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The periodic service network design problem with regular and express deliveries under demand uncertainty

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Abstract

This paper studies the routing of multiple commodities (shipments) through a network with the aim to minimize the total cost. To transport these commodities from their origin to their destination hub, a combination of different services can be used, including scheduled trucks (following a dedicated trajectory, similar to bus routes) and express delivery. Each commodity starts its itinerary at its origin hub and needs to arrive at its destination hub before its deadline. The following cost factors are considered in the model: a fixed cost as well as a distance-based travel cost for the scheduled truck services, a cost for express delivery between each pair of hubs based on the size of the commodity, and the inventory holding cost at each hub.

We first define the problem as a mixed-integer linear program (MILP). To solve this MILP, we apply a branch-and-price algorithm that relies on column generation. In a second phase, we extend our model formulation to also deal with demand uncertainty (i.e., the size of each shipment varies) and present a two-stage, scenario-based stochastic model which we also solve using the branch-and-price algorithm. To generate the scenarios for the stochastic model, we apply Sample Average Approximation (SAA). Extensive computational experiments, including a sensitivity analysis are presented.

Keywords: network design problem, vehicle routing, branch-and-price, column generation, sample average approximation, stochastic optimization.

1 Introduction and research context

In today's international context, the planning and coordination of all necessary logistics operations within a supply network is a tedious task. As supply chain partners often established strong dependencies on each other — with the aim to improve overall efficiency of the network —, any delay or disruption in the transport flows between these partners will create a significant impact on the underlying operations [3, 21, 14].

To plan and execute all required logistical operations within the supply chain, companies rely on third party logistics service providers (3PLs). These 3PLs manage the flow of goods between the different supply chain entities by either dispatching their own vehicles or by subcontracting logistics service providers to execute the required transportation requests [45, 41, 31].

The problem presented in this paper is motivated by a case study in which a 3PL is responsible for coordinating all material flows that belong to the supply network of a large construction company within Europe (company names are confidential). The network consists of multiple hubs, which either take the form of transhipment points within the supply network or represent a local supply or demand node (potentially uniting multiple suppliers / customers within a certain region for simplicity). The flow density between each pair of hubs varies significantly over time (some connections are used only seldom, others have high volumes every day) and is uncertain (exact volumes are only known last-minute). As the 3PL does not have its own fleet of trucks, it relies on — often local — subcontractors (carriers) to execute the transports.

We distinguish two types of agreements between the 3PL and its subcontractors. First, there is a long-term agreement to establish a *periodic fixed capacity* on some of the network connections. For example, a truck is chartered every Monday and Thursday to drive a fixed trajectory. As these long-term commitments are valuable to the carriers (these provide a predictable income), competitive prices can be negotiated for the service. However, sufficient flow should be guaranteed over the link, as one should always pay for a full truckload, independent from the actual load. Second, the 3PL can book an *ad-hoc express delivery* on the spot market. This service is more flexible and its cost depends solely on the volume and trajectory of the actual load at a particular time (i.e., there is no long-term commitment here).

To model the decisions faced by the 3PL, we use a *service network design problem* (SNDP) formulation. SNDPs mainly support tactical decisions (e.g., fleet size, transport modes, ...) for the routing of commodities (such as goods, data, people, ...) within

a network that consists of interconnected hubs and where the transport of a commodity occurs between its source and destination node. Variants of the SNDP have been successfully applied to many problems in, e.g., road transportation planning [34, 17, 44], railway planning [8, 32, 5, 57], flight scheduling [27, 10, 35] and telecommunication [39, 38, 36].

This paper contributes to the academic literature in the following ways. First, we consider a service network design model over time to allow differentiation between the (periodic) scheduled truck services and the ad-hoc express delivery option. To the best of our knowledge, we are the first to distinguish these two transport modes with their individual cost structure. Second, we enrich the current state-of-the-art formulations by accounting for hub capacities and manage inventory levels accordingly. Third, we develop competitive solution approaches based on a branch-and-price algorithm with a column generation algorithm in each node to solve this realistic variant of the SNDP. Moreover, we extend our models and results to a setting with uncertain demand and present a two-stage, scenario-based stochastic model which is solved using the sample average approximation method. Finally, a broad range of managerial insights have been generated by means of an extensive sensitivity analysis.

The remainder of the paper is organized as follows. In Section 2, the relevant literature is discussed. We present a formal problem statement and a mathematical model formulation in Section 3. Section 4 details a column generation solution approach for the deterministic problem variant. This model is extended towards stochastic demands in Section 5. The implementation of the models and an extensive set of computational experiments are presented in Section 6, after which we summarize the main conclusions and limitations of our research in Section 7.

2 Literature review

2.1 Service network design problem

In this section, we review the literature on the service network design problem (SNDP) as we identified this problem is most closely related to the main topic of this manuscript. Early research on the service network design problem dates back to Crainic and Rousseau [16] and Farvolden and Powell [22]. Since then, many researchers have been attracted to extend these models to incorporate more realistic problem features [47].

Since our focus is on exact solution approaches, we will limit ourselves to contributions from the literature in which such methods have been presented. For an overview of the

state-on-the-art on heuristic and metaheuristic solution procedures, we refer the interested reader to Salimifard and Bigharaz [47].

Within the exact solution approaches, we distinguish two main research directions. First, there are methods that rely on *branching strategies*, such as branch-and-bound, branch-and-price(-and-cut) and column generation (see, e.g., Andersen et al. [4], Sarubbi et al. [49], Akyüz, Öncan, and Altınel [2], Boccia et al. [11] and Canel et al. [13]). Second, there are the contributions that focus on *decomposition-based methods* (see, e.g., Teypaz, Schrenk, and Cung [51], Oğuz, Bektaş, and Bennell [42], Rahmaniani et al. [43], Çakır [12] and Moradi, Raith, and Ehrgott [40]). In what follows, we highlight the most related and relevant contributions.

Boccia et al. [11] propose a *multi-commodity location routing problem* which they solve using a branch-and-cut algorithm. Given a set of potential facility locations and a set of demands (commodities), the multi-commodity location routing problem is about deciding how many and which of these facilities to open in order to minimize the total cost (i.e., a fixed cost for opening a facility and a variable cost based on the routing of the commodities) while covering all demand.

Wang et al. [56] propose a *service network design model* in which the routes for a heterogeneous fleet of vehicles should be determined, given a set of delivery points with predefined demand. The authors present both arc-based and path-based mathematical formulations to model the problem. To solve the problem, a hybrid algorithm is used that combines exact and heuristic techniques (including column generation, cutting planes and local search) to solve large-scale instances. The exact solvers within the algorithm are responsible for providing lower bounds and feasible solutions, whereas local search is used to generate feasible upper bounds. The presented computational experiments demonstrate the potential of a heterogeneous fleet in tactical planning, as this provides higher vehicle loading rates less unused capacity. In contrast to this paper, the authors did not consider delivery times (deadlines), capacities in the hubs and demand uncertainty.

The *capacitated multi-commodity network design problem* is presented by Katayama [28]. In this problem variant, the arcs in the network have limited capacity. The decision maker also decides which arcs (and thus which of their corresponding capacities) to make available within the network. The total cost of the network — which is to be minimized — is given by the routing cost for shipping all commodities from their source to their destination and the activation of an arc. The authors present a path-based formulation augmented with strong inequalities. They use column generation in combination with an arc capacity

scaling (i.e., a linear approximation on the use of arc capacity) and local branching (i.e., improve the quality of the relaxed model by also considering neighbouring solutions) to solve the problem. In this paper, we do not restrict the arc capacity — even though individual vehicles do have capacities, we do not restrict the amount of vehicles that can travel on a certain arc. However, we do consider capacity restrictions in the hubs. As the opening of hubs is a long-term (strategic) decision, we do not consider the opening/closing of hubs nor flexibility in the available capacity.

Trivella et al. [53] develop a mathematical path based model formulation for the *multi-commodity network flow problem with soft transit times*. The model explicitly discourages the use of long commodity routes by means of a penalty for delays. The authors present a column generation approach to solve the problem. The economic implications on costs and delays for different definitions of the penalty functions are discussed within a context of the liner shipping industry.

In Çakır [12], the authors use Benders decomposition to solve the *multi-commodity, multi-mode distribution planning problem*. In this multi-commodity flow problem, commodities do not have a dedicated source node but some nodes are labeled as general source node. Consequently, the demand of each destination node can be fulfilled from any source node.

2.2 Demand uncertainty and stochastic models

In this section, we review the most relevant research contributions concerning stochastic network design and network flow optimization models under uncertainty.

Most stochastic models that focus on demand uncertainty make use of *two-stage stochastic programming* [15]. In a first stage — before the realization of the stochastic demand — these models (partially) set the values of some of the decision variables that are not directly influenced by the uncertainty while considering also the expected cost of the second stage model (i.e., after the realization of the stochastic demand). Two-stage stochastic programming was introduced by Dantzig [19] and has been applied successfully to tackle different supply chain problems [30, 6].

The *multi-commodity redistribution problem* with stochastic supply, demand and network is studied by Gao and Lee [23]. The authors focus on the redistribution of commodities to respond to different realizations of the demands. To solve the problem, the authors make use of a two-stage, scenario-based stochastic programming model. In the first stage, the authors minimize the total dissatisfaction cost (unmet demand and oversupply) over

different demand and supply scenarios. In the second stage, the authors vary the network availability and minimize the total response time.

Barbarosoğlu and Arda [9] use a two-stage stochastic programming model to optimize the transport of first-aid commodities to disaster-affected areas. A multi-commodity, multi-modal network flow formulation is developed to describe the flow of material over an urban transportation network. The random variables in this study are dedicated to the resource requirements, which are assumed uncertain after a disaster has occurred.

Hamdan and Diabat [25], then, apply a two-stage stochastic model to plan the production, inventory and location decisions in a red blood cell supply chain under demand uncertainty. Similarly, Dillon, Oliveira, and Abbasi [20] propose a two-stage stochastic model for inventory management in a blood supply chain by considering uncertain demand.

The generation of scenarios — as well as determining the optimal number of scenarios — largely affects the performance of stochastic programming models. These decision should therefore be taken with care. Löhndorf [33] review the most common methods for scenario generation in the context of stochastic programming, including the *quasi-Monte Carlo method, moment matching, methods based on probability metrics*, and the *Voronoi cell sampling method*. The Monte Carlo method — better known as *Sample Average Approximation (SAA)* — is a well-known approach to reduce the size of stochastic optimization problems by considering a subset of (preferably independent) scenarios after which a deterministic problem is solved for each of these. For a clear guide to SAA, we refer the interested reader to Kim, Pasupathy, and Henderson [29].

Within the context of supply chain network design, Santoso et al. [48] make use of the SAA scheme. In combination with an accelerated Benders decomposition algorithm, they can compute solutions to large-scale problems with a huge number of scenarios.

Sörensen and Sevaux [50] study a stochastic vehicle routing problem. The authors propose a method to combine a sampling-based approach to estimate the robustness or flexibility of a solution with a metaheuristic optimization technique, which allowed them to solve large problems with more complex stochastic structures.

Mendoza et al. [37] propose a bi-objective multi-commodity vehicle routing problem with stochastic demand. The goal is to simultaneously minimize the total expected cost of a set of routes and the coefficient of variation. The authors use chance constraints to make sure that the probability of a route duration is less than its maximum given threshold. Monte Carlo simulation is applied for the feasibility check of these chance constraints.

Based on the presented literature review, we conclude that only few mathematical programming models for variants of the service network design problem have been proposed. Moreover, these models lack the inclusion of important real-life problem features such as heterogeneous vehicles (more specifically the inclusion of the possibility to use express delivery services), multi-commodity problems over time in which commodities have dedicated release times and deadlines, capacitated hubs and periodicity of the planning over time. The current manuscript aims to fill this research gap by considering the mentioned elements in the context of a multi-commodity service network design problem.

3 Single-period service network design problem

In this section, we formally present the *multi-commodity service network design problem with regular and express deliveries for a single period* (typically a week). This period is subdivided in multiple time intervals (e.g., days) for each of which a decision has to be made regarding the capacity on the individual network links and the flow over each link. For now, we will limit ourselves to a deterministic variant of the problem.

3.1 Mathematical notation and model assumptions

We are given a network, represented by the complete graph $\boldsymbol{G}(\boldsymbol{V}, \boldsymbol{A})$ in which $\boldsymbol{V}$ is the set of vertices (hubs) and $\boldsymbol{A}$ the set of arcs. For each $(i, j) \in \boldsymbol{A}$, c_{ij} and τ_{ij} represent the cost associated with traversing the arc and the travel time, respectively.

Inside each hub, we distinguish two different processes: storage and cross-docking. Storage refers to the possibility to store shipments over multiple time intervals (i.e., the arrival time interval of the shipment is different from the departure time interval). For each vertex $i \in \boldsymbol{V}$, the storage capacity is limited and denoted by Q_i^V. Cross-docking refers to the process of receiving, sorting, recombining and dispatching incoming shipments within the same time interval, usually within a few hours [46]. As these activities do not make use of the internal storage space of the hub, we do not limit these by the hub capacity.

Let $\boldsymbol{K}$ be the set of all shipments (commodities) that should be served by the network. For each shipment $k \in \boldsymbol{K}$, O_k and D_k denote the source (origin) and destination node, respectively. The volume of the shipment is denoted by q_k. We allow the splitting of this volume such that partial customer orders can be transported via a different route through the network, if desirable. Furthermore, each shipment has a release time l_k, defined as the time at which the shipment becomes available at its source, and a dispatching time

u_k, at which the shipment will be send from its destination hub to the customer. This dispatching time can be interpreted as a hard deadline for the transport activities within the network related to this shipment. In case the shipment arrives at the destination hub before its dispatching time, it will be temporarily stored in inventory.

To execute the necessary transportation requests, the logistics service provider can choose between the following three transport options:

1. *Scheduled truck service*: based on long-term contracts, a dedicated capacity is available on certain routes in the network. Because of the long-term stability of these routes, competitive prices can be negotiated for installing the capacity. Let F denote the fleet of scheduled trucks, each with a capacity Q^F. For this service, we incur both a fixed cost for establishing a truck connection and a distance-based variable cost, denoted by c^F and c_{ij}, respectively. There should not be a one-to-one relationship between a shipment and a scheduled truck service as a shipment can switch to another truck at any hub.

2. *Express delivery*: The full transport of the shipment can be outsourced to a third-party logistics provider at a fixed rate based on the origin – destination as well as the volume of the shipment. This option is more expensive than using the capacity of the scheduled truck service, but offers more flexibility.

3. *Mixed scenario*: To execute the required transport operations, a combination of scheduled truck services and express delivery can be used. This means that for certain connections the scheduled truck service will be used, whereas other parts of the itinerary will be covered using the express service.

The goal is to decide on the required capacity for the scheduled truck service and design the corresponding routes for these vehicles. Here, the decision maker trades-off installing more capacity on the scheduled truck service versus accepting the (higher) costs of express delivery. Over high demand connections, we likely prefer the scheduled truck service as loading rates can be high and the fixed cost of establishing the connection can be divided over a larger volume. For low demand connections, it might not be worth installing a scheduled truck service and an express delivery will then be preferred.

3.2 Mixed Integer Linear Programming formulation

We now model the deterministic network design problem with express deliveries as a mixed integer linear problem. Assuming that each period is identical, we will focus on a single period with T time intervals. For example, T can represent one week, which can be subdivided in 7 days, denoted by $t = \{1, \ldots, 7\}$. By imposing that the status of the network (i.e., amount of truck available in each hub) at the end of the period equals the initial status, the logistics plan can easily be repeated for each consecutive period.

The following decisions have to be made:

1. The total *number of scheduled trucks* available at hub i at time t, denoted by f_{it}.

2. The *routes covered by the scheduled trucks*, represented by decision variable z_{ijt}, denoting the number of scheduled trucks traversing arc (i, j) at time t.

3. The *itinerary for each shipment* $k \in \boldsymbol{K}$, based on the following decision variables:

   - The quantity of shipment k shipped over arc (i, j) using a scheduled truck service at time t denoted by x^k_{ijt}.
   - The quantity of shipment k shipped over arc (i, j) using the express delivery service at time t, denoted by e^k_{ijt}.

4. *Inventory decisions within each hub*, given by the quantity of shipment k kept in inventory at hub i at time t, denoted by I_{ikt}.

To allow tractability of the model and avoid (unnecessary) complexity, the model is built according to the assumption that no partial shipments can be handed over to the next period. This means that all shipments must be handled within the period under consideration and — consequently —- that all shipments are assumed to have a release time and delivery date within the current period T. Within our model formulation, this also means that all inventory levels will equal zero at the start and the end of the period T. The assumption can be justified by the fact that the presented model has the purpose to support tactical (or even strategical) decisions with respect to the long-term contracts and required capacities for the scheduled truck services. In this respect, shipments can be generated (e.g., based on historical traffic data) such that they represent the partial trips typically covered within a single period. For operational decision support (e.g., the day-to-day dispatching of shipments), other methods can be used that take the established

Table 1: *Mathematical notation for the MILP formulation of the deterministic multi-commodity network design problem with express deliveries.*

Sets	
$\boldsymbol{V}$	The set of all vertices (hubs) in the network.
$\boldsymbol{A}$	The set of all arc (i, j), with $i, j \in \boldsymbol{V}$.
$\boldsymbol{K}$	The set of all shipments that should be served by the network.
$\boldsymbol{T}$	The set of all time intervals.
Parameters	
Q_i^V	The storage capacity of hub i.
h_i	Inventory cost per time interval per unit of volume at hub i.
c_{ij}	The cost to traverse arc (i, j) with a scheduled truck.
τ_{ij}	The travel time over arc (i, j) for a scheduled truck.
τ_{ij}^E	The travel time for shipping by express over arc (i, j).
O_k	The source node (origin) for shipment k.
D_k	The destination node for shipment k.
q_k	The volume of shipment k.
l_k	The release time of shipment k.
u_k	The time at which the shipment will be dispatched from its destination hub to the customer.
c_{ij}^E	The cost per volume-unit to use express delivery on arc (i, j).
Q^F	Capacity of a scheduled truck.
c^F	Fixed cost for establishing a scheduled truck.
Decision variables	
f_i^0	The number of scheduled trucks available at hub i at the beginning of the time horizon.
f_{it}	The total number of scheduled trucks that remain at hub i at the end of time t.
z_{ijt}	The number of scheduled trucks traversing arc (i, j) at time t.
e_{ijt}^k	The volume of shipment k shipped by express mode over arc (i, j) at time t
x_{ijt}^k	The volume of shipment k shipped over arc (i, j) at time t.
I_{ikt}	The volume of shipment k kept in inventory at hub i at time t.

capacity of the scheduled truck service as given and optimize loading rates and costs based on, e.g., a rolling time window approach.

The full MILP formulation of the deterministic network design problem with express deliveries is given below. We summarize all notation in Table 1.

$$\min \left[\sum_{i \in \boldsymbol{V}} c^F f_i^0 + \sum_{t \in \boldsymbol{T}} \left(\sum_{(i,j) \in \boldsymbol{A}} c_{ij} z_{ijt} + \sum_{(i,j) \in \boldsymbol{A}} \sum_{k \in \boldsymbol{K}} c_{ij}^E e_{ijt}^k + \sum_{i \in \boldsymbol{V}} \sum_{k \in \boldsymbol{K}} h_i I_{ikt} \right) \right] \tag{1}$$

s.t.

$$\sum_{k \in \boldsymbol{K}} x_{ijt}^k \leq Q^F z_{ijt} \qquad \forall (i,j) \in \boldsymbol{A}; \forall t \in \boldsymbol{T} \tag{2}$$

$$f_{it} = f_{i(t-1)} + \sum_{(j,i) \in \boldsymbol{A} | t - \tau_{ji} \geq 1} z_{ji(t-\tau_{ji})} - \sum_{(i,j) \in \boldsymbol{A}} z_{ijt} \qquad \forall i \in \boldsymbol{V}; \forall t \in \boldsymbol{T} \backslash \{1\} \tag{3}$$

$$f_{i1} = f_i^0 - \sum_{(i,j) \in \boldsymbol{A}} z_{ij1} \qquad \forall i \in \boldsymbol{V} \tag{4}$$

$$f_i^0 = f_{iT} \qquad \forall i \in \boldsymbol{V} \tag{5}$$

$$I_{ikt} - I_{ik(t-1)} + \underbrace{\left(\sum_{(i,j) \in \boldsymbol{A}} x_{ijt}^k - \sum_{(j,i) \in \boldsymbol{A}} x_{ji(t-\tau_{ji})}^k \right)}_{scheduled\,truck} + \underbrace{\left(\sum_{(i,j) \in \boldsymbol{A}} e_{ijt}^k - \sum_{(j,i) \in \boldsymbol{A}} e_{ji(t-\tau_{ji}^E)}^k \right)}_{Express}$$
$$= \begin{cases} 0 & \forall k \in \boldsymbol{K}; \forall t \in [l_k, u_k]; \forall i \in \boldsymbol{V} \backslash \{O_k, D_k\} \\ q_k & \forall k \in \boldsymbol{K}; t = l_k; i = O_k \\ -q_k & \forall k \in \boldsymbol{K}; t = u_k; i = D_k \end{cases} \tag{6}$$

$$I_{ikt} = 0 \qquad \forall i \in \boldsymbol{V}; \forall k \in \boldsymbol{K}; \forall t \notin [l_k, u_k] \tag{7}$$

$$\sum_{k \in \boldsymbol{K}} I_{ikt} \leq Q_i^V \qquad \forall i \in \boldsymbol{V}; \forall t \in \boldsymbol{T} \tag{8}$$

$$e_{ijt}^k, x_{ijt}^k, I_{ikt} \geq 0 \qquad \forall i,j \in \boldsymbol{V}; \forall t \in \boldsymbol{T}; \forall k \in \boldsymbol{K} \tag{9}$$

$$f_i^0, f_{it}, z_{ijt} \in \mathbb{N} \qquad \forall i,j \in \boldsymbol{V}; \forall t \in \boldsymbol{T} \tag{10}$$

Objective function. The goal is to minimize the total cost of running the network over the full planning horizon T, given in Equation (1). This objective function contains the following terms: the sum of the fixed and variable cost related to the scheduled trucks, the cost of using express deliveries for the (partial) shipments that are not transported using the scheduled truck and the inventory holding costs at the hubs. Note that the total amount of scheduled trucks established in the network is given by the sum of all trucks initiated at the hubs at the start of the period, denoted by $\sum_{i \in \boldsymbol{V}} f_i^0$.

Constraints. To ensure feasibility of the network, the following constraints with respect to the truck routes, the itineraries of the commodities and the inventories in the hubs should be satisfied. Constraints (2) ensure that for each arc at each time the total flow dedicated to the scheduled truck service does not exceed the scheduled truck capacity available on the arc. Constraints (3) take care of the allocation of scheduled trucks over the different hubs in the network at each time interval. The number of trucks available at hub i at the end of time t is given by the amount trucks stationed at this hub at the end of $t-1$ plus the incoming trucks minus the outgoing trucks. The initial allocation of trucks at the start of the period is given by constraints (4). To allow the schedule to be repeated over time, the starting configuration is set equal to the ending configuration in constraints (5).

The inventory levels are controlled by constraints (6). These constraints define the (partial) amount of shipment k in different hubs (potential transshipment points, origin and destination) over time. Once the shipment has been released into the system ($t \geq l_k$), this amount equals the total volume of the shipment received in each hub minus what has left the hub either via a scheduled truck or express delivery. By means of constraints (7), we explicitly set all inventory levels to zero for times that the shipment is not active in the network (i.e., before its release time and after its dispatching time). Constraints (8) control the storage capacity of each hub.

Finally, the domain of the decision variables is set by constraints (9) and (10).

4 Branch-and-price algorithm

Branch-and-price (BP) algorithms embed dynamic column generation into a branch-and-bound framework to solve a MILP. We apply a *best-first branching strategy* on the number of trucks on each arc, denoted by z_{ijt}.

In each node of the search tree, we apply the column generation algorithm presented above to solve the linear problem relaxation (relaxing the integrality constraint on the z_{ijt} variable). Each time no additional columns (routes) improve the master problem and the LP-relaxed solution does not satisfy the integrality conditions we use bounding on each branch and solve two separate column generation algorithm for each branch as follows: $z_{ijt} < \lfloor z_{ijt} \rfloor$ and $z_{ijt} \geq \lceil z_{ijt} \rceil$.

In what follows, we will go deeper on the column generation approach that is at the core of each node in our branch-and-price algorithm.

The presented MILP model presented above aims to integrate the routing decisions for

the scheduled trucks and — if desirable — the use of the express delivery service with the individual (partial) routes for each commodity flowing through the network. As a result, it easily becomes intractable, even for small instances.

To decouple the complexity of finding good routes for the vehicles from the routes of the commodities, we will rely on a *Dantzig-Wolfe decomposition* [58] and solve the problem using a column generation framework in which a master problem and sub-problem (the pricing problem) are solved in an iterative way.

To initialize the column generation procedure, we start with the situation in which no scheduled truck routes are established and solve the master problem. As a result, all shipments will be sent directly from source to destination via a dedicated express delivery. Even though this solution is feasible, it is likely not optimal as no bundling opportunities are seized, even not for shipments with the same origin and destination.

The sub-problem aims to find promising routes for each individual shipment for which a scheduled truck service can be used. By focusing solely on the most promising routes for an individual shipment, the size of the problem is kept as small as possible and many high-quality routes can be added to the master problem in each iteration. Once routes have been generated by the sub-problem, the master problem is run again with the aim to route all shipments through the available network in an optimal way (i.e., select a combination of routes, potentially complemented with one or multiple express connections, for each shipment). This procedure is iterated until no more routes (columns) with negative reduced cost can be found.

4.1 Master problem

The master problem determines the flow of all shipments through the network using a combination of scheduled trucks or express delivery — defined as routes. These routes, denoted by $\boldsymbol{R}$, are dedicated to specific shipments (i.e., the set of routes available for shipment k is denoted by $\boldsymbol{R}_k \subset \boldsymbol{R}$) and generated by the sub-problem.

Each route $r \in \boldsymbol{R}_k$ is characterized by a binary parameter w_{ij}^{rt}, denoting whether the route r runs over the link (i, j) at time t. The quantity of shipment k transported using this route r is given by X_r^k. We summarize the additional notation for the master problem in Table 2.

The master problem is defined mathematically as follows:

Table 2: *Additional mathematical notation for the master problem.*

Sets	
$\boldsymbol{R}$	The set of all routes in the master problem.
$\boldsymbol{R}_k \subset \boldsymbol{R}$	The set of all routes of scheduled trucks for shipment k.
$\boldsymbol{A}_r$	The set of all arcs in route r.
Parameters	
w_{ij}^{rt}	Binary parameter denoting whether route r runs over the link (i,j) at time t.
Decision variables	
x_r^k	The amount of shipment $k \in K$ shipped via route $r \in R_k$.

$$\min \left[\sum_{i \in \boldsymbol{V}} c^F f_i^0 + \sum_{t \in \boldsymbol{T}} \left(\sum_{(i,j) \in \boldsymbol{A}} c_{ij} z_{ijt} + \sum_{(i,j) \in \boldsymbol{A}} \sum_{k \in \boldsymbol{K}} c_{i,j}^E e_{ijt}^k + \sum_{i \in \boldsymbol{V}} \sum_{k \in \boldsymbol{K}} h_i I_{ikt} \right) \right] \tag{11}$$

$s.t.$

$$\sum_{k \in \boldsymbol{K}} \sum_{r \in \boldsymbol{R}_k} x_r^k w_{ij}^{rt} \leq Q^F z_{ijt} \qquad \forall (i,j) \in \boldsymbol{A}; \forall t \in \boldsymbol{T} \tag{12}$$

$$I_{ikt} - I_{ik(t-1)} + \sum_{r \in \boldsymbol{R}_k} x_r^k \left(\sum_{(i,j) \in \boldsymbol{A}_r} w_{ij}^{rt} - \sum_{(j,i) \in \boldsymbol{A}_r} w_{ji}^{r(t-\tau_{ji})} \right) + \left(\sum_{(i,j) \in \boldsymbol{A}} e_{ijt}^k - \sum_{(j,i) \in \boldsymbol{A}} e_{ji(t-\tau_{ji}^E)}^k \right)$$
$$= \begin{cases} 0 & \forall k \in \boldsymbol{K}; \forall t \in [l_k, u_k]; \forall i \in \boldsymbol{V} \backslash \{O_k, D_k\} \\ q_k & \forall k \in \boldsymbol{K}; t = l_k; i = O_k \\ -q_k & \forall k \in \boldsymbol{K}; t = u_k; i = D_k \end{cases} \tag{13}$$

$$x_r^k \geq 0 \qquad \forall k \in \boldsymbol{K}; \forall r \in \boldsymbol{R} \tag{14}$$

Constraints (3), (4), (5), (7), (8), (9) and (10).

Objective function. The objective function of the master problem is equal to the objective function of the global MILP formulation, presented in Section 3.2. The function minimizes the total cost of the network, including the fixed and variable cost of the scheduled trucks, the cost for all express deliveries and the inventory holding cost in the hubs.

Constraints. To comply with the route-based formulation required for connecting the master and its sub-problem, we slightly adapted some of the constraints from the global

MILP formulation.

Constraints (12) set the required amount of scheduled trucks that drive over arc (i, j) at time t, given by z_{ijt}, based on the total flow over the routes that make use of this arc. Similar to our global MILP, we assume that each shipment can be split in a continuous way. In other words, we do not consider any bin packing formulation for splitting the (partial) shipments over multiple trucks.

Constraints (13) are the flow balancing constraints in which we account for the inventory at the hubs. The third term of the equation accounts for changes in the inventory related to the shipment flowing through the available scheduled truck routes. The last (fourth) term on the left-hand side of the equation accounts for express deliveries of the shipment from the current hub to other hub(s) in the network.

Finally, constraints (14) set the domain for the newly added decision variable x_r^k.

4.2 Route generation sub-problem

The aim of the sub-problem is to generate additional routes that can be added to the set $\boldsymbol{R}$ and considered by the master problem. A route is defined as a path of one or multiple arcs in our network. To generate many promising routes fast, we run the sub-problem for each shipment separately.

Let z be the objective function of Master Problem (MP). Moreover, let μ_{ijt} be the dual variables corresponding to the capacity constraint (12) and γ_{ikt} the dual variables for the flow balancing constraints (13).

We also define y_{ijt} as a binary variable that takes the value 1 if arc (i, j) is used at time t in the generated route, 0 otherwise. For modelling purposes, we also introduce S_{it} and E_{it} to represent the starting and ending node of the route, respectively. As the final itinerary of a shipment in the master problem can be defined as a combination of routes — potentially also including one or multiple express arcs —, we do not impose that the starting (or ending) hub of the generated routes coincide with the source (or destination) hub of the shipment.

Finally, let I_{it}^B be an auxiliary binary variable that takes the value 1 if there is a positive inventory in hub i at time t, 0 otherwise. Again, we summarize the additional mathematical notation in Table 3.

The *total reduced cost* for shipment k is computed using equation (15), which accounts for all reduced costs for the master problem constraints with the non-basic variable x_r^k, i.e., constraints (12) and (13).

Table 3: *Additional notation for the sub-problem.*

Parameters	
μ_{ijt}	Dual variable for constraints (12) of the master problem (capacity constraint).
γ_{ikt}	Dual variable for constraints (13) (inventory and flow balancing constraint).
Decision variables	
y_{ijt}	Binary variable that equals 1 if arc (i,j) is traversed at time t, 0 otherwise.
I^B_{it}	Binary variable that equals 1 if there is a positive inventory in hub i at time t ($\sum_{k\in\boldsymbol{K}} I_{ikt} > 0$), 0 otherwise.
S_{it}	Binary variable that equals 1 if the current route starts in hub i at time t, 0 otherwise.
E_{it}	Binary variable that equals 1 if the current route ends in hub i at time t, 0 otherwise.

$$Z^k = \sum_{t\in\boldsymbol{T}} \left(- \sum_{(i,j)\in\boldsymbol{A}} y_{ijt}\mu_{ijt} - \sum_{(i,j)\in\boldsymbol{A}} y_{ijt}\gamma_{ikt} + \sum_{(j,i)\in\boldsymbol{A}} y_{ji(t-\tau_{ji})}\gamma_{ikt} \right) \tag{15}$$

Then, the route generation sub-model is given by the following mathematical program.

$$\min \left[Z^k\right] \tag{16}$$

$$s.t.$$

$$\sum_{i\in\boldsymbol{V}} \sum_{t=(l_k+\tau^E_{O_k i})}^{(u_k-\tau^E_{iD_k})} S_{it} = 1 \tag{17}$$

$$\sum_{i\in\boldsymbol{V}} \sum_{t=(l_k+\tau^E_{O_k i})}^{(u_k-\tau^E_{iD_k})} E_{it} = 1 \tag{18}$$

$$S_{it} + E_{it} \leq 1 \qquad \forall i \in \boldsymbol{V}; \forall t \in \boldsymbol{T} \tag{19}$$

$$I^B_{it} - I^B_{i(t-1)} - S_{it} + E_{it} = \sum_{(j,i)\in\boldsymbol{A}} y_{ji(t-\tau_{ji})} - \sum_{(i,j)\in\boldsymbol{A}} y_{ijt} \qquad \forall i \in \boldsymbol{V}; \forall t \in \boldsymbol{T} \tag{20}$$

$$y_{ijt}, I^B_{it}, S_{it}, E_{it} \in \{0,1\} \qquad \forall i,j \in \boldsymbol{V}; \forall t \in \boldsymbol{T} \tag{21}$$

Objective function. The objective function of the sub-problem is the minimization of the reduced cost. If this optimal reduced cost is negative, the corresponding route will be added to the set of routes considered by the master problem.

Constraints. Constraints (17) and (18) ensure that routes have exactly one starting and one ending node, which are visited within a feasible time window for the shipment under

consideration. Constraints (19)) force the starting hub and time to be different from the ending hub and time.

The generated route should not only represent a path from start to end node, also the time at which different links are used should be consistent by considering intermediate storage in a hub if necessary, as seen in constraints (20).

Finally, constraints (21) take care of the domain constraints for the decision variables.

Once the sub-problem is solved for all shipments, the generated routes will be added to the master problem. This is done via the parameters w_{ij}^{rt}, which represent the y_{ijt} variables for each route.

5 Multi-period service network design problem

In the previous sections, we determined scheduled truck routes for a single period (e.g., week) with multiple time intervals. As these routes are established through long-term collaboration with dedicated carriers, they will be repeated every period. For example, if a scheduled truck connection is installed between two hubs during the first time interval of the period (e.g., Monday), this service will be provided every Monday. In this Section, we therefore extend the problem definition to a multi-period time horizon.

As demand is not constant, but might differ between periods, we need to establish the scheduled truck routes such that the overall long-term cost is minimized. If for most Mondays, e.g., the demand for connection A-B is rather low, we rather not install a scheduled truck over this connection on Monday (as this leads to large unused capacities during most weeks) but cover the demand with occasional express deliveries. If, on the other hand, demand is consistently high on Mondays, it might be beneficial to install a (less expensive) scheduled truck service that covers this connection.

To determine the optimal configuration of the scheduled truck services over multiple periods, we make use of a stochastic optimization model to account for the variability (and thus uncertainty) in the demand over time. More specifically, we employ a *two-stage scenario-based stochastic programming model* for this purpose.

5.1 Two-stage scenario-based stochastic programming model

The main idea behind the two-stage stochastic programming model is that we separate decision variables that depend directly on the scenario (i.e., what will remain constant over the different periods) from the decision variables that are impacted directly by the

realization of the demand (i.e., what will change every period). The first set of variables are related to the scheduled truck routes, as these are part of a long-term collaboration and thus cannot be altered every period. The second set of variables relates to the volumes transported via the express delivery service as well as the inventories at the different hubs, as these will vary every period depending on the demand scenario.

The formulation of the two-stage problem assumes that the second-stage data (i.e., the demand realization) can be modelled as a random vector with a known probability distribution which remains constant over time. Consequently, one may reliably estimate the underlying probability distribution after which the optimization on the expected value could be justified by the law of large numbers [55, 24, 52].

We will model the demand realization by means of a finite set of scenarios. Let $\boldsymbol{\Omega}$ be the set of scenarios (indexed by s). The probability of each scenario is denoted by $P(s)$, $\forall s \in \boldsymbol{\Omega}$. Additionally, we extend the decision variables denoting the flow in the network with an index s to account for the differences in demand for each shipment k between each scenario. For an overview of the additional notation, we refer to Table 4.

Table 4: *Additional notation for the two-stage scenario-based stochastic programming model.*

Sets	
$\boldsymbol{\Omega}$	The set of all scenarios, indexed by s.
Parameters	
q_{ks}	The total volume of shipment k under scenario s.
Decision variables	
e^k_{ijts}	The (partial) volume of shipment k shipped by express mode over arc (i,j) at time t under scenario s.
x^k_{ijts}	The (partial) volume of shipment k shipped over arc (i,j) at time t under scenario s.
I^k_{its}	The (partial) volume of shipment k kept in inventory at hub i at time t under scenario s.

We extend our original mathematical problem to a two-stage stochastic programming model as follows.

$$\min \left[\underbrace{\left(\sum_{i \in \boldsymbol{V}} c^F f_i^0 + \sum_{(i,j) \in \boldsymbol{A}} \sum_{t \in \boldsymbol{T}} c_{ij} z_{ijt} \right)}_{\text{First stage}} + E\left[\varphi\right] \right] \tag{22}$$

$s.t.$

$$\sum_{k\in\boldsymbol{K}} x^k_{ijts} \leq Q^F z_{ijt} \qquad \forall (i,j)\in\boldsymbol{A}; \forall t\in\boldsymbol{T}; \forall s\in\boldsymbol{\Omega} \quad (23)$$

$$I^k_{its} - I^k_{i(t-1)s} + \left(\sum_{(i,j)\in\boldsymbol{A}} x^k_{ijts} - \sum_{(j,i)\in\boldsymbol{A}} x^k_{ji(t-\tau_{ji})s}\right) + \left(\sum_{(i,j)\in\boldsymbol{A}} e^k_{ijts} - \sum_{(j,i)\in\boldsymbol{A}} e^k_{ji(t-\tau^E_{ji})s}\right)$$
$$= \begin{cases} 0 & \forall k\in\boldsymbol{K}; \forall t\in[l_k,u_k]; \forall i\in\boldsymbol{V}\backslash\{O_k,D_k\}; \forall s\in\boldsymbol{\Omega} \\ q_{ks} & \forall k\in\boldsymbol{K}; t=l_k; i=O_k; \forall s\in\boldsymbol{\Omega} \\ -q_{ks} & \forall k\in\boldsymbol{K}; t=u_k; i=D_k; \forall s\in\boldsymbol{\Omega} \end{cases} \quad (24)$$

$$I^k_{its} = 0 \qquad \forall i\in\boldsymbol{V}; \forall k\in\boldsymbol{K}; \forall t\notin[l_k,u_k]; \forall s\in\boldsymbol{\Omega} \quad (25)$$

$$\sum_{k\in\boldsymbol{K}} I^k_{its} \leq Q^V_i \qquad \forall i\in\boldsymbol{V}; \forall t\in\boldsymbol{T}; \forall s\in\boldsymbol{\Omega} \quad (26)$$

$$e^k_{ijts}, x^k_{ijts}, I^k_{its} \geq 0 \qquad \forall i,j\in\boldsymbol{V}; \forall t\in\boldsymbol{T}; \forall k\in\boldsymbol{K}; \forall s\in\boldsymbol{\Omega} \quad (27)$$

Constraints (3), (4), (5), and (10).

Objective function. Equation (22) minimizes the cost associated with the first stage variables (i.e., the scheduled truck service) plus the expected value for the second stage cost. The latter cost is defined as a weighted sum of the cost of the outcome for each scenario multiplied by its respective probability, such that

$$E[\varphi] = \sum_{s\in\boldsymbol{\Omega}} P(s)\varphi_s. \quad (28)$$

The second stage cost of scenario s, denoted by φ_s, is given by the following equation. The first term accounts for the cost of all required express deliveries for the shipments that could not be transported entirely by the scheduled truck services. The second term relates to the inventory holding costs in each hub.

$$\varphi_s = \sum_{t\in\boldsymbol{T}} \left(\sum_{(i,j)\in\boldsymbol{A}} c^E_{ij} e^k_{ijts} + \sum_{i\in\boldsymbol{V}}\sum_{k\in\boldsymbol{K}} h_i I^k_{its}\right) \quad (29)$$

Constraints. Some constraints of the original model are updated to account for the different demand scenarios. Equation (23) ensures that under no scenario, the capacity of the arcs (with respect to the installed scheduled truck capacity) is violated. Constraints (24) connect the flows over the network links — both using scheduled trucks as well as express

delivery — with the inventory at the different hubs over time. The inventory is initialized to zero for all time intervals a shipment is not in the system by constraints (25). Constraints (26) ensure the capacity of the hub is never exceeded. Finally, the domain constraints for the newly added decision variables are denoted by constraints (27).

5.2 Scenario generation using Sample Average Approximation

As the size of the network, the amount of shipments and the number of time intervals per period increase, the number of required scenarios to realistically represent the possible demand outcomes grows fast. To keep the model tractable, we make use of *Sample Average Approximation* (SAA). This technique relies on the generation of scenarios by means of Monte Carlo simulation [54]. Assuming that each scenario occurs with the same probability, we can rewrite equation (28) to

$$E\left[\varphi\right] = \frac{1}{|\Omega|} \sum_{s \in \mathbf{\Omega}} \varphi_s. \tag{30}$$

To construct our scenarios, we draw the volume of each shipment k, denoted by q_{ks}, from a normal probability $N(\mu, \sigma)$.

Following Verweij et al. [54], Bagaram and Tóth [7], and Ahmed, Shapiro, and Shapiro [1], the steps that are considered within our SAA implementation are summarized in Table 5.

To test the relationship between the number of scenarios in the two-stochastic optimization model and the optimality gap, we conducted a computational experiment which is discussed in detail in Section 6.2.

6 Model implementation and computational experiments

6.1 Test instances

The algorithms presented in this paper are tested extensively on a variant of the *Canad problem instances*[1] for the multi-commodity network design problem [18, 26]. In total, 41

[1]The original instances can be downloaded via `https://commalab.di.unipi.it/datasets/mmcf/#Canad` or `https://zenodo.org/record/4050442`.

Table 5: *Different steps of the SAA implementation.*

Step 1.	Initialize the number of independent samples (SAA replications), denoted by M, as well as the sample size n. For defining the value of the M, we used the method presented in Ahmed, Shapiro, and Shapiro [1] which is based on the probability of the best improvement in the objective value. The sample size dictates the number of scenarios you will consider. The larger this value, the better the accuracy (lower optimality gap) at the expense of larger computing times. n is set to a small number such that $n << N$, in which N is defined as the largest sample size for which the stochastic model is tractable.
Step 2.	Generate M independent samples — each of size n — and solve the two stage stochastic problem for each sample.
Step 3.	Compute the mean and the variance of the results obtained in step 2. The average objective value is used as a lower bound for the stochastic problem.
Step 4.	Solve the stochastic model with N scenarios to find a (close to) optimal solution $\hat{x}$. Use this solution to set the first stage variables of the two-stage stochastic model for the M independent samples. Solve these models and again take the average objective function value now as an upper bound for the stochastic problem.
Step 5.	Compare the lower and upper bound computed in steps 3 and 4, respectively by determining the optimality gap.
Step 6.	If the optimality gap found in step 5 is small enough, you stop. Otherwise, increase the sample size n and return to step 2.

instances are considered (from which 14 R and 27 C instances), which we altered to comply with our problem definition.

More specifically, we added a release and dispatching time (deadline) for each shipment as follows: First, we scale the time horizon to ensure that no dispatching time (deadlines) falls beyond the length of the period T. Then, for each shipment, we randomly select a release time such that the time difference the release time and deadline is at least the transit time given in Hellsten et al. [26].

Furthermore, we also changed the cost structure of the instances to match the difference between the scheduled truck service and the express delivery option in the following way: the express cost per volume unit per arc is set in such a way that if the shipped volume is less than half of the truckload, then transporting the commodity by express mode is cheaper than shipping it by scheduled truck and vice versa. The scheduled truck capacity is fixed to 30 tons (based on long-haul transportation trucks). In Hellsten et al. [26], the fixed costs are given for each arc. To transform these into a fixed cost for a vehicle, we compute the shortest path for each shipment and take the average of the corresponding fixed costs of the respective arcs. For the variable cost, we take the unit flow costs given in the benchmark instances. The express costs, finally, is set in such a way that the following

equality holds:

$$\sum_{(i,j)\in \boldsymbol{A}_k} c_{ij}^E = \frac{c^F + \sum_{(i,j)\in \boldsymbol{A}_k} c_{ij}}{0.5Q^F}$$

Here, $\boldsymbol{A}_k$ is the set of all arcs in the shortest path of shipment k. As a result of this procedure, the costs are instance-dependent.

In all instances, a single period (week) contains 7 consecutive time intervals (days), so $t = \{1, 2, \ldots, 7\}$. The hub capacity is assumed to be 1000, and inventory cost per unit per day is 2.

All instances are solved using CPLEX 12.8 with default parameters on a MACBOOK AIR with an APPLE SILICON M1 CHIP and 16GB of RAM. Computational time is limited to 7200 seconds (2 hours).

6.2 Impact of the number of scenarios on the optimality gap

To test the impact of the number of scenarios on the optimality gap, we conducted a small computational experiment. We set M (the number of independent replications) equal to 30 and, given our hardware setup, N was found to be around 60 scenarios. We then varied the sample size n from 5 to 60. The results are summarized in Figures 1 and 2.

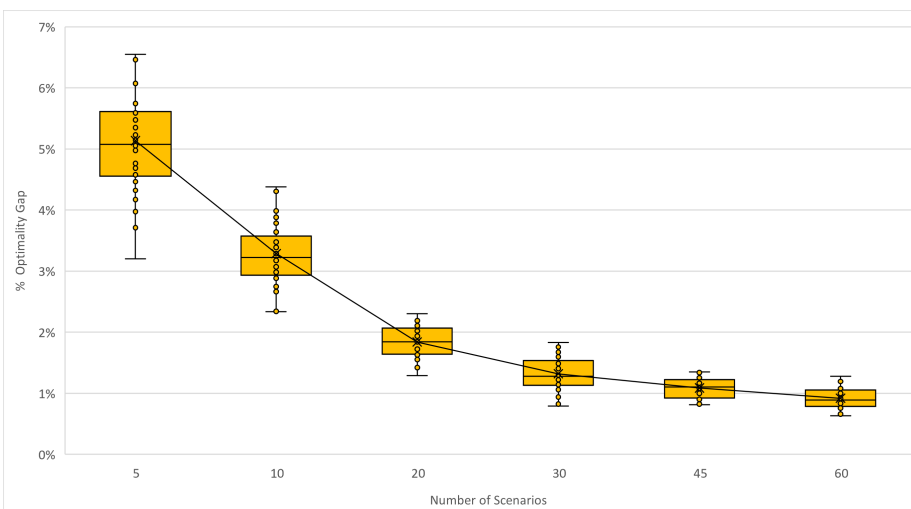


Figure 1: *Gap percentage for different number of scenarios n (with $N = 60$ and $M = 30$).*

Figure 1 shows the average optimal gap value for all the instances for different numbers of scenarios. The figure shows that as the number of scenarios increases, the solutions

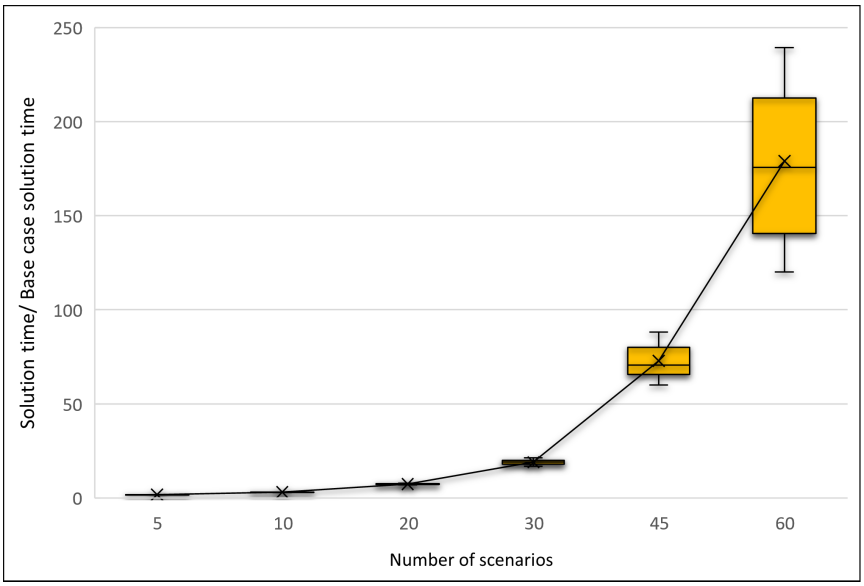


Figure 2: *(Relative) solution time vs number of scenarios*

converge toward the optimal value, meaning that larger number of scenarios result in a lower optimality gap. However, as shown in Figure 2, larger sample sizes lead to a significant increase in solution time. The value on the y-axis represents the average solution time for all the instances (stochastic variant), divided by the objective function of the deterministic model (with only one scenario).

Based on this experiment, we conclude for our experiments that solving with up to 30 scenarios is sufficient to obtain close-to-optimal solutions (1.5% gap on average).

6.3 Results for the deterministic single-period problem variant

6.3.1 Performance analysis of the branch-and-price algorithm

We analyse the performance of the branch-and-price algorithm on the different instances. A detailed overview of the results is presented in Tables 6 and 7.

Each instance is characterized by the number of hubs (nodes in the network), the number of arcs, and the number of shipments, denoted by $|\boldsymbol{V}|$, $|\boldsymbol{A}|$ and $|\boldsymbol{K}|$, respectively.

Next to the objective function value, we report on the number of columns added to the model (**#col**), the number of nodes in the branch-and-price tree (**#nodes**), the optimality gap (**Gap(%)**) and the computation time (**Time (s)**). The optimality gap is computed as follows:

$$\text{Optimality Gap (\%)} = \frac{\text{OptValue} - \text{Lower bound}}{\text{OptValue}} * 100$$

Table 6: *Results for the branch-and-price algorithm on the deterministic single-period problem variant (R instances).*

Inst.	$\lvert V \rvert$	$\lvert A \rvert$	$\lvert K \rvert$	**Obj.**	**#col**	**#nodes**	**Gap (%)**	**Time (s)**
R04.1	10	60	10	10200.95	148	216	0%	182.3
R07.1	10	82	10	10402.83	387	414	0%	489.4
R05.1	10	60	25	24442.39	500	392	0%	448.6
R08.1	10	83	25	22084.3	671	531	0%	232
R09.1	10	83	50	37401.92	498	706	0%	1318.2
						# optimal	5/5	
						Average	0%	534.1
R10.1	20	120	40	32728.53	1100	1822	0%	2193.6
R13.1	20	220	40	31497.12	2672	4263	0%	4058.5
R16.1	20	314	40	32549.78	3049	2544	0%	6244.7
						# optimal	3/3	
						Average	0%	4165.6
R11.1	20	120	100	81768.7	1290	3074	0%	6916.9
R14.1	20	220	100	74208.35	1033	935	2.6%	7200
R17.1	20	318	100	74266.11	2841	5029	4.8%	7200
						# optimal	1/3	
						Average	2.47%	7105.6
R12.1	20	120	200	152209.11	1528	1118	1.7%	7200
R15.1	20	220	200	133881.56	2924	3812	5%	7200
R18.1	20	315	200	132956.13	2002	3684	4%	7200
						# optimal	0/3	
						Average	3.57%	7200
						# optimal	**9/14**	
						Average	**1.29%**	**4148.87**

Based on the results in Tables 6 and 7, we see that the smaller instances (with ≤ 20 hubs and ≤ 50 shipments) are all solved to optimality within the two-hour time limit. Except for instance C36, a 3% optimality gap remains.

For the larger R instances (see Table 6) with more than 50 shipments, 1 instance (out of 6) is still solved to optimality (R11.1). An average optimality gap of 2.47% and 3.57% is found for the R instances with 100 and 200 shipments, respectively. With an overall average optimality gap of 1.29%, we obtain competitive results for the R instances.

For the larger C instances (see Table 7) we see that the branch-and-price model is viable for most instances with 20–30 hubs and 100–200 shipments with optimality gaps below or around 10%. The average results are impacted heavily by the high optimality gaps for instance C48 (108%) and C59 (129%). The two-hour time limit is clearly insufficient to solve instances with ≥ 30 hubs or ≥ 200 shipments to optimality.

Table 7: *Results for the branch-and-price algorithm on the deterministic single-period problem variant (C instances).*

Inst.	$\|V\|$	$\|A\|$	$\|K\|$	**Obj.**	**#col**	**#nodes**	**Gap (%)**	**Time (s)**
C33	20	228	40	148638.88	2343	3574	0%	3752
C35	20	230	40	113698.45	1222	926	0%	4020.7
C36	20	230	40	139436.73	1217	1603	3%	7200
C41	20	288	40	137455.48	3112	5161	0%	3085.8
C42	20	294	40	161605.96	2533	2117	0%	3102.9
C43	20	294	40	137794.95	3205	5008	0%	3371.4
C44	20	294	40	153048.34	1844	1962	0%	3241.4
						# optimal	6/7	
						Average	0.43%	3967.74
C37	20	228	200	12172.44	1170	2409	5%	7200
C38	20	230	200	14931.77	1966	3277	0%	4422.6
C39	20	229	200	14436.66	1233	1493	6.9%	7200
C40	20	228	200	12703.92	1890	2855	4.5%	7200
C45	20	294	200	13589.56	2736	2719	10%	7200
C46	20	292	200	14197.69	2349	3901	13%	7200
C47	20	291	200	14376.92	1721	1355	4.4%	7200
C48	20	291	200	25285.64	2439	3984	108%	7200
						# optimal	1/8	
						Average	18.98%	6852.83
C49	30	518	100	14318.39	6928	13722	9.5%	7200
C50	30	516	100	13897.58	5821	7407	11%	7200
C51	30	519	100	14973.29	3829	3365	7.1%	7200
C52	30	517	100	16721.7	3271	7691	3.8%	7200
C57	30	680	100	13810.24	7419	21502	5.4%	7200
C58	30	680	100	13771.14	5516	7921	5.2%	7200
C59	30	687	100	26641.45	4528	11829	129%	7200
C60	30	686	100	13855.5	7709	22640	5.9%	7200
						# optimal	0/8	
						Average	22.11%	7200
C53	30	520	400	63779.25	4080	10428	169.2%	7200
C54	30	520	400	66407.1	2592	6116	184.1%	7200
C55	30	516	400	67828.75	6982	15208	187.5%	7200
C56	30	518	400	65287.05	4090	10382	178.4%	7200
						# optimal	0/4	
						Average	179.8%	7200
						# optimal	**7/27**	
						Average	**38.92%**	**6259.14**

For the largest instances with 30 hubs and 400 shipments, we even notice very large optimality gaps. From this point onwards, the branch-and-price algorithm becomes really intractable.

6.3.2 Impact of express delivery option

A unique feature of our problem formulation is the possibility to make use of an express delivery service if the costs for establishing all required capacities within the network becomes too high. In this section, we study the impact of including the express option by varying the express cost. To simulate changes in the express cost, we multiply its value by a coefficient which we vary between 0.2 and 2.

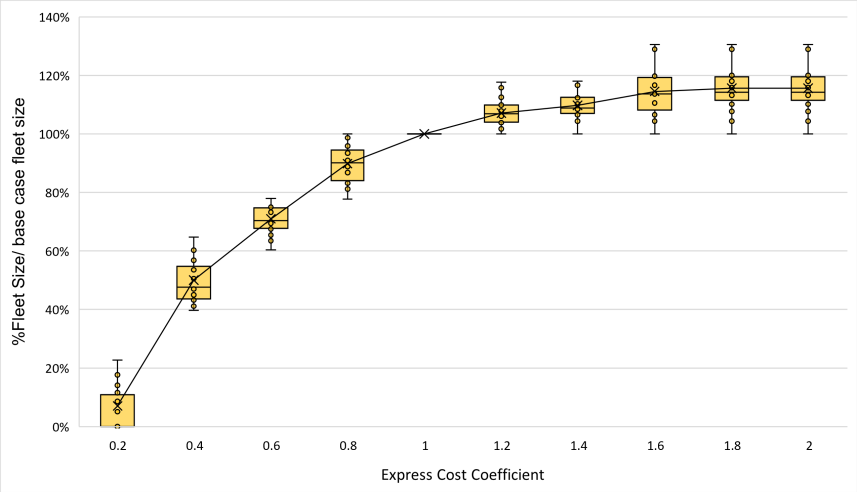


Figure 3: *Number of scheduled trucks for different values of the express cost coefficient.*

Figure 3 reveals that with high express costs, the decision maker is reluctant to use it as installing a scheduled truck service will be cheaper although its capacity will not be used efficiently (see below). As a result, more scheduled trucks will be installed such that all hubs are connected to the scheduled truck network. However, if express costs are low, the volume shipped via express will increase and scheduled trucks will only be established on the connections where loading rates are very high (up to the point where no scheduled trucks will be installed as they are never competitive against the express service).

The relationship between the capacity utilization of the scheduled truck service and the express cost is visualized specifically in Figure 4. In the case express costs are too high, and therefore the service is hardly used, we observe that the unused capacity of the scheduled truck service ranges between 10 and 30 percent (on average slightly below 20%). By making

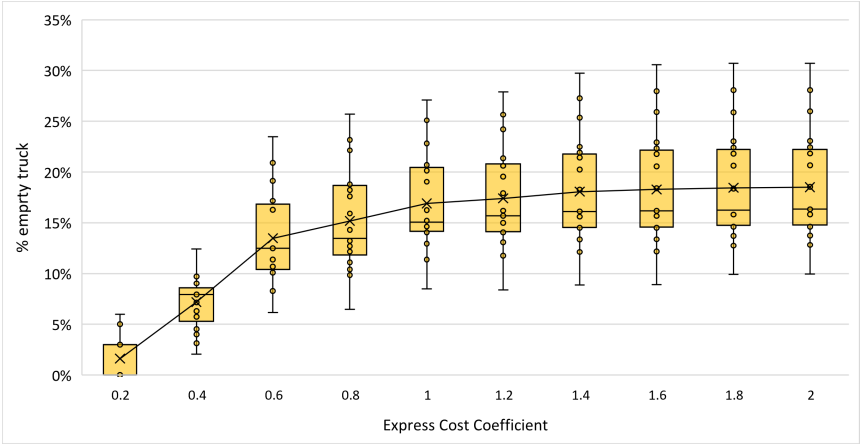


Figure 4: *Capacity utilization of the scheduled truck service, measured as the percentage of unused volume.*

the express service more attractive, inefficient scheduled truck transports are replaced by express delivery up to the point where we see a close to 100% capacity utilization ($\geq 95\%$) of all scheduled trucks in the system.

6.3.3 Impact of hub capacity and inventory holding cost

Another unique feature of our model is the consideration of capacity restrictions in the hubs. We expect that the more we restrict the capacity of the hubs, the higher the operational cost will be as it is more likely that shipments will have to deviate from their shortest / cheapest route from source to destination to avoid capacity violations in the hubs.

For each instance, we first determine the *maximum hub capacity* Q^V_{MAX}. This is the minimum capacity for which the capacity constraints become non-binding (i.e., capacity is no longer a constraint in our optimization model and the solution matches the solution with infinite capacity in the hubs). We now run different simulation experiments in which we set the hub capacity equal to a percentage of the maximum hub capacity.

Figure 5 visualises the relationship between Q^V_{MAX} and the total network cost. The base line is given by the scenario with infinite capacity. The more we restrict the hub capacity, the higher the total network cost. We see that the total cost increases slightly, with an average cost increase of around 10% if only one fifth of the non-binding hub capacity is available in the network.

Further analysis reveals that this increase is mainly due to an increase in fleet size for the scheduled truck service and a slightly higher utilization of the express delivery option (see Figure 6). The reason is twofold. First, the lack of capacity requires shipments

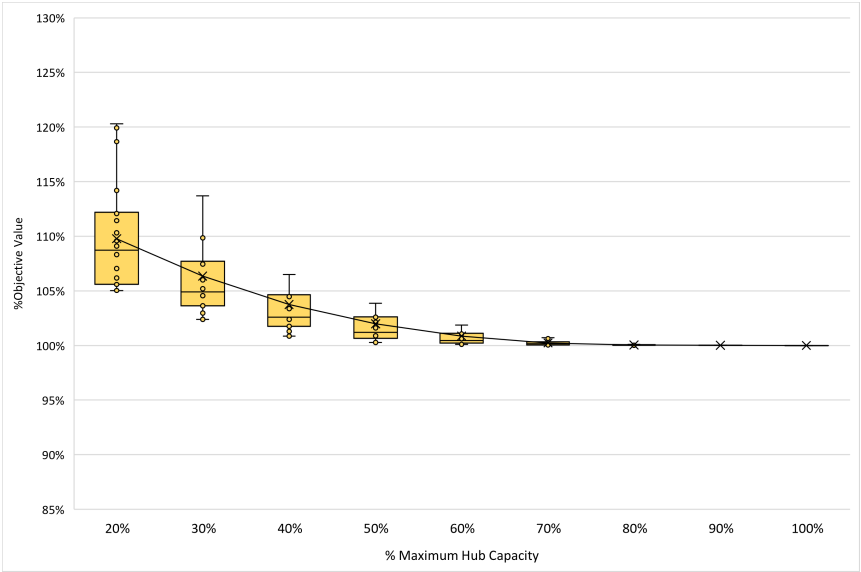


Figure 5: *Total network cost as a function of the hub capacity, measured as a percentage of the maximum hub capacity* Q^V_{MAX}.

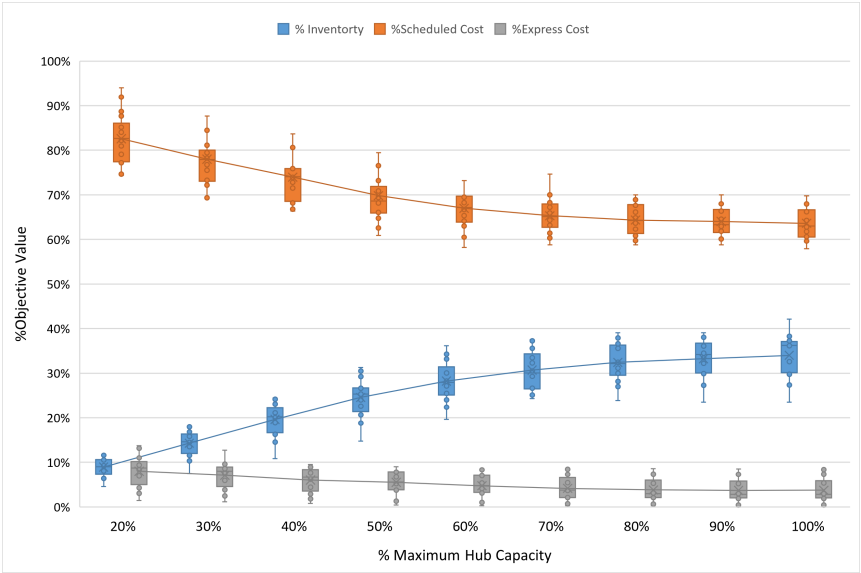


Figure 6: *Different cost factors as a function of the hub capacity, measured as a percentage of the maximum hub capacity* Q^V_{MAX}.

to deviate from their shortest path more often. To accommodate these detours, more capacity is required in the scheduled truck service. Second, these detours increase the cost of a shipment when shipped via the scheduled truck service. Consequently, the express delivery option becomes more attractive to cover certain connections.

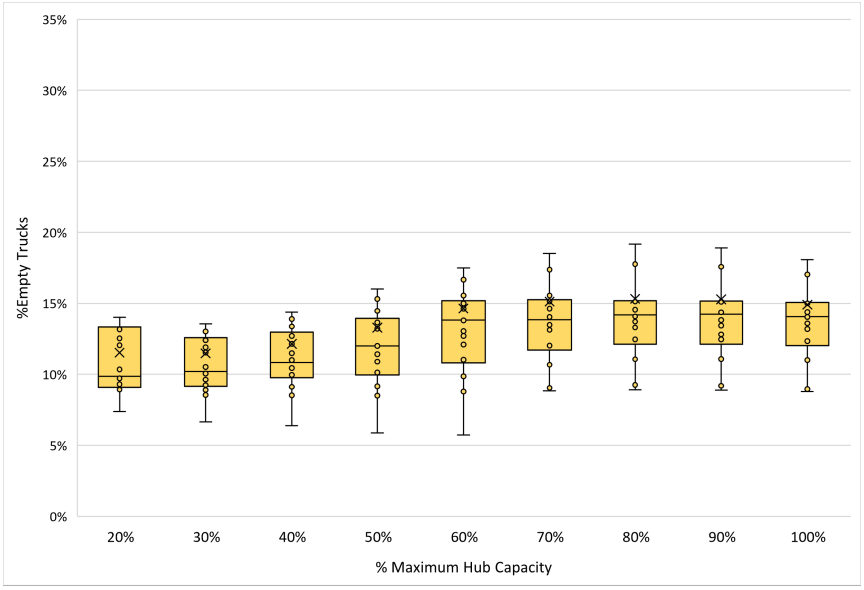


Figure 7: *Vehicle utilization of the scheduled truck service as a function of the hub capacity, measured as a percentage of the maximum hub capacity* Q^V_{MAX}.

The fact that the additional truck capacity installed when the hub capacities are very restrictive is mainly used to accommodate additional transport operations is also corroborated by the relationship between the capacity utilization of the scheduled trucks (measured as the percentage of unused volume) and the maximum hub capacity Q^V_{MAX} given in Figure 7. The Figures shows that despite the increase in fleet size, the vehicle utilization increases slightly, from around 85% to close to 90%. This shows that due to limited hub capacities, it becomes more attractive to have the shipments 'stored' during transport.

Similar conclusions are found when increasing the inventory holding costs. For increasing values of the holding cost, keeping inventory in the hubs becomes less attractive and more costly. As such, the same decision will be made as when inventory capacity is restricted by the model. We prefer keeping shipments moving on the road by installing a larger fleet of scheduled trucks to bridge the gap between their release time and dispatching time and are willing to accept express deliveries from source to destination more often as no intermediate inventory costs occur then.

6.4 Results for the stochastic multi-period problem variant

6.4.1 Impact of demand variance

To generate the instances, the stochastic demand for shipment k under scenario s was generated based upon a normal distribution as follows,

$$q_{ks} = N(q_k, \alpha q_k)$$

in which q_k represent the average demand of shipment k. The standard deviation is defined as a proportion α of the mean value. In this section, we will vary the variability in the data by changing the value for α within the interval $[0, 0.5]$.

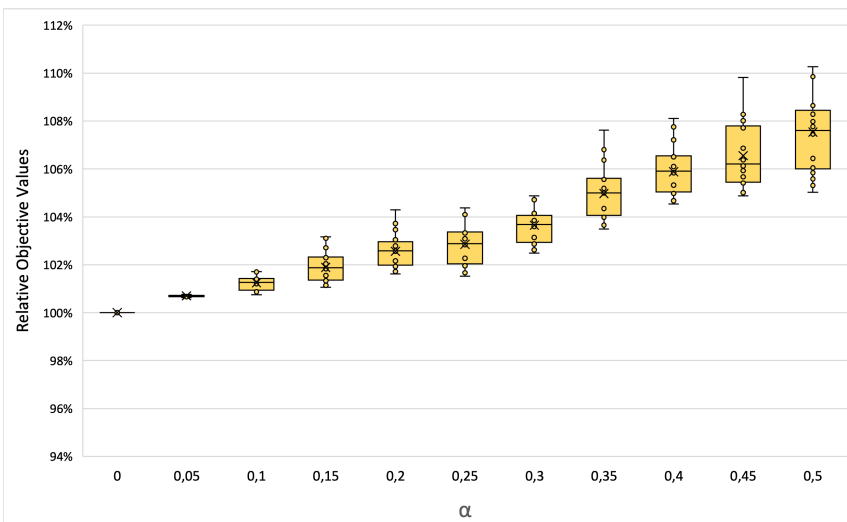
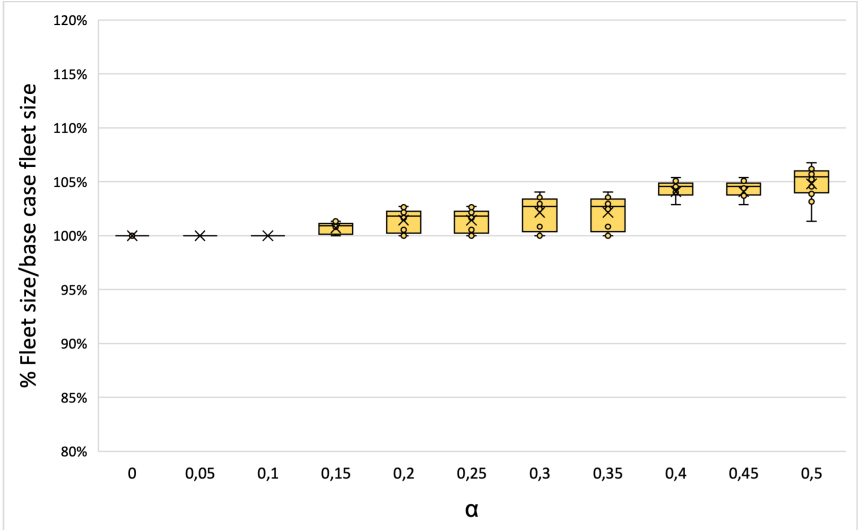


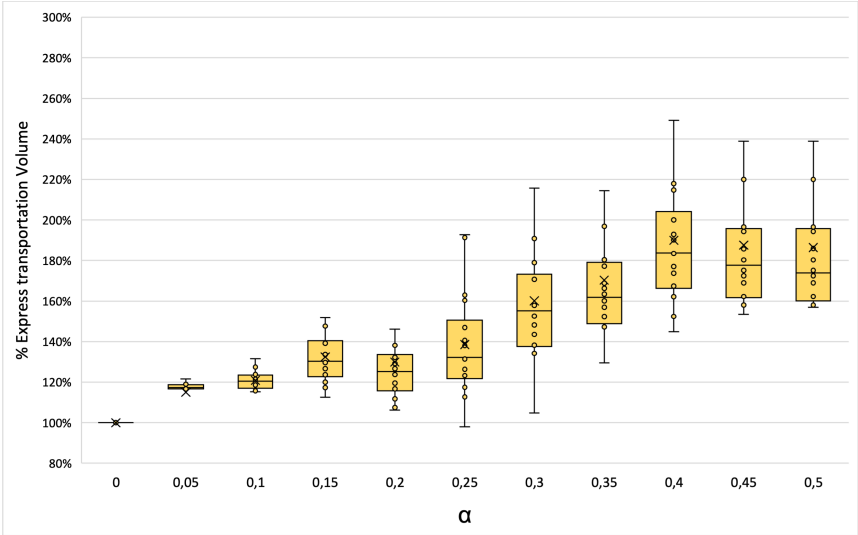
Figure 8: *Total network cost as a function of the standard deviation of the demand, denoted by α.*

The relationship between the total network cost and the value of α is visualized in Figure 8. When α equals zero — our baseline scenario— , there is no variability in the demand (i.e., there is only one scenario with the demand for each shipment equal to q_k). As expected, the total network cost grows for increasing values of α, but the increase remains relatively small (with up to 10% cost increase on average for α equal to 0.5).

Investigating the relationship between α and the network configuration, we see that the demand uncertainty mainly impacts the need for express delivery. The volume shipped via express delivery increases fast, even for small values of α. It is nice to see that our simulation results align with the original motivation for considering express delivery as an alternative transport mode. Whereas the scheduled truck service provides a baseline



(a) *Optimal fleet size under different standard deviations*



(b) *Expected express transportation volume under different standard deviations*

Figure 9: *Commodity variance analysis for Canad benchmark cases*

capacity on the links of the network where a considerable flow is guaranteed, express delivery offers the flexibility to absorb the variations above this baseline capacity.

6.4.2 The Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS)

expected value: You run the model only once, for an average scenario. -¿deterministic solution perfect information: Compute the model for each individual scenario and obtain the corresponding objective function value. Then take the average over all objective function values found. Recourse = benchmark. two-stage stochastic model

In this section, we compare the performance of our two-stage stochastic model with decision making based upon expected values or under perfect information. In decision making based upon expected value, we solve the model only once for a single (average) scenario, i.e., we would consider solely the scenario in which

$$q_k = \frac{1}{|\mathbf{\Omega}|} \sum_{s \in \mathbf{\Omega}} q_{ks}.$$

To solve the model under perfect information, we first compute the total network cost of each individual scenario. Then, we take the average over all objective function values found as the expected cost under perfect information (recall that we assume each scenario to occur with the same probability).

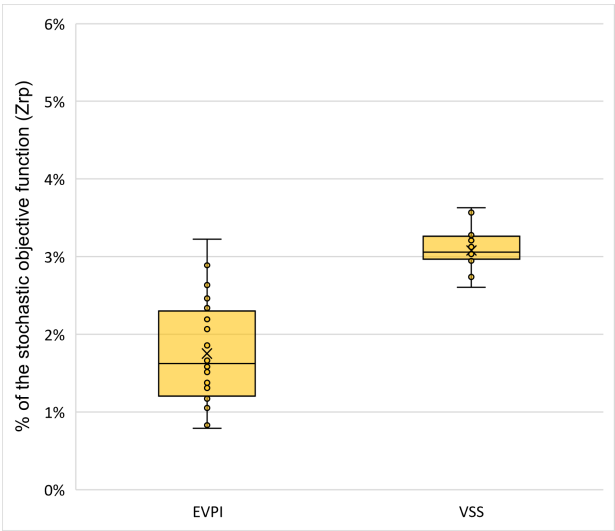


Figure 10: *VSS and EVPI Percentage with respect to the* Z_{RP}

Figure 10 summarizes our main results. Here, we plot the *expected value under perfect*

information (EVPI) as well as the *value of stochastic solution* (VSS). Both measures are computed relative to the total network cost obtained when applying the two-stochastic model. The value of perfect information relates to the decrease in total network cost once the decision maker no longer faces uncertainty on the demand for each shipment. In other words, having perfect information will lead to a decrease of 1.8% on average in total network cost.

The value of stochastic solution compares decision making under expected value with the stochastic model. In other words, if the decision maker would ignore the demand variability and solely optimize for the expected values for q_k, the average cost would be around 3.2% higher.

7 Conclusion

Motivated by a real case study from the industry, We presented a periodic multi-commodity service network design problem to model the decisions of a 3PL when managing all logistics operations of a supply network using both scheduled truck services (representing long-term agreements with carriers to provide regular capacities on specific network links) and ad-hoc express delivery. Next to the multi-modal approach, we also include the time dimension, hub capacities and account for stochastic demand.

Our computational experiments show that our proposed exact model performs very well. For the small instances with up to 20 hubs and 50 commodities, the model finds the optimal solution within the time limit. For the average instances with 20-30 hubs and up to 100 commodities, our exact model gives very promising solutions with the maximum of 10% optimal gap, and for the large instances (30 hubs and more than 100 commodities) the optimal gap within the time limit is 20% on average.

Adding the option of express delivery as an alternative to the scheduled truck service leads to a lower network cost. This is due to the fact that express deliveries can replace low-volume connections where installing a fixed capacity is not cost-effective. This is similar to passenger transport, were bus services are replace by on-demand bus lines or taxi rides in rural areas with very low demand.

Furthermore, we show that limiting the available hub capacity increases the fleet size for the scheduled truck service and express delivery. At the same time, it leads to better vehicle utilization for the scheduled truck service. Similar results were found when the inventory holding cost is increase. In both scenarios, we observe that the available scheduled truck

capacity is used as inventory capacity during transport to bridge the gap between release time and dispatching time. Moreover, adding the express delivery improves the objective function and decreases the total costs in comparison to the case where there exists no express delivery and all the commodities are transported by the scheduled trucks.

We extend the deterministic single-period model to a stochastic multi-period variant in which the variation in demand over the different periods in included explicitly in the model. In contrast to a deterministic case, based on the average demand solely, the inclusion of stochastic demand leads to a 3.2% network cost reduction on average.

As we present an exact solution method, based upon the principles of branch-and-price, we see that the model lacks some scalability towards large (potentially more realistic) instances. The development of an efficient heuristic and sample strategy that allows good convergence to the optimal solution is a short computation time would be valuable.

Further promising extensions of the model left for further research are the addition of a delivery time window (instead of a fixed dispatching time), a heterogeneous fleet for the scheduled truck service (e.g., large trailers vs small(er) vans), social constraints related to the drivers (e.g., breaks, route duration, etc.), and additional sources of uncertainty (e.g. stochastic travel times,etc.). Another itinerary for further research could be the modelling of the pricing decision of the scheduled truck services between the 3PL and the carrier or set of potential carriers each covering a certain part of the envisaged network.

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