

# Cooperative games and mechanisms for division problems

Citation for published version (APA):

Gong, D. (2022). *Cooperative games and mechanisms for division problems*. [Doctoral Thesis, Maastricht University, Northwestern Polytechnical University]. Maastricht University. <https://doi.org/10.26481/dis.20220927dg>

## Document status and date:

Published: 01/01/2022

## DOI:

[10.26481/dis.20220927dg](https://doi.org/10.26481/dis.20220927dg)

## Document Version:

Publisher's PDF, also known as Version of record

## Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

## General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.umlib.nl/taverne-license](http://www.umlib.nl/taverne-license)

## Take down policy

If you believe that this document breaches copyright please contact us at:

[repository@maastrichtuniversity.nl](mailto:repository@maastrichtuniversity.nl)

providing details and we will investigate your claim.

# Impact of the thesis

Division problems are common in the real world, and most relevant work is devoted to find a fair allocation when the allocated resource is insufficient for the agents with specific demands. This thesis focuses on two classes of valuable division problems: bankruptcy problems and division problems with single-dipped preferences. Bankruptcy problems study the situations where an insufficient estate is allocated among several claimants, each of whom has a claim on the estate. For example, when a company goes bankrupt, how should an authority liquidate the residual assets of the company among its creditors. Another example is how to allocate one's heritage among his heirs, when their debts are totally more than the amount of inheritance. Division problems with single-dipped preferences consider the problems of allocating one unit of an infinitely divisible commodity among agents with single-dipped preferences. A single-dipped preference has a worst point, the dip, and preference strictly increases in both directions away from the dip. Such a preference may arise, for instance, when allocating time between two types of work and an agent prefers spending time on only one of the two instead of on a combination – think of research versus teaching at a university. Another example is a two-goods exchange economy with fixed prices and a strictly quasi-convex utility function, which induces a single-dipped preference on the budget line.

This thesis provides new mechanisms to deal with these division problems. Following these mechanisms, some reasonable allocations can be achieved in Nash equilibria of the induced non-cooperative games. In bankruptcy problems, the allocation of the constrained equal awards rule is implemented. In division problems with single-dipped preferences, a selection of the Pareto social choice correspondence is implemented, namely picking the Pareto optimal allocations that are characterized by so-called maximal coalitions: this means that outside agents

prefer getting zero over equally sharing the one unit with the agents in the coalition, whereas for agents in the coalition the opposite holds.

Moreover, the new class of two-bound core (cooperative) games is introduced in this thesis, where the core is nonempty and can be described by a lower bound and an upper bound on the allocations. Many games are two-bound core games, including additive games, all balanced games with at most three players, unanimity games, bankruptcy games, 1-convex games, big boss games, clan games, compromise stable games, and reasonable stable games. The core, the nucleolus, and the egalitarian core are studied on this new domain. On the one hand, new expressions of these solutions are provided, which make the calculations of these solutions easier. On the other hand, based on associated reduced game properties, new axiomatic characterizations of these solutions on the new domain are provided.