# Evidence on an endogenous growth model with public R&D

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Abstract The empirical investigation of properties of an endogenous growth model by Huang, Lai, and Peretto (2023) in this paper confirms important assumptions and results of the model for OECD countries. Labour-augmenting technical change is enhanced through private and public R&D stocks in FMOLS and DOLS mean-group estimations, and pooled mean-group (PMG) estimation, also when adding the number of enterprises. The CES spillover functions in the growth models functions for R&D stock dynamics are supported through nonlinear estimation under the assumptions of identical or different spillover parameters for private and public R&D. We suggest strong public-to-private spillovers and weak private-to-public spillovers as well as high elasticities of substitution for private-public R&D stocks for private R&D processes and low CES for public R&D processes. We confirm the existence of private-public researcher interaction effects in the private R&D knowledge growth function and provide tentative evidence for the linear relation between public researchers and firm-level R&D and the hump-shaped relation between public and private researchers (both as % labour force). A vector-autoregressive (VAR) panel model in growth rates produces results, which are in accordance with the impact of public R&D cuts on the steady state and the transitional dynamics of the HLP model.
Keywords: Endogenous growth; public R&D; evidence. JEL code: O41; O38; O47.

## 1. Introduction

Huang, Lai, and Peretto (2023) (henceforth HLP) are the first to have extended the most recent version of endogenous growth models to include public R&D leaving a model with only private R&D as special case. In this paper, the purpose is to estimate and test some of the properties of their model for samples of OECD countries. The major elements of the model are (i) a technical change function depending on stocks of private and public R&D and the number of firms; (ii) dynamic equations for private and public R&D containing business and government researchers respectively, CES spillover functions with stocks of knowledge, and private-public labour interaction terms; (iii) a hump-shaped relation between government and business researchers, and (iv) an analysis of the model dynamics after a cut of the number of government researchers.

The contribution of this paper is as follows. (i) We estimate the technical change function, and (ii) also the private and public R&D accumulation functions nonlinearly under the assumptions of identical and non-identical parameters of the CES spillover functions. (iii) We build a VAR from GMM (orthogonal deviations version) in growth rates of all just mentioned variables (because of country-specific time trends leading to fixed effects after differencing) to compare the empirical effects of public R&D cuts on private and public R&D capital, government and business researchers, the number of firms, and technical change to those

[1] I am grateful for useful comments from Chien-Yu Huang.

of the HLP model; the reason is that the steady-state properties and transitions to a new steady state after a cut on public R&D explain the working of the theoretical model.

The evidence is preliminary in the sense that not all modern econometric methods can be used because the available data series are very short and nonlinear estimation has its own difficulties known from the estimation of CES production functions. Earlier literature on R&D spillover functions has used linear or log-linear models, mostly Cobb-Douglas or translog functions; one exception is the use of a generalized CES (briefly 'VES') function, staying in the comfortable realm of linear econometrics by way of using log-log estimates based on first-order conditions (Ziesemer 2021a).[2] The R&D accumulation functions in this paper are only partially linear (see Greene 2012) and therefore cannot avoid the nonlinear estimation of CES spillover functions. We indicate the problems in due course. However, the nonlinear estimation of R&D spillover CES functions is an innovation in this paper as much as its theoretical modelling of HLP is an innovation.

## 2. Theoretical and empirical Modelling

### 2.1 The theoretical model

Labour productivity of the firms, $T_i$, in eq (7) of HLP (2023), $Z_i^{\theta} D_i^{\gamma}$, is log linear with *Z* as private R&D capital stock and *D* as public R&D capital stocks for each firm *i = 1, …, N*.

$$T_i = Z_i^{\theta} D_i^{\gamma} \tag{1}$$

In order to go to the macroeconomic level, we write (1) in natural logarithms, replace $D_i$ by *D/N*, and $Z_i$ by *Z/N* and add *logN* on both sides. This yields

$$logT = \log T_i + logN = \theta logZ + \gamma logD + (1-\theta-\gamma)logN \tag{1'}$$

In case of decreasing returns in (1), the function for the firm level cannot be just rewritten for the macroeconomic level as it has been done in the empirical literature, but rather the theoretical model of HLP suggests having the number of firms on the right-hand side of (1').

The firms' R&D dynamics with CES spillovers from (8) and (9) in HLP leads to (2) and (3)

$$\dot{Z} = \alpha f(s_G)\left[\chi K_i^{\eta} + (1-\chi)B_i^{\eta}\right]^{1/\eta} L_Z \tag{2}$$

$$\dot{D} = \left[(1-\chi)K_i^{\delta} + \chi B_i^{\delta}\right]^{1/\delta} L_G \tag{3}$$

This is obtained when summing equations (8) and (9) in HLP over all *N* firms with index *i*, under the simplifying assumption that all terms before the labour variables $L_Z$ and $L_G$ for business and government researchers, as well as the left-hand side are identical for all firms. $f(s_G)$ is a government-labor interaction term explained below. $\chi$ is the own private to private and public-to-public spillover parameter. (1-$\chi$) is the cross spillover parameter

[2] Belderbos and Mohnen (2020) discuss spillovers in detail.

from public to private and private to public R&D. Symmetry in the spillover parameters is assumed for simplicity and will be generalized in the estimation later.

Next, we divide the first equation by *Z* and the second by *D*. The result is as follows.

$$\frac{\dot{Z}}{Z} = \alpha f(s_G)\left[\chi K_i^{\eta} + (1-\chi)B_i^{\eta}\right]^{1/\eta}\frac{L_Z}{Z} \quad (2')$$

$$\frac{\dot{D}}{D} = \left[(1-\chi)K_i^{\delta} + \chi B_i^{\delta}\right]^{1/\delta}\frac{L_G}{D} \quad (3')$$

Productivity growth according to the theoretical model (1) or (1'), (2'), and (3') then is

$$\hat{T} = \hat{T}_i + \hat{N} = \theta\hat{Z}_i + \gamma\hat{D}_i + \hat{N} = \theta(\hat{Z} - \hat{N}) + \gamma(\hat{D} - \hat{N}) + \hat{N} \quad (1'')$$

$$= \theta\alpha f(s_G)\left[\chi K_i^{\eta} + (1-\chi)B_i^{\eta}\right]^{1/\eta}\frac{L_Z}{Z} + \gamma\left[(1-\chi)K_i^{\delta} + \chi B_i^{\delta}\right]^{1/\delta}\frac{L_G}{D} + (1-\theta-\gamma)\hat{N}$$

## 2.2 Specification Issues and the empirical Model

To get a regression form we have to specify the function $f(s_G)$. We use a polynomial of the third degree because of its flexibility:

$$f = \xi_0 + \xi_1 s_G + \xi_2 s_G^2 + \xi_3 s_G^3 \quad (4)$$

The construct simplifies as in the HLP model if the squared and cubic terms are statistically insignificant, but they may be helpful in the estimation especially if the linear part is statistically insignificant or has an unexpected sign. HLP use

$$s_{G,a} = L_G/LF,$$

where LF is the labour force. The interaction HLP have in mind then is that of $L_G$ with $L_Z$ in (2) and (2'). For the private-public interaction we explore two additional ideas. First, personnel-interaction, s($L_G$*$L_Z$) is modelled as an econometric interaction term specified as

$$s_{G,b} = L_Z L_G$$

The essence of the exercise is getting $\xi_i$ to capture the curvature of *f($s_G$)(1-$s_G$)* in (16) of HLP (2023), which is a version of (2) which is modified by inclusion of othr parts of the model. Second, $s_G$ could be defined more broadly as money going from the government to firms, which we take as percentage of BERD flows financed by government, GB. The theory of HLP (2023) can be interpreted as seeing a log-log effect of a function of GB = S/BERD, from BERD being split up into BERD = R+S (private and government money sources of BERD), on the change or growth rate of BERDST in (2) and (2'):

$$s_{G,c} = GB, or\ logGB, \text{or GB squared}$$

If, instead, researchers would go from the government to the firms as during projects of universities executed for private business then money goes the opposite direction from firms to governmental research institutions, leaving open which sign is more plausible, and resulting in the risk of statistical insignificance.

Using versions of GB or $L_G*L_Z/(LF)^2$ never yields interesting results.

We set $K_i = Z/N$ and $B_i = D/N$ in the spillover functions of (2') and (3') to bring the idea of domain specific knowledge in HLP to the macro level. Knowledge in turn is measured as accumulated private and public R&D capital stock. Finally, we add the rate of depreciation of 0.15 to the growth rates on the left-hand side as in the process of making stock data explained in the next section.

Taking natural logarithms his leads to the following system of equations with $g_Z, g_N$ as growth rates of $Z$ and $N$.

$$\log(g_Z + 0.15) =$$

$$log\alpha + \log[f(s_G)] + (1/_\eta)log\left[\chi\left(\frac{Z}{N}\right)^\eta + (1-\chi)\left(\frac{D}{N}\right)^\eta\right] + \tau logL_Z - \beta logZ \qquad (2'')$$

$$\log(g_D + 0.15) = (1/_\delta)log\left[(1-\chi)\left(\frac{Z}{N}\right)^\delta + \chi\left(\frac{D}{N}\right)^\delta\right] + \varphi logL_G - \varepsilon logD \qquad (3'')$$

Allowing for non-unit coefficients of $logZ$ and $logD$ suggests that these terms do not just come in through the division by $Z$ and $D$ in (2') and (3'), but rather not only labour is a factor of production but also $Z$ and $D$, similar to a Cobb-Douglas case but without the constraint that $logZ$ and $logD$ have the same coefficients as labour up to the sign. The estimated results for $f(s_G)$ and $s_G$ can be used in the short-run general equilibrium growth rate equation (16) in HLP (2023) for private R&D to calculate the factor $f(1\text{-}s_G)$ and perhaps compare its values to the possible short-run maximum.

## 3. Data

Translating it to the macro level we use *log Z = LBERDST* and log *D = LPUBST*, where *L* stands for the natural logarithm and *ST* for stock, BERD is business R&D, and PUB is the flow of GERD minus BERD both taken from OECD MSTI until 2017. More recent years in OECD MSTI are incomplete or characterized as provisional or estimated. Stocks are constructed using the perpetual inventory method with a depreciation rate of 0.15 (see Hall et al. 2010 for a survey).[3] Labour productivity data, LTH07, have been constructed by Ziesemer (2023a) for alternative elasticities of substitution of CES production functions including human capital. We use values calculated under the assumption of a CES = 0.7. These data typically have more than forty yearly data points per country.

In the dynamic R&D equations, $L_Z$ denotes business enterprise researchers (FTE); $L_G$ government researchers (FTE); $s_G$ is either (i) $L_G/LF$, with labour force data from WDI, which start only in 1990, or (ii) the (log of) the percentage of BERD financed by government from OECD-MSTI, $s_G = GB$, with values between 0.8 and 32 percent and entered as GB/100 in the regressions, or (iii), focusing on the idea of personnel-interaction, $s_G = L_G*L_Z$, which has a panel maximum of 9.77E+10; therefore we divide

[3]We are grateful to ANONYMOUS for providing the R&D data.

$L_G*L_Z$ *by 1E+11 = 100 billion (one billion = 1E+9)* to make sure that 1- $s_G > 0$. When using *GB*, we drop Austria and Sweden from the set of 17 OECD contries; when using the labour interaction we drop Austria, Finland, and USA because they both have a very low number of observations. For *GB*, $L_Z$, $L_G$ we have then fourty data points per country with some gaps, and a total of unbalanced observation for *GB = 590*, $L_Z = 573$, $L_G = 569$. For the number of firms, $N$ in (1') and its growth rate $g_N$ in (2''), we use 'Number of enterprises' from 'OECD SSIS: Structural Statistics of Industry and Services, 05_82_LESS_K: Business economy, except financial and insurance activities'. These data typically have 19 yearly data points with a total of balanced observation of 250 when no lags reduce it.

**Table 1: Growth rate of number of enterprises for 17 OECD countries**

| | | | | | |
|---|---|---|---|---|---|
| AUT | 0.0157 | FIN | 0.0106 | NLD | 0.0798 |
| BEL | 0.0449 | FRA | 0.0261 | NOR | 0.0141 |
| CAN | 0.0069 | GBR | 0.0235 | PRT | -0.0095 |
| DEU | 0.0318 | IRL | 0.0875 | SWE | 0.0232 |
| DNK | 0.0031 | ITA | -0.0054 | USA | -0.0011 |
| ESP | -0.0050 | JPN | -0.0182 | Average | 0.0193 |

Source: OECD SSIS: Structural Statistics of Industry and Services, 05_82_LESS_K: Business economy, except financial and insurance activities.

The unweighted average over the countries' growth rates of the number of firms in Table 1 is 0.0193, with significantly negative rates for Spain and Italy, slightly negative growth rates for USA and Japan, and slightly positive rates for Denmark and Norway. Ireland and the Netherlands have the highest growth rates. Table 1 shows more details on the number of firms. Table 2 provides a data description for all variables.

**Table 2 Data description for 17 OECD countries: Coefficients (standard errors)**

| Variable | Business researchers $L_Z$ (FTE) | Government researchers $L_G$ (FTE) | Number of enterprises ETP | LTH07 (c) | Private R&D LBERDST | Public R&D LPUBST | Labour force |
|---|---|---|---|---|---|---|---|
| (a) Panel average | 89551 (7182) | 55996 (2618) | 1316163 (76508) | 1.9 (0.006) | 10.02 (0.012) | 9.74 (0.0133) | 24,54mio |
| Std. dev. | 171925 | 62459 | 1209696 | 0.339 | 1.894 | 1.573 | 35.9 mio |
| Maximum | 1201000 | 295864 | 4326720 | 2.93 | 14.42 | 13.5 | 1.67E+08 |
| Minimum | 607 FTE | 1507 | 81264 | 0.658 | 4.658 | 5.8 | 1.44 mio |
| Average trend (b) | 0.047 (0.0026) | 0.035 (.0016) | 0.015 (0.0026) | 0.014 (8E-3) | 0.05 (0.003) | 0.04 (.0016) | 0.0077 (0.0006) |
| Periods | 40, 1981-2020 | 40, 1981-2020 | 19, 2002-2020 | 55, 1963-2017 | 55, 1963-2017 | 55, 1963 -2017 | 31, 1990-2020 |
| Unb. obs. | 573 | 569 | 250 | 935 | 911 | 902 | 527 |

(a) From regression of variable on a constant. Standard error of estimation in parentheses. (b) From fixed effects regression with cross-section weights of log of variable on a trend. Standard error of estimation in parentheses. (c) Labour augmenting technical change, log level.

As the number of enterprises has the lowest number of observations, we first conduct analyses where they are not necessary or ignored as in earlier research that was not based on endogenous growth theory. When using the number of enterprises in preliminary country-specific regressions for the system (2'') and (3'') together with the numbers of researchers, we get information on the number of observations available per country. For most countries we have 13 observations per equation or 26 total observations, leaving us with a system total of 414 observations or 212 per equation, which is in the order of magnitude of the number of observations for firms, which is 250. We have 16 (32) observations for Canada; 8 (16) for the USA, 6 (12) for Japan, and 11 (22) for Norway. We form three panel data set; one including all 17 countries and two not including those with small numbers of observations: in the second dropping Japan, Norway, and USA, and in a third one including Norway, but not Japan and the USA.

## 4. Econometric Aspects

The data for LTH07, LBERDST, and LPUBST have panel (near) unit roots (see Table A.1). Therefore, we use cointegration methods for the estimation for 17 OECD countries[4]. We use three different estimation methods, which have been developed to deal with non-stationarity and endogeneity: group-mean versions of FMOLS and DOLS, which are consistent estimators (Pedroni 2001), as well as PMG/ARDL (pooled mean group estimator) (Pesaran et al. 1999). As the PMG/ARDL method yields different results than FMOLS and DOLS when assuming one cointegrating equation for the three variables of (1), we get somewhat suspicious in regard to this assumption. In the time-series literature, the ARDL method underlying the PMG for each country requires having only one cointegrating equation (Pesaran and Shin 1999) and two cointegrating relations of two variables are generally held to be more informative than one of three variables (Kilian and Lütkepohl 2017). Therefore, we also test for the number of cointegrating relations using the Johansen-Fisher panel cointegration tests for the cointegration rank. As the number of firms has a small number of observations, we cannot use the Fisher-Johansen test and FMOLS estimation. For estimation we lean on DOLS, and PMG/ARDL. For cointegration testing we use the Pesaran CIPS[5] and the Bai and Ng (2004) PANIC[6] test (with MQF meaning to allow for a VAR(p) with p>1 in the testing for the number of (non)stationary common factors found according to the average of the Bai/Ng criteria), and in connection with PMG/ARDL estimates, also the bounds test.

Results for the spillover parameters in the dynamic R&D functions (2'), (3') should be in the unit interval. Therefore, we set $\chi = \frac{e^c}{1+e^c}$ or $\chi = \frac{1}{e^{-c}+1}$. For any estimated value of the parameter *c* this yields a value of $\chi$ in the unit interval. Without this specification we find values outside the unit interval in country-specific explorations (not shown).

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[4] AUT,BEL,CAN,DEU,DNK,ESP,FIN,FRA,GBR,IRL,ITA,JPN,NLD,NOR,PRT,SWE,USA.

[5] Cross-sectionally Augmented IPS.

[6] 'PANIC' abbreviates 'Panel Analysis of Nonstationarity in Idiosyncratic and Common Components'.

(2'') and (3'') are partially linear equations. The nonlinear part requires a choice of starting values for the iterative nonlinear estimation procedure that makes it possible to get nonlinear estimates. Otherwise, we get no result and only messages like 'no valid observation in equation …' or 'log of non-positive number'. In setting initial values for the search procedure for the nonlinear parameters, we use the information from the calibration in HLP (2023) and from similar estimates preceding the current one (Greene 2012). When we do not add least-squares dummy variables (LSDV) to (2'') and (3'') we can get results only when using full-information maximum likelihood (FIML) as estimation method. When introducing least-squares dummy variables (LSDVs) we can get estimation results only by using the nonlinear, iterative least-squares method. All other ideas and efforts for estimation lead to no result because of the difficulties with nonlinear estimation: for some regressions we could not use fixed effects; for no regression we could extend the analysis to using lagged regressors as instruments. As problems of potential endogeneity and omitted variables may go into serial correlation, we will save the residuals of the equations and add their lags to the regressions. This procedure is typically followed in serial correlation tests based on the Frisch-Waugh-Lovell theorem (Davidson and MacKinnon 2004). Here we use it as a pragmatic serial correlation correction, being aware of the most well-known advice on serial correlation correction, which is 'don't' (Greene 2012); but that leads to no additional information. We also deal with serial correlation by way of extending the LSDV model with $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t,$ autoregressive processes of order two for the residuals; we drop them if they are statistically insignificant. For $\rho_2 = 0$, this is denoted as ar(1), and for $\rho_1 = 0$ this is denoted as ar(2), and if none of them is zero the notation is [ar(1), ar(2)]. When regressions are more important than just for the sake of comparison and an intuitive development of an argument, we test for cross-section dependence using the Pesaran CD test. It over-rejects the null of independence but the new, extended developments of Pesaran and Xie (2023) are not yet implemented in the programs. Testing for panel cointegration could be done by residual based tests. Those considering cross-section dependence like the Pesaran CIPS test, or the Bai&Ng PANIC test require many observations and are used again if possible. When these are not available, we use the older tests as a rough indication instead of an exact test. ECM based tests are criticized by Pedroni (2019) as missing important terms and therefore are not implemented in all econometrics packages and should be considered with scepticism otherwise. Banerjee and Carrion-i-Silvestre (2017) have improved the CIPS test but provide critical values only for the regressions carried out with pooled data and not those from group-mean estimation.

In trying to test for a hump-shaped relation for business researchers and government researchers, both as percent of the labour force, we explore FMOLS, DOLS, and PMG/ARDL, as all variables have panel unit roots in all tests with time trends (see Table A.2).

For the sake of comparison with steady-state and transitional dynamics of the HLP model we consider a panel VAR with slope homogeneity and fixed effects from the perspective of chapter 5 in Hsiao (2022). The recommendation there is to difference the model to get rid of the fixed effects and estimate using GMM or maximum likelihood with trend coefficient transformation. When the equations still have fixed effects, this points to country specific trends, whose coefficients appear as fixed effects after differencing. Differencing again leads to a VAR in changes of growth rates with zero intercepts which is stable in changes of growth rates but not in growth rate. Therefore, we pursue two avenues. First, we estimate the equations separately using the orthogonal deviation version of System GMM by Arellano and Bover (1995) and integrate them into a VAR. Second, we assume that the fixed effects have limited differences because the underlying country-specific trends may be similar if not identical. We analyze the effects of R&D cuts in both models and put the results for the latter into an appendix.

## 5. Estimation results for the technical progress function

Table 3 shows preliminary results under the assumption of having one log-linear cointegrating relation as in (1). This is closest to the traditional approach, which does not include the number of firms. The estimates using FMOLS and DOLS show positive signs, low standard errors and low p-values suggesting statistical significance. This is preliminary evidence in favour of the Cobb-Douglas function (1) of HLP (2023). The elasticity of production for private R&D relative to public R&D is 1.5 to 2 times as large, which is similar to the EU policy idea of having two thirds of R&D privately and one third public R&D (Meisters and Verspagen 2004). Linking back to the model this would suggest $\theta = 0.3$ and $\gamma = 0.18$ as the average of the two estimates. The PMG/ARDL estimator in the last column of Table 3 has a negative sign for public R&D stocks. This contradiction from the PMG results motivates us to test for the number of cointegrating equations, because the underlying time-series ARDL method is made for only one cointegrating equation and two pairs may be more informative than one triple of variables.

**Table 3 Regression coefficients for labour-augmenting technical change explained by private and public R&D stocks for 17 OECD countries 1964-2017**

| *Method* | FMOLS (b) (group mean) | DOLS (b) (group mean) | Pooled mean group/ARDL (c) |
|---|---|---|---|
| *Dependent→ Regressors ↓ (a)* | *LTH07* | *LTH07* | *LTH07* |
| *LBERDST (log Z)* | 0.313 (0.048; 0.00) | 0.284 (0.057; 0.00) | 0.413 (0.052; 0.00) |
| *LPUBST (log D)* | 0.229 (0.06; 0.00) | 0.148 (0.068; 0.03) | -0.089 (0.05; 0.075) |
| *Total observations* | 885 | 863 | 868 |
| *S.E. of regression* | 4.53 | 3.968 | Log-likelih.: 2310 |

(a) Constant and trends included in data transformations for FMOLS and DOLS, or, for PMG, outside long-term relation. Std. errors and p-values in parentheses.

(b) Long-run covariance estimates: Prewhitening with lags from AIC maxlags = -1, Bartlett kernel, Newey-West automatic bandwidth, NW automatic lag length.

(c) Selected model: PMG(1,2,2) indicates number of differenced terms per variable in the equation including the dependent variable on the left-hand side; unrestricted mean-group trend. Bounds test for cointegration not passed except for Italy and Sweden (AUT, PRT, NLD inconclusive); p=0.94 for similarity with mean-group estimator.

We assume that exact multicollinearity excluded by Pesaran et al. (1999) in assumption 5 is not a problem for public and private R&D as shown in Table 4.

**Table 4**

**Regression coefficients for labour-augmenting technical change explained by private and public R&D stocks for 17 OECD countries 1964-2017 in two cointegrating equations.**

| *Method →* *Variable (a)* | FMOLS (b) (group mean) | | DOLS (b) (group mean) | | Pooled mean group/ARDL (c) | |
|---|---|---|---|---|---|---|
| LTH07 | 1 | - | 1 | - | 1 | - |
| *LBERDST (log Z)* | 0.387 (9.47) | 1 | 0.358 (13.2) | 1 | 0.385 (10.7) | 1 |
| *LPUBST (log D)* | - | 0.543 (9.45) | - | 0.564 (9.97) | - | 0.628 (5.94) |
| *Trend* | yes | yes | yes | yes | -0.014 (-6.11) | 0.0093 (3.46) |
| *Tot. observ.* | 894 | 885 | 882 | 865 | 877 | 843 |
| *S.E.E. or Loglikelhood* | 3.865 | 7.39 | 2.96 | 7.213 | PMG(1,2) 2231.1 | PMG(4,0) 2856.3 |
| *p-val. 'No coint for all countr.' (d)* | 0.002 | 0.0000 | 0.415 | 0.0000 | 0.0000 | 0.0000 |
| *Indirect eff. LPUBST (e)* | LTH07 = 0.387* (0.543logD... ) ... | | LTH07= 0.358* (0.5643logD…) | | LTH07 = c - 0.014t + 0.385*(0.628logD…) | |
| *New tech production function (f)* | 2LTH07 = 0.387 *(0.543logD ...) + 0.387logZ ... | | 2LTH07= 0.358 *(0.564logD...) + 0.358logZ.. | | 2LTH07 = $c_1$ - 0.014t + 0.385* (0.628 log D + 0.0093t + $c_2$) + $c_1$ + 0.385logZ - 0.014t | |
| *Cobb-Douglas form of row 9* | TH07 = $AZ^{0.193}D^{0.105}$ | | TH07 = $AZ^{0.179}D^{0.101}$ | | TH07 = $Be^{-0.012t}Z^{0.179}D^{0.12}$ | |

(a) Constant and trends included in data transformations for DOLS and FMOLS or, for PMG, in long-term relation. t-statistic in parentheses (all p=0.0000). No cointegration for all countries.

(b) Long-run covariance estimates (Prewhitening with lags from AIC maxlags = -1, Bartlett kernel, Newey-West automatic bandwidth, NW automatic lag length). Regressions using FMOLS and DOLS do not show constant and trend.

(c) Selected models: PMG(1,2) indicates number of differenced terms per variable including the dependent; restricted trend (unrestricted not significant at 5% level but at 10% level; results do not change when dropping trend in second equation). PMG output does not show a constant in the long-term relation. Adjustment coefficients are -0.071 and -0.022 with p-values 0.0050 and 0.0003. Five (five) countries pass the bounds (Bai/Ng PANIC) test in the first equation and three (nine) in the second equation.

(d) Null: No cointegration for all countries. Bai and Ng (2004) PANIC test on the residuals with MQF meaning to allow for p>1 in a VAR(p) in the testing for the number of nonstationary common factors found according to the average of the Bai/Ng criteria with time and cross-section demeaned variables.

(e) Obtained from insertion of second equation into the first. Two or three dots (…) mean that constant and trend have been ignored.

(f) Obtained from adding the first cointegrating equation to the indirect effect of public R&D of row 8.

The Johansen-Fisher panel tests and a Johansen test for a VECM (vector-error correction model) with pooled data reject the assumptions of no cointegration or one cointegrating equation. This suggests having two cointegrating relations for the panel and for almost all countries (see Table A.3). We re-run the estimations of Table 3 assuming instead two cointegrating equations obtained separately, shown in Table 4.

In columns 1 and 2 of Table 4 we show results from using FMOLS, in column 3 and 4 for DOLS, in column 5 and 6 for PMG/ARDL. All methods are using a time trend, but for FMOLS and DOLS this is not shown in the regression output because they are included in a variable transformation. Columns 1, 3, and 5, explain labour-augmenting technology, *LTH07.* They show a positive effect of private R&D, denoted as *LBERDST* (log *Z* in (1)). This effect has coefficients of similar size in all three dynamic regression methods between 0.35 and 0.39. In columns 2, 4, and 6, we show the effect of public R&D, *LPUBST* (log*D* in (1)), and a time trend on private R&D. The coefficient of public R&D is positive, between 0.54 and 0.63. The time trend in column 5 is negative, which is in line with literature on negative research productivity growth (see Ziesemer 2021a).

The p-values for the Bai-Ng PANIC test for 'no cointegration for all countries' are low for FMOLS and PMG/ARDL estimates. For DOLS the first equation has a high probability for 'no cointegration for all countries'. This would suggest that cointegration is confirmed for the FMOLS and PMG/ARDL estimates.

We merge the two relations to get a Cobb-Douglas function for each method. Writing the first equation as $logTH07 = a + blogZ$, and the second as $logZ = c + flogD$, omitting the time trend for a moment, insertion of *logZ* from the second equation into the first yields $logTH07 =a+b(c+flogD) = a +bc + bflogD$, which is the indirect effect of public R&D on technical change. The numerical results for this, when including the time trend where possible, are shown in Table 4, row 8. Adding up the first cointegrating equation and the equation for the indirect effect of public R&D on the technology level in row 8 we get $2logTH07 = 2a +bc +bflogD+blogZ$. With *b* and *f* positive we have a log-linear function TH07 with positive exponents. This is shown numerically in row 9 of Table 4, and in standard Cobb-Douglas form in row 10, which is comparable with the function (1) of HLP (2023).[7] The coefficient *A* in the first two functions would include the time trend used implicitly in the data transformations in FMOLS and DOLS estimation. *B* is a constant in the last two functions because the time trends are shown explicitly.

The R&D elasticities of technology functions in levels are remarkably similar for the three methods. Using two cointegrating equations for three variables makes estimation results from FMOLS, DOLS and PMG/ARDL similar to each other, whereas they were different in Table 3. The PMG/ARDL result should be preferred according to Pesaran et al. (1999) who claim that their method is superior to FMOLS and DOLS,[8] and it is in line with the literature on decreasing research productivity, which we could not see though for FMOLS and DOLS

[7] Alternatively, one could interpret the regressions as first-order condition related to a generalized CES function (see Ziesemer 2021a).

[8] For the time-series analogue, Pesaran and Shin (1999) state that the ARDL approach dominates the FMOLS method when the signal-to-noise ratio is low, and vice versa.

estimates. Overall, these results are very similar to and provide support for the formulation of the technology function of HLP (2023).

The model of HLP suggests including the number of firms in the regressions. In (1') this would have an impact because of decreasing returns to private and public R&D. The data series is too short to repeat the Fisher-Johansen test for the number of cointegrating equations. Log(etp) is stationary for most unit root tests and has a low p-value for others (see Table A.2). Therefore, we assume having two cointegrating equations again, now for R&D stock variables per firm. From the DOLS estimate in column 1 and 2 of Table 5 we do not get explicit coefficients for constants and trends. For the PMG-ARDL method we are limited to allow for not more than two lags of the regressors, because of the small number of observations for the number of firms, which is likely to have an impact on the results; constants are country specific and therefore outside the long-term relation. For the second equation the trend is also country-specific and therefore outside the long-term relation.

**Table 5 Regressions including the number of firms, labour-augmenting technical change, private and public R&D stocks for 14 OECD countries 1964-2017. (a)**

| *Method →* *Variable ↓* | DOLS (b) | | PMG-ARDL (c) | |
|---|---|---|---|---|
| LTH07 | 1 | - | 1 | - |
| letp | - | 1 | - | 1 |
| *LBERDST-letp* | 0.253 (2.62) | -0.342 (-5.21) | -0.182 (-4.344) | -0.521 (-11.77) |
| *LPUBST-letp* | -0.243 (-3.53) | -0.503 (-8.70) | 0.168 (3.17) | -0.47 (-12.47) |
| *constant* | yes | yes | 0.766/-0.36 | 3.17/-0.32 |
| *Trend* | yes | yes | 0.0048 (8.745) | 0.0075/-0.32 |
| *Tot. observ.* | 171, 2003-2017 | 171, 2003-2017 | 157, 2007-2017 | 143, 2007 -2017 |
| *S.E.E. (d)* | 1.666 | 1.283 | - | - |
| *Loglikelhood* | - | - | 560.2488 | 842.6964 |
| *Tech progress function (e)* | logT - logetp = 0.596 (LBERDST-letp) + 0.26(lpubst-letp) + … | | logT - logetp = 7.778 + 0.028t + 0.339 (LBERDST-letp) + 0.302 (lpubst-letp) | |
| *CD form of row 8* | $T/N = B(Z/N)^{0.6}(D/N)^{0.26}$ | | $T/N = 2388e^{0.028t}\,(Z/N)^{0.34}(D/N)^{0.3}$ | |

(a) Long-term relations only. Short-term relations and adjustment coefficients not shown. T-values in parentheses. 14 countries.

(b) Panel method: Grouped estimation. Cointegrating equation deterministics: C, TREND. Automatic leads and lags specification (based on AIC criterion, max=*). Long-run variances (Prewhitening with lags from AIC maxlags = -1, Bartlett kernel, Newey-West automatic bandwidth, NW automatic lag length) used for individual coefficient covariances.

(c) For both equations: Dependent lags: 3 (Automatic). Automatic-lag linear regressors (2 max. lags): LPUBST-LOG(ETP); LBERDST-LOG(ETP). Deterministics: Unrestricted constant divided by adjustment coefficient and restricted trend. Model selection method: Akaike info criterion (AIC). Number of models evaluated: 27 and 18. Selected models: PMG(2,2,2) and PMG(2,2,2).

(d) DOLS and PMG estimates consist of two equations estimated separately.

(e) Obtained through subtraction of second from first equation. Two or three dots (…) mean that constant and trend have been ignored because they are included in the variable transformation and do not appear in the regression output.

We report the average value of country-specific coefficients divided by the adjustment coefficient as appropriate when they are taken into the long-term relation after the estimation. In row 9 we show the Cobb-Douglas functions for technical change. The results are likely to be sensitive to having very short T. FMOLS does not even work when the number of firms is included, and for PMG we had to limit the maximum number of lags. The bounds test indicates critical values only for at least thirty periods, but the F-statistic is based only on 11 observations where critical values would be higher. Cointegration tests considering cross-section dependence suffer from the same problem. Panel unit root tests on the residuals series of the equations in Table 5 ignoring csd (LLC, ADF Fisher and PP-Fisher chi-sq) with null 'unit root' have p=0.0000 for the residuals of the PMG-ARDL estimates and p < 0.005 for the DOLS estimates, indicating absence of unit roots in the residuals and thereby a rough suggestion of cointegration. Table 3, 4, and 5 all support the Cobb-Douglas form in the model of HLP, but they differ in the size of the coefficients because of the differences in length bringing the regression more or less away from being a cross-section regression as N/T ratios matter for panel results (Smith and Fuertes 2016; p.4). Coefficients in Table 3 and 5 are higher than those of Table 4. The time trend is strongest in the PMG estimate of Table 5.

On average, the elasticities of production for firm level variables are 0.28 for public R&D (per firm) and 0.47 for private R&D (per firm), which is higher than the estimates in Table 4 when when variables are not considered per firm. The ratio of the coefficients is slightly less than twice as large for private relative to public R&D. Methodologically, the production function for technical change has been derived from two cointegrating equations, one of which is explaining the number of firms as suggested by the theoretical model of HLP (2023). As this had been done above also without the number of firms, the more general lesson here is that estimation of a production function is not necessarily based on estimation of one equation but can be based on merging several cointegrating equations. The number of firms and the number of cointegrating equations may be aspects that help explain why early literature did not converge to numerically clear values of the elasticities (Hall et al. 2010). Again, this provides support for the HLP model, but no clear instruction for the choice of parameter values for the calibration is obtained, which is dependent also on the choices made for other parts of the model.

## *6. Estimation results for dynamic private and public R&D functions with CES spillovers from pooled panel data*

In this section we present results from the estimation of the dynamic processes of private and public R&D, (2'') and (3''), from panel data analysis with slope homogeneity and no fixed effects for which we could not get estimates because of the nonlinearity problems indicated above. We start in Table 6 with Full Information Maximum Likelihood estimates using the HLP version of government spillovers $s_{G,a} = L_G/LF$. In columns 1 to 4 we present estimates assuming identical spillover functions for private and public R&D processes. In column 5 and 6 we allow them to differ. In column 1 and 2 we do this for 17 OECD countries and in column 3 to 6 for the 14 OECD countries for which we have slightly more observations, dropping

## Table 6 Regression coefficients of dynamic private and public R&D equations with government R&D-labour share and CES constant-returns-to-scale spillover functions (a)

| Panel, model → Variables ↓ | OECD 17, identical | OECD17, ident., resid. augm. | OECD14 (b) ident. | OECD14 (b) ident., resid augm | OECD 14 (b), asym. | OECD 14 (b), asym., Resid augm. |
|---|---|---|---|---|---|---|
| α = $c_1$ | 0.042 (3.56) | 0.24 (16.3) | 3222(2.73) | 0.036 (14.3) | 0.726 (3.91) | 0.76 (15.0) |
| $L_G$/LF , $c_2$ | 23.06 (2.31) | 15.49 (3.8) | - | - | 27.0 (2.51) | 17.7 (3.57) |
| $(L_G/LF)^3$, $c_7$ | - | - | 757747.5 (4.32) | 560011.7 (6.77) | - | - |
| $c_{14}$ in $\chi_1 = \frac{e^{c(14)}}{1+e^{c(14)}}$ | 0.72 (9.75) | 0.79 (-8.2) | 0.63(4.58) | 0.67 (4.95) | -2.03 (-3.26) | -2.42 (-8.7) |
| $\chi_1$ | 0.67 | 0.69 | 0.65 | 0.662 | 0.116 | O.082 |
| $c_{18}$ in $\chi_2 = \frac{e^{c(18)}}{1+e^{c(18)}}$ | 0.72 (9.75) | 0.79 (-8.2) | 0.63 (4.58) | 0.67 (4.95) | 2.986 (5.97) | 33.46(216) |
| $\chi_2$ | 0.67 | 0.69 | 0.65 | 0.662 | 0.952 | 1 |
| η, $c_{13}$ | -2.93 (-5.75) | -3.65 (-10.8) | -5.95 (-4.1) | -6.89 (-7.8) | 0.52 (0.49) | 0.95 (1.89) |
| δ, $c_{17}$ | -2.93 (-5.75) | -3.65 (-10.8) | -5.95 (-4.1) | -6.89 (-7.8) | -4.61 (-2.24) | -70.4 (-11.7) |
| $\sigma_1 = 1/(1-\eta)$ | 0.254 | 0.215 | 0.144 | 0.127 | 2.085 | 18.02 |
| $\sigma_2 = 1/(1-\delta)$ | 0.254 | 0.215 | 0.144 | 0.127 | 0.178 | 0.014 |
| LOG($L_Z$), $c_8$ | 0.140 (4.09) | 0.112 (7.28) | 0.086 (2.4) | 0.067 (3.4) | 0.106 (3.05) | 0.093 (4.77) |
| LBERDST(-1),$c_{10}$ | -0.419 (-11.9) | -0.40 (-30.14) | -0.326 (-7.057) | -0.312 (-18.2) | -0.31 (-8.45) | -0.302 (-17.85) |
| LOG($L_G$), $c_9$ | 0.126 (4.73) | 0.123 (12.27) | 0.148 (5.25) | 0.136 (11.0) | 0.09 (3.35) | 0.076 (6.10) |
| LPUBST(-1), $c_{11}$ | -0.395 (-12.1) | -0.396 (-45.93) | -0.36 (-9.76) | -0.351 (-29.3) | -0.286 (-8.84) | -0.276 (-22.68) |
| Intercept 1 (c) | 7.51 (1131.6) | 5.887 (2239.9) | -4.003 (-166.46) | 7.6 (2857.7) | 4.01 (243.0 | 4.05 (894.2) |
| Resid 1 (-1) | - | 0.953 (95.2) | - | 0.95 (72.3) | - | 0.955 (79.6) |
| Intercept 2 | 4.24 (17.08) | 4.28 (90.6) | 3.72 (58.1) | 3.76 (68.3) | 3.603 (14.9) | 3.66 (69.1) |
| Resid 2 (-1) | - | 0.946 (97.2) | 0.93 (11.4) | 0.94 (94.8) | - | 0.950 (93.8) |
| Obs per equation (f) | 207 2002-17 | 189 2003 -17 | 185 2002-17 | 171 2003-17 | 185 2002-17 | 171 2003-17 |
| Iterations to convg. | 59 | 66 | 59 | 69 | 56 | 83 |
| Log likelihood | 703.9 | 1107.8 | 642.63 | 983.62 | 665.96 | 995.2 |
| DW statistic (d) 1st, 2nd equation | 0.026 0.03 | 1.65 1.94 | 0.0297 0.034 | 1.64 1.92 | 0.03 0.038 | 1.656 1.957 |
| Pesaran CD: p-val. (e) (resid of 1st, 2nd eq) | 0.044, 0.0002 | Na, na | 0.3068, 0.0234 | 0.0000, 0.1832 | 0.3777, 0.0173 | 0.0000, 0.0720 |
| Unit root LLC t* (resid of 1st, 2nd eq) | 0.0996 0.2579 | 0.0000 0.0000 | 0.1551 0.3049 | 0.0000 0.0000 | 0.2042 0.3290 | 0.0000 0.0000 |
| ADF-Fisher Chi sq. (resid of 1st, 2nd eq) | 0.1615 0.0288 | 0.0000 0.0000 | 0.1443 0.0141 | 0.0000 0.0000 | 0.2341 0.0228 | 0.0000 0.0000 |
| PP-Fisher Chi square (resid of 1st, 2nd eq) | 0.0655 0.0183 | 0.0000 0.0000 | 0.0448 0.0036 | 0.0000 0.0000 | 0.1613 0.0096 | 0.0000 0.0000 |

(a) Estimation Method: Full Information Maximum Likelihood (BFGS /Marquardt steps); coefficient covariance computed using the Huber-White method; z-values (= coefficient/ std. error) in parentheses. Results depend on initial values. Identical or asymmetric CES and spillover parameters in the private and public R&D production functions. (b) Excluding Japan, Norway, USA. (c) In the first regression we have log($c_1$) + $c_{24}$, which is a combination of a non-linear and a linear specification of an intercept allowed only under ML estimation (Greene 2012). (d) The Durbin-Watson statistic is used only as descriptive information, not as a test (see Epple and McCallum 2006). (e) Null: No cross sect.dep. (f) time span determined by CAN with most observation; loss of observation per dropped country is only 6 or 7.

Japan, Norway, and USA. In columns 2, 4, and 6 we add the residuals of the previous regressions in order to correct for serial correlation.

In columns 1 to 4 for identical spillover functions we find values for the linear own-spillover parameters $\chi$ between 0.65 and 0.69, which is close to the value of 0.7 assumed in the calibration of HLP, and it implies a cross-spillover, 1- $\chi$, from private to public R&D and vice versa of 0.35 to 0.31. The elasticity of substitution between private and public R&D capital is between 0.12 and 0.26, calculated from the estimation of the CES parameter between -2.9 and -6.9. The labour production terms have the expected positive sign and the R&D capital terms have the expected negative sign. The government labour share has a positive sign in all columns. The function $f = 1+\xi(s_G)^\alpha$ with $\alpha = 1$ or 3 runs up (in the data range going to $s_G$ = 0.0074) to 1.17 for a coefficient of $\xi$ = 23.06 in column 1, to 1.115 for a coefficient of $\xi$ = 15.49 in column 2, to about 1.3 for column 3, and 1.25 for column 4. As $s_G < 0.0074$, the expression $f(1- s_G)$ in formula (16) of HLP also is at almost the same values as those for f just indicated. By implication, the share of government researchers clearly enhances growth also at this level of the analysis. This confirms that HLP make reasonable assumptions regarding the CES spillover and labour interaction terms in the dynamic R&D function. In column 5 and 6 we now allow the CES spillover functions to differ between private and public. The private own spillover goes to about 0.1 implying a cross-spillover from public to private R&D of 0.9. The public own spillover goes to 0.95 or even 1, implying a cross-spillover from private to public of 0.05 or even zero. The elasticities of substitution go to remarkably high values for private R&D and very low values for public R&D. The labour share interaction effect is about the same as in column 1 and 2. There is obviously some panel heterogeneity (comparing columns 1 and 2 with 3 and 4, and an impact from serial correlation correction comparing even and odd numbered columns. The presence of panel heterogeneity suggests estimation on a country by country basis if there is an interest in getting all the parameters more exactly. The fourth last row of Table 6 tries to indicate the p-values for the hypothesis of no cross-section dependence (csd). These are 'non-available' or close to this again. In each case one of the equations has csd at the five percent level and the other has not. Residual augmentation turns around which one has (no) csd. Periods are too short to allow for panel unit root analysis considering csd. Given the overrejection of the null of independence by the Pesaran CD test (see Pesaran and Xie 2023), we may have more independence than indicated.

Panel unit root tests without considering cross-section dependence for the residuals are shown in the last three lines. In the equations without serial correlation correction there are common and individual unit roots according to the LLC and the ADF Fisher Chi square test, and, less likely, according to the PP Fisher Chi square test. This would suggest having no cointegration. However, in equations with serial correlation correction, unit roots vanish with the serial correlation correction. For other well-known tests considering csd we do not get any test output because the data series is too short. Therefore, our conclusion on cointegration is more intuitive than statistically exact.

Next, we replace the labour share of government researchers by the labour interaction term, $s_G = L_G{*}L_Z/\mathrm{E}{+}11$. Table 7, column 1 and 2, shows results for 17 OECD countries and in column 3 and 4 for 14 OECD countries leaving out Japan, Norway, and the USA because of the small number of observations. In column 1 and 3 we assume that private and public R&D equations both have the same spillover parameters, $\chi_1 = \chi_2$, and the same CES parameters of the spillover function, δ = η, and in columns 2 and 4 we allow these parameters to differ for the two equations. In Table 8 we add the serial correlation correction to these estimates.

In column 1, we have χ = 0.69, and the CES parameter is δ = η = -3.199 leading to an elasticity of substitution for private and public R&D in the spillover function of σ = 0.238 for the 17 OECD countries. The parameters for the *f* function in private R&D dynamics, $c_2$, $c_6$, $c_7$, imply the possibility that labour interaction goes (within the data range) to almost 65% beyond the model without interaction as shown in Figure 1.

**Table 7 Regression coefficients of dynamic private and public R&D equations with R&D-labour-interaction terms and CES spillover functions with constant returns to scale (a)**

| Panel → Variables ↓ | OECD 17 identical | OECD 17(e) asymmetric | OECD 14 (b) identical | OECD 14 (b) asymm. |
|---|---|---|---|---|
| $\alpha = c_1$ | 0.554 (2.29) | 0.303 (2.55) | 0.464 (2.19) | 0.443 (2.02) |
| $L_G*L_Z$, $c_2$ | 1.481 (4.44) | 0.963 (3.15) | -0.573 (-0.552) | 0.93 (0.76) |
| $(L_G*L_Z)^2$, $c_6$ | -1.045 (-3.47) | -0.887 (-2.97) | 14.73 (2.60) | 4.92 (0.78) |
| $(L_G*L_Z)^3$, $c_7$ | 0.22 (2.835) | 0.221 (2.79) | -28.74 (-2.95) | -13.49 (-1.31) |
| $c_{14}$ in $\chi_1 = \frac{e^{c(14)}}{1+e^{c(14)}}$ | 0.81 (9.38) | -1.61 (-4.59) | 0.7 (4.56) | -2.55 (-5.54) |
| $\chi_1$ | 0.69 | 0.166 | 0.668 | 0.072 |
| $c_{18}$ in $\chi_2 = \frac{e^{c(18)}}{1+e^{c(18)}}$ | 0.81 (9.38) | 3.29 (3.96) | 0.7 (4.56) | 4.07 (2.82) |
| $\chi_2$ | 0.69 | 0.964 | 0. 668 | 0.983 |
| η, $c_{13}$ | -3.199 (-5.985) | 1.22 (1.95) | -7.61 (-3.14) | 1.428 (2.265) |
| δ, $c_{17}$ | -3.199 (-5.985) | -3.64 (-3.06) | -7.61 (-3.14) | -3.42 (-3.278) |
| $\sigma_1 = 1/(1-\eta)$ | 0.238 | -4.545 | 0.116 | -2.336 |
| $\sigma_2 = 1/(1-\delta)$ | 0.238 | 0.2155 | 0.116 | 0.226 |
| LOG($L_Z$), $c_8$ | 0.091 (3.054) | 0.113 (3.82) | 0.0715 (1.96) | 0.08 (2.25) |
| LBERDST(-1), $c_{10}$ | -0.481 (-13.46) | -0.444 (-14.09) | -0.379 (-8.55) | -0.381 (-9.85) |
| LOG($L_G$), $c_9$ | 0.1135 (04.54) | 0.08 (3.34) | 0.1 (3.34) | 0.107 (3.375) |
| LPUBST(-1), $c_{11}$ | -0.386 (14.1) | -0.347 (-13.5) | -0.303 (-8.895) | -0.281 (-9.235) |
| Intercept 1 (c) | 6.629 (583.3) | 6.58 (687.8) | 6.03 (438.0) | 5.83 (383.7) |
| Trend 1 | -0.01 (1.364) | -0.0078 (-1.13) | -0.00878 (-1.09) | -0.0048 (0.64) |
| Intercept 2 | 4.84 (15.9) | 4.756 (16.78) | 4.143 (12.43) | 3.966 (11.82) |
| Trend 2 | -0.011 (1.76) | -0.00956 (-1.58) | -0.01 (1.483) | -0.0072 (7.83) |
| Obs per equation | 207 | 207 | 185 | 185 |
| Log likelihood | 727.2485 | 757.1327 | 656.6719 | 677.3284 |
| Pesaran CD: p-val. (d) | 0.9752, 0.5146 | 0.9577, 0.9867 | 0.8336, 0.3693 | 0.7967, 0.9078 |
| Unit root LLC t* (e) | 0.1930, 0.1680 | 0.4067, 0.1317 | 0.3615, 0.1564 | 0.3870, 0.2404 |
| ADF-Fisher Chi sq. (e) | 0.0261, 0.0066 | 0.4010, 0.0292 | 0.2854, 0.0252 | 0.2924, 0.0541 |
| PP-Fisher Chi sq. (e) | 0.1459, 0.0041 | 0.3505, 0.0095 | 0.1344, 0.0122 | 0.1759, 0.0603 |

(a) Estimation Method: Full Information Maximum Likelihood (BFGS /Marquardt steps); z-values (= coefficient/ std. error) in parentheses. Identical or asymmetric CES spillover parameters in the private and public R&D production functions. (b) Excluding Japan, Norway, USA. (c) In the first regression we have log($c_1$) + $c_{24}$, which is a combination of a non-linear and a linear specification of an intercept, allowed only under ML estimation (Greene 2012). (d) Tests applied to residuals of 1st and 2nd eq; null: No cross sect. dep. (e) Null: unit root.

In column 2, we allow the parameters to be different in private and public R&D dynamics. The own spillover parameter is now low in the private R&D function, $\chi_1 = 0.166$, implying a high public-to-private R&D cross spillover $1-\chi_1 = 0.834$, and it is high in public R&D own spillovers, $\chi_2 = 0.964$ implying a low private-to-public cross spillover of $1-\chi_2 = 0.036$. From the CES parameters we get $\sigma_1 = -4.545$, sign change indicating complementarity in private R&D dynamics, and $\sigma_2 = 0.2155$ indicating low substitutability in public R&D dynamics. The *f* function (similar to Figure 1 and not shown) would go to almost 1.4 in the maximum at $L_Z L_G = 0.756$.

In column 3, imposing identical functions again, now for 14 OECD countries, the own spillover parameter now is $\chi = 0.67$, a bit smaller than for 17 countries, and the elasticity of substitution is 0.116, which is about half of that for 17 countries, both together indicating panel heterogeneity. The *f* function (similar to Figure 1 and not shown) again remains near 1.4 in its maximum, which is located again slightly below $L_Z L_G = 0.8$.

In column 4 for 14 OECD countries, we again allow both functions to have different parameters. Again, own spillovers in private R&D are low at $\chi_1 = 0.072$ and high in public R&D $\chi_2 = 0.983$. Elasticities of substitution change sign again in private R&D dynamics and are low in public R&D, at $\sigma_1 = -2.336$ and $\sigma_2 = 0.226$ repectively. The *f* function peaks with a maximum below 1.3 at a value of LL slightly larger than 0.3, which is much lower than under identical parameters and lower than for 17 countries in column 2.

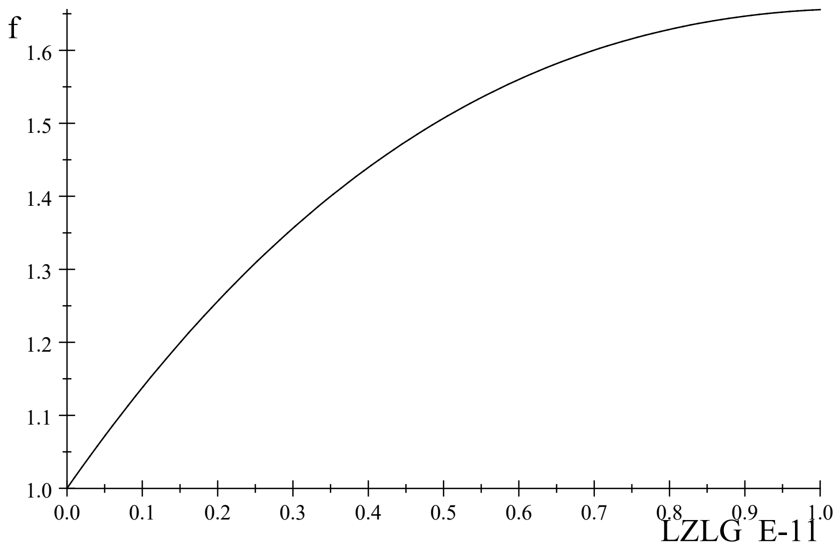


Figure 1 The impact of private-public research-labour interaction on private R&D growth

In spirit and even numerically, the results of Table 7 are close to those of Table 6 using the government research labour share. The fourth but last row shows high probabilities for cross-section independence in the residuals of all equations. The panel unit root tests for the residuals show high probabilities for unit roots suggesting lack of cointegration. In column 1 to 4 we therefore have very low Durbin-Watson statistics near zero (not shown). We have saved the residuals of the equations and added their lags to the regressions.

**Table 8 Regression coefficients of dynamic private and public R&D equations with R&D-labour-interaction terms, CES spillover functions with constant returns to scale, and residual augmentation (a)**

| Panel → Variables ↓ | OECD 17, identical | OECD 17, asymmetric | OECD 14 (b), identical | OECD 14 (b) asymmetric |
|---|---|---|---|---|
| $\alpha = c_1$ | 0.234 (4.22) | 1.877 (3.35) | 0.567 (6.07) | 0.475 (7.9) |
| $L_G*L_Z$, $c_2$ | 1.787 (8.79) | 1.114 (6.56) | 0.456 (0.814) | 1.549 (2.45) |
| $(L_G*L_Z)^2$, $c_6$ | -1.252 (-5.045) | -0.897 (-4.47) | 10.56 (3.365) | 1.955 (0.558) |
| $(L_G*L_Z)^3$, $c_7$ | 0.26 (3.40) | 0.203 (3.34) | -21.3 (-3.998) | -7.82 (-1.41) |
| $c_{14}$ in $\chi_1 = \frac{e^{c(14)}}{1+e^{c(14)}}$ | 0.954 (8.075) | -1.604 (-8.57) | 1.087 (3.77) | -2.278 (-6.413) |
| $\chi_1$ | 0.722 | 0.167 | 0.748 | 0.093 |
| $c_{18}$ in $\chi_2 = \frac{e^{c(18)}}{1+e^{c(18)}}$ | 0.954 (8.075) | 33.3 (184.1) | 1.087 (3.77) | 31.68 (52.65) |
| $\chi_2$ | 0.722 | 1 | 0.748 | 1 |
| $\eta$, $c_{13}$ | -4.31 (-7.977) | 0.881 (2.38) | -12.04 (-3.51) | 0.597 (0.815) |
| $\delta$, $c_{17}$ | -4.31 (-7.977) | -63.69 (-15.73) | -12.04 (-3.51) | -54.2 (-15.15) |
| $\sigma_1 = 1/(1-\eta)$ | 0.188 | 8.41 | 0.077 | 2.48 |
| $\sigma_2 = 1/(1-\delta)$ | 0.188 | 0.0155 | 0.077 | 0.018 |
| LOG($L_Z$), $c_8$ | 0.054 (2.76) | 0.083 (4.41) | 0.029 (1.367) | 0.049 (2.50) |
| LBERDST(-1), $c_{10}$ | -0.469 (-28.05) | -0.433 (-26.28) | -0.377 (-23.69) | -0.376 (-24.5) |
| LOG($L_G$), $c_9$ | 0.136 (9.646) | 0.093 (6.74) | 0.13 (9.84) | 0.124, (10.76) |
| LPUBST(-1), $c_{11}$ | -0.363 (-33.5) | -0.408 (-30.1) | -0.33 (-26.3) | -0.323 (-30.6) |
| Intercept 1 (c) | 7.73 (41.06) | 4.94 (18.6) | 6.21 (1226.35) | 6.01 (1478.0) |
| Trend 1 | -0.0097 (-4.90) | -0.0075 (-3.84) | -0.008 (-5.669) | -0.0047 (-3.399) |
| Resid 1 (-1) | 0.952 (78.64) | 0.962 (72.3) | 0.945 (77.26) | 0.962 (78.6) |
| Intercept 2 | 4.83 (46.56) | 4.80 (46.8) | 4.132 (58.1) | 0.124 (10.76) |
| Trend 2 | -0.01 (-6.046) | -0.0111 (-5.47) | -0.01 (-8.65) | -0.008 (-6.669) |
| Resid 2 (-1) | 0.934 (67.9) | 0.95 (68.84) | 0.928 (86.77) | 0.949 (88.2) |
| Obs per equation | 189 | 189 | 171 | 171 |
| Log likelihood | 1105.047 | 1114.284 | 985.5202 | 996.8732 |
| Adj R-sq | 0.379, 0.252 | 0.39, 0.256 | 0.374, 0.239 | 0.387, 0.245 |
| DW statistic (d) | 1.66, 1.89 | 1.691, 1.951 | 1.644, 1.871 | 1.677, 1.93 |
| Pesaran CD: p-val. (e) | Na, na | Na, na | 0.0000, 0.1184 | 0.0000, 0.0845 |
| Unit root LLC t*(f) | 0.0000, 0.0000 | 0.0000, 0.0000 | 0.0000, 0.0000 | 0.0000, 0.0000 |
| ADF-Fisher Chi sq. (f) | 0.0000, 0.0000 | 0.0000, 0.0000 | 0.0000, 0.0000 | 0.0000, 0.0000 |
| PP-Fisher Chi sq. (f) | 0.0000, 0.0000 | 0.0000, 0.0000 | 0.0000, 0.0000 | 0.0000, 0.0000 |

(a) Estimation Method: Full Information Maximum Likelihood (BFGS /Marquardt steps); z-values (= coefficient/ std. error) in parentheses. Results depend on initial values; mostly we find the highest likelihood when initial values from the calibration are used. Identical or asymmetric CES and spillover parameters in the private and public R&D production functions. (b) Excluding Japan, Norway, USA. (c) In the first regression we have log($c_1$) + $c_{24}$, which is a combination of a non-linear and a linear specification of an intercept allowed only under ML estimation (Greene 2012). (d) The Durbin-Watson statistic is used only as descriptive information, not as a test (see Epple and McCallum 2006). (e) Tests applied to residuals of 1st and 2nd eq; null: No cross sect. dep. (f) Null: unit root.

Therefore, we report both results in order to make the sensitivity visible. The results for residual augmentation are shown in Table 8.[9]

The result for the residual augmentation of Table 7, column 1, is shown in column 1 of Table 8. The spillover parameter is now $\chi = 0.72$, which is almost identical to the calibration of HLP (2023) and similar to that of Table 7, column 1. The elasticity of substitution is now about 0.188, which is below that from the CES parameter assumptions of $\delta = \eta = 0.287$ in the calibration of HLP, which leads to a CES above unity. The *f* function (not shown) goes to about 1.8, which is slightly higher than without added residuals, and has a maximum at $L_Z L_G$E-11 = 1.07.

The result for the residual augmentation of column 2 of Table 7 is shown in column 2 of Table 8. The own-spillover parameters are now $\chi_1 = 0.167$ and $\chi_2 = 1$, implying again high(er) public-to-private spillovers and no private-to-public spillovers, which is more extreme than in the earlier results above. The elasticities of substitution are now 8.41 and 0.0155, high(er) in private R&D and low(er) in public R&D. The *f* function goes slightly beyond 1.4 and reaches a maximum at $L_Z L_G$E-11 = 0.888.

The result for the augmentation of column 3 of Table 7 is shown in column 3 of Table 8. The spillover parameter is now $\chi = 0.748$, slightly higher than the result in column 1 and the calibration of the theoretical model of HLP. The elasticity of substitution is now 0.077, which is very low. The *f* function goes to about 1.5, which is slightly higher than without added residuals, and has a maximum at $L_Z L_G = 1.07$ as in column 1.

The result for the residual augmentation of column 4 of Table 7 is shown in column 4 of Table 8. Spillover parameters again are very low for private, $\chi_1 = 0.093$, and very high for public R&D functions, $\chi_2 = 1$, implying the opposite for the cross spillovers: a strong effect from public to private and none for private to public. The labour interaction function *f* again peaks around 0.35 with a maximum of about 1.4.

Spillover parameters are near the calibration values of 0.7 in HLP(2023) in all equations of Table 7 and 8 when imposing that the spillover functions are identical for private and public R&D capital growth processes, and they get very low for private and very high for public R&D when we allow them to differ between private and public R&D processes, implying that public R&D has strong spillovers to private R&D but private R&D has (almost) no spillovers to public R&D. Elasticities of substitution do not change signs anymore when lagged residuals are added in Table 8. They are low when assuming identical ones for private and public R&D. They get higher for private R&D and even lower for public R&D when the spillover functions are allowed to differ. Residual augmentation improves the DW statistic and makes unit roots vanish and therefore reduces the suspicion of non-cointegration. This could change under cross-section dependence, which remains present in Table 8. However, the test is (close to) unavailable when countries with less (more) observations are included and it rejects the null of independence too often (Pesaran and Xie 2023).

[9] One plausible reason is the omission of foreign R&D because the motivating model by HLP (2023) is a closed economy model. Its inclusion can be done in future research.

In all equations of Table 7 and 8 we see a coefficient for labour much below unity and for R&D capital much less negative than minus one. This may indicate that the linear labour part of the function, $L_Z/Z$ or $L_D/D$, could be replaced by a Cobb-Douglas function. The parameters would suggest that they have decreasing returns. In addition, we have small negative time trends roughly between one percent or a half, which may not only be mere detrending but rather indicate productivity decreases in dynamic R&D capital production functions.

The appearance of decreasing returns together with technical change in R&D production functions is similar to and familiar from classical economics and has entered neoclassical economics in agriculture-industry models in the 1960s (Jorgenson 1961) and later for the case of positive technical change. The idea that high elasticities of substitution (or even complementarities) can have the same effect as technical change has been discussed more recently by Klump and De La Grandville (2000) and the related literature (see Ziesemer 2023a). This idea may be even more important if technical change is negative at the macro-level, perhaps through structural shifts to sectors with low productivity growth. Decreasing returns, technical change and the elasticity of substitution are therefore closely related, and it is hardly surprising that they all appear in R&D production functions again.

It remains to be clarified in future research whether the assumed linearity and the calibrated substitution in the spillovers function in the model of HLP (2023) is more or less exactly equivalent to the empirically low substitution with decreasing returns and negative time trends shown above. Perhaps simulation analysis can help show in the future how similar they are.

**7. Panel data analysis: Country fixed effects and autoregressive processes**

In this section we try to add country-specific fixed effects in the LSDV form. Only the nonlinear, iterative least squares estimation method for the case of identical CES spillover functions for both processes give a regression output, which is presented in Table 9. Again, we need to start with the calibration values of the theoretical growth model. For Japan and the USA, we have an insufficient number of observations. Therefore, we include the other 15 countries, for which we typically have 13 observations per equation of each country (11 for Norway, 16 for Canada). When we add autoregressive (ar) processes for the residuals, we lose an observation per lag in each equation for each country leaving us with only eleven observations for most of the equations. Results are shown in Table 9 for varying assumptions on ar processes.

Because of the small number of observations and fixed effects absorbing degrees of freedom, many coefficients are statistically insignificant. CES parameter $\eta$ is statistically significant at the five or one percent level with values between -0.64 and -0.7 leading to an elasticity of substitution of about $\sigma = 0.6$. Again, this provides support for the idea of a CES spillover function. The spillover parameter $c_{14}$ in the e-functions are between 0.375 and 0.86, leading to $\chi$ between 0.59 and 0.7. However, they are statistically insignificant. Setting $c$ to zero yields $\chi = \frac{e^0}{1+e^0} = 0.5$. In column 2 and 3, in the private R&D equation an ar(2) process is significant, and in the public R&D equation an ar(1) process is significant at all standard levels, and an ar(2) process is significant at the 10 percent level. Introducing the ar processes turns the unexpected negative sign of the government labour term in the public R&D equation (column 1) into a positive one (column 2 and 3). Unlike Table 7 and 8, the coefficients of the labour-interaction terms in the private R&D equation are statistically insignificant. This does not change when taking one or two of them out, which is a major difference with the previous tables, raising the question whether the interaction specification or the collaboration idea are false, or significance would come about with more observations and degrees of freedom. When ar processes are included the functions peak at f = 1.875 and 1.89 both at LL = 0.41, and a minimum with f < 1 at LL = 0.047. The corresponding function f(1-s) is above unity for the range 0.17439 < s < 0.45360 for the estimate of column 2, and the range 0.17369 < s < 0.45680 for the estimate of column 3; both ranges are within the data range for s, (0,1). A positive minimum of interaction is required to get a nonnegative effect. We were unable to get results when the government labour share was used instead of an interaction term.

**Table 9 Regression coefficients of dynamic private and public R&D equations with R&D-labour-interaction terms, fixed effects, CES spillover functions with constant returns to scale, and autoregressive-processes (a)**

| Panel →<br>Variables ↓ | OECD 15 | OECD 15 | OECD 15 |
|---|---|---|---|
| $L_G$*$L_Z$, $c_2$ | -5.69 (1.54) | -2.277 (-0.335) | -2.373 (-0.36) |
| $(L_G*L_Z)^2$, $c_6$ | 29.6 (1.91) | 26.84 (0.849) | 27.6 (0.892) |
| $(L_G*L_Z)^3$, $c_7$ | -36.1 (-1.77) | -39.2 (-0.89) | -40.28 (-0.934) |
| $c_{14}$ in $\chi = \frac{e^{c(14)}}{1+e^{c(14)}}$ | 0.375 (0.38) | 0.858 (0.481) | 0.606 (0.347) |
| $\chi$ | 0.593 | 0.7 | 0.647 |
| η, $c_{13}$ | -0.643 (-2.77) | -0.70 (-2.019) | -0.66 (-3.14) |
| σ =1/(1-η) | 0.61 | 0.588 | 0.602 |
| LOG($L_Z$), $c_8$ | 0.496 (4.58) | 0.525 (3.87) | 0.525 (3.966) |
| LBERDST(-1),$c_{10}$ | -1.43 (-5.70) | -2.654 (-5.54) | -2.616 (-5.43) |
| ar 1st eq: ar(2) | - | 0.243 (2.83) | 0.240 (2.84) |
| LOG($L_G$), $c_9$ | -0.082 (-0.664) | 0.213 (1.845) | 0.214 (1.788) |
| LPUBST(-1), $c_{11}$ | -0.277 (-0.91) | -1.877 (-2.439) | -2.36 (-3.13) |
| Trend per country (b) | yes | yes | yes |
| ar 2nd eq: ar(1); ar(2) | - | 0.597 (c)<br>(7.34) | 0.6; -0.148<br>(6.32) (-1.68) |
| Period (d) | 2002-2017 | 2003-2017 | 2004-2017 |
| Obs (e) | 392 | 347 | 332 |

(a) Estimation Method: Iterative Least Squares; t-values (= coefficient/std. error) in parentheses. Identical CES and spillover parameters in the private and public R&D production functions. Excluding Japan, USA. Country specific intercepts and trends in both equations; slope homogeneity for ar processes. (b) Results are shown in Table 10. (c) ar(1) only. (d) 13 observations per country and equation, 11 for NOR, 16 for CAN; periods indicated for CAN, all others three less, NOR 5 less. (e) 15 obs lost per ar lag.

For the three equations in Table 9, the results for country-specific time trends per country are shown in Table 10. Averaging over the countries, only public R&D has a negative trend in public research productivity in column 2 of Table 10 belonging to column 1 of Table 9, where we have no autoregressive processes. When ar processes are introduced productivity trends become positive on average for these 15 OECD countries for data of this millennium until 2017.

However, time trends may also simply detrend variables on the right-hand side of the estimated equations. Unfortunately, we do not get results with fixed effects and ar-terms when allowing for different parameters in the two R&D functions or when using different estimation methods. Moreover, using lags as instrumental variables has not led to any regression output, perhaps because of losing one more year of observations when using lagged instrumental variables.

**Table 10**
**Research productivity trends per country: Coefficients of time trend per equation with fixed effects**

| Specification (a) | OECD 15 no ar process | | OECD 15 1st eq.: ar(2); 2nd eq. ar(1) | | OECD 15, 1st eq.:ar(2);2nd eq.ar(1), ar(2) | |
|---|---|---|---|---|---|---|
| Equation → Country↓ | DLBERDST | DLPUBST | DLBERDST | DLPUBST | DLBERDST | DLPUBST |
| AUT | 0.0140 | -0.0159 | 0.0711 | 0.03177 | 0.0700 | 0.0467 |
| BEL | 0.0441 | 0.0250 | 0.0735 | 0.0722 | 0.0720 | 0.0842 |
| CAN | - 0.0109 | -0.0268 | - 1.7531e-05 | 0.0103 | - 0.0012 | 0.0236 |
| DEU | - 0.0109 | 0.0171 | 0.0307 | 0.0473 | 0.0290 | 0.0661 |
| DNK | - 0.01305 | -0.0153 | 0.0185 | 0.0295 | 0.0169 | 0.0534 |
| ESP | - 0.033 | -0.07374 | - 0.00105 | -0.01516 | - 0.0023 | 0.0025 |
| FIN | - 0.0241 | -0.0306 | - 0.0330 | -0.00269 | - 0.0337 | 0.0102 |
| FRA | + 0.0074 | +0.01323 | 0.0150 | 0.0210 | 0.0140 | 0.0294 |
| GBR | + 0.0163 | 0.0055 | 0.0241 | 0.02045 | 0.0232 | 0.0235 |
| IRL | 0.0723 | 0.0174 | 0.0685 | 0.0092 | 0.0680 | 0.0169 |
| ITA | 0.0012 | -0.0320 | 0.0177 | -0.0347 | 0.0179 | -0.0257 |
| NLD | 0.0625 | 0.0603 | 0.08735 | 0.0789 | 0.0869 | 0.0856 |
| NOR | 0.0199 | -0.00134 | 0.0550 | 0.0419 | 0.0538 | 0.0549 |
| PRT | - 0.0560 | -0.0517 | - 0.0181 | -0.0370 | - 0.0170 | -0.0512 |
| SWE | - 0.00166 | 0.0021 | - 0.0070 | 0.0388 | - 0.0084 | 0.0563 |
| Average | 0.0059 | -0.0071 | 0.027 | 0.021 | 0.0259 | 0.0318 |

(a) Corresponding to the three equations and columns of Table 9.

## 8. A VAR model in growth rates for 14 OECD countries with cuts in public R&D

### 8.1 The panel VAR model

In this section we compare the dynamic properties of the HLP model with those of a VAR (vector autoregressive) model in growth rates for the 14 OECD countries with sufficiently many observations. We consider the growth rates in terms of log differences (dL) of technical progress TH07, private R&D stock, BERDST, public R&D stocks, PUBST, number of enterprises, ETP, number of business researchers, BR, and number of government researchers, GR. VARs in log-levels of pooled data with lag length eight or less are all unstable. The corresponding VECMs are mostly unstable, but for some constellations VECMs are stable for some combinations of lag length and number of long-term relations. But then the equations turn out to have fixed effects as in Hsiao (2022), chapter 5.2.1.1, special case iv. The recommended procedure then is to take differences of the underlying VAR to get rid of the fixed effects. This leads us to a VAR in growth rates, for which use of maximum likelihood estimation, or GMM with lagged regressors as instrumental variables is recommended. It turns out that all equations have fixed effects again. The reason for this may be that differencing leads to country-specific coefficients stemming from the time trend, which in Hsaio's textbook model has slope homogeneity. Finding these fixed effects suggests that time trends do have slope heterogeneity as in the single-country VECM estimates of Soete et al. (2022). To get rid of the fixed effects, we may take differences again, leading to a VAR in changes of growth rates without constants that have vanished through the differencing. A steady state then requires zero changes of growth rates. The maximum number of lags according to Schwert's formula is eight. VARs with eight or seven lags are unstable. Therefore, we try using a VAR with lag length six, which is stable and recommended by standard length criteria AIC (preferred by Kilian and Lütkepohl 2017), HQ, LR, and FPE, whereas SIC suggests 2 lags, leaving probably more (risk of) serial correlation in the model. However, while stable in changes of growth rates the model generates growth rates some of which get exorbitantly large when simulating forward to 2100, also for any other lag length. Therefore, we have to go back to a VAR in growth rates, for which we have two options: (i) under the assumption that the bias from fixed effects is small as coefficients from trends cannot differ strongly, we can ignore the fixed effects and estimate a VAR in growth rates using maximum likelihood estimation; (ii) we can take fixed effects into account, leading us to panel VAR with equations estimated separately using the orthogonal deviation method of Arellano and Bover (1995). Both versions generate comparable results supporting the HLP model.

For the VAR in growth rates of pooled data, among the stable models with lag length up to 7, we choose the model with four lags, which is suggested by the Hannan-Quinn criterion (when the maximum number of lags is 6), which is known to be consistent.[10] We then get rid of cross-section dependence. Models with lags three or four have no negative growth rates in the forward simulation until 2100; allowing for more lags, leads to several negative growth rates in the long run, which is unrealistic as the corresponding variables would run to negative values in the long run. We report results from an ML estimate of a VAR in growth rates with four lags (ignoring fixed effects) in an appendix.

For the VAR with GMM-OD (orthogonal deviations) there are two approaches. Abrigo and Love (2016) have developed a VAR version for STATA where the complete system is estimated simultaneously. Alternatively, we can estimate the equations separately and then import them into a simultaneous equation system (Chu et al. 2021) and estimate the intercepts using the seemingly unrelated regression (SUR) method. This implies a loss of efficiency. However, we use four lags in line with the lag length tests from the VAR based on

[10] When allowing for a maximum of seven lags, HQ suggests three lags and we get cross-section dependence in the residuals.

the HQIC and, in line with the VAR 'culture' we do not drop insignificant ones when there is low significance because this could stem from collinearity.

As a VAR with six equations, each with six variables with four lags has 24 estimated slope coefficients in each equation, we have summarized the results for the orthogonal deviation

**Table 11 System GMM (orthogonal deviations) for growth rates: Sums of coefficients (a)**

| *Regressors\Dependent* | *D(LOG(TH07))* | *D(LBERDST)* | *D(LPUBST)* | *D(LOG(ETP))* | *D(LOG(BR))* | *D(LOG(GR))* |
|---|---|---|---|---|---|---|
| *D(LOG(TH07(-j)))* *sum of c(j)* | -0.554 (-1.671) | 0.092 (0.763) | 0.087 (0.776) | 0.138 (0.224) | 0.900 (0.868) | -1.852 (-2.597) |
| *D(LBERDST(-j))* *sum of c(j)* | **0.074 (0.409)** | **0.562 (5.659)** | 0.017 (0.245) | -1.266 (-2.866) | -0.458 (-0.553) | 0.431 (0.790) |
| *D(LPUBST(-j))* *sum of c(j)* | -0.383 (-1.648) | **0.252 (2.263)** | **0.627 (6.536)** | -0.258 (-0.502) | 0.887 (1.008) | -1.187 (-2.024) |
| *D(LOG(ETP(-j)))* *sum of c(j)* | -0.173 (-2.63) | **0.085 (3.33)** | -0.011 (-1.20) | -0.225 (-1.213) | **0.497 (2.114)** | **0.829 (4.789)** |
| *D(LOG(BR(-j)))* *sum of c(j)* | -0.035 (-0.573) | **0.016 (4.84)** | 0.015 (0.890) | 0.007 (0.043) | -0.453 (-1.467) | -0.168 (-0.907) |
| *D(LOG(GR(-j)))* *sum of c(j)* | **0.129 (1.416)** | 0.019 (0.554) | -0.041 (-1.61) | -0.151 (-0.851) | **0.503 (1.819)** | 0.223 (1.108) |
| *Periods* | 10 | 10 | 10 | 11 | 11 | 11 |
| *Total panel (unbalan.) observations* | 101 | 101 | 101 | 115 | 114 | 114 |
| *Mean dependent var* | -0.0016 | 0.0026 | 0.0044 | 0.0031 | -0.0018 | 0.0069 |
| *S.D. dependent var* | 0.0165 | 0.0108 | 0.0081 | 0.0368 | 0.0591 | 0.0720 |
| *GMM IV lagged dep.(c)* | -2, -7 | -2, -6 | -3, -7 | -2, -7 | -3, -8 | -2, -5 |
| *S.E. of regression* | 0.0141 | 0.0062 | 0.0047 | 0.0378 | 0.0618 | 0.0411 |
| *Sum squared resid* | 0.0133 | 0.0026 | 0.0015 | 0.1144 | 0.3019 | 0.1336 |
| *J-statistic* | 51.797 | 37.065 | 31.721 | 55.578 | 62.793 | 33.227 |
| *Instrument rank* | 72 | 66 | 66 | 78 | 80 | 64 |
| *Prob(J-statistic)* | 0.067 | 0.247 | 0.481 | 0.095 | 0.0408 | 0.2687 |
| *Pesaran CD p-value (b)* | 0.1256 | 0.0701 | 0.0815 | 0.4503 | 0.0745 | 0.0716 |
| *Constant (p-val.) (d)* | 0.026 (0.00) | 0.001 (0.25) | 0.011 (0.00) | 0.105 (0.00) | 0.039 (0.00) | 0.050 (0.00) |

(a) Method: Panel Generalized Method of Moments. Transformation: Orthogonal Deviations, also for period dummies except 1st eq with levels not transformed to slightly improve CD test. 2SLS instrument weighting matrix. Cross-section weights (PCSE) standard errors & covariance (d.f. corrected). Sample (adjusted): 2008 to 2017 or 2018. 1 to 4 lags for all regressors. T = coefficient/stddev values in parentheses; stdev calculated as root of variance, which equals the sum of all elements of the coefficient covariance matrix (Davidson and MacKinnon 2004, formula 3.68) of the four lags of a regressor.

(b) Null hypothesis: No cross-section dependence in residuals. Test employs centered correlations computed from pairwise samples. Cross-sections included: 14.

(c) First and last lag for GMM style instrumental variable for lagged dependent variables. Tested using Sargan chi-sq difference test against less or more regressors. Regressors serve as their own instrument for all other variables.

(d) Slope results from SGMM-OD imported to simultaneous equation system for SUR estimation of constant.

version of System GMM[11] in Table 11 reporting the sum of coefficients over all four lags for each variable.

Technical change in column 1 is reduced by 55% of the sum of its lagged values. The number of public R&D workers has positive effects. Surprisingly, the growth of private researchers has a small negative effect in terms of growth rates. Public and private R&D expenditure stocks have negative effects on changes in rates of technical change because this captures the costs in the presence of R&D workers.[12]

In column 2, all regressors enhance the growth rate of BERD stocks with the largest coefficients coming from their own lags. In particular, public R&D is positively correlated because it leads to an incentive to invest in private R&D, which indicates a lagged complementarity. Technical change needs to be mastered and generates the means through growth and therefore leads to higher growth of private R&D expenditure. Business and government researchers have a small positive effect, respectively.

According to column 3, public R&D growth reacts positively to its own lags. All other coefficients are very small. Public and lagged private R&D capital stocks are almost unrelated in terms of growth rates. Regarding the question of complementarity versus substitutability of R&D it becomes clear here that we have complementarity in column 2 and 3 (much smaller) and with lags. Public R&D growth reacts slightly negatively to the lagged number of firms and the number of government researchers and positively to that of business researchers. The weak and negative impact of the lagged number of government researcher on the growth of public R&D expenditure stocks suggests that after rapid growth of hirings, growth of public R&D slows down with or without an increase in wages.[13] Technical change drives growth and thereby earns the money for public R&D expenditure growth.

The number of firms in column 4 reacts negatively to its own lags. Technical change creates growth and extends the market, leading to more firms' growth. Business R&D are fixed costs requiring a lower number of firms. Public R&D expenditures also have a negative effect. Past growth rates of private researchers lead to more firms, according to the theory through inventions. Government researchers also reduce the growth of the number of firms through resource competition.

[11] An alternative version of system GMM is that of Blundell and Bond (1998). It has a level equation with lagged differences and a difference equation with lagged levels as instruments, where the later typically has weak instruments and therefore the level equation is added to avoid this. The orthogonal deviation version of Arellano and Bover (1995) does not use the equation with differenced residuals but rather replaces it by a version with Helmert transformation, where the average of the future residuals is subtracted from the current residuals. This version is less sensitive to missing observations.

[12] When only cost or only output indicators are included in a regression the results may be dubious but becoming clear when the counterpart is also included. Blankenau et al. (2007) show this for taxation and education. Soete et al. (2022) use accumulated R&D expenditure (cost) in line with the literature (OECD 2017) as we do here in the early sections all allowing for the interpretation of knowledge indicator as in HLP. The results are clearly positive with only a few exceptions because R&D has remarkably high rates of return when dynamic methods are used (Ziesemer 2023b) making the positive aspects dominate.

[13] David and Hall (2000) explain closely related possibilities why the expenditure and the number of researchers may go opposite ways.

The number of business researchers reacts negatively in column 5 to business R&D stock growth, suggesting that money also goes to wage increases leading to less growth of the number of researcher. The number of firms, and earlier growth of the numbers of government researchers (generating start ups?) increases the number of business researchers. The latter is therefore a complementarity, here with lags, in line with the HELP model. Lagged numbers of business researchers have a negative effect. Public R&D capital growth has positive effects, indicating complementarity and perhaps even policy action coordinating firm and government behavior. Lagged technical change has a strongly positive effect suggesting that past success encourages hiring researchers.

The number of government researchers reacts in column 6 negatively to the past change of growth of business researchers, indicating here a lagged substitutability relation between the number of private and public researchers. The number of firms increases the growth of government researchers a bit more than it increased demand for business researchers. Weakness in the growth of technical change, and private R&D expenditures have positive effects on the change of growth of government R&D workers perhaps because of more spending for projects at technical universities or other government institutes. Public R&D has a strongly negative effect on the growth rate of government researchers indicating that they also drive up wages (Goolsbee 1998) and slow down the growth of hirings, a partial effect.

Many variables have low t-values, which do not necessarily mean that these variables should be dropped but rather that R&D is risky. As all variables have positive growth rates according to Table 2, a look at the positive coefficients in the system can tell us which of the variables are the major drives of growth are. There are two variables which do not have a major driver, the growth rate of the number of firms and of public R&D expenditure stocks, which has persistence with coefficient of 0.63. Public R&D has a positive impact on private R&D, which in turn has a positive impact (with low t-value indicating risk) on technical change; this is the standard main line of argumentation in the literature (OECD 2017; Soete et al. 2022). The number of firms suggested by the HLP model drives demand for private and government researchers (the latter with positive feedback to private researchers); private researchers drive private R&D stocks in terms of costs and experience and government researchers drive technical change directly. The five statistically significant positive constants indicate that part of the growth is exogenous, with an insignificant coefficient for business R&D suggesting full endogeneity.

The p-values for the J-statistic indicates that the probability of being chi-square distributed is higher than 5 percent with one exception. The Sargan difference test (for adding or taking out lagged dependent regressors at the early and the late end; not shown) suggests that many lags increase the J-statistic and decrease its p-value not too much to be in the chi-square distribution (Davidson and MacKinnon 2004) but also not too little to have no effect from the additional instruments (Roodman 2009). We have not added instruments when the p-value of the difference test is far outside the interval (0.75, 0.25) or when the p-value of the Hansen-Sargan test goes to very low values through adding an instrument. The last row of Table 11 suggests absence of cross-section dependence for all variables with probability higher than five percent.

We have imported the slope estimates from the GMM-OD approach into a simultaneous equation model to get the constant, which are not contained in the GMM-OD regression output, through a SUR estimate shown in the last row of Table 11. We use the simultaneous equation model, a VAR with GMM-OD slopes, then for simulations in order to compare them to results of the HLP model.

### 8.2 Public R&D cuts and the dynamic properties of the GMM-VAR and the HLP model compared

In the data we find a negative 1.1% *change* in growth rates for government researchers during 2006-2017, which also has the strongest standard deviation; this is the most negative change of growth rates in our data set. The level series goes down after 2015. HLP show the dynamic properties of their model for the problem of cutting down the number of government R&D researchers. In doing so they distinguish two modelling constellations: first, no cross-fertilization spillovers, 1-χ = 0, and second, positive cross-spillovers 1- χ = 0.3.

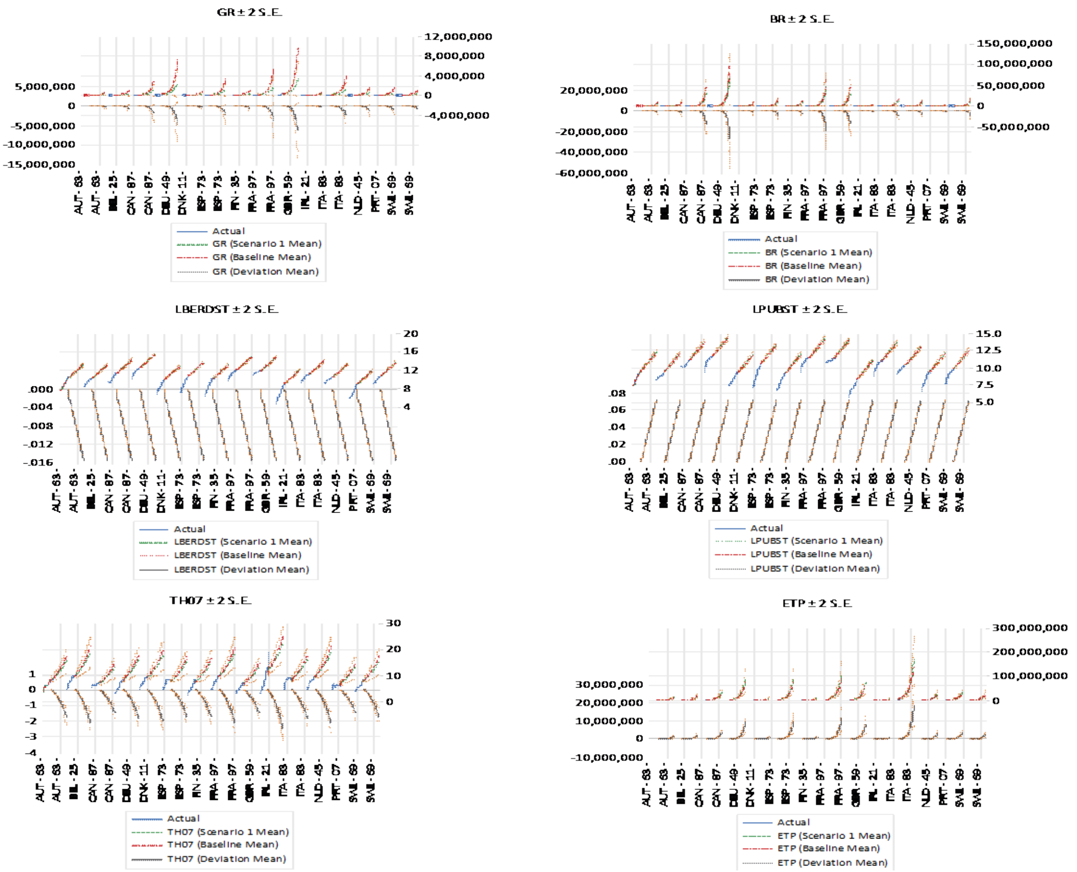


Figure 2 Effects in terms of (log) levels of variables from a negative shock to the change of the growth rate of government researchers of (-0.012) for the period 2010-2100. The upper set of curves in each graph shows the baseline mean and the scenario mean measured on the right vertical axis. The lower set of curves shows the deviation of scenario from baseline measured on the left vertical axis.

Our estimation results have indicated the possibility for a third constellation where public-to-private spillovers are positive but private-to-public spillovers are absent or weak. We will therefore compare the results from shocks on public R&D researchers with those from both settings in HLP. In the VAR model the spillovers are not modeled explicitly and thereby can include the constellations only implicitly.

We impose a negative shock of (-0.012) on the intercept of the growth rate equation for the number of public researchers in order to see the effects on all variables. We show effects for log(levels) during the period 2010-2100 for all countries in Figure 2 and for growth rate effects in Figure 3. For the econometrics of this approach see Lau (1997).

In the graphs of Figure 2 the upper set of curves shows actual observations, the baseline means, and the scenario means measured on the right vertical axis, which are so close to each other that they are hard to distinguish. The lower set of curves shows the deviation of shock scenario from baseline measured on the left vertical axis. We see in Figure 2 that the number of government researchers goes down for all countries.

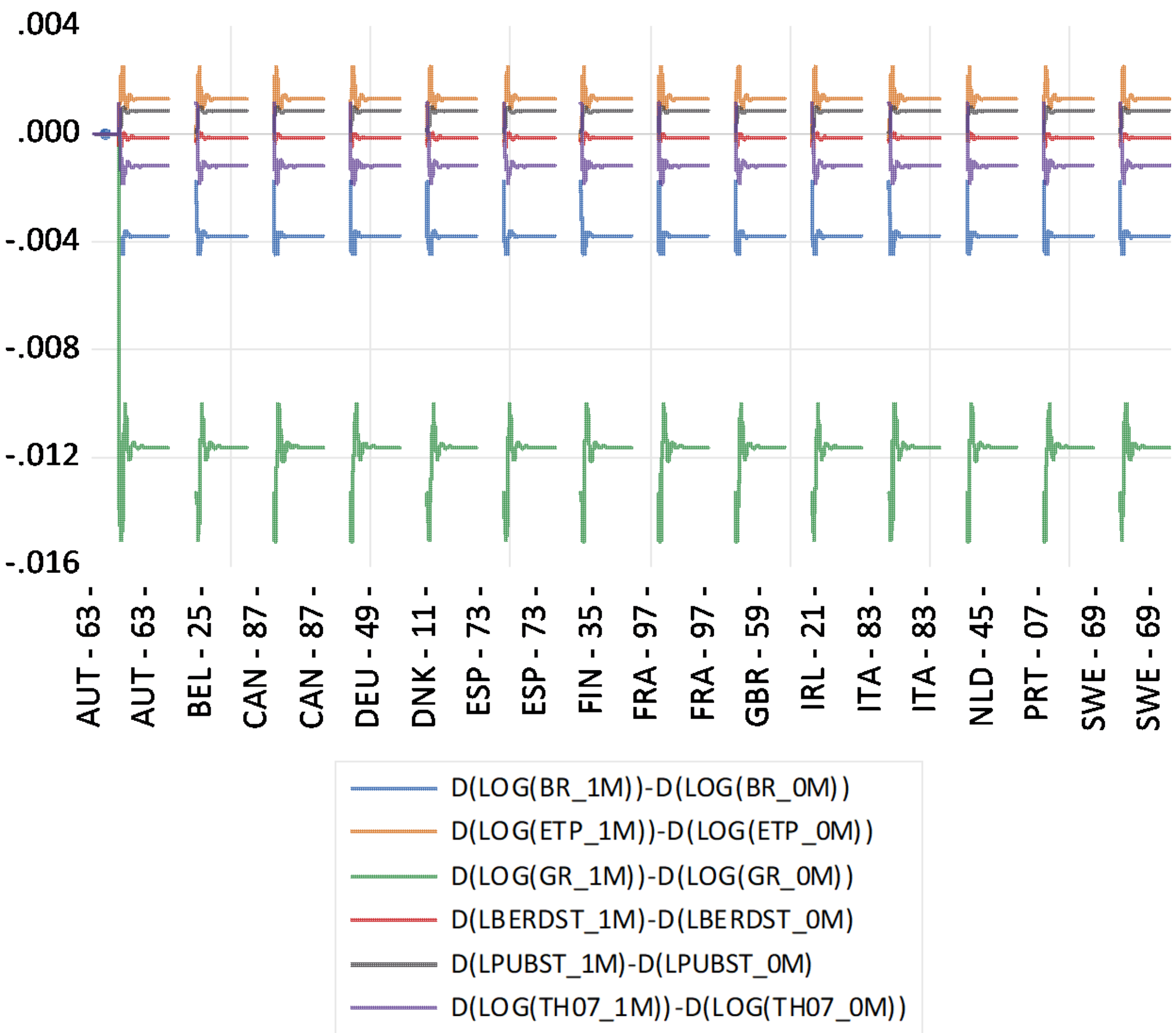


Figure 3 Long-run effects, 2011-2100, in terms of growth rates as deviation from baseline from a negative shock to the number of government researchers of (-0.012).

In Figure 3, this effect for the panel average growth rate is shown as the lowest line of the deviation from baseline by -0.0116 in the long run, which is roughly the size of shock. The number of business researchers is also going down in Figure 2 in levels and in Figure 3, second line from below, in growth rates by -0.0038. This indicates that the complementarity of business and government researchers shown in Table 11, column 5, is dominating the substitutability of Table 11, column 6. In line with results of column 1 of Table 11, the level of labour productivity goes down according to Figure 2, and its growth rate in Figure 3, third line from below, by about -0.0012 in the long run. Private R&D expenditure stocks go down with the number of government researchers in terms of log-levels in Figure 2 and in terms of growth rates in Figure 3, third line from above, by -0.0002. In contrast, public R&D expenditures go up in terms of log-levels in Figure 2 and in terms of growth rates in Figure 3, second line from above, by -0.0013. The number of firms goes up in line with the fixed costs from BERD going down in terms of levels and growth rates. Public R&D goes up in both figures in spite of the reduction in the number of researchers, -0.0008 in terms of growth rate difference. As wages are unlikely to fall through the negative shock on government researchers this would mean that relatively more money goes to public R&D (lab equipment) capital. Except for both types of researchers all growth rate effects are small, about one tenth of the shock by -0.012 and of the long-run effect on government researchers.

In line with the deterministic model of HLP, we see the following results in terms of levels or growth rates of expected values of our simulations. Under the *assumption of no cross-spillovers,* HLP find that public R&D cuts

(i) increase the steady state number of firms (for a given population growth path),
(ii) have no effect on technical change (growth rate),
(iii) do not change firm level R&D in terms of researchers (in levels).

For the number of firms, we see a positive effect in Figure 3, highest line, described above in terms of a higher *growth rate*; the number of firms in *levels* is higher in the R&D-cut scenario for all periods 2013-2100 (see Figure 2, upper left graph). This supports (i) above. Technical change growth rates and the level of firm researchers are shown to fall in Figure 3 and Figure 2 and are not constant. Therefore, results that hold under no cross-spillover assumptions in the theoretical model have no empirical support from our GMM-OD VAR in regard to (ii) and (iii). This is not surprising because we found positive cross-spillovers in the earlier sections and HLP use the no-cross-spillover assumption for reasons of simplification in a pedagogic step of explaining the model.

Under the *assumption of positive cross-spillovers*, HLP found the following results for changing $s_G = gr/lf$.

(i) Technical change in terms of growth rates falls under a public R&D cut, $\frac{\partial \hat{T}}{\partial s_G} > 0$.
(ii) Business researchers *per firm* get lower under a public R&D cut, $\frac{\partial L_Z}{\partial s_G} > 0$.

(iii) The growth of private knowledge stocks decreases under a public R&D cut, $\frac{\partial \hat{z}}{\partial s_G} > 0$.

These theoretical results are meant to hold for the *steady state* values in HLP.

The corresponding empirical results from our impulse response analysis using the GMM-OD VAR model are shown in Figure 3 for the growth of labour productivity and of BERDST. The result (i) for technical change is confirmed in Figure 3. The result (ii) for business researchers *per firm* is confirmed in Figure 4. This plot compares to HLP's Fig. 5, upper-left panel, which has more business researchers in the phase shortly after the shock, but then is comparable to the falling pattern shown here. The result (iii) for business R&D knowledge stock growth holds empirically in the long run of shown in Figure 3. Result (iii) has therefore confirmation from our empirical GMM-OD VAR.

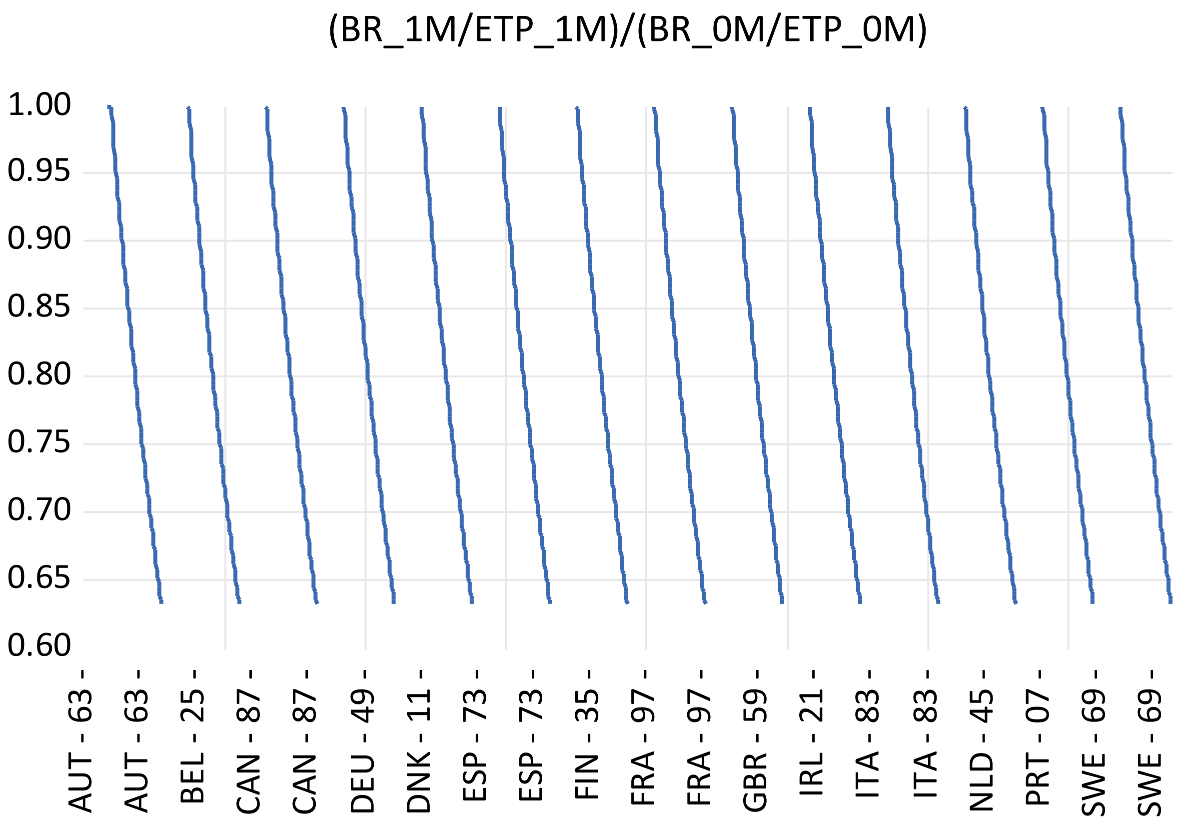


Figure 4: Effects of public R&D cut on firm-level R&D, 2010-2100. 0M indicates baseline expected panel average value; 1M indicates expected panel average value from shock scenario. Each line represents one country.

The growth reduction of firm knowledge shown in Figure 3 is also present in the *transition* phase of the theoretical model (HLP, section 4.2). This holds also for technical change (after a theoretically unclear impact effect), which is also approximately visible in Figure 3.

Overall, we consider this to be convincing evidence in favour of properties (i) to (iii) of the steady state of the HLG model; transitional results are also confirmed so far. Results upon impact are hard to compare because the GMM-OD VAR has lags which the theoretical growth model does not have.

*The dynamic relation between public and private R&D*

In the HLP model, firm level R&D, $L_Z(t)/N(t)$, reacts to the negative R&D shock on public R&D researchers, as far as the interaction effect is concerned, with an ambiguous impact effect, and then falls to its new lower steady state. The corresponding GMM-OD VAR result is shown in Figure 4. There is no impact or early effect on average; the simulation for the shock goes below that of the baseline scenario as in the theory. In particular, private R&D per firm in Figure 4 and public R&D in terms of researchers in Figure 2 go down together as compliments. The availability of more data allowing for (panel) time-series analysis and the progress in econometrics in this millennium has lead to strong evidence for this complementarity (David et al. 2000; Becker 2015; Ziesemer 2021b).

'The cut yields the percentage change in the steady-state mass of firms per capita' (HLP, p.15). This increase is visible after some periods in Figure 2 for an unaffected population growth path.

Having shown in Figure 4 that under a public R&D cut firm-level R&D goes down and in Figure 2 that the number of firms goes up, the question is how exactly private R&D (not per firm) goes together with public R&D, both in terms of number of researchers. HLP suggests that this is a hump shape form because firm level R&D goes together with public R&D but the number of firms doing R&D goes the opposite way.

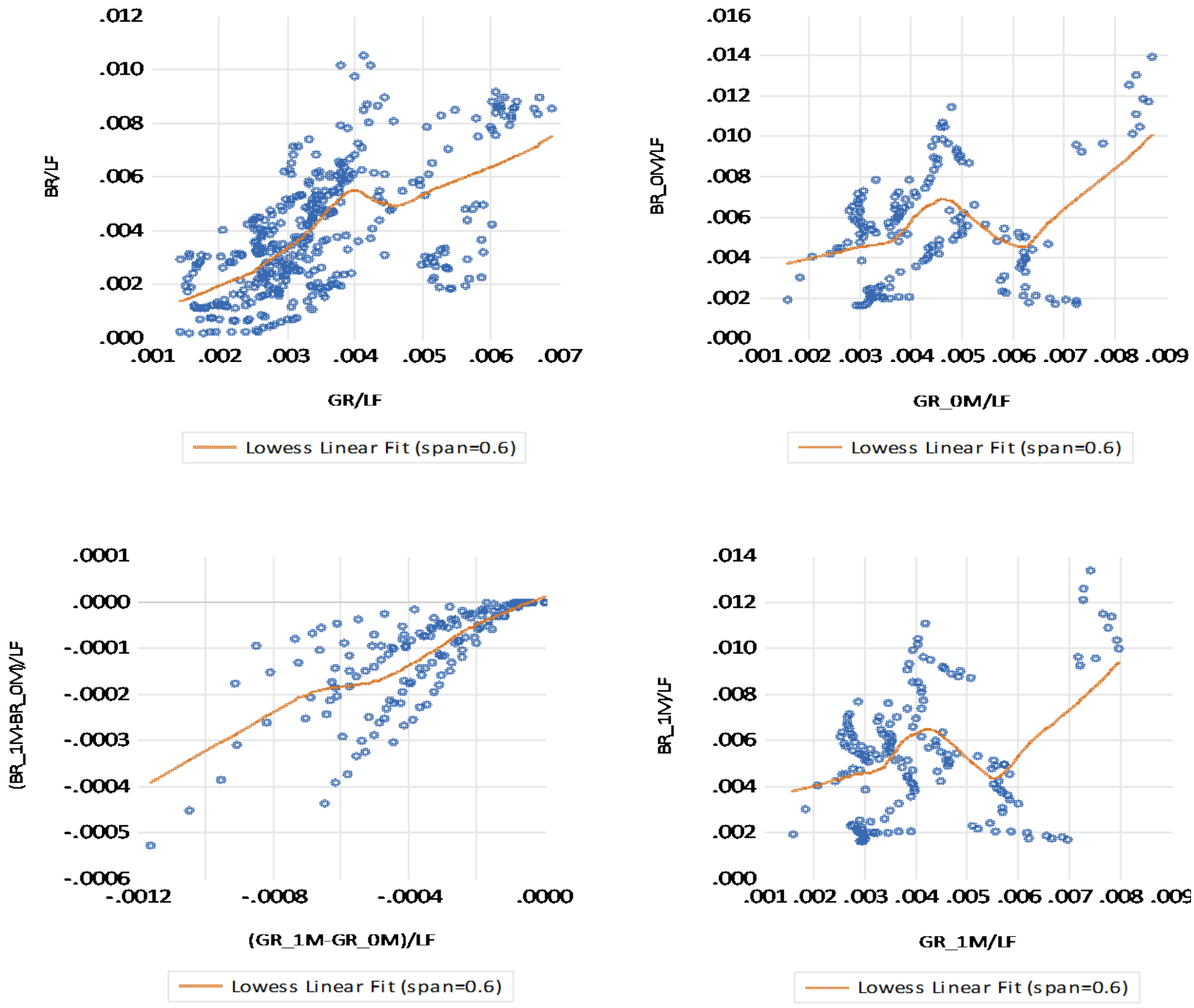


Figure 5 Decreasing change of business researchers through more government researchers.

We take both variables as a share of the exogenous labour force data. Figure 5 shows the relation between government researchers per person in the labour force, *gr/lf*, and business researchers as a share of the labour force, *br/lf*, in terms of the nearest-neighbour or lo(w)ess fit, using 60% of all data to generate a point, attributing it to the middle observation, and then shifting one point further to repeat the procedure. Using 90% or 30% of the data per regression would lead to a similar graph. The upper-left part of Figure 5 shows the decreasing slope between *gr/lf* and *br/lf* in terms of data. The upper-right part shows this also in terms of points of the baseline simulation of the GMM-OD VAR (indicated as 0m). The lower-right part shows the decreasing slope for the public-R&D-cut scenario (indicated as 1m). The lower-left shows their change  from the baseline to the public R&D cut scenario  in the VAR model (indicated as 1_0m), where business R&D is reduced much less than public R&D; this is ‘a tiny amount’ in the steady state in HLP, and here between -0.0000 and -0.0006 for the period 2013-2020. All figures suggest that business R&D reacts less strongly to public R&D at higher values of public R&D. In all four graphs of Figure 5 the data points most far away from the origin are those of Finland; they spoil the idea of an inverse u-shaped curve by bending the downward line upward. The outlier position of Finland and perhaps other countries suggests the use of fixed effects methods for Figure 5.

To underpin these results with more sophisticatedly testing regression methods than the loess fit of Figure 5, we use pooled and weighted DOLS, because the variables have panel unit roots according to all tests (see Table A.2) and are stationary after differencing (not shown). The long-term relation is (with constants digested in the modified variables and not appearing in the regression output;[14] p-values in parentheses)

$$BR/LF = 274.1(GR/LF)^2 - 18615.84(GR/LF)^3$$
$$(0.0000) \qquad (0.0625)$$

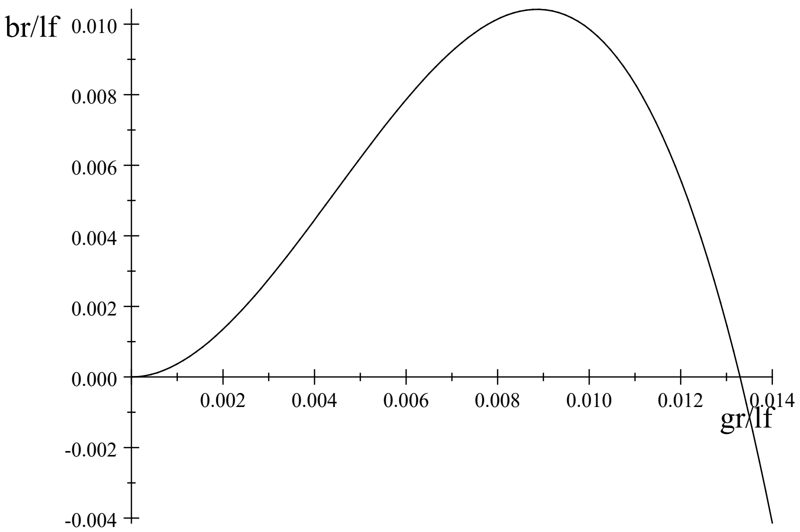


Figure 6: The HLP hump shape relation. Plot of DOLS regression relation between government and business researchers (% labour force, lf).

Plotting the equation leads to Figure 6 with data in the range of the upward sloping part as can be seen from comparison with the data in Figure 5, where they are below 0.008, in the upward sloping part as noted by HLP. This provides some support for the idea of a hump-shape form of HLP. However, we cannot test the shape in connection with all desirable properties of mean-group estimators with the limited number of observations available.[15]

[14] Sample (adjusted): 1982-2020. Periods included: 39. Cross-sections included 14. Total panel (unbalanced) observations: 420. Panel method: Weighted estimation. Cointegrating equation deterministics: C (no time trend as variables have to stay in the unit interval). Automatic leads and lags specification (based on AIC criterion, max=*). Long-run variance weights (Prewhitening with lags from AIC maxlags = -1, Bartlett kernel, Newey-West automatic bandwidth, NW automatic lag length). Adjusted R-squared: 0.84. S.E. of regression: 0.001010.

[15] The related (pooled) mean group estimators of DOLS or PMG/ARDL require more lags than the data allow here. A PMG/ARDL estimate, which looks similar to Figure 6 is $BR/LF = 397.8780(GR/LF)^2 -29927.32 (GR/LF)^3$. This is the best available PMG/ARDL estimate. It uses only observations 2007-2020; the required similarity with the mean group estimator has p = 0.43. The selected model then is PMG(3,2,2) with 3 as the maximum number of differenced terms of the dependent variable indicating that more lags might be desirable, but they are not feasible with the limited number of observations. Moreover, we have no constants, neither restricted nor

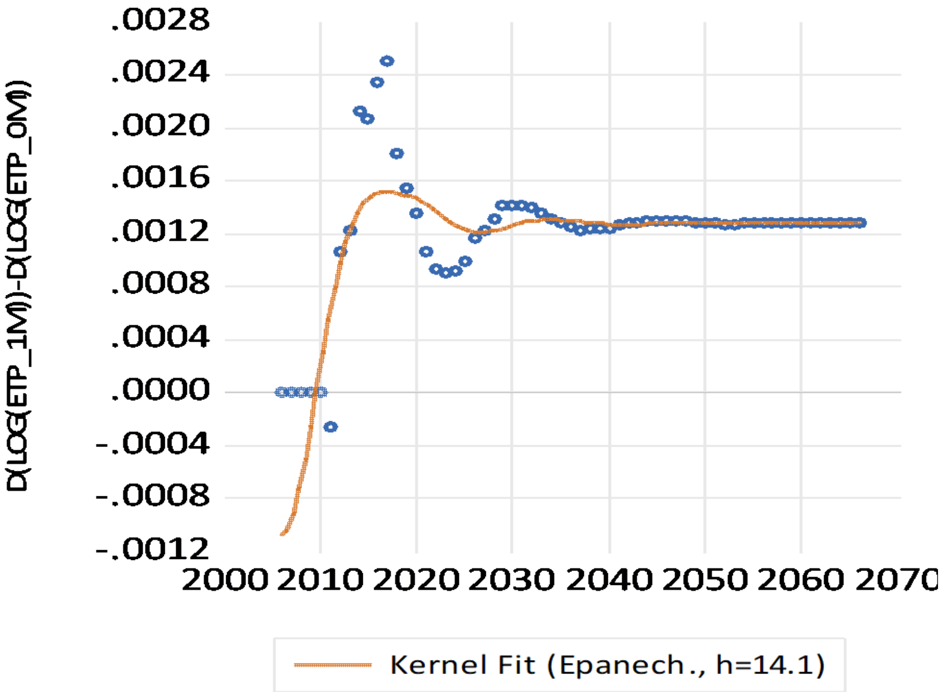


Figure 7 The growth rate of the number of firms is lower in the shock scenario than in the baseline scenario in the first year after the shock and higher later.

In the model of HLP, the growth rate of the number of firms (% labour force) jumps down after the public R&D cut and then quickly increases beyond its baseline of zero to which it returns in the long run (see HLP, Figure 6). In Figure 7, differing from Figure 3 through the use of a Kernel fit, which goes more to the initial and final observations, we show a similar result: the growth rate of the number of firms (% labour force) starts at zero in 2006, is lower in 2011, directly at the public R&D cut in 2010, and higher in all periods afterwards. In the HLP model, impact effects are strong compared to our VAR model in which the lags in principle smooth the early effects, but here only by one period.

The adjustment process of the complete HLP model is conducted in terms of the number-of-firms/labour ratio and the public/private knowledge ratio. In Figure 8 we show that the public/private knowledge stock ratio is going down relative to baseline (until 2100 indeed) as in the HLP analysis. The number-of-firms/labour ratio evolves as in the theoretical analysis of HLP, first going slightly down and then up.

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unrestricted, also because of the limited number of observations, and no cointegration according to the bounds test and the insignificance of the adjustment coefficient. Cointegration perhaps requires linking to more variables, which might also repair the other properties and preserve the shape, which is the major point of interest here.

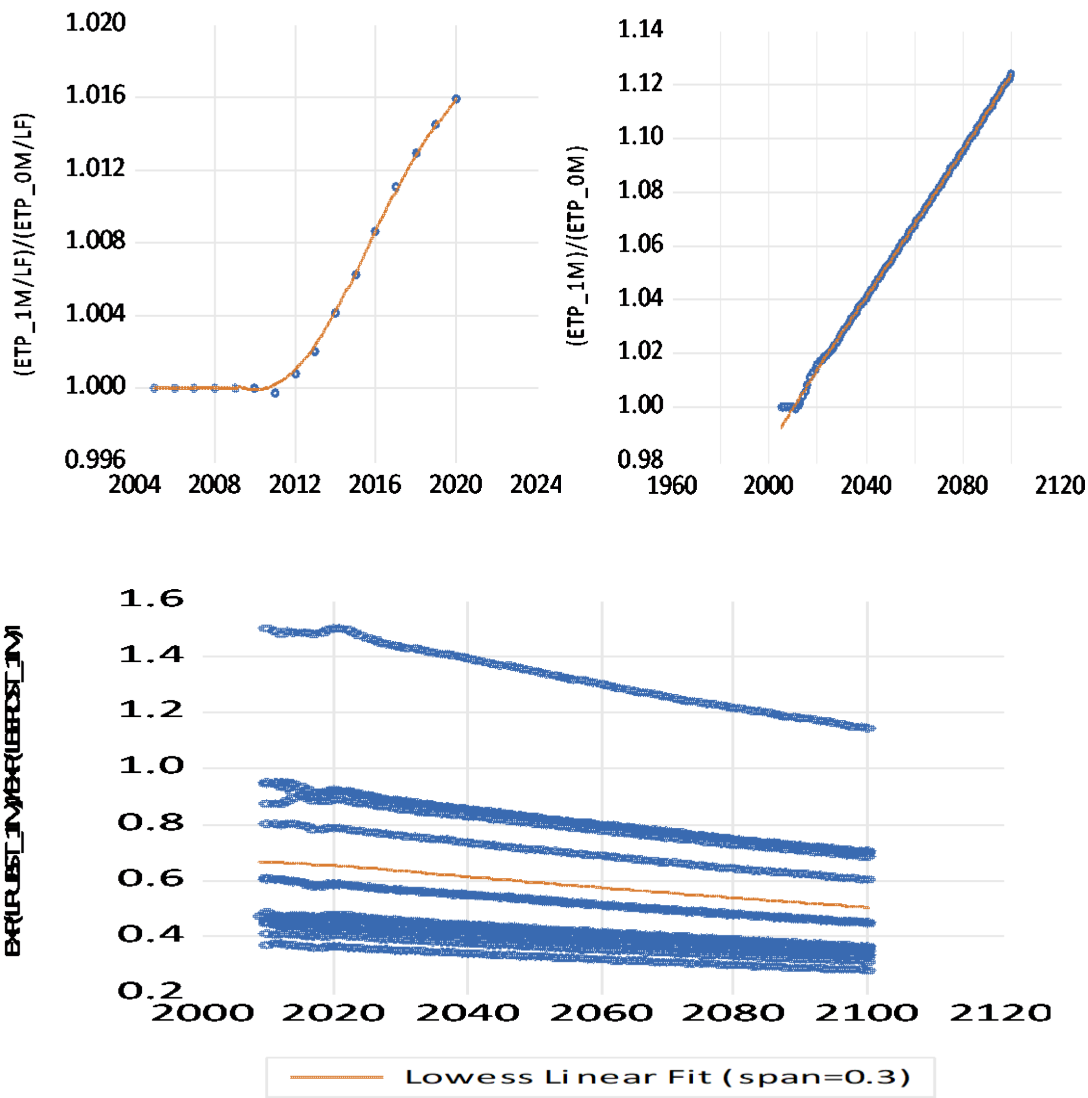


Figure 8: The adjustment process after a public R&D cut. Upper part: The number of firms (% labour force) first goes down and then goes up, in the data period and also until 2100. Lower part: the public-private knowledge ratio is decreasing.

Although Figure 3 shows that the growth rate of the public knowledge stock goes up after the shock and that of private knowledge goes down, Figure 8 implies that the growth rate of private R&D remains higher than that of public R&D.

## 9. Summary and conclusion

We have derived a Cobb-Douglas function for technical progress depending on public and private R&D knowledge stocks from two cointegrating equations for each of three estimation methods. Standard results from the literature are shown to hold also when including the number of firms as in the HLP model.

For the dynamic R&D capital growth functions of the HLP model, results obtained through full information maximum likelihood (FIML) estimation *without fixed effects* are summarized as follows. The research productivity of the private R&D process is enhanced through collaboration of business and government researchers by 17-80% above baseline. The CES spillover functions in their R&D growth functions have linear own (cross) spillover distribution parameters between 0.65 and 0.75 (0.35-0.25) if private and public R&D processes are restricted to have the same CES spillover function as assumed in HLP's calibration of the theoretical model. Elasticities of substitution between private and public R&D stocks in the spillover function are low under this assumption. When spillover functions are allowed to be different between private and public R&D, (i) own spillover (distribution) parameters get lower for private R&D processes and close to unity in public R&D functions, implying almost no spillovers from private to public R&D and strong spillovers from public-to-private R&D; (ii) CES parameters for spillover functions get positive for private R&D functions and more negative for public R&D functions. Generalization of the linear labour argument in the R&D growth functions to a Cobb-Douglas function in labour and R&D capital shows decreasing returns to labour and R&D capital. A negative research productivity trend in both R&D growth functions is found when the R&D labour interaction term is modelled as product of government and business researcher, but not if it is modelled as government researchers as a share of the labour force as in HLP.

*Fixed effects* in combination with autoregressive processes in nonlinear, iterative least squares estimation for the case of identical CES spillover functions in dynamic R&D functions yields a spillover parameter range of 0.59-0.7 and a CES of 0.6.

A VAR model turns out to have not only fixed effects as in Hsiao's (2022) textbook model but also country-specific time trends which turn up as fixed effects after differencing. We have assumed that these fixed effects are small relative to each other and do cause only negligible biases in results shown in an appendix. In contrast, considering fixed effects using the orthogonal deviation version of system GMM we find the slopes for all equations separately and import them into a simultaneous equation model, leading to a GMM-OD VAR. The results from a public R&D cut in the VAR are compared to those in the theoretical model of HLP. All steady-state and adjustment properties of the theoretical HLP model are in line with those of the VAR model. The VAR ignoring fixed effects in the appendix has comparable results.

A suggestion for further research for theory with calibration is to allow for asymmetric private and public R&D dynamics, with strong public-to-private spillovers and weak private-to-public spillovers in the CES spillover function. The HLP model is the ideal basis for the analysis of the consequences of asymmetric CES spillover functions.

Overall, we provide an empirical investigation that stays close to the endogenous growth model and confirms the assumptions and results of the HLP model.

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**APPENDIX: Table A.1-3**

**Table A.1 Panel unit root tests with cross-section dependence for three variables**

| Coun-try | LTH07 | | | | LBERDST | | | | LPUBST | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (a) Pesaran-CADF / CIPS | | Bai&Ng PANIC (b) | | Pesaran-CADF/ CIPS | | Bai&Ng PANIC | | Pesaran-CADF/CIPS | | Bai&Ng PANIC | |
| | t-stat | p-val. | t-stat | p-val. | t-stat | p-val. | t-stat | p-val. | t-stat | p-val | t-stat | p-val |
| AUT | -2.06 | >=0.1 | -2.22 | 0.127 | -3.98 | <0.05 | -1.27 | 0.62 | -1.88 | >=0.1 | -0.93 | 0.86 |
| BEL | -3.88 | <0.05 | -0.75 | 0.94 | -1.71 | >=0.1 | -2.86 | 0.02 | -1.39 | >=0.1 | -0.94 | 0.85 |
| CAN | -1.64 | >=0.1 | -1.09 | 0.75 | -3.30 | >=0.1 | -2.73 | 0.036 | -1.95 | >=0.1 | -2.45 | 0.08 |
| DEU | -3.26 | >=0.1 | -2.38 | 0.09 | -2.83 | >=0.1 | -1.06 | 0.768 | -3.20 | >=0.1 | -2.22 | 0.13 |
| DNK | 0.76 | >=0.1 | -1.65 | 0.37 | -3.69 | <0.1 | -1.70 | 0.34 | -3.99 | <0.05 | -2.84 | 0.03 |
| ESP | -2.38 | >=0.1 | -1.08 | 0.75 | -4.15 | <0.05 | -1.46 | 0.49 | -4.27 | <0.05 | -5.11 | 0 |
| FIN | -1.25 | >=0.1 | -1.65 | 0.37 | -1.54 | >=0.1 | -3.34 | 0.006 | -1.83 | >=0.1 | -2.97 | 0.02 |
| FRA | -1.69 | >=0.1 | -0.97 | 0.82 | -1.75 | >=0.1 | -0.81 | 0.91 | -5.07 | <0.01 | -1.57 | 0.42 |
| GBR | -2.52 | >=0.1 | -1.51 | 0.46 | -1.34 | >=0.1 | -2.71 | 0.04 | -1.93 | >=0.1 | -3.58 | 0.00 |
| IRL | -3.36 | >=0.1 | -0.82 | 0.91 | -1.96 | >=0.1 | -2.26 | 0.116 | -3.76 | <0.1 | -0.84 | 0.90 |
| ITA | -1.56 | >=0.1 | -1.46 | 0.49 | -1.53 | >=0.1 | -1.2 | 0.67 | -0.11 | >=0.1 | -1.69 | 0.35 |
| JPN | -3.05 | >=0.1 | -2.73 | 0.035 | -1.75 | >=0.1 | -0.86 | 0.89 | -1.49 | >=0.1 | -0.94 | 0.85 |
| NLD | -2.13 | >=0.1 | -0.93 | 0.854 | -3.95 | <0.05 | -1.36 | 0.56 | -2.75 | >=0.1 | -3.29 | 0.01 |
| NOR | -4.73 | <0.01 | -1.21 | 0.665 | 1.04 | >=0.1 | -2.13 | 0.16 | -2.38 | >=0.1 | -3.32 | 0.01 |
| PRT | -1.96 | >=0.1 | -1.17 | 0.69 | 0.09 | >=0.1 | -2.77 | 0.03 | -3.50 | <0.1 | -3.21 | 0.01 |
| SWE | 0.44 | >=0.1 | -1.14 | 0.72 | -4.27 | <0.05 | -2.30 | 0.103 | -3.71 | <0.1 | -1.34 | 0.58 |
| USA | -1.44 | >=0.1 | -0.73 | 0.95 | -4.89 | <0.01 | -1.12 | 0.73 | -1.27 | >=0.1 | -2.3 | 0.11 |
| Panel | -2.10 | >=0.1 | -0.84 | 0.40 | -2.44 | >=0.1 | 2.98 | 0.003 | -2.62 | >=0.1 | +/-Inf | 0.0 |

(a) Null: unit root; Cross-sections: 17. Sample: 1963-2020. Balanced observations: 54, 51, 47. Total observations: 918, 867, 799. Deterministics: Constant and trend. Max lag, AIC: 7.

(b) Data demeaning: Time, Cross-sections. Data standardization: Time, Cross-sections. Maximum factors: 7 (Schwert). Criterion: Average of criteria.

For LPUBST in DNK, ESP and PRT the unit root hypothesis is rejected in both tests. These are rare exceptions. At least one of the two tests supports a unit root for all other country-variable combinations. Under the Bai/Ng unit root tests, all t-values have negative signs indicating expected roots less than unity also called near unit roots.

**Table A.2 Panel unit root tests for number of firms, government and business researchers (% labour force) in 14 OECD countries**

| *Variable →* <br> *Test ↓* | *GR/LF* <br> *p-val.* | <br> *Obs.* | *BR/LF* <br> *p-values* | <br> *Obs.* | *Log(etp)* <br> *p-values* | <br> *Obs.* |
|---|---|---|---|---|---|---|
| Levin, Lin & Chu t* (a) | 0.2695 | 424 | 0.9095 | 403 | 0.0000 | 202 |
| Breitung t-stat (a) | 0.6743 | 410 | 1.0000 | 389 | 0.2508 | 188 |
| Im Pesaran Shin W-stat (a) | 0.3760 | 424 | 0.8968 | 403 | 0.0000 | 202 |
| ADF - Fisher Chi-square (b) | 0.5827 | 424 | 0.6657 | 403 | 0.0000 | 202 |
| PP - Fisher Chi-square (b) | 0.8686 | 437 | 0.9989 | 444 | 0.1081 | 211 |
| Truncated CIPS (c) | P >=0.10; t = -.2.49 | | P >=0.10, t = -1.87 | | Insufficient obs | |

(a) Exogenous variables: Individual effects, individual linear trends; no cross-section dependence. Automatic selection of maximum lags. Automatic lag length selection based on AIC: 0 to 3. Newey-West automatic bandwidth selection and Bartlett kernel. Null: Unit root (assumes common unit root process).
(b) As in (a) but Null: Unit root (assumes individual unit root process). Probabilities for Fisher tests are computed using an asymptotic Chi-square distribution. All other tests assume asymptotic normality.
(c) Null hypothesis: Unit root. With constant and trend. Maximum of two lags feasible. According to the Bai/Ng test 13 (12) of 14 countries have a unit root for gr/lf (br/lf) (not shown).

**Table A.3 P-values of Trace Test for Johansen-Fisher Panel Cointegration rank tests for LTH07, LBERDST, LPUBST**

| Lags | Lag 2 (SIC) | | | Lag 3 (HQ) | | | Lag 7 (AIC, FPE, LR) | | |
|---|---|---|---|---|---|---|---|---|---|
| Rank (a) | No ce | 1 ce | 2 ce | No ce | 1 ce | 2 ce | No ce | 1 ce | 2 ce |
| AUT | 0 | 0.0044 | 0.4181 | 0 | 0.0063 | 0.5124 | 0 | 0.0045 | 0.3242 |
| BEL | 0.001 | 0.0373 | 0.2183 | 0 | 0.0272 | 0.4106 | 0 | 0.0101 | 0.0449 |
| CAN | 0.043 | 0.2509 | 0.5985 | 0.01 | 0.2534 | 0.4051 | 0 | 0 | 0.0632 |
| DEU | 0.004 | 0.6291 | 0.6249 | 0.0003 | 0.2799 | 0.2333 | 0 | 0.0002 | 0.0107 |
| DNK | 0.046 | 0.2098 | 0.2739 | 0.0006 | 0.0363 | 0.1416 | 0 | 0.0208 | 0.0976 |
| ESP | 0 | 0.0147 | 0.9445 | 0.0025 | 0.4753 | 0.7671 | 0 | 0.0117 | 0.1908 |
| FIN | 0.009 | 0.1139 | 0.299 | 0.0008 | 0.0215 | 0.4791 | 0 | 0.0051 | 0.1143 |
| FRA | 0.187 | 0.5666 | 0.875 | 0.0021 | 0.0597 | 0.6733 | 0 | 0.0092 | 0.0836 |
| GBR | 0.038 | 0.2115 | 0.5194 | 0.0138 | 0.1086 | 0.4328 | 0 | 0.0991 | 0.7617 |
| IRL | 0.211 | 0.6629 | 0.6353 | 0.1522 | 0.6553 | 0.9252 | 0 | 0.0017 | 0.33 |
| ITA | 0.022 | 0.1509 | 0.4298 | 0.0092 | 0.203 | 0.2786 | 0 | 0 | 0.0031 |
| JPN | 0.029 | 0.1332 | 0.1983 | 0.0029 | 0.1678 | 0.5754 | 0 | 0.0122 | 0.1308 |
| NLD | 0.076 | 0.355 | 0.8099 | 0.1616 | 0.4813 | 0.923 | 0 | 0.0495 | 0.4922 |
| NOR | 0.05 | 0.1388 | 0.4035 | 0.0064 | 0.0834 | 0.289 | 0 | 0 | 0.0259 |
| PRT | 0.062 | 0.492 | 0.4918 | 0 | 0.0094 | 0.4092 | 0 | 0 | 0.0022 |
| SWE | 0 | 0.0046 | 0.0575 | 0.0032 | 0.0423 | 0.2973 | 0.0001 | 0.0073 | 0.0409 |
| USA | 0.034 | 0.3494 | 0.2513 | 0.0089 | 0.1032 | 0.1317 | 0 | 0.0001 | 0.0024 |
| FisherStat | 0 | 0.0003 | 0.5951 | 0 | 0 | 0.6183 | 0 | 0 | 0 |
| VECM | 0 | 0.0002 | 0.1118 | 0 | 0.0046 | 0.4213 | 0 | 0.0019 | 0.7368 |

(a) Rank indicates number of cointegrating equations (ce); null: at most r ce, r = 0, 1, or 2. Sample: 1963-2020. Included observations: 986. Trend assumption: Linear deterministic trend (restricted). MacKinnon-Haug-Michelis (1999) p-values.

We have estimated a pooled VAR for the three variables of interest: Lth07, LBERDST, LPUBST. The VAR favours lag length 3, 4, or 8 for the criteria SIC, HQ, and AIC (and also FPE, LR). For the Johansen-Fisher test, which is based on country-specific VEHM models we then use one lag less as models are formulated in differences. For these given lags, Table A'S shows that

- for two lags, 'no ce' is acceptable only for two countries (at the ten percent level), 'one ce' is rejected for four countries and for more when considering low power. 'Two ce' is not rejected for any country under the five percent level. The Fisher statistic for the whole panel suggests '2 ce'.
- For three lags, only for two countries 'no ce' is acceptable (at the ten percent level); 'one ce' is acceptable for at least six countries and two ce for all countries. For the Fisher statistic $-2\sum_{i=1}^{17}\ln \pi_i$, which follows a chi-square distribution, zero or one ce is rejected in the second but last row. It suggests 'two ce'.
- Only for the case of 7 lags, the test would favor full rank, r=3, of the underlying VAR. Seven lags are not applied later.
- When applying the Johansen test to a pooled VAR the trace test also favors '2 ce' under the three lag length assumptions as shown in the last row of Table A'S.

**APPENDIX Panel VAR with pooled data on growth rates.**

For the maximum likelihood estimation of the VAR with pooled data, we do not repeat the text but rather emphasize only the difference with the GMM-OD version in the main text, Table 11. Technical change in column 1 is self-perpetuating with coefficient 0.61 far as its own feedback from lags is concerned, which is a big contrast with the GMM version. Only the number of firms and public R&D workers have positive effects.

### Table A1
### Panel VAR model in changes of growth rates 2007-2017: Sums of coefficients (a)

| *Regressors\Dependent* | *D(LOG(TH07))* | *D(LBERDST)* | *D(LPUBST)* | *D(LOG(ETP))* | *D(LOG(BR))* | *D(LOG(GR))* |
|---|---|---|---|---|---|---|
| *D(LOG(TH07(-j)))* *sum of c(j)* | 0.610 | 0.094 | 0.065 | 0.134 | -0.086 | -0.970 |
| *D(LBERDST(-j))* *sum of c(j)* | -0.135 | 0.800 | -0.009 | -0.647 | 0.310 | 0.412 |
| *D(LPUBST(-j))* *sum of c(j)* | -0.0002 | 0.016 | 0.927 | 0.007 | -0.595 | 0.085 |
| *D(LOG(ETP(-j)))* *sum of c(j)* | 0.062 | 0.076 | -0.020 | 0.170 | 0.772 | 0.490 |
| *D(LOG(BR(-j)))* *sum of c(j)* | -0.078 | 0.005 | 0.007 | 0.0105 | -0.121 | -0.312 |
| *D(LOG(GR(-j)))* *sum of c(j)* | 0.275 | -0.006 | -0.0180 | -0.022 | 0.226 | 0.251 |
| *constant* | 0.004 | 0.003 | 0.001 | 0.039 | 0.041 | 0.001 |
| *Mean dependent* | 0.009 | 0.024 | 0.020 | 0.018 | 0.043 | 0.021 |
| *S.D. dependent* | 0.029 | 0.017 | 0.017 | 0.045 | 0.070 | 0.075 |
| *Adj. R-squared* | 0.633 | 0.829 | 0.896 | 0.031 | 0.150 | 0.614 |
| *Pesaran CD p-value (b)* | 0.1445 | 0.5036 | 0.2356 | 0.5117 | 0.2933 | 0.9822 |

(a) Estimation by Full Information Maximum Likelihood (FILM). Sample (adjusted): 2007-2017. 1 to 4 lags. No root lies outside the unit circle. VAR satisfies the stability condition.

(b) Null hypothesis: No cross-section dependence in residuals of six equations. Test employs centered correlations computed from pairwise samples. Periods included: 11; cross-sections included: 14; total panel (unbalanced) observations: 115 for each residual.

In column 2 the sign of government workers differs from the GMM version, and the coefficient of public R&D is much smaller here. In column 3, the negative sign of the growth of BERDST is the main difference with the GMM version; however, both coefficients are small. In column 4 the number of firms reacts mildly positively to their own lags in contrast to the GMM version where this effect is negative. Another different result is that here public R&D stock has a positive coefficient although close to zero; R&D performance used in our data overlaps with R&D financing; permanent subsidies may make more new and old firms profitable, but the effect is small here and negative in the GMM version. In column 5, growth of BERDST, PUBST and labour productivity have the opposite signs compared to the GMM version; the coefficient for the own lags of business researchers is much less negative here. The number of firms, business and government researchers are the (last three) arguments which work in the same positive direction; the first three argument have signs opposite to the GMM version. In column 6 the major difference with the GMM version is that public R&D stock has a positive sign and a small coefficient.

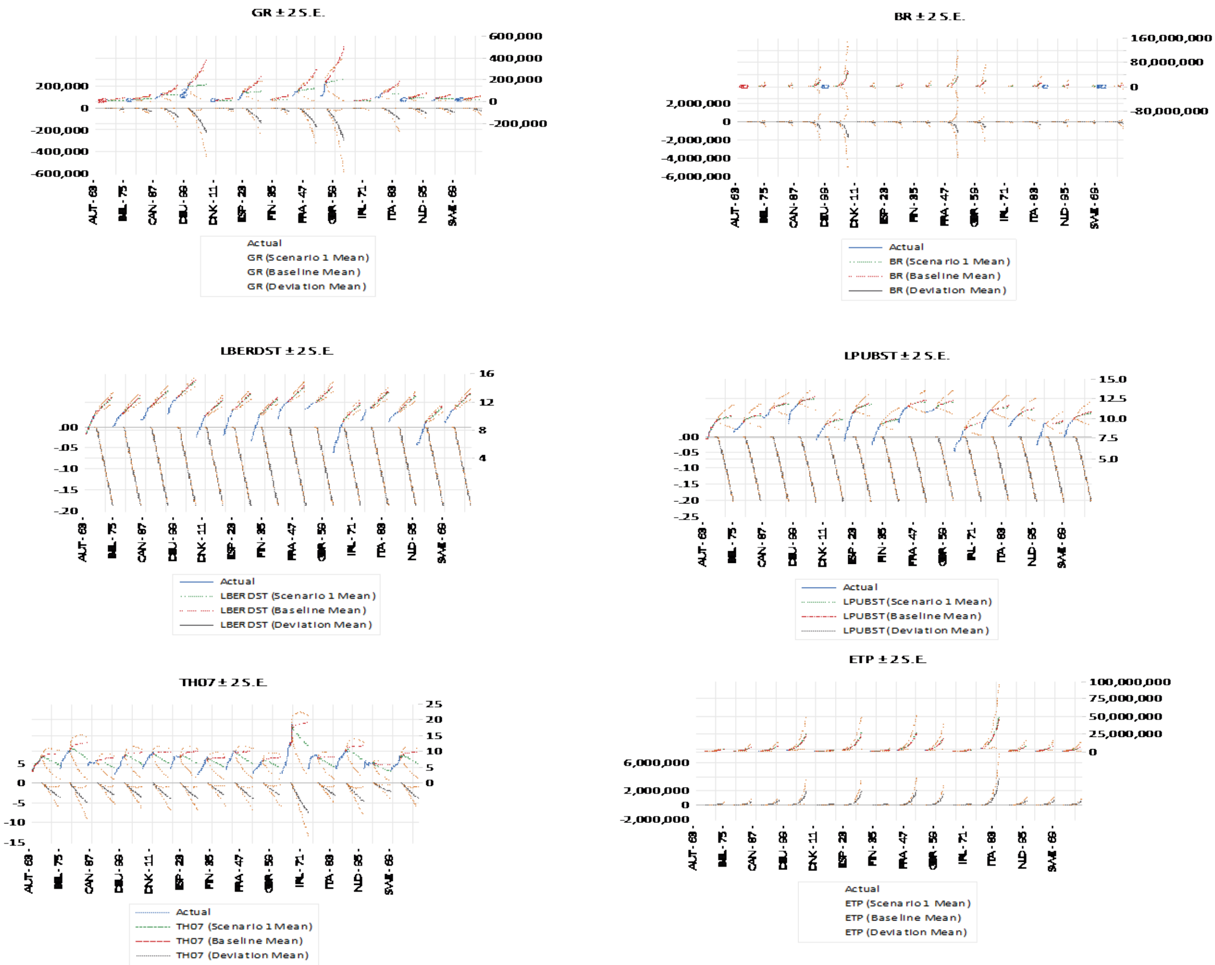


Figure A.2 Effects in terms of (log) levels of variables from a negative shock to the change of the growth rate of government researchers of (-0.012). The upper set of curves in each graph shows the baseline mean and the scenario mean measured on the right vertical axis. The lower set of curves shows the deviation of scenario from baseline measured on the left vertical axis.

VAR models are normally seen as reduced form of structural models. However, when reactions take a years time, they can be interpreted in a causal way similar to structural models as we have tentatively tried above. Stepwise deletion of insignificant lags of variables does not reduce the number of variables very much and thereby cannot help easing the exposition here.

Results for log(levels) are shown in Figure A.2. The major difference with the GMM-OD version in the main text is the fall in public R&D stocks which here is more in line with the fall of the number of researchers.

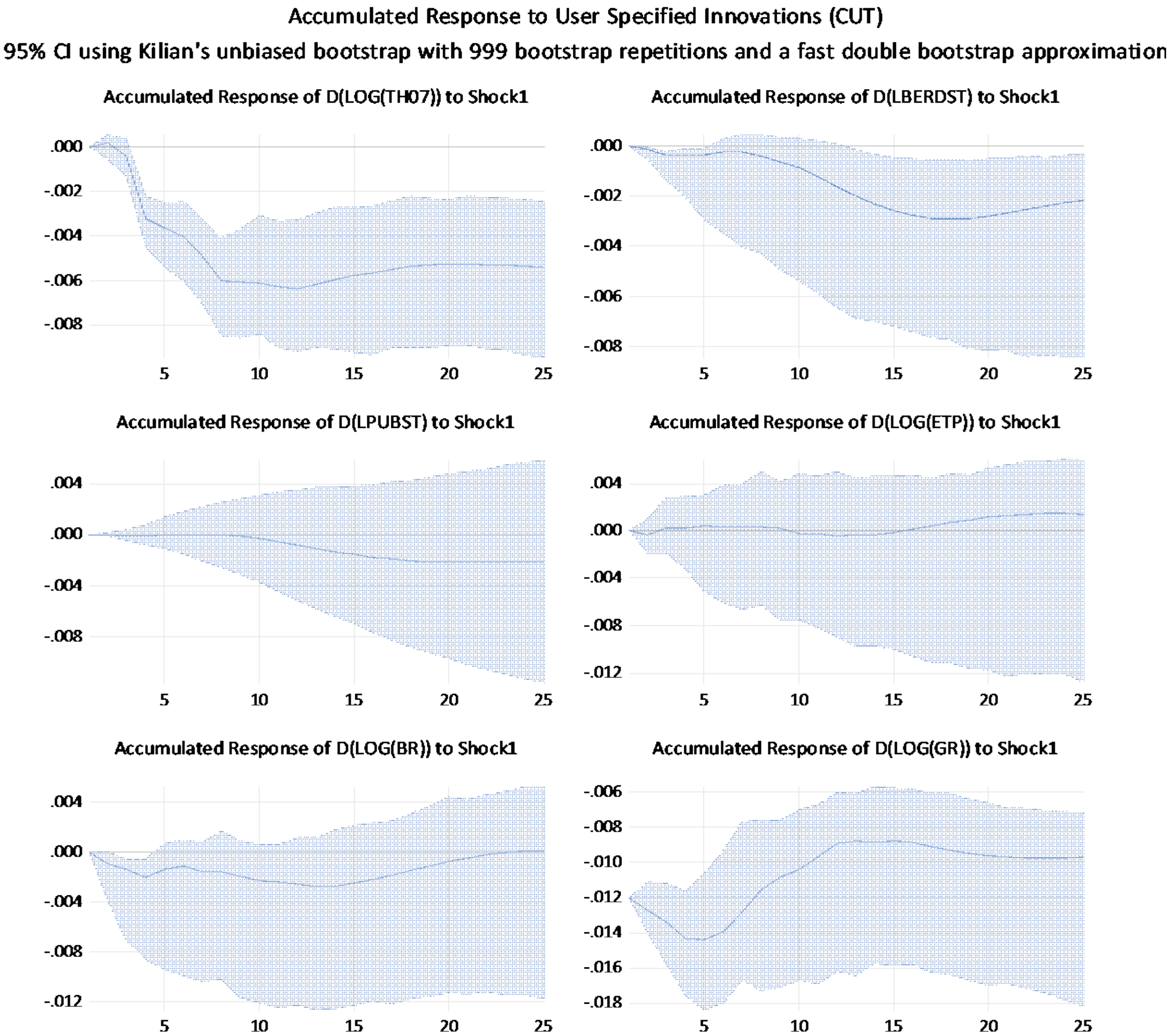


Figure A.3 Long-run effects, 2013-2038, in terms of growth rates as deviation from baseline from a negative shock to the number of government researchers of (-0.012).

In Figure A.3 we choose Kilian's unbiased confidence intervals that have been developed for VARs with small numbers of observation. The number of business researchers is also going down in line with the two-sided lagged complementarity between business and government researchers shown in Table 11, column 5, but this effect is statistically significant for the panel average in Figure A.3 only for four periods because it is mitigated through the substitutability of column 6. In line with results of column 1 of Table 11, the level of labour productivity goes down after two periods according to Figure A.3 by about -0.0054 in the long run similar to the drop from 2 to 1.44 percent in HLP, and this is significant after four years. Private R&D expenditures go down with the number of government researchers in log-levels in Figure A.2 with statistical insignificance from periods 6 to 12 for growth rates in Figure A.3. In contrast, public R&D expenditures go down in in log-levels in Figure A.2, and in growth rates in Figure A.3, in a statistically insignificant way and weakly so in the first eight years. The number of firms goes down for two years and then up and down in statistically insignificant ways. We see a positive effect in Figure A.3 described above in terms of a higher growth rate only in the late period and it is insignificant. The hoped for crowding in of new firms, here in terms of growth rates, is preceded by two periods with less firms and becomes a bit stronger between periods 20 and 25, but it is also statistically insignificant. For the last three variables the signs differ from those in the main text.

The number of firms, etp, difference between the R&D cut scenario and the baseline scenario 2013-2100, starting from value 1 differs from the GMM-OD version of Figure 2 by having smaller numbers on the vertical axis of Figure A.2.

The result (ii) (for HLP results with cross spillovers) for business researchers per firm is confirmed in Figure A.4. This plot compares to HLP's Figure 4, upper-left panel, which has more business researchers in the phase shortly after the shock, but then is comparable to the falling pattern shown here. The result (iii) for business R&D knowledge stock growth holds empirically for 23 year, is significant only for period 2, and does not hold in the long run in Figure A.3. Result (iii) has therefore only limited confirmation from this empirical VAR.

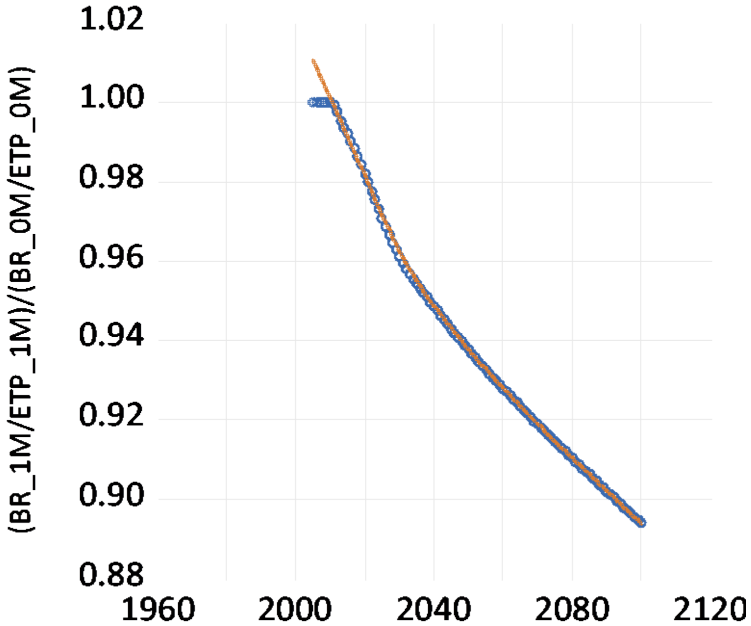


Figure A.4: Effects of public R&D cut on firm-level R&D, 2010-2100. 0M indicates baseline expected panel average value; 1M indicates expected panel average value from shock scenario. Each dot represents fourteen countries.

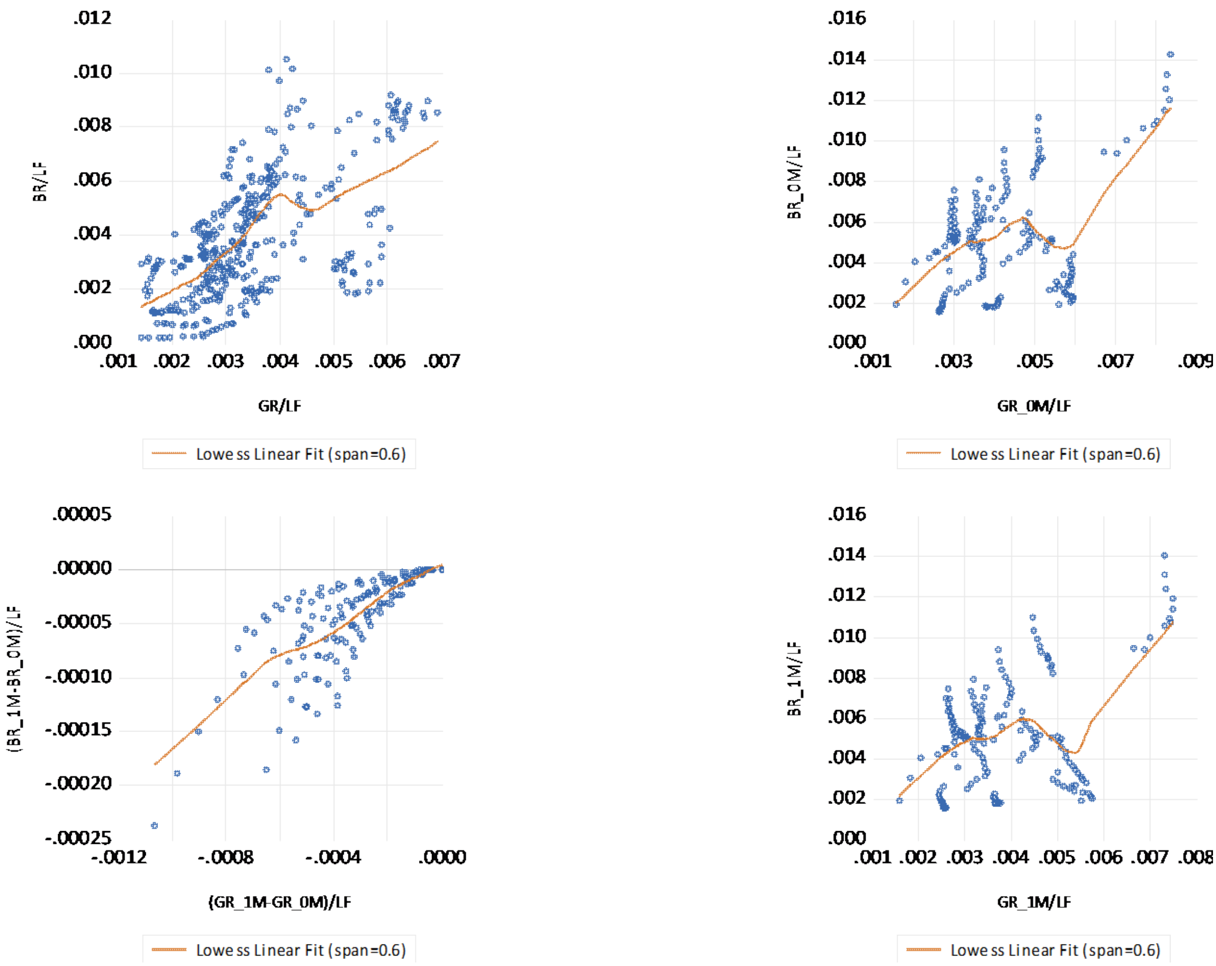


Figure A.5 Decreasing change of business researchers through more government researchers.

In the change from the baseline to the public R&D cut scenario in the VAR model (indicated as 1_0m), business R&D is reduced much less than public R&D; this is 'a tiny amount' in the steady state in HLP, and here between -0.0000 and -0.00025 for the period 2013-2020, which is smaller than the effect for the GMM-OD VAR in the main text.

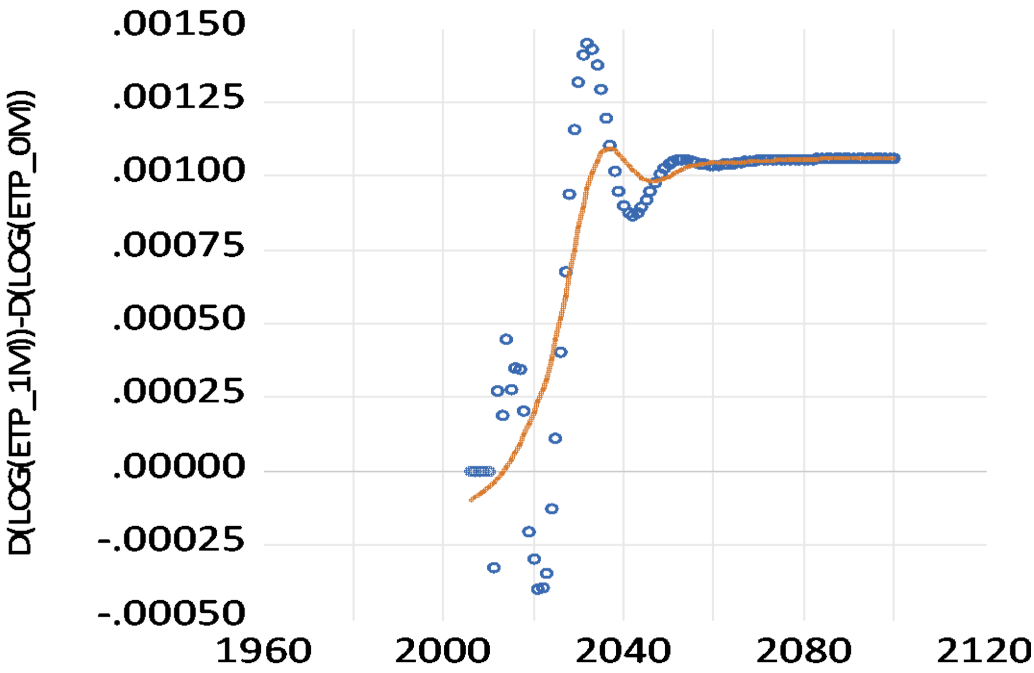


Figure A.7 The growth rate of the number of firms is lower in the shock scenario than in the baseline scenario in the early phase and higher later.

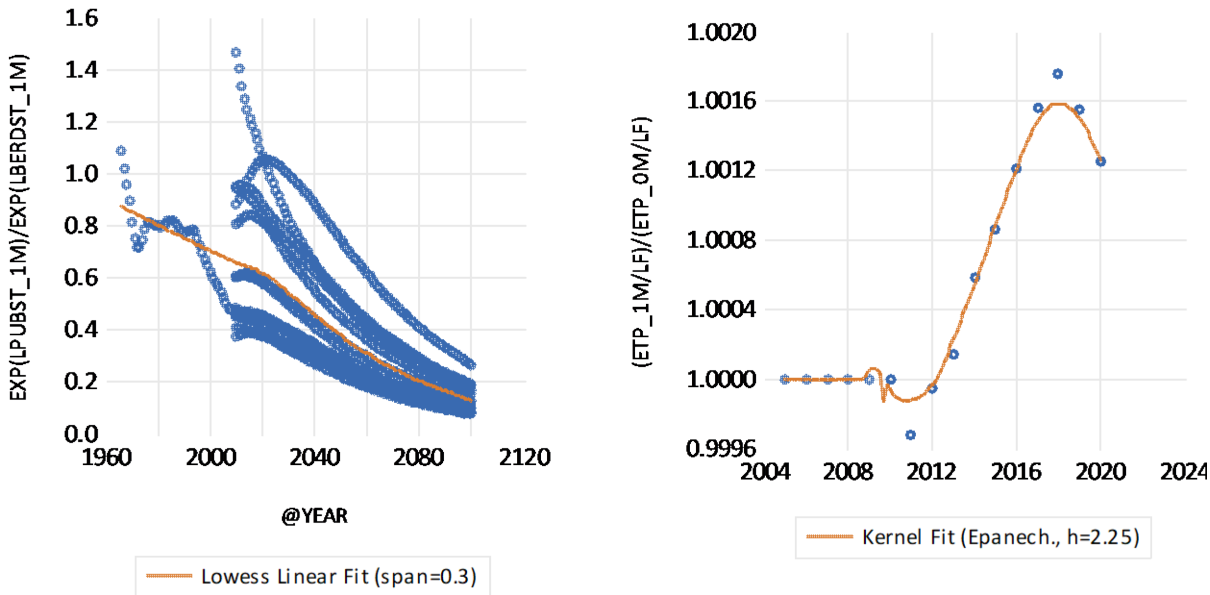


Figure A.8: The adjustment process after a public R&D cut: the number of firms (% labour force) first goes down and then goes up; the public-private knowledge ratio is decreasing.

The last falling part of Figure A.8 for the number of firms would hold for a theoretical constellation crossing the isoclines two times more often than drawn in their Fig.1. This would imply a falling phase for the public/private R&D capital ratio though, which we do not find empirically in our VAR. However, adding the labour force to the VAR, which we have not done in line with the theoretical model, the shock would yield non-decreasing growth rates of the number of firms, and statistically significantly decreasing growth rates of the labour force. This would imply that the number of firms would always increase relative to the labour force. This constellation would be obtained when the moment after the shock has both variables below the isoclines.

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