

*MERIT-Infonomics Research Memorandum series*

*Monopolistic Competition, Search  
Unemployment, and  
Macroeconomics*

*Thomas Zieseemer*

*2002-019 (revised version of 2001-006)*



*MERIT – Maastricht Economic Research  
Institute on Innovation and Technology*

PO Box 616  
6200 MD Maastricht  
The Netherlands  
T: +31 43 3883875  
F: +31 43 3884905

<http://meritbbs.unimaas.nl>  
e-mail: [secr-merit@merit.unimaas.nl](mailto:secr-merit@merit.unimaas.nl)



*International Institute of Infonomics*

c/o Maastricht University  
PO Box 616  
6200 MD Maastricht  
The Netherlands  
T: +31 43 388 3875  
F: +31 45 388 4905

<http://www.infonomics.nl>  
e-mail: [secr@infonomics.nl](mailto:secr@infonomics.nl)

***MONOPOLISTIC COMPETITION, SEARCH UNEMPLOYMENT, AND  
MACROECONOMICS \****

Thomas Zieseemer, Department of Economics and MERIT, University of Maastricht, P.O. Box 616,  
NL-6200 MD Maastricht. Phone: +31-43-3883872. Fax: +31-43-3884905. E-mail:  
T.Zieseemer@algec.unimaas.nl.

*Revised version July 2002.*

*Abstract.* The monopolistic competition model of Dixit and Stiglitz for the goods market and the search unemployment model of Pissarides are combined. The Pissarides part loses its Walrasian goods market and the Dixit-Stiglitz part loses its Walrasian labour market. Pissarides' results now also depend on the degree of competition and in the Dixit-Stiglitz part the size and number of firms as well as aggregate output now also depend on aggregate hiring costs, tightness and unemployment, and real wages are not fixed. Some partial results of comparative static properties of the original models survive. New results concerning the effects of changes in labour (goods) market parameters on goods markets (the labour market) variables are obtained and compared to the literature on macroeconomic theory, empirical results and policy issues. *Keywords:* *Monopolistic competition, unemployment.* *JEL-code:* *E13, E 24.*

# ***MONOPOLISTIC COMPETITION AND SEARCH UNEMPLOYMENT, AND MACROECONOMICS***

*Abstract.* The monopolistic competition model of Dixit and Stiglitz for the goods market and the search unemployment model of Pissarides are combined. The Pissarides part loses its Walrasian goods market and the Dixit-Stiglitz part loses its Walrasian labour market. Pissarides' results now also depend on the degree of competition and in the Dixit-Stiglitz part the size and number of firms as well as aggregate output now also depend on aggregate hiring costs, tightness and unemployment, and real wages are not fixed. Some partial results of comparative static properties of the original models survive. New results concerning the effects of changes in labour (goods) market parameters on goods markets (the labour market) variables are obtained and compared to the literature on macroeconomic theory, empirical results and policy issues.

## **I. Introduction**

In his theory of search unemployment Pissarides (1990) links the essentials of his theory to the neo-classical production function and the neo-classical growth model. In this paper we link Pissarides' theory to the monopolistic competition model of Dixit and Stiglitz (1977).

Many economists believe that fixed costs and product differentiation are very attractive features of the Dixit-Stiglitz model, which have made it a useful tool in new trade and growth theory, macroeconomics and regional economics. In particular, in economic policy debates the number and size of firms is often related to the

unemployment issue. However, the number and size of firms are not determined in models using the neo-classical production function, and search unemployment does not appear in the Dixit-Stiglitz model. As many economists argue, the search theory of unemployment has received much empirical support. It seems to be a worthwhile effort to link Pissarides' theory to the monopolistic competition model, thus joining two of the workhorse models in economic theory.

Pissarides' model – after dropping capital - can be seen as a special case of ours where the fixed costs of a firm are zero and marginal revenue is unity. The major new results of our model compared to those of Pissarides (1999, 2000) are the following:

- (i) Marginal costs now also include expected hiring costs, which link the goods and the labour market. Real wages are not fixed. These are two major differences in comparison to the Dixit-Stiglitz model.
- (ii) All changes of labour market parameters, which enhance the rate of unemployment, now also have an impact on the size of the firm and also on the number of firms.
- (iii) We add comparative static results for taxes (financing unemployment benefits) and *net* wages: Increases in bargaining power and unemployment benefits increase gross wages as usual but also the taxes necessary to finance unemployment benefits because unemployment gets higher. Net wages may fall if increases in total unemployment benefits from this latter effect are larger than increases of gross wages.
- (iv) Marginal labour productivity in Pissarides' model is replaced by marginal revenue times the marginal product of labour, because marginal revenue is no longer equal to the price of goods, which is the numéraire in both models (Marginal revenue is equal to the parameter of the constant-

elasticity-of-substitution utility function and its deviation from unity indicates the strength of monopoly power).

- (v) Larger monopoly power yields a lower tightness ratio, lower hiring costs, more unemployment, higher unemployment taxes, lower gross and net wages. The impact on the number of firms, which is positive in the Dixit-Stiglitz model because increased love-of-variety induces a smaller firm size, is now reduced by the higher unemployment and lower hiring costs.
- (vi) A higher marginal product of labour – unlike the Dixit-Stiglitz model where it has no impact on the number of firms – now has the direct effect of decreasing the number of firms because marginal hiring costs are decreased, but increases the number of firms via a decreased probability of filling a vacancy, and because it yields a lower rate of unemployment.
- (vii) Changes in the fixed costs parameter have no impact on the labour market and aggregate output but only on the size and number of firms.

Other results are presented as propositions below and are summarised in Tables 1 and 2 and section 6. Section 5 compares the results of the model to those of some other macroeconomic models. Which of the results are better – ours, or those in the literature - is an empirical question that goes beyond the scope of this paper.

## **II. The model<sup>ii</sup>**

### *Trade in the labour market*

From the Pissarides (1990) model we use the matching function  $m L = m(uL, vL)$ , where  $L$  is the total number of employed and unemployed workers,  $u$  is the

unemployment rate,  $v$  is the rate of vacancies and  $mL$  is the number of matches produced by this function. The function is assumed to be increasing in both arguments, concave and linearly homogenous.<sup>iii</sup> Defining labour market tightness as  $\theta \equiv v/u$ , division of the matching function by  $vL$  yields  $q(\theta) = m(u/v, 1)$  as the probability of a firm to find a worker for a vacancy with  $q' \equiv \partial q / \partial \theta < 0$ , and  $\theta q(\theta) = m/u = m(1, v/u)$  as the probability of an unemployed worker to find a job. A shock is a percentage rate  $s$  at which  $(1-u)L$  employed workers loose their job by assumption in every period. Therefore  $s(1-u)L$  workers go from a job into unemployment every period. On the other hand  $\theta q(\theta)uL$  unemployed workers expect to find a job each period. A labour market steady state equilibrium is defined as a situation where the numbers of workers going into and out of unemployment are equal and expectations turn out to be true, i.e.  $s(1-u)L = \theta q(\theta)uL$ . Solving this equation for  $u$  yields the Beveridge or UV curve (lower indices referring to variables indicate partial derivatives):

$$u = \frac{s}{s + \theta q(\theta)}, u_s > 0, u_\theta < 0 \quad (1)$$

Multiplying equation (1) by  $\theta$  yields an equation for the vacancy rate because  $u\theta = uv/u = v$ :

$$v = \frac{s}{s/\theta + q}, v_s > 0, v_\theta > 0 \quad (1')$$

Equation (1) and (1') are drawn in the lower right quadrant of figure 1.

*Government and unemployment benefits*

The government is assumed to pay unemployment benefits  $z$  to each unemployed worker. Total expenditures of the government for unemployment benefits are  $zuL$ . It will turn out that the incentives are ultimately unchanged if both the employed and the unemployed pay a tax or unemployment premium to finance the unemployment benefits. Revenue then is  $tL$ . From the balanced budget assumption we make, it follows that  $tL = zuL$  and therefore  $t = zu$ . Workers therefore receive  $w-t = w - zu$  and unemployed benefits are  $z - t = z - zu$ . As  $z$  is considered to be a policy variable, the budget equation determines the value of  $t$ , whereas  $u$  is determined in the general equilibrium part of the model below. Introducing the taxes explicitly allows us to calculate the effects of comparative static changes of all parameter changes on net wages in proposition seven below besides making the system of budgets explicitly consistent.<sup>iv</sup>

### *Households and workers*

Households are assumed to have love-of-variety preferences of the CES type,

$$y = \left[ \int_{i=0}^n c_i^\alpha di \right]^{\frac{1}{\alpha}}, \text{ with } 0 < \alpha < 1, \text{ on a continuum of goods with index } i, \text{ ranging from}$$

zero to  $n$ , the (integral measure of) the number of firms.<sup>v</sup> The market for goods is assumed to have no search frictions. It is well known that this specification of preferences leads to a constant elasticity of the inverse demand function,  $\alpha-1$ , and to relative demand of goods independent of the income earned by employed or unemployed persons. If the temporary utility function is discounted and integrated we may get an inter-temporal utility function for which it is well known from endogenous growth theory or the theory of optimal growth that, in the absence of a rate of permanent productivity growth, the steady-state value of consumption will be

stationary and the interest rate will equal the discount rate. This seems to be the shortest way to determine the interest rate.<sup>vi</sup> The problem of a household with an infinite time horizon then is to choose the values of  $c_i$  and of savings such that the

choice maximizes  $\int_{\tau=0}^{\infty} e^{-\rho\tau} \left[ \int_{i=0}^n c_i^\alpha di \right]^{1/\alpha} d\tau$  subject to the budget constraint

$$\dot{W} = I - \int_{i=0}^n p_i c_i di + rW \text{ and } W(0) = W_0 \geq 0, \text{ where } W \text{ is current wealth, a dot indicates a}$$

time derivative,  $r$  is the interest rate,  $p_i$  is the price of good  $i$ , and current non-interest income is  $I = (1-u)w + uz - t$ . The assumption here is that a household gets the wage  $w$  with probability  $(1-u)$  and is unemployed and gets benefits  $z$  with probability  $u$ , but pays taxes  $t$  in both cases. As the utility function exhibits risk neutrality there are no complications from the uncertainty. A second interpretation could be that every household is representative in the sense that the same share  $1-u$  ( $u$ ) of its members is (un-) employed as in the total labour force of the economy.<sup>vii</sup> In the first interpretation the (ex-post) employed workers lend money to (ex-post) unemployed workers allowing the latter to smooth consumption under the assumption of a perfect capital market. In the second interpretation this happens within the households and lending among identical households must be zero in equilibrium. In the appendix we show that the inverse price elasticity is  $\alpha - 1$  and the interest rate in a steady state with a constant number of firms is  $r = \rho$ . All results henceforth are steady-state results.

The present value, discounted at rate  $r$ , of the expected income stream of an unemployed and an employed worker,  $U$  and  $E$  respectively, are:  $U = [z - zu + \theta q(\theta)(E-U)]/r$  and  $E = [w - zu + s(U-E)]/r$ .  $E-U$  is the income difference an unemployed worker can gain by finding a job with probability  $\theta q(\theta)$ .  $U-E$  is the

corresponding loss by a worker from losing his job with probability  $s$ . These two equations can be solved for  $E$  and  $U$  explicitly:

$$U = \frac{(r+s)z + \theta q(\theta)w}{r+s+\theta q(\theta)} / r - zu / r, \quad E = \frac{sz + [r + \theta q(\theta)]w}{r+s+\theta q(\theta)} / r - zu / r$$

### *Firms*

The present-discounted value of a vacancy is  $V = [-\gamma + q(\theta)(J-V)]/r$ . It consists of the hiring costs  $\gamma$  and the net return of transferring the vacancy  $V$  into a job with value  $J$  expected with probability  $q(\theta)$ .  $r$  is the discount rate. As the value of the vacancy is zero in equilibrium, we get  $J = \gamma / q(\theta)$ : the value of a job is equal to the vacant job costs  $\gamma$  multiplied by the expected duration of the vacancy. When considering the firms' hiring costs we must consider that the occupied job may be separated from the worker again with probability  $s$ . The current value of the expected value of a job therefore is  $(r+s)J = (r+s)\gamma/q(\theta)$ . These are labour costs that are added to the real wage received by the worker. Labour costs then equal  $w + (r+s)\gamma/q(\theta)$ . Pissarides (1990) links the above to the neo-classical production function.<sup>viii</sup> Here we link it to the model by Dixit and Stiglitz (1977).

Technologies are defined by  $l_i = f + a x_i$ , with  $a, f > 0$ . The left side represents demand for labour to produce good  $i$ ,  $f$  is the fixed part and  $a x_i$  is the variable part of labour demand, where  $x_i$  is the output of a firm. As all goods are assumed to be identical in the utility function and in the production technology, their prices and quantities will be the same.

Total labour demand is  $nl_i = n(f + a x_i)$ . Equating this to the employment  $(1-u)L$  yields  $(1-u)L = nl_i = n(f + a x_i)$ . Solving the latter equation we find the rate of unemployment linked to the number of firms as:

$$u = (L - nl_i) / L = [L - n(f + ax_i)] / L \quad (2)$$

There is a partial negative relation between the rate of unemployment and the number of firms: The larger the number of firms, the lower the unemployment rate (*ceteris paribus*), or the lower the unemployment rate the more firms can be in the market. The interesting question to be answered below is whether a higher number of firms induces a lower rate of unemployment or the causality goes the other way around.

Present-discounted value of the firm's expected profits<sup>ix</sup>, which has a current value of zero in every period in equilibrium, are defined in nominal terms as:

$$\Pi_i = \int_0^{\infty} e^{-rt} \{ p(x_i)x_i - W(f + a x_i) - p\mathcal{W}_i \} dt \quad (3)$$

$W$  is the nominal wage rate and real hiring costs for vacancies,  $\mathcal{W}_i$ , are made nominal by multiplying their real value with the price. The assumption is that *nominal* hiring costs are given from the labour market; monopoly pricing then has no impact on the value of hiring costs. The firm maximizes profits as defined in equation (3) through choice of the quantity  $x$  and the number of vacancies  $V_i$  using the dynamic concept of the large firm from Pissarides (2000, chap.3). The dynamics comes from the fact that the firm can post a number of  $V_i$  vacancies, which increase employment with

probability  $q(\theta)$  and costs  $p\gamma V_i$ . On the other hand, the firm loses workers  $sl_i$ . The expected change in employment then is  $\dot{l}_i = q(\theta)V_i - sl_i$ . From  $l_i = f + ax_i$  and  $dl_i = a dx_i$  we get the corresponding change in the quantity as

$$\dot{x}_i = q(\theta)V_i / a - s(f / a + x)$$

The current value Hamiltonian for each firm's decision problem then is:

$$H = p(x_i)x - W(f + ax_i) - p\gamma V_i + \lambda[q(\theta)V_i / a - s(f / a + x)]$$

The first-order condition for the number of vacancies determines the value of the co-state variable as marginal hiring costs:

$$\partial H / \partial V_i = -\gamma p + \lambda q(\theta) / a = 0, \text{ or, } \lambda = \gamma p a / q(\theta)$$

The other canonical equation is

$$-\dot{\lambda} = \partial H / \partial x = -\{p'x + p - aW - \lambda s\} = \dot{x} - r\lambda$$

Insertion of  $\lambda$  from the previous first-order condition and setting its change equal to zero in the steady state, and noticing that the price elasticity  $p'x/p = \alpha - 1$  this latter first-order condition yields<sup>x</sup>:

$$p\alpha = a[W + p(r + s)\gamma / q(\theta)] \quad (4)$$

In the steady state the change of employment also must be zero and therefore we get the number of vacancies as a function of the quantity produced:

$$V_i = s(f + ax)/q(\theta) \quad (5)$$

The solution for the quantity and the tightness ratio will be derived below.

### *Wages*

There are two sorts of rents in Pissarides' model: on occupied jobs, indexed  $j$ , employed workers do not have to search and therefore have an income rent of  $E_j - U$  and firms do not have to incur hiring costs and therefore have a rent  $J_j - V$ . Bargaining these rents is assumed to determine real wages. This is done by choosing the real wage by maximising the function  $(E_j - U)^\beta (J_j - V)^{1-\beta}$  with  $\beta$  as the bargaining power of workers and  $1-\beta$  that of firms,  $V=0$ ,  $E_j = [w_j - zu + sU]/(r+s)$ ,  $U$  according to the explicit solution given above, and  $J_j = \gamma/q(\theta) = (\alpha/a - w_j)/(r+s)$  where the last equality stems from the solution of (4) for expected hiring costs.  $E$ ,  $U$  and  $V$  are as in Pissarides (1990). The value for  $J$  differs from Pissarides' model because we have replaced the neoclassical production function by elements of the Dixit-Stiglitz model: as we have increasing returns on the firm level, the value of an occupied job is the present-discounted value not of the *average* but rather the *marginal* profit from a worker gross of hiring costs. The result of the maximisation with respect to the real wage in its general form is identical to that of Pissarides in that workers get a share  $\beta$  of the sum of the rents to be distributed:  $E_j - U = \beta(E_j - U + J_j - V)$ . Insertion of the values for  $E_j$ ,  $U$ ,  $J_j$  and  $V$  yields the solution for real wages.<sup>xi</sup>

$$w_j = (1 - \beta)(rU + zu) + \beta \frac{\alpha}{a}$$

Insertion of  $E_j - U = \beta(J_j - V)/(1 - \beta)$  from the general form of the bargaining result and  $J = \gamma q(\theta)$  into  $rU = [z - zu + \theta q(\theta)(E - U)]$  yields  $rU = z - zu + \theta \beta \gamma / (1 - \beta)$ . Insertion of  $rU$  into the above wage result yields<sup>xii</sup>

$$w_j = (1 - \beta)z + \beta \left( \frac{\alpha}{a} + \theta \gamma \right) \quad (6)$$

The last term indicates that workers participate in the hiring costs saved on occupied jobs compared to vacancies. The second but last term is net revenue per worker as in Pissarides (1990) - where the output-per-worker term is  $f(k) - (r + \delta)k$  -, but here without capital cost as in the Dixit-Stiglitz model. The unemployment tax,  $zu$ , has dropped out only in the very last step of the calculation yielding (6). This shows that the Pissarides approach is consistent with an explicit financing scheme for the unemployment benefit if both unemployed and employed workers have the same reduction of their gross payments  $w$  and  $z$  respectively. Then the difference of going from a status of unemployed to employed workers is unchanged and incentives of benefits are exactly as in Pissarides' model. This equation is drawn as the BB curve in the upper right quadrant of Figure 1.

This model is kept as simple as the basic workhorse models were. We resist the temptation to endogenize the bargaining power parameter, or distinguish between the love-of-variety and the price elasticity parameters, or endogenize the mark-up. These and other extensions can be taken on board when applications require doing so.

### III. The equilibrium solution: Existence and uniqueness

Equations (1)-(6) – with the index  $j$  dropped because we consider the general equilibrium now - determine the six variables of the model when goods produced serve as numéraire ( $p=1$ ):  $V_i$ ,  $u$ ,  $n$ ,  $x$ ,  $\theta$  and  $w$ . Inserting (5) for the number of vacancies into (3), equations (3), (4) and (6) can be solved for  $x$ ,  $w$  and  $\theta$ ; then (1) determines  $u$  and (2) determines  $n$ . Insertion of wages per worker from (4) and the number of vacancies into the current profit function contained in (3) allows to solve for the zero-profit<sup>xiii</sup>- equilibrium quantity:

$$x = \frac{\left[\frac{\alpha}{a} - \frac{r\gamma}{q(\theta)}\right]f}{1 - \alpha + \frac{ar\gamma}{q(\theta)}}, \partial x / \partial \theta < 0 \quad (7)$$

Using equation (7) we can calculate the labour demand per firm as

$$l_i = f + a x_i = \frac{f}{1 - \alpha + \frac{ar\gamma}{q(\theta)}}, \partial l_i / \partial \theta < 0$$

Both, output and labour demand depend negatively on hiring costs via the probability  $q(\theta)$  because an increase in the tightness ratio increases expected marginal hiring costs. Each firm knows that it will be separated from the worker with probability  $s$ , resulting in  $sl_i$  separations, and can fill a vacancy with probability  $q(\theta)$ . A flow equilibrium of the firm - allowing the firm to keep the labour demand, which allows producing the profit maximizing output level - then requires that expected separations

equal expected hiring,  $sl_i = q(\theta)V_i$ . The number of vacancies the firm will post to satisfy its labour demand,  $l_i$ , then is calculated from this equilibrium flow condition as<sup>xiv</sup>

$$V_i = f[s/q(\theta)]/[1-\alpha + ar\gamma/q(\theta)] \quad (8)$$

The equilibrium output quantity of the model is directly dependent of the labour-market parameters  $r$  and  $\gamma$  and indirectly on all those having an impact on the tightness ratio stemming from Pissarides' part of the model (unemployment benefit  $z$ , hiring costs  $\gamma$ , unemployment rate  $u$ , vacancies  $v$ , separation rate  $s$ , power parameter  $\beta$  and interest  $r$ ; see below). Clearly, this result is due to the fact that the firm part of the Dixit-Stiglitz model is changed by adding hiring costs (per vacancies actually filled) to the wage rate: these terms, the wage and the expected hiring costs constitute marginal costs and therefore have an impact on the quantity, the employment and the vacancies posted.

Using (7) to replace  $x$  in (2), we get:

$$n = L(1-u) \frac{1-\alpha + ar\gamma/q(\theta)}{f} \quad (2')$$

This is a function  $n(\theta)$ . A higher tightness ratio increases expected hiring costs, decreases the firm size and the unemployment rate and therefore increases the number of firms. Aggregate output can be found by multiplying the solutions for the output and the number of firms, equations (7) and (2'):

$$nx = (1-u)L[\alpha/a - r\gamma/q(\theta)]$$

Although there are internal economies of scale on the firm level, aggregate output has constant returns in the size of the economy  $L$ , and employment  $L(1-u)$  for a given tightness ratio. An increase in the marginal value product of labour,  $d\alpha/a < 0$ , increases  $nx$  directly because it appears in the numerator but will be shown below to have an indirect impact on the tightness ratio, hiring costs and the unemployment rate. Using the result for the number of firms from equation (2'), we can calculate the *total* number of vacancies from equation (8) as  $vL = n V_i = n(s/q)l_i = (s/q)L(1-u)$ . Cancelling  $L$  and dividing by  $(1-u)$  yields  $v/(1-u) = s/q = \theta u/(1-u)$ . This equation corresponds to equation (3.14) in Pissarides and can be re-transformed into equation (1) by solving for  $u$ .

**Proposition 1:** *The equilibrium quantity and employment of the firm, the total number of firms and vacancies as well as aggregate output of the modified Dixit-Stiglitz model are all dependent on marginal hiring costs, which link it to the labour market variables via the tightness ratio.*

To solve the system the next steps serve to get a second equation – besides (6) – relating the real wage and the tightness ratio. Dividing (4) by the price and solving for the real wage yields:

$$w = \frac{\alpha}{a} - \frac{r+s}{q(\theta)} \gamma \quad (4')$$

Larger hiring costs  $(r+s)\gamma/q(\theta)$  imply lower wages according to (4') as in Pissarides' model when interest is given. Here the model resembles Pissarides' because the zero-profit condition in his model implies constant labour costs as long as  $r = f'(k) - \delta$  and therefore  $k$  are constant. By implication wages  $w$  always move in the opposite direction of hiring costs,  $(r+s)\gamma/q(\theta)$ , in Pissarides' model and in ours.<sup>xv</sup> Equation (4') is drawn as a function  $w(\theta)$  in the upper right quadrant of figure 1, indicated as the MM curve. The MM curve is rotated downward by increases in  $r$ ,  $s$  and  $\gamma$ , and shifted downward by decreases of  $\alpha/a$ . It is also drawn in the upper left quadrant of figure 1 with wages as a function of hiring costs.

INSERT FIGURE 1 OVER HERE

The intersection of lines BB and MM determines the wage and the tightness rate in the upper right quadrant, and hiring costs in the upper left quadrant. Given the rate of tightness thus determined, the solution for the rates of unemployment and vacancies can be found in the lower right quadrant.

Equations (4) and (6) are two functions  $w(\theta)$ . Equating (4) and (6) we get<sup>xvi</sup>:

$$\alpha = a \left[ z + \frac{1}{1-\beta} \frac{r+s}{q(\theta)} \gamma + \frac{\beta\gamma\theta}{1-\beta} \right] \quad (9)$$

The left side is marginal revenue and the right side is marginal cost. In figure 2 both functions are drawn. The left side is denoted as MR and the right side as MC in figure 2. MC is increasing in  $\theta$  and may have a negative second derivative in  $\theta$ .<sup>xvii</sup>

INSERT FIGURE 2 OVER HERE

The MC curve starts at  $az$  if  $\theta=0$  and  $\lim_{\theta \rightarrow 0} q = \lim_{\theta \rightarrow 0} m(1/\theta, 1) = \lim_{\theta \rightarrow 0} m(\infty, 1) = \infty$ ; otherwise it starts above  $az$ . As  $\theta$  goes to infinity the MC curve also goes to infinity. Thus, the MC curve either intersects once or not at all. Therefore we have a unique or no equilibrium.

**Proposition 2:** *The existence of a unique equilibrium is guaranteed if  $z < \alpha/a$ . This implies a positive equilibrium value for the tightness ratio  $v/u = \theta$ . The fixed cost parameter  $f$  and the size of the economy,  $L$ , have no impact on the value of  $v/u = \theta$ .*

If, however,  $z \geq \alpha/a$ , the tightness ratio is zero, there are no vacancies and unemployment is 100% according to equation (2). With no output,  $z$  cannot be paid. Therefore this cannot be an equilibrium situation.

#### IV. Comparative static analysis

All comparative static changes could be done using Figure 1. However, this treatment, although preferable in classroom teaching with extensive interpretations, takes more space than the one used below.

The following changes in the determinants of labour costs drive up the MC curve on the RHS of equation (9), but leave the MR curve unchanged:  $d\beta > 0$ ,  $d\gamma > 0$ ,  $dr > 0$ ,  $dz > 0$ ,  $ds > 0$ . This decreases the value of the tightness ratio in figure 2. The UV equation (1), drawn as  $u(\theta)$  in the lower right quadrant of figure 1, shifted to higher  $u$  in case of  $ds > 0$ , then implies that the rate of unemployment goes up. When labour

market parameters are changed in this model, the causality goes from tightness and employment to the number of firms - not the other way around as is often hoped for in justifications for policies based on the impression that having more firms implies getting more employment.<sup>xviii</sup>

For a given  $z$ , all increases of unemployment require more unemployment benefits,  $zuL$ . As all persons pay the tax  $zu$ , this is increased whenever the rate of unemployment is increased. If the benefit  $z$  is increased too, there is a second reason for getting a higher tax rate.

From (1'), which does not contain  $\beta$  and  $z$ , we can conclude that increased bargaining power and unemployment benefits lead to less vacancies because of the fall in the tightness ratio. In figure 1 this can be seen as only the BB curve shifts (for  $dz$ ) and also rotates upward (for  $d\beta$ ) but all other curves do not change their position. This leads to higher wages and lower hiring costs. The size of firms is increased through lower hiring costs and the number of firms is going down because of the higher rate of unemployment and the lower hiring costs.

In the upper right quadrant of figure 1, increasing the rate of interest and the strength of separation shocks rotates the MM curve downward and leaves the BB curve unchanged. We can conclude that increasing the rate of interest decrease wages and the tightness ratio.<sup>xix</sup> The impact on the size and number of firms remains unclear, because the decrease in the tightness ratio decreasing hiring costs increases firm size, but the increase in the interest rate has the direct effect of decreasing firm size.

The impact of the change in hiring costs,  $d\gamma > 0$ , is to decrease tightness according to (9) and vacancies according to (1'). In terms of Figure 1 it rotates the MM curve downward and increases the slope for the BB curve. Its effect on wages depends on the strength of these two movements. In equations (4') and (6)  $\gamma$  and  $\theta$

have opposite effects on wages. In equation (6) the question is whether  $\gamma\theta$  is increasing or decreasing. Applying the implicit function rule to (9) and dividing by  $\theta$  and multiplying by  $\gamma$  yields

$$\frac{\partial\theta/\theta}{\partial\gamma/\gamma} = -\frac{(r+s)(\gamma/q) + \beta\theta\gamma}{(r+s)\gamma(-1)q^{-2}q_{\theta}\theta + \beta\gamma\theta}$$

The numerator and the denominator have identical terms up to  $-q_{\theta}\theta/q$ .<sup>xx</sup> This term is smaller than one<sup>xxi</sup>. By implication the percentage decrease of  $\theta$  is larger in absolute terms than the increase of  $\gamma$ , leading to a fall in wages according to (6). In terms of figure 1 this means that the upward rotation of BB is weaker than the downward rotation of MM.

**Proposition 3:** *With increases in worker's bargaining power  $\beta$ , hiring costs  $\gamma$ , the discount rate  $r$  and unemployment benefits  $z$  and the size of shocks  $s$  we confirm Pissarides' results of decreasing tightness and expected hiring costs and increasing unemployment. Wage rates go up with bargaining power and benefits and go down when interest and separation rates or hiring costs increase. We add the following results: (i) Lower employment requires an increasing unemployment tax or premium. (ii) There is an increasing size and a decreasing number of firms from increasing bargaining power, benefits and separation rates with unclear effects on aggregate output. (iii) Increases in hiring costs  $\gamma$  and interest rates  $r$  have direct effects on the number and size of firms and aggregate output, which are opposite to those of the decreasing tightness ratio and lead to no clear results for these variables.*

Changes in fixed costs,  $df < 0$ , which is one of two possible versions of exogenous productivity increases, do not change the tightness ratio and therefore the unemployment and vacancy rates, and marginal labour costs are unchanged.<sup>xxii</sup> Firm size  $x$  is decreased and the number of firms is increased as in Dixit-Stiglitz. Aggregate output,  $nx$ , is unaffected.

A decrease in marginal costs via  $da < 0$ , decreases the slope and intercept of the MC curve in figure 2 and shifts up the MM curve in figure 1. The result is a larger value for the tightness ratio,  $v/u = \theta$ . This reduces the unemployment rate and increases the rate of vacancies according to equation (1) and (1'). The direct impact is to decrease the number of firms and increase its size and aggregate output. However, the indirect effect through larger tightness is to increase hiring costs: This decreases firm size and aggregate output and increases the number of firms. Moreover, a lower unemployment rate increases the number of firms and aggregate output. Wages are increased according to equation (6).

**Proposition 4:** *(i) Productivity increases in the fixed cost parameter,  $df < 0$ , when we leave labour market variables unchanged, decrease the size of firms while increasing the number of firms as in the Dixit-Stiglitz model. Aggregate output, however, is unchanged. (ii) Pissarides' productivity results appear in our model and are caused by a decrease in the variable labour demand parameter,  $da < 0$ . Wages, tightness and employment increase. The number and size of firms and aggregate output do not have clear effects anymore as they did in Dixit-Stiglitz because unemployment and hiring costs change with the tightness ratio.*

By implication, policies that try to enhance the number of firms in order to decrease unemployment should - according to this model with a constant elasticity of substitution - not try to reduce fixed costs of firms but rather variable costs, or, the *personal* fixed costs of entrepreneurs for setting up a firm according to the model of Fonseca et al. (2001).

A decrease in love-of-variety,  $d\alpha > 0$ , which means that the elasticity of substitution is getting larger in absolute terms and competition is increased, shifts up the MR curve in figure 2 and the MM curve in figure 1. The tightness ratio and expected hiring costs increase, the unemployment rate falls and the number of vacancies goes up.<sup>xxiii</sup> Wages increase according to (6). Firm size grows through the direct effect but decreases through the increase in expected hiring costs according to (6). The effect on the number of firms is ambiguous because the direct effect is decreasing the number of firms whereas the indirect effects via the rate of unemployment and expected hiring costs are increasing the number of firms. This latter aspect is analysed formally in an appendix for the case of constant hiring costs and summarized at the end of the following proposition.

**Proposition 5:** *A preference induced increase in substitution and competition increases employment, the tightness ratio and wages. Unlike the Dixit-Stiglitz model, the effect on the number is ambiguous even for a negligible change in hiring costs: a decrease in monopoly power  $d\alpha > 0$  decreases (increases) the number of firms if the measure of scale economies is larger (smaller) than the elasticity of employment with respect to monopoly power. The size of firms and aggregate output are increased if the increase in hiring costs is sufficiently weak.*

For the decrease of unemployment through a higher degree of competition,  $d\alpha > 0$ , an increase in the number of firms is inessential. What matters are the larger marginal revenue and the larger marginal value product of labour, which increases labour demand.

A transition from monopolistic to perfect competition can be made by setting fixed costs  $f=0$  and removing product differentiation setting  $\alpha=1$ . This increases the tightness ratio and vacancies and decreases unemployment. By implication unemployment is lower under perfect competition than under monopolistic competition.<sup>xxiv</sup>

Integrating two identical economies doubles  $L$ . This leaves the tightness ratio, the rates of unemployment and vacancies unchanged. The number of firms and varieties is doubled as in the Dixit-Stiglitz model and so are the number of unemployed workers and vacant jobs,  $uL$  and  $vL$ .

**Proposition 6:** *Doubling the size of the market doubles the number of firms, the number of unemployed people and the number of vacant jobs, but leaves the rate of unemployment unchanged. Doubling the number of varieties is a gain from trade and integration of two economies.*

All changes in parameters inducing the above comparative static results have to be interpreted as stemming from perfectly non-anticipated shocks that are expected to be permanent with probability one because the Pissarides part of the model uses steady-state present values.<sup>xxv</sup>

The economic mechanism of causation (in the sense of finding the solution of the model) of the comparative-static changes as summarised in Figure 1 always goes

from the exogenous change to its impact on real wages, hiring costs and the tightness ratio and from there to the rate of unemployment and the number of firms.

Changes in the fixed cost parameter have no impact on aggregate output. The positive cross-country correlation of employment *changes* and start ups found by Fonseca et al. (2001, Fig.4) therefore may have a two way causality: on the one hand it is plausible that countries with a lower start-up cost index have more start ups and therefore lower unemployment; on the other hand it may also be the case that changes in productivity and competition,  $d\alpha/a$ , and labour market parameters did reduce the rate of unemployment and thereby did increase the number of firms.

The results of this section are summarized in Tables 1 and 2.

#### Table 1 ABOUT HERE

The first four columns of table 1 clearly show that Pissarides' results for parameter changes in the labour market carry over to our model. The fifth column adds results concerning the number of firms added by this model: Whenever unemployment is increased, the number of firms is decreased unless there are dominant offsetting effects from interest and expected hiring costs. The sixth column indicates that firm size increases when tightness decreases, but direct effects from increases of interest  $r$  and hiring costs  $\gamma$  may be stronger. Finally, the change of the unemployment tax or premium has the same sign as the change of the unemployment rate.

#### TABLE 2 ABOUT HERE

In Table 2 the column for wages is qualitatively exactly the same as in Dixit-Stiglitz, but real wages now depend also on hiring cost and the effects therefore differ in quantity. The first three columns summarize the effects of goods market parameters from the Dixit-Stiglitz model on the labour market variables. These are the new results: Country size and fixed costs have no impact on labour market variables; increasing monopoly power increases unemployment and decreases tightness and the number of vacancies. The results for marginal productivity are the same as those of Pissarides for productivity, and country size also had no impact on the solution of his perfect competition model. The column concerning the number of firms has different results from Dixit-Stiglitz concerning marginal costs and marginal revenue because the impact via the unemployment rate and hiring costs changes their results because the direct effect  $d\alpha/a > 0$  increases the size of the firm which has a negative effect on the number of firms and increases employment, which has a positive impact on the number of firms. Moreover, hiring costs change. Aggregate output is increased by  $d\alpha/a > 0$  unless hiring costs increase too much. The last column adds results from the unemployment tax or premium  $t$ , which again follows those of the unemployment rate  $u$ .

Liu and Yang (1999) observe that there is declining average size (employment) of the firm, growth of per capita income and productivity in the data of some countries. If recent shifts of parameters are assumed to be  $d\alpha > 0$ , because of an increase in competition,  $da < 0$  and  $df < 0$ , both because of productivity improvements, our model generates exactly these observed changes if the change in the fixed costs dominates the others in regard to the number and size of firms.<sup>xxvi</sup>

In regard to net wages,  $w-zu$ , we can say that they will be increased whenever gross wages are increased and unemployment is decreased, that is, by lower hiring

costs  $\gamma$ , interest rates  $r$ , separation rates  $s$  and marginal costs  $a$ , and by higher  $\alpha$ , i.e. lower monopoly power. These results follow directly from the definition of net wages and the results in the tables. The impact of changes in bargaining power  $\beta$  and benefits  $z$  are more difficult to get. We show the following results in an appendix.

**Proposition 7:** *An increase in bargaining power  $\beta$  increases (decreases) net wages if the effect of the induced decrease of the tightness ratio increases unemployment multiplied by constant benefits less than it decreases rents from expected hiring costs. An increase in benefits  $z$  decreases net wages if (sufficient) the induced decrease of the tightness ratio (multiplied by initial benefits) increases unemployment more than it decreases rents from expected hiring costs.*

## V. A comparison with some macroeconomic models

Table 3 summarizes literature on imperfect competition and *non-monetary*<sup>xxvii</sup> theories of unemployment, which comes closest to our contribution. The first column lists models of the Dixit-Stiglitz type. Unemployment in this column is based on pessimistic expectations in Weitzman (1985) and Dutt and Sen (1997). In Weinrich (1993), using the Blanchard and Kiyotaki (1987) variant of the Dixit-Stiglitz model, unemployment is based on the use of an effort function that is continuous in wages and unemployment. In Matusz (1996, 1998) it is based on the Shapiro-Stiglitz model of efficiency wages. In deGroot and van Schaik (1998, 2000) dual labour markets with unemployment arise because of an efficiency wage relation. In de Groot (2001) dual labour markets arise because of union bargaining. The last model mentioned in the first column of the table is the focus of this paper.

### TABLE 3 ABOUT HERE

In the second column, models with other forms of monopolistic competition than that of the Chamberlain-Dixit-Stiglitz type are listed. Weitzman (1982) uses a Hotelling model and unemployment is based on pessimistic expectations of the self-fulfilling prophecy type. Bräuninger (2000) considers unemployment based on bargaining. Bargaining is about profits and rents from being employed. Unlike the Pissarides model there are no hiring costs creating rents for firms but rather – unlike the Dixit-Stiglitz model - profits are positive because the model has no entry but rather a large, exogenous number of firms and no fixed costs. Blanchard and Giavazzi (2000) in a similar set up have fixed costs and also consider entry. There have been other attempts to relate *search unemployment* and monopolistic competition in the literature. One group of attempts is related to the Aghion and Howitt (1994, 1998 chap.4) quality ladders approach. In that approach there is vertical product differentiation; each firm has the exogenous size of one worker and because unemployment is endogenous the number of firms is also endogenous, whereas in our approach both, the number and the size of firms, are endogenous. We use the Dixit-Stiglitz approach of horizontal differentiation because it has an endogenous number and size of firms. Bean and Pissarides (1993) adjust the Pissarides (1990) model to an overlapping-generations growth model by fixing the employment duration to last one period. Bean and Pissarides then go to analyse the effect of savings shocks on employment and growth; this topic is not considered here. In their model, firms play Cournot in each of the markets for differentiated products. Table 3 indicates which

gap in the literature this paper fills: combining the search model of the labour market with the goods market model of Dixit-Stiglitz.

In this paper, we did investigate to what degree the solution of the model and the comparative *static* results of Pissarides (1990) and Dixit-Stiglitz (1977) are unchanged and which results on the number and size of firms and real wages can be added. We limit our comparison with the literature to that of other *static* models. Results from literature that uses either a different theory of unemployment or a different model of monopolistic competition can be compared with our results more straightforwardly.

There are two qualifications concerning our result that decreasing fixed costs have no impact on the rate of tightness and unemployment. First, Blanchard and Giavazzi (2000) show in a framework without matching imperfections and hiring costs, that the increase in the (integral measure of) the number of firms obtained from the fixed costs reduction decreases unemployment *if* the CES parameter, which is also the degree of competition, changes with the number of firms. However, if that number of firms is large - as is the assumption built into the continuum of goods in the Dixit-Stiglitz model – the model does not have this effect. The question then is, which model is best representing the macro economy. Second, fixed *personal* start-up costs of people becoming entrepreneurs may have an impact on the unemployment rate according to Fonseca et al. (2001). Policy therefore should focus on these fixed personal start-up costs or on variable costs.

Matusz (1998) has linked the Dixit-Stiglitz model to efficiency-wage unemployment of the Shapiro-Stiglitz type. In the non-shirking constraint the number of varieties appears. Therefore country size or international trade, increasing the market size and yielding more variety, relaxes the non-shirking constraint. By

implication the unemployment needed to deter shirking is lower. In Matusz (1996) an increase in market size increases productivity because of an increased number of intermediate products, which allows for higher wages and less equilibrium unemployment again via a non-shirking constraint. In this paper we find no market size effects of international trade on the unemployment percentage rate for several reasons. First, with differentiated consumer goods in our model the productivity effect of Matusz (1996) is absent in our set up. Second, having no non-shirking constraint in the Pissarides part of our model, variety cannot have such a prominent role in relation to unemployment as it has in Matusz (1998). Two other channels - discussed verbally by Matusz (1996) – which might in principal affect the rate of unemployment, are also absent here. First, the price elasticity of demand in his model as in ours is independent of the number of firms. If it were dependent on the number of firms as it is in models with strategic interaction, entry induced by larger market size might affect firms' size and decrease average cost, which might be (similar to) a productivity effect. However, it needs a proof to see the interaction of market size, firms' size, productivity and unemployment. Second, if there were an impact of more variety on the search intensity, the rate of unemployment might be affected. In our set up following Pissarides (1990) this effect is absent because the bargaining is on income and not on utility, and the utility function and the Nash-product are specified independently of each other. These channels are also not modelled here because there is hardly any empirical indication that they are relevant. I agree with Matusz that it is an empirical question whether or not these channels matter quantitatively.

In the models by Matusz the size of the firms equals that of the Dixit-Stiglitz model and therefore is independent of the rate of unemployment whereas our model

has larger hiring costs when steady-state unemployment is lower and therefore the size of firms is lower.

Concerning an increase in worker bargaining power Dutt and Sen (1997) find a different result in a demand-constrained model of monopolistic competition cum unemployment of the Weitzman (1985) type. In their model bargaining power raises wage rates. As workers save less than profit earners, demand is increased and so is employment. The authors show that the result can neither be proven nor rejected if a decreasing marginal product of labour exists. Unlike Dutt and Sen (1997) we do not find that increased bargaining power of unions increases demand and therefore employment. The reason is that our model, unlike the Weitzman (1985) type of model used by Dutt and Sen, is not demand constrained via pessimistic expectations and allows for real-wage flexibility and entry.

In the model by Pissarides (1990) the neo-classical way to get full employment with decreasing real wage costs is blocked by search frictions. In most monopolistic competition models without hiring costs this road is absent anyway because real wages are fixed as in (4) (see Weitzman (1985), Blanchard and Kiyotaki (1987), Matusz (1996, 1998). The Dixit-Stiglitz road to full employment used to be entry whenever profits are positive, according to equation (2) with  $u=0$ . In Weitzman (1982)  $u>0$  comes in exogenously via the assumption that firms expect aggregate demand to be (in symbols of our model)  $u^e wL$ , where all the three variables are given exogenously. With this expected demand, a self-fulfilling prophecy yields the unemployment rate  $u=u^e >0$ . In our model, however, the Dixit-Stiglitz road to full-employment is present but limited by Pissarides' search friction, resulting in an endogenous value of unemployment  $u > 0$  incorporated in equation (2).

In Weitzman (1982) the rate of unemployment is negatively related to real wages: if the (expected) rate of unemployment goes up real wages decrease, because lower expected demand moves the firm up its average cost curve. This is interpreted as a pro-cyclical movement of real wages. In our model a positive or a negative relation may exist depending on the reason that drives up wages: if the exogenous change in question increases (decreases) tightness the unemployment rate goes down (up). For example, an increase in the rate of interest or the separation rate, which decreases tightness and wages and increases unemployment, yields a negative relation as in Weitzman's model. On the other hand, an increase in unemployment benefit  $z$  or bargaining power  $\beta$  increases wages, decreases the tightness ratio and increases unemployment. A similar negative relation  $w(u)$  can be found in Weinrich (1993) based on a linear transformation of a Cobb-Douglas effort function that generates constant effort. On this relation  $u$  is determined through the use of an equation similar to our equation (4'), which *fixes* the real wage in the absence of hiring costs (see Weinrich 1993, p.545). Moving the latter equation along the former would result in a negative relation between  $w$  and  $u$ . Specific *combinations* of parameter shifts could also generate a positive relation between  $w$  and  $u$  in the comparative static manner in our model. In our model there is no fixed wage.

With the exception of Vogt (1996) all the other *static* monopolistic competition models mentioned above do not take hiring costs into account. Vogt has criticised monopolistic competition models because of their property to translate wage increases completely into price increases (see Weitzman (1985), Blanchard and Kiyotaki (1987), Matusz (1996, 1998)) for empirical reasons. He develops a potential competition model in which sunk costs deter entry of a potential Bertrand competitor resulting in a demand curve with a kink at a limit price. This ensures that prices,

quantity and employment are independent of cost increases within certain limits. In this paper, the problem is avoided by introducing search unemployment into a model of monopolistic competition. Our combination of the models by Pissarides and Dixit-Stiglitz generates another, though less strict, separation of wages from prices because costs, price and employment are still interdependent. Wage increases can either increase prices or decrease hiring costs as in Pissarides (1990).

Our model has higher employment under perfect than under monopolistic competition because the marginal value product of labour is larger. In Blanchard and Kiyotaki (1987) this result is achieved by an aggregate demand externality based on real balances in the utility function and endogenous labour supply.

## **VI. Summary and conclusion**

Linking Pissarides' (1990) search theory of unemployment to the Dixit-Stiglitz (1977) model rather than to the neo-classical production function yields six groups of results. First, the rates of tightness is linked to the size of firms via expected hiring costs and via the rate of unemployment also to the number firms. Every variable that has an impact on the rate of unemployment and hiring costs also has an impact on the number and size of firms. Thus, aggregate output is affected via the number and size of firms.

Second, the introduction of love-of-variety preferences has an impact as well. A decrease in the love-of-variety parameter,  $\alpha$ , decreases the elasticity of substitution and competition, decreases marginal revenue and the tightness ratio, increases unemployment and hiring costs, decreases vacancies and wages and - deviating from the Dixit-Stiglitz model - has an ambiguous effect on the number of firms. It also

decreases the size of the firms unless the decrease in hiring costs increases the size of firms. By implication employment is higher under perfect than under monopolistic competition.

Third, some other comparative static results of Pissarides' model and the model of Dixit and Stiglitz survive. However, as the equilibrium quantity of firms now depends on marginal hiring costs changes of interest and hiring cost parameters have unclear effects on the size and number of firms. Similarly, increases in the marginal value product of labour would increase the size and decrease the number of firms but the new effect of hiring costs on the size and number of firm can invalidate this result if changes in hiring costs are large. By implication the introduction of hiring costs in both cases decreases wages received by workers.

Four, decreases in bargaining power or benefits decrease gross wages but do not necessarily decrease net wages because the resultant decrease in unemployment – if it is sufficiently strong - may increase them.

Five, unlike earlier monopolistic competition models, real wages have some flexibility because of the hiring costs depending on the rate of tightness. The model allows for a negative or a positive relation between real wages and the rate of unemployment. The negative relation occurs provided the shocks come from hiring costs, interest and separation rates, the CES parameter or the marginal cost parameter. The positive relation occurs when there is variation in bargaining power and benefits.

Finally, the fixed cost parameter has no impact on the solution for the tightness ratio and other labour market variables and influences only the size and number of firms, but not aggregate output. Doubling market size doubles the number of firms, and the *number* but not the *rate* of unemployed workers and vacant jobs. It

increases the gains from trade only via variety. Increases in marginal productivity increase employment but have an ambiguous effect on the number and size of firms.

The Pissarides-Dixit-Stiglitz model of this paper has the desirable properties of both its underlying models, and also some which have been stated as desirable in macroeconomics such as flexible real wages and the size of firm being dependent on the steady-state level of activity, unemployment and tightness. The combination of the basic models yields some new results as well such as the dependence of the number and size of firms on the activity indicators of the economy such as the rates of tightness and unemployment. Moreover, it is simple enough to allow for extensions in many directions. One of the possible extensions is to introduce energy-input coefficients and treat the issues of environmental policy under endogenous unemployment. A second possible extension is to treat information and communication technology as technical progress in the matching function and as a decrease of the variable labour-cost coefficient at the cost of increasing the parameter for fixed costs. A third one is to derive the central planners optimum without and with acceptance of the bargained-wage relation and derive optimal policies.

## **References**

Aghion, P. and P. Howitt (1994), 'Growth and unemployment', *Review of Economic Studies* 61, pp.477-494.

\_\_\_\_\_ (1998), *Endogenous Growth Theory*, MIT Press, 1998.

Bean, C. and C. Pissarides (1993), 'Unemployment, Consumption and Growth', *European Economic Review*, 37, No.4, pp.837-855.

Blanchard, O. and F. Giavazzi (2000), 'Macroeconomic Effects of Regulation and Deregulation in Goods and Labor Markets', Massachusetts Institute of Technology, Department of Economics Working Paper: 01/02, January 2000, pp. 41.

Blanchard, O.J. and N. Kiyotaki (1987), 'Monopolistic Competition and the Effects of Aggregate Demand', *The American Economic Review*, September, pp.647-666.

Boone, J. (2000), 'Technological Progress, Downsizing and Unemployment', *The Economic Journal* 110, July, pp.581-600.

Bräuning, M. (2000), 'Wage Bargaining, Unemployment, and Growth', *Journal of Institutional and Theoretical Economics* 156/4, pp.646-660.

Dixt, A. and J. Stiglitz, (1977), 'Monopolistic Competition and Optimum Product Diversity', *The American Economic Review*, 67(3), pp.297-308.

Dutt, A.K. and A. Sen (1997), 'Union Bargaining Power, Employment, and Output in a Model of Monopolistic Competition with Wage Bargaining', *Journal of Economics (Zeitschrift für Nationalökonomie)*, Vol.65, No.1, pp.1-17.

Fonseca, R., P. Lopez-Garcia, C.A. Pissarides (2001), 'Entrepreneurship, start-up costs and employment', *European Economic Review* 45, pp.692-705.

Groot de, Henri, (2000), '*Growth, Unemployment and Deindustrialization*', [Edward Elgar](#), Cheltenham, 2000.

\_\_\_\_\_, (2001), 'Unemployment, Growth and Trade Unions', *Growth and Change* 32(1), Winter, pp.69-91.

\_\_\_\_\_, and A.B.T.M. van Schaik (1998), 'Unemployment and Endogenous Growth', *Labour*, 12, pp.189-219.

\_\_\_\_\_, 'Growth and Unemployment in a Dual Economy', in: *The Unemployment Debate: current issues*, edited by P. de Gijsel, R. Olthoff, T. Zwick, Metropolis Verlag, Marburg 2000, pp.39-72.

Kirchesch, K. (2001), 'Employment and Investment of the West German Industry 1968-1998: A Descriptive Analysis', *Jahrbücher für Nationalökonomie und Statistik*, 221(5-6), October, pp. 473-90.

Liu, P.-W. and X. Yang (1999), 'Division of Labour, Transaction Cost, Emergence of the Firm and Firm Size', *CID Working paper* No.10, April, 22 p.

Lordon, F. (1997), 'Endogenous structural change and crisis in a multiple time-scale model', *Journal of Evolutionary Economics* 7, pp.1-21.

Loveman, G. and W. Sengenberger (1991), 'The Re-emergence of Small Scale Production: An International Comparison', *Small business Economics* 3: 1-37.

Matusz, S.J. (1996), 'International trade, the division of labour and unemployment', *International Economic Review* 37(1), February, pp.71-84.

\_\_\_\_\_ (1998), 'Calibrating the Employment Effects of Trade', *Review of International Economics*, 6(4), pp.592-603.

Pissarides, C. (1990), '*Equilibrium Unemployment Theory*', Basil Blackwell.

\_\_\_\_\_ (1998), 'The impact of employment tax cuts on unemployment and wages; the role of unemployment benefits and tax structure', *European Economic Review* 42, pp.155-183.

Shapiro, K. and J.E. Stiglitz (1984), 'Equilibrium Unemployment as a Worker Discipline device', *American Economic Review*, June 1984, Vol.74, No.3, pp.433-444.

Vogt, W. (1996), 'Accelerating Inflation or Rising Unemployment--Is There an Alternative?' *Metroeconomica* 47(2), June, pp.197-230.

Weinrich, G. (1993), 'A Business cycle Model Based on Efficiency Wages, Monopolistic Competition and Nondecreasing Returns', *Economic Notes* 22 (3), pp.535-551.

Weitzman, M.L. (1982), 'Increasing Returns and the Foundations of Unemployment Theory', *Economic Journal* 92, December, pp.787-804.

\_\_\_\_\_, (1985), 'The Simple Macroeconomics of Profit Sharing', *American Economic Review*, December, pp.937-953.

**Table 1**      *Comparative static results of labour market parameters*

*Effect on →*    *tightness  $\theta$*     *unempl.  $u$*     *vac.  $v$*     *wage  $w$*     *firms  $n$*     *size  $x$*     *premium  $t$*   
*from increases of ↓*

Bargaining power $\beta$	-	+	-	+	-	+	+
Hiring costs $\gamma$	-	+	-	-	?	?	+
Interest rate $r$	-	+	-	-	?	?	+
Benefit $z$	-	+	-	+	-	+	+
Separation rate $s$	-	+	-	-	-	+	+

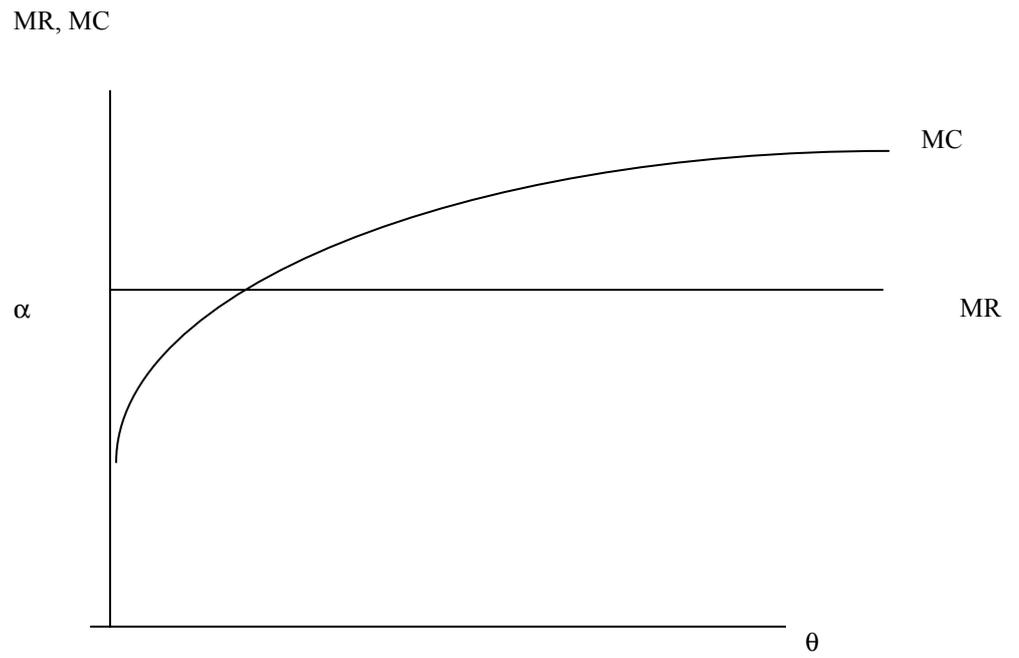
**Table 2 Comparative static results of goods market parameters**

*Effect on  $\rightarrow$  tightness  $\theta$ , unempl.  $u$ , vac.  $v$ , firms  $n$ , wage  $w$ , size  $x$ , outp.  $nx$ , prem  $t$   
from increases of  $\downarrow$*

Fixed costs $f$	0	0	0	-	0	+	0	0
Marginal costs $a$	-	+	-	?	-	?	?	+
Marginal revenue $\alpha$	+	-	+	?	+	?	?	-
Country size $L$	0	0	0	+	0	0	+	0

**Table 3**      *Theories of unemployment and imperfect competition.*

<i>Imperfect competition:</i>	<u>Dixit-Stiglitz</u>	<u>others</u>
<u>(Un)employment theory</u>		
Pessimistic expectations	Weitzman 1985 Dutt/Sen 1997	Weitzman 1982
Walrasian labour market	Blanchard/Kiyotaki 1987	
Effort function	Weinrich 1993	
reservation wage cum hiring costs		Vogt 1996
Shapiro-Stiglitz	Matusz 1996, 1998	
Dual labour market	de Groot/ v.Schaik (1998, 2000), de Groot (2001)	
Union bargaining		Bräuninger (2000) Blanchard/Giavazzi (2000)
Search models	This paper	Aghion/Howitt 1998 Bean/Pissarides 1993



*Figure 2.* Marginal revenue, MR at value  $\alpha$ , is equal to marginal cost, MC. The intersection of both lines determines the tightness ratio  $\theta$ .

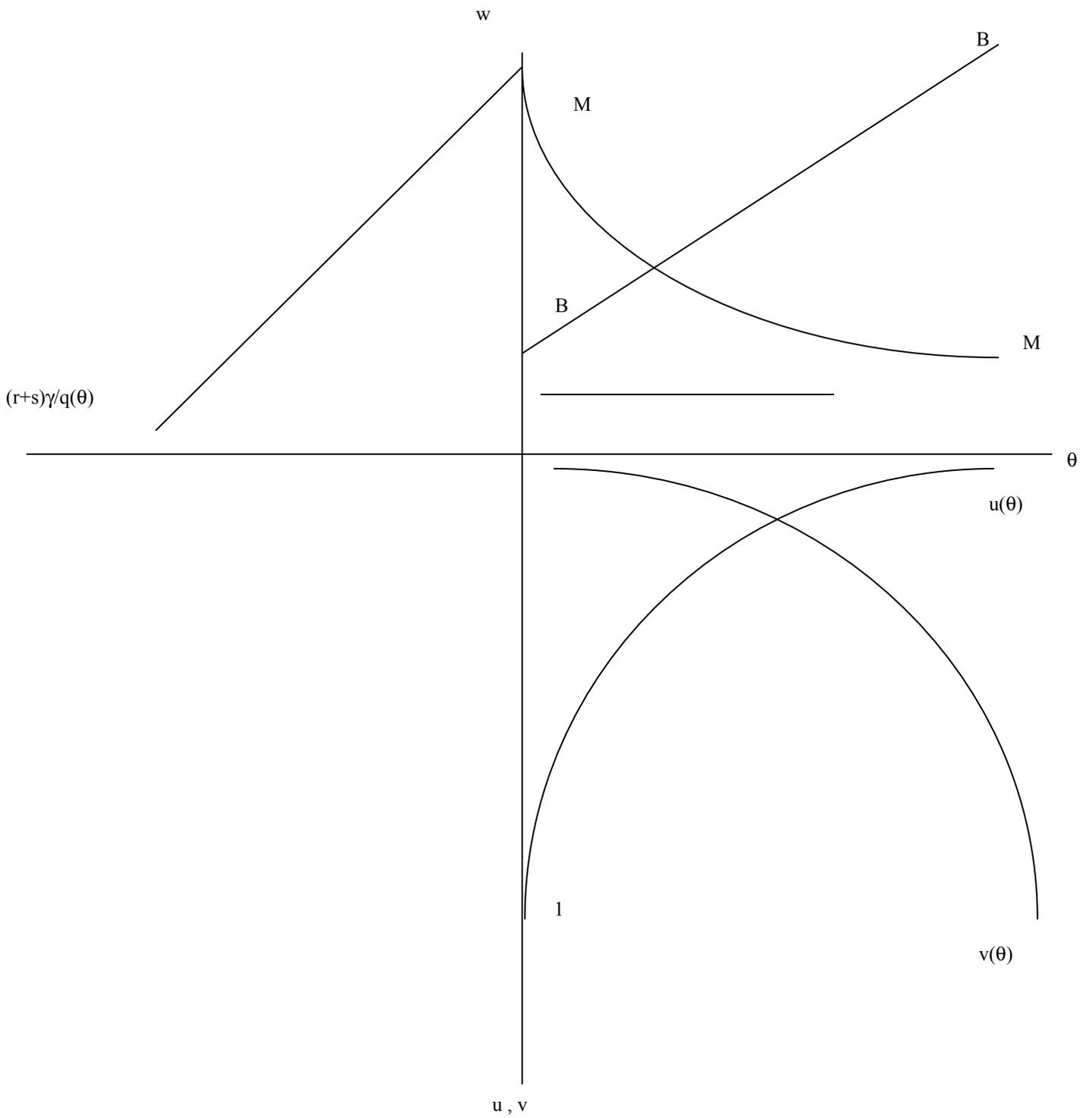


Figure 1. The wage bargaining result, BB, and the profit maximising real wage, MM, determine the real wage and the tightness ratio in the upper right quadrant. This implies a solution for the unemployment rate  $u$  and vacancies  $v$  in the lower right quadrant. Each result for wages implies a result for hiring costs in the upper left quadrant.

---

<sup>i</sup> I am grateful to Steven Brakman, Michael Bräuninger, Georg Götz, Henri de Groot, Karin Kamp, Gerald Nekker, Erik de Regt, Johannes Schneider, Bas ter Weel, Gerd Weinrich and Seminar participants in Maastricht and Groningen for useful comments. Responsibility for remaining errors is entirely mine.

<sup>ii</sup> Subsections are titled as in Pissarides 1990. The search part is explained in greater detail there.

<sup>iii</sup> Pissarides (1998, p.167, footnote 15) refers to estimates of the matching function using a Cobb-Douglas functional form, which justifies the assumptions made in the text.

<sup>iv</sup> Policy and financing constraints are discussed more extensively in Pissarides (2000), Chap.9.

<sup>v</sup> By implication we only consider the case of a large number of firms in which no strategic behaviour takes place.

<sup>vi</sup> Shapiro and Stiglitz (1984, p.435, fn. 5) also follow this procedure.

<sup>vii</sup> See Pissarides (2000), section 3.4 for this interpretation.

<sup>viii</sup> In Pissarides (1990) this leads to the zero-profit condition  $f(k) - (r + \delta)k - w - (r + s)q(\theta) = 0$ . Here  $f(k)$  is the output per unit of labour and  $\delta$  is the rate of depreciation.

<sup>ix</sup> This equation corresponds to equation 3.2 in Pissarides (2000).

<sup>x</sup> Equation (4) corresponds to equation (3.7) in Pissarides (2000).

<sup>xi</sup> This result corresponds to equation 1.18 in Pissarides 1990. Note that with  $\beta=1$ , the negotiation result would require  $V=J=q=q(m/v)=0$ , which could only hold for  $v=0$  without additional assumptions on the matching function. However, with  $v=0$  we also have  $\theta=0$  and therefore no vacancies and hiring costs. Equation (5) would imply that wages equal revenue per worker.

<sup>xii</sup> This result corresponds to equation 1.19 in Pissarides 1990 or to equation 1.20 in Pissarides (2000).

<sup>xiii</sup> Note that if the sum of all present-discounted profits is zero, in a steady state with all terms in the profit function constant - except for time in the discount factor - it follows from carrying out the integration that current profits have to be zero.

<sup>xiv</sup> This equation corresponds to equation (3.8) in Pissarides (2000), p.69.

<sup>xv</sup> In Pissarides (2000) this equation corresponds to equation (1.9) there.

<sup>xvi</sup> Dividing both sides by 'a' and replacing the left-hand side by  $f(k)-(r+\delta)k$  shows that this equation is essentially present already in Pissarides. We do not claim that it is new but rather it is modified to contain the degree of competition and it serves to solve for the tightness ratio.

<sup>xvii</sup> To get a negative second derivative of the MC curve it is sufficient to assume that the matching function is of the Cobb-Douglas type.

<sup>xviii</sup> See Kirchesch (2001) and the literature cited there for a recent contribution on this issue.

---

<sup>xix</sup> The impact on vacancies remains unclear here. Pissarides' result is achieved if the direct effect of an increase of  $s$  dominates.

<sup>xx</sup> It is the formal analogue to the share of capital for a neo-classical production function  $f(k)$ .

<sup>xxi</sup> Using the properties of the matching function presented on the first page and Euler's theorem we get (with lower indices indicating partial derivatives):

$$-q_{\theta} \frac{\theta}{q} = \frac{m_1 \frac{1}{\theta}}{m_1 \frac{1}{\theta} + m_2} < 1$$

<sup>xxii</sup> Effects of changes in fixed costs may be quite different in endogenous growth models. See de Groot (2000)

<sup>xxiii</sup> This result can also be found in the quality ladders literature (see Boone 2000, Aghion and Howitt 1998). However, Boone's model is one of a partial equilibrium and in Aghion and Howitt each firm consists of one worker thus fixing firm size making employment equal to the number of firms.

<sup>xxiv</sup> The number and size of firms then become indeterminate according to equations (2') and (7).

<sup>xxv</sup> Bean and Pissarides (1993) show that it can be adjusted for use on short-run issues as well.

<sup>xxvi</sup> Loveman and Sengenberger (1991, section V) also report the decrease in the trend concerning the size of firms. The decrease in fixed costs in our model can be associated as a short cut for "... the new breed of 'flexible' capital ... and R&D ... less costly ... to small firms." The authors present many more complementary arguments to explain the phenomenon. Liu and Yang emphasize transaction costs and Lordon (1997) emphasizes changes in taste to explain these phenomena.

<sup>xxvii</sup> The papers by Vogt and Blanchard and Kiyotaki are of course monetary but the monetary part is inessential for our purposes but not for theirs.

---

**Appendix: Household's utility maximization (not for publication)**

The Hamiltonian for the dynamic optimisation problem of the household indicated in the text is

$$H = \left[ \int_{i=0}^n c_i^\alpha di \right]^{1/\alpha} + \lambda[(1-u)w + uz - t - \int_{i=0}^n p_i c_i di + rW]$$

$\lambda$  is a co-state variable. The first-order conditions of dynamic optimisation are the following:

$$\partial H / \partial c_i = \left[ \int_{i=0}^n c_i^\alpha di \right]^{1/\alpha} c_i^{\alpha-1} - \lambda p_i = 0 \text{ for all goods } i.$$

$$-\partial H / \partial W = \cancel{\rho} \lambda = -r\lambda$$

The canonical transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda = 0$  need not be stated here

because the differential equation for the co-state variable implies  $\hat{\lambda} = \rho - r$ . For any positive interest rate the transversality condition is redundant. The first-order

condition for the goods  $i$  implies that for any two goods  $k$  and  $j$  we get  $\left( \frac{c_j}{c_k} \right)^{\alpha-1} = \frac{p_j}{p_k}$ .

Therefore the inverse partial price elasticity is  $\alpha-1$ . Multiplying the first-order conditions for  $c_i$  by  $c_i$  and summing over all goods yields

---


$$\left[ \int_{i=0}^n c_i^\alpha di \right]^{1/\alpha} = \lambda \int_{i=0}^n p_i c_i di$$

As all goods in the model have the same costs and therefore the same prices we set the prices equal to unity using them as numéraire. Quantities then also will be identical because goods enter the utility function symmetrically. The last equation then can be written as  $n^{(1/\alpha)-1} = \lambda$ . In any steady state where the number of firms is constant, the co-state variable  $\lambda$  must also be constant and its growth rate  $\rho - r$  must also be zero.

Therefore  $r = \rho$  in a steady state with a constant number of firms.

***Appendix: Effects of changes in bargaining power and benefits on net wages (not for publication)***

The equations (4') and (5'') can be rewritten in the following way:

$$w - \alpha/a + \gamma(r + s)/q(\theta) = 0$$

$$w - (1-\beta)z - \beta(\alpha/a + \theta\gamma) = 0$$

Differentiating totally with respect to  $w$ ,  $\theta$ ,  $\beta$  and  $z$  yields the following:

$$\begin{bmatrix} 1 & (r+s)\gamma q^{-2}(-1)q' \\ 1 & -\beta\gamma \end{bmatrix} \begin{bmatrix} dw \\ d\theta \end{bmatrix} = - \begin{bmatrix} 0 \\ z - \alpha/a - \theta\gamma \end{bmatrix} d\beta - \begin{bmatrix} 0 \\ -(1-\beta) \end{bmatrix} dz - \begin{bmatrix} -1/a \\ -\beta/a \end{bmatrix} d\alpha$$

Cramer's rule yields (with  $D = -\beta\gamma(r+s)\gamma q^{-2}(-1)q' < 0$ ):

---


$$\partial w / \partial \beta = (z - \alpha / a + \theta \gamma)(r + s) \gamma q^{-2} (-1) q' / D > 0$$

$$\partial w / \partial z = -(1 - \beta)(r + s) \gamma q^{-2} (-1) q' / D > 0$$

$$\partial \theta / \partial \beta = (-z + \alpha / a + \theta \gamma) / D < 0$$

$$\partial \theta / \partial z = (1 - \beta) / D < 0$$

$$\partial \theta / \partial \alpha = [(\beta - 1) / a] / D > 0$$

Moreover, we have  $\partial u / \partial \theta = -s[s + \theta q(\theta)]^{-2} (q + \theta q') < 0$ . For the change in net wages,  $w^n - zu$ , we get after insertion of the above results:

$$\partial w / \partial \beta - z \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial \beta} = \underbrace{(z - \alpha / a + \theta \gamma)}_{-} \{ (r + s) \gamma q^{-2} (-1) q' - z s [s + \theta q(\theta)] (q + \theta q') \} / \underbrace{D}_{-}$$

$$\partial w / \partial z - u - z \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial z} = -u + \underbrace{\frac{\beta - 1}{1 - \beta}}_{+} \{ (r + s) \gamma q^{-2} (-1) q' - z s [s + \theta q(\theta)]^{-2} (q + \theta q') \}$$

The first expression has the sign of the curly brackets, which contains the value of the decrease of expected rents minus that of the increase of the unemployment rate multiplied by benefits  $z$ . If the increase in the rate of unemployment is lower (higher) than that of the decrease in rents, net wages can be increased (decreased) because the increase in taxes reduces the increase in gross wages only to a small extent. The

second expression is negative if (sufficient) the change in unemployment dominates the change in rents.

**Appendix: The impact of the marginal revenue on the number of firms (not for publication)**

The change in the number of firms –ignoring hiring costs - is  $dn = [-L(1-u)/f] d\alpha - [L(1-\alpha)/f] du$  with

$$du = \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial \alpha} d\alpha = \{-s[s + \theta q]^{-2} (q + \theta q') [(\beta - 1) / a] / D\} d\alpha$$

The last equality uses results from the previous appendix. Insertion of  $du$  into the definition of  $dn$  yields:

$$dn = (-L / f) d\alpha \left\{ (1-u) + (1-\alpha) \frac{(-u)}{s + \theta q} (q + \theta q') [(\beta - 1) / a] / D \right\}$$

As this result does not lend itself to a nice interpretation we go back to the first formulation:

$$dn = \frac{-L}{f} d\alpha \left[ 1-u + (1-\alpha) \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial \alpha} \right] = \frac{-L}{f} d\alpha (1-u) \frac{1-\alpha}{\alpha} \left[ \frac{\alpha}{1-\alpha} + \frac{1}{1-u} \underbrace{\frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial \alpha}}_{- \quad +} \alpha \right] =$$

$$\frac{-L}{f} d\alpha (1-u) \frac{1-\alpha}{\alpha} \left[ \frac{a}{f} - \varepsilon_{1-u, \alpha} \right]$$

For the last step we have made use of the fact that (i) in zero-profit equilibrium  $\alpha/(1-\alpha) = a/f$ , the ratio of marginal to fixed costs or the measure of scale economies, and of

---

the fact that (ii) the last expression in square brackets is the negative value of the elasticity of employment in regards to monopoly power  $\alpha$ . By implication, an increase in monopoly power  $d\alpha < 0$  increases (decreases) the number of firms if the measure of scale economies is larger (smaller) than the elasticity of employment with respect to monopoly power.

***List of symbols (not for publication)***

$\alpha$	<i>marginal revenue, CES parameter</i>
$a$	<i>marginal costs</i>
$\beta$	<i>bargaining power</i>
$\gamma$	<i>hiring costs</i>
$f$	<i>fixed costs</i>
$L$	<i>country size</i>
$n$	<i>number of firms</i>
$nx$	<i>aggregate output</i>
$r$	<i>interest rate</i>
$s$	<i>separation rate</i>
$t$	<i>unemployment premium or tax</i>
$\theta$	<i>tightness ratio <math>v/u</math></i>
$u$	<i>unemployment rate.</i>
$v$	<i>vacancy rate</i>
$w$	<i>wage</i>
$x$	<i>firm size in terms of output quantity</i>
$y$	<i>utility</i>
$z$	<i>unemployment benefit</i>