

Manzan, S. (2011). Differential interpretation in the survey of professional forecasters. Journal of Money, Credit and Banking. https://doi.org/10.1111/j.1538-4616.2011.00404.x

Marinacci, M. (2015). Model uncertainty. Journal of the European Economic Association. https://doi.org/10.1111/jeea.12164

Milani, F. (2012). The modeling of expectations in empirical DSGE models: A survey. Advances in Econometrics. https://doi.org/10.1108/S0731-9053(2012)0000028004

Ng, S., & Wright, J. H. (2013). Facts and challenges from the great recession for forecasting and macroeconomic modeling. Journal of Economic Literature. https://doi.org/10.1257/jel.51.4.1120

Nowzohour, L., & Stracca, L. (2020). More than a feeling: Confidence, uncertainty, and macroeconomic fluctuations. Journal of Economic Surveys. https://doi.org/10.1111/joes.12354

Ozoguz, A. (2009). Good times or bad times? investors' uncertainty and stock returns. Review of Financial Studies. https://doi.org/10.1093/rfs/hhn097

Peterman, W. B. (2016). Reconciling micro and macro estimates of the frisch labor supply elasticity. Economic Inquiry. https://doi.org/10.1111/ecin.12252

Piccillo, G., & Van Den Hurk, J. (2020). The surprising effect of social distancing on our perception: Coping with uncertainty. CEPR Press.

Poledna, S., Miess, M. G., Hommes, C., & Rabitsch, K. (2023). Economic forecasting with an agent-based model. European Economic Review. https://doi.org/10.1016/j.euroecorev.2022.104306

Pulford, B. D. (2009). Short article: Is luck on my side? optimism, pessimism, and ambiguity aversion. Quarterly Journal of Experimental Psychology. https://doi.org/10.1080/17470210802592113

Ruge-Murcia, F. (2012). Estimating nonlinear DSGE models by the simulated method of moments: With an application to business cycles. Journal of Economic Dynamics and Control. https://doi.org/10.1016/j.jedc.2012.01.008

Ruge-Murcia, F. (2020). Estimating nonlinear dynamic equilibrium models by matching impulse responses. Economics Letters. https://doi.org/10.1016/j.econlet.2020.109624

Ruge-Murcia, F. J. (2007). Methods to estimate dynamic stochastic general equilibrium models. Journal of Economic Dynamics and Control. https://doi.org/10.1016/j.jedc.2006.09.005

Sims, C. A. (2003). Implications of rational inattention. Journal of Monetary Economics. https://doi.org/10.1016/S0304-3932(03)00029-1

Slobodyan, S., & Wouters, R. (2012). Learning in a medium-scale DSGE model with expectations based on small forecasting models. American Economic Journal: Macroeconomics. https://doi.org/10.1257/mac.4.2.65

Smets, F., & Wouters, R. (2007). Shocks and frictions in US business cycles: A bayesian DSGE approach. American Economic Review. https://doi.org/10.1257/aer.97.3.586

Theodoridis, K. (2011). An efficient minimum distance estimator for DSGE models. Bank of England working papers. https://doi.org/10.2139/ssrn.1951938

Tuckett, D., & Nikolic, M. (2017). The role of conviction and narrative in decision-making under radical uncertainty. Theory & Psychology. https://doi.org/10.1177/0959354317713158

Wieland, V., & Wolters, M. H. (2011). The diversity of forecasts from macroeconomic models of the US economy. Economic Theory. https://doi.org/10.1007/s00199-010-0549-7

Impact paragraph

Since the Global Financial Crisis, there has been an increased interest in examining uncertainty and its impact on the economy. Empirical studies have shown that uncertainty diminishes economic outputs but macroeconomic models have difficulty in accurately capturing these effects, particularly during recessions. This can be partially attributed to the complex relationship between macroeconomic uncertainty and people's beliefs, as well as the technical limitations of solving and estimating macroeconomic models. This dissertation addresses these two issues, in order to improve our understanding of the economy under uncertainty and enhance the performance of macroeconomic models. The positive implications of this study will not only be felt across the economic literature but are also likely to inform policymakers and ultimately benefit society.

In Chapter 2, we study the effects of macroeconomic uncertainty on individual expectations of income and GDP growth using four panel datasets of professional forecasters and households. We quantify these expectations by the mean and subjective uncertainty of the forecast distributions provided by survey respondents. Our findings suggest that people become more pessimistic when macroeconomic uncertainty rises, but they can either be more uncertain or more certain about their beliefs. This chapter makes two main contributions to the existing literature. First, we contribute to the body of research on survey-based measures of subjective uncertainty in professional forecasters (Clements, 2014; Giordani & Söderlind, 2003; Glas & Hartmann, 2016; Manzan, 2021) and firms (Altig et al., 2020; Bachmann et al., 2021). We use both simple standard deviation and a fitted Beta distribution as done in Fermand et al. (2023) to measure subjective uncertainty in households. Second, we present direct evidence on the relationship between macroeconomic uncertainty and individual expectations which are assumed in many macroeconomic models. This is significant since the effects of macroeconomic uncertainty on the economy are highly dependent on the mechanisms specific to the models.

Chapter 3 studies a real business cycle model featuring smooth ambiguity preferences based on Altug et al. (2020). The model can reproduce the effects of macroeconomic uncertainty on individual expectations as found in Chapter 2 and other empirical studies. We make two primary contributions to the existing literature. Firstly, while a substantial amount of macroeconomic models have been exploring uncertainty in the form of a time-varying volatility or risk (Born & Pfeifer, 2021; Fernández-Villaverde & Guerrón-Quintana, 2020; Lhuissier & Tripier, 2021), we incorporate uncertainty taking the form of ambiguity. This type of uncertainty has been used in business cycle models by, for example, Bianchi et al. (2018), Ilut and Schneider (2014, 2022) and Altug et al. (2020). We depart from these models as our representative household is uncertain whether the economy will be in recession or normal growth, and macroeconomic uncertainty has asymmetrical impacts on the two scenarios. Lastly, we provide an analytical framework which links the empirical findings to decision-making theories under uncertainty. This framework shows that the smooth ambiguity model can replicate the empirical stylized facts regarding the relationship between macroeconomic uncertainty and individual expectations. Moreover, ambiguity aversion plays an important role in shaping these relationships.

Chapter 4 brings the theoretical discussions in Chapter 3 to real world data. We show that the smooth ambiguity model can fit the output growth data of the U.S. and major European countries better than the benchmark model without uncertainty, especially in recessions. Our estimations also suggest that ambiguity aversion amplifies the effect of macroeconomic uncertainty through individual beliefs which is consistent with the findings in Chapters 2 and 3. This paper contributes to the literature in the techniques of solution and estimation, and the applications of smooth ambiguity models. First, to solve the model, we apply a parameterized expectation algorithm to preserve the nonlinearity in the transmission mechanism. This approach allows us to determine the expectations of the two scenarios without imposing an additional decision rule for each scenario. Second, while smooth ambiguity models have been utilized to fit financial asset returns (Collard et al., 2018; Gallant et al., 2019), their application to estimate macroeconomic variables has been scarce. We estimate the model by employing nonlinear least squares, minimizing the distance between model-implied and actual output growth. We demonstrate that the level of ambiguity aversion is time-varying and associated with economic crises. To the best of our knowledge, this is the first study that has measured the level of ambiguity aversion utilizing macroeconomic data.

Throughout this dissertation, we gain a deeper understanding of how uncertainty can affect the economy and demonstrate that this can improve the performance of macroeconomic models, especially in times of recession. This can have important contributions to policymaking in three main ways. First, distinguishing the presence of ambiguity or risk can be beneficial in the policymaking process. For instance, Chapter 2 points to the fact that households and professional forecasters respond differently to macroeconomic uncertainty. One of the possible reasons for this could be that households are more exposed to uncertainty in the form of ambiguity, while professional forecasters may not be. This finding could help policymakers to better communicate and explain their decisions to the public, increasing accountability and transparency.

Second, a deeper comprehension of how crises influence individuals can enhance policy effectiveness. In Chapter 4, we find that ambiguity aversion of US representative households remained constant throughout the Dot-com crisis but structurally increased after the Global Financial crisis. This suggests that the expansionary policies which were effective during the Dot-com crisis could be inadequate to address the Global Financial Crisis since people had become more pessimistic on account of heightened ambiguity aversion. This kind of insight could be useful for policymakers in formulating policies more effectively.

Finally, Chapter 3 and 4 demonstrate that the smooth ambiguity model has the potential to be used as a forecasting tool in situations of high uncertainty. Forecasts are essential for monetary policy, as they can give information about future output growth, inflation and unemployment. Central bankers can then use this information to determine appropriate interest rates and other policies to help them achieve their targets.

Summary

This dissertation studies the effect of uncertainty on the economy, considering the role of individuals' beliefs using empirical data and a theoretic macroeconomic model.

Chapter 2 documents the empirical effects of macroeconomic uncertainty on individual expectations using four different panel datasets. We find evidence that macroeconomic uncertainty reduces the mean expectation of income when using professional forecasters' data. This aligns with the assumption in most theoretic macroeconomic models that predicts a negative relationship between uncertainty and expected utility. However, we also show that macroeconomic uncertainty does not always increase subjective uncertainty. This finding is inconsistent with most models, which assume higher individual subjective uncertainty as the microfoundation for the impact of uncertainty on agents' decisions. We use these empirical findings as a ground to develop our theoretical model.

Chapter 3 develops a theoretical macroeconomic model by imposing the microfoundation based on the finding in Chapter 2. In our model, the household holds smooth ambiguity preferences and is uncertain on which scenario the economy will be in during the next period: whether it will be a normal growth period or a recession period. Assuming that uncertainty affects the household's utilities asymmetrically, we anchor the ratio of expected utilities between the two scenarios through the macroeconomic uncertainty index. The higher the macroeconomic uncertainty, the deeper the recession that the household is expecting. Our simulations show that the smooth ambiguity model can reproduce the empirical relationship between macroeconomic uncertainty and people's beliefs found in Chapter 2. Using data from the US, we show that when calibrated with a relevant degree of ambiguity aversion, the smooth ambiguity model captures economic

recessions better than the benchmark (no uncertainty) and ambiguity neutral models. This result suggests that ambiguity aversion in combination with periods of higher uncertainty could be a relevant channel to explain the disproportionately large output declines in economic crises.

Chapter 4 examines the empirical performance of the smooth ambiguity model under uncertainty. We solve the model with a parameterized expectations algorithm, which takes into account the properties of two scenarios described in Chapter 3 and preserves the nonlinear effect of uncertainty. In addition, we estimate the model with a nonlinear least square estimator using macroeconomic and survey data of the US and major European countries. Our estimations show that the smooth ambiguity model outperforms the benchmark model (no uncertainty) in predicting output growth, particularly during recessions. This holds true even in out-of-sample forecasts. Additionally, we observe that the Global Financial Crisis is associated with an increase in the US representative household's aversions to risk and ambiguity, whereas the Dot-com Crisis affected only risk aversion. Lastly, our estimates of the models of major European countries reveal that the representative households in Italy and Spain are ambiguity averse, while those in Germany and France are close to ambiguity neutral. This suggests that the experience of severe economic crises may contribute to increased ambiguity aversion.

In Chapter 5, we outline the main limitations of our research, the future research agenda, and our position in the economic literature. Throughout the study, we have emphasized the distinction between risk and ambiguity, which, however, is difficult to observe from the data. The macroeconomic uncertainty indices used in this dissertation do not adequately differentiate between the two. Moreover, the smooth ambiguity model focuses on demand-side uncertainty and does not take into account supply-side uncertainty, such as that which was experienced during the Corona crisis. These limitations prompt future research to develop suitable ambiguity and risk indices and to expand the smooth ambiguity model to study other prominent topics. Lastly, this study is one of many research endeavors aimed at improving the macroeconomic model's performance in fitting with data. At the same time, other literature emphasizes, for instance, the deviation from rational expectations (adaptive learning) or the interactions between heterogeneous agents (agent-based models). Our study elaborates on this literature by focusing on uncertainty and its transmission mechanism.

This study examines the effects of uncertainty, in the form of ambiguity, on the economy through empirical and theoretical approaches. Our findings contribute to the economic literature which seeks to explore economic decisions in periods of high uncertainty, as well as to policymakers whose policy decisions are heavily exposed to uncertainty and economic forecasts. Further research is necessary in order to effectively accommodate the implications of ambiguity in macroeconomic models and provide appropriate recommendations for policymaking.

Samenvatting

Dit proefschrift bestudeert onderzoekt het effect van onzekerheid op de economie en de rol die individuele verwachtingen hierin spelen met behulp van data een theoretische macro-economisch model.

Hoofdstuk 2 documenteert de empirische effecten van onzekerheid op individuele verwachtingen in vier verschillende paneldatasets. We vinden bewijs dat macro-economische onzekerheid het gemiddelde verwachte inkomen verlaagt wanneer professionele prognosedata wordt gebruikt in de analyse. Dit komt overeen met de aanname in de meeste theoretische macro-economische modellen die een negatieve relatie tussen onzekerheid en verwacht nut voorspellen. We laten echter zien dat macro-economische onzekerheid niet altijd de subjectieve onzekerheid verhoogt. Deze bevinding komt niet overeen met de aanname gemaakt in de meeste modellen, waaraan ten grondslag ligt dat hogere individuele subjectieve onzekerheid van invloed is op het gedrag van individuele agenten. We gebruiken deze bevindingen als basis om ons theoretische model te ontwikkelen.

Hoofdstuk 3 ontwikkelt een theoretisch macro-economisch model op basis van micro grondslagen, gebaseerd op de bevindingen in hoofdstuk 2. In ons model houdt het huishouden er *smooth ambiguity preferences* op na en is het onzeker in welk scenario, laag of hoogconjunctuur, de economie zich tijdens de volgende periode zal bevinden: een periode van normale groei of een recessieperiode. Ervan uitgaande dat onzekerheid het nut van het huishouden asymmetrisch beïnvloedt, verankeren we de verhouding van het verwachte nut tussen de twee scenario's met behulp van de macro-economische onzekerheidsindex. Hoe hoger de macro-economische onzekerheid, hoe dieper de recessie die het huishouden verwacht. Onze simulaties laten zien dat het *smooth ambiguity* model de empirische relatie tussen macro-economische onzekerheid en de overtuigingen van mensen, gevonden in hoofdstuk 2, kan reproduceren. Met behulp van data van de VS, laten we zien dat wanneer het model wordt gekalibreerd met een relevante ambiguïteitsaversie, het smooth ambiguity model de recessies beter weergeeft dan het basismodel (zonder onzekerheid) en het ambiguïteit neutrale model. Dit resultaat suggereert dat ambiguïteitsaversie in combinatie met perioden van grotere onzekerheid een relevant mechanisme zou kunnen zijn om de grote productiedalingen in economische crises te verklaren.

In hoofdstuk 4 wordt de empirische validiteit onderzocht van het *smooth ambiguity* model onder onzekerheid. We lossen het model op met een parametrisch verwachtingsalgoritme dat rekening houdt met de eigenschappen van de twee scenario's genoemd in hoofdstuk 3 en het non-lineaire effect van onzekerheid behoudt. Daarnaast schatten we het model met een non-lineaire kleinste kwadraten methode met behulp van macro-economische data van de VS en de belangrijkste Europese landen. Uit onze schattingen blijkt dat het smooth ambiguity model beter presteert dan het basismodel (zonder onzekerheid) in het voorspellen van de productiegroei, met name tijdens recessies. Dit geldt zelfs voor out-of-sample voorspellingen. Tevens stellen we vast dat de wereldwijde kredietcrisis gepaard ging met een toename van risico- en ambiguïteitsaversie voor een representatief huishouden in de VS, terwijl de dotcom crisis enkel risicoaversie beïnvloedde. Tot slot blijkt uit onze schattingen van de modellen voor de belangrijkste Europese landen dat representatieve huishoudens in Italië en Spanje afkerig zijn van ambiguïteit, terwijl die in Duitsland en Frankrijk bijna ambiguïteitsneutraal zijn. Dit suggereert dat de ervaring van een ernstige economische crisis kan bijdragen aan een grotere afkeer van ambiguïteit.

In hoofdstuk 5 schetsen we de belangrijkste beperkingen, de toekomstige onderzoeksagenda en onze positie in de economische literatuur. In het hele onderzoek hebben we het onderscheid tussen risico en ambiguïteit benadrukt, wat echter moeilijk uit de beschikbare data op te maken is. De macro-economische onzekerheidsindices die in dit proefschrift worden gebruikt, maken onvoldoende onderscheid tussen de twee. Bovendien richt het smooth ambiguity model zich op onzekerheid aan de vraagzijde en houdt het geen rekening met onzekerheid aan de aanbodzijde, zoals die zich voordeed tijdens de Corona-crisis. Deze beperkingen zetten aan tot toekomstig onderzoek om geschikte ambiguïteits- en risico-indices te ontwikkelen en om het smooth ambiguity model uit te breiden om andere prominente onderwerpen te bestuderen. Tot slot is deze studie maar een van de vele onderzoeksinspanningen die gericht zijn op het verbeteren van de prestaties van het macro-economische model. Een deel van deze literatuur legt de nadruk op bijvoorbeeld de afwijking van rationele verwachtingen (adaptief leren) of de interacties tussen heterogene agenten (agent-based model). Onze studie borduurt hierop voort met de focus op het begrijpen van onzekerheid en het transmissiemechanisme ervan.

Deze studie onderzoekt de effecten van onzekerheid, in de vorm van ambiguïteit, op de economie door middel van empirische en theoretische benaderingen. Onze bevindingen dragen bij aan de economische literatuur die economische beslissingen in perioden van grote onzekerheid onderzoekt, en verschaft inzichten en instrumenten aan beleidsmakers wier beleidsbeslissingen sterk afhankelijk zijn van onzekerheid en economische voorspellingen. Verder onderzoek is nodig om de implicaties van ambiguïteit effectief te kunnen verwerken in macro-economische modellen en passende aanbevelingen te kunnen doen voor beleidsvorming.

About the author

Poramapa Poonpakdee, also known as Paun, was born on 1st April 1990 in Nakhon Sri Thammarat, Thailand. In 2008, she received the Korean Government Scholarship, which allowed her to study economics at Ewha Womans University in Seoul, South Korea. She completed her bachelor's degree in 2013 and went on to pursue a master's degree in mathematics of finance at Columbia University in New York City. In 2016, Poramapa joined the central bank of Thailand (BOT) as a junior economist. After 2.5 years, Poramapa decided to continue her education and moved to Maastricht University to pursue her doctoral studies in Economics. She is supervised by Prof. Clemens Kool and Dr. Giulia Piccillo, and is also a recipient of a BOT scholarship for her doctoral research.

During her PhD, Poramapa taught three undergraduate courses at the School of Business and Economics, Maastricht University. These courses included Macroeconomics, Economics and Society in Contemporary Asia, and Emerging Market in the Global Economy. She also had the opportunity to present her research papers at several conferences, including the Annual Dynare conference, Mathematics and Subjective Probability conference, and the Young Economist Seminar hosted by the Croatia Central Bank. In June 2023, Poramapa began a PhD internship at De Nederlandsche Bank, the central bank of the Netherlands, further enhancing her expertise. Upon completion of her doctoral study, she will return to the BOT, where she aims to deepen, broaden, and share her valuable insights and knowledge gained during her time in the Netherlands.