Cooperative games and mechanisms for division problems

Summary

This thesis involves cooperative games, non-cooperative games, and mechanism design. We introduce a new class of cooperative games, and study several important solutions in this domain. Then, we design mechanisms for division problems, and consider equilibria in the induced non-cooperative games.

Quant et al (2005) studied the class of compromise stable games where the core coincides with the core cover (Tijs and Lipperts, 1982). The core cover is the set of pre-imputations between a specific pair of bounds. We generalize the approach of Quant et al (2005) to all games where the core equals the set of pre-imputations between an arbitrary pair of bounds, which we call two-bound core games. We show that the core of each two-bound core game can be described equivalently by the pair of exact core bounds (Bondareva and Driessen, 1994), and study to what extent the exact core bounds of a two-bound core game can be stretched while retaining the core description. We provide explicit expressions of the nucleolus (Schmeidler, 1969) and the egalitarian core (Arin and Iñarra, 2001) for two-bound core games in terms of the exact core bounds. We also show that the egalitarian core for two-bound core games is a single-valued solution. Then, we study Davis-Maschler reduced games of two-bound core games. Based on associated reduced game properties, we axiomatically characterize the core, the nucleolus, and the egalitarian core for two-bound core games.

In addition, we design mechanisms to solve bankruptcy problems and division problems with single-dipped preferences. We consider a sequential partition method for bankruptcy problems. The idea of this method is that claimants gather and successively partition the estate in a given order. On the basis of the ascending order of claims, a divide-and-choose mechanism and a divide-and-object

mechanism are designed. For each non-cooperative game induced by our mechanisms for bankruptcy problems, we show that the unique Nash equilibrium outcome is consistent with the allocation of the constrained equal awards rule. Then, we consider a mechanism for division problems with single-dipped preferences, which allocates one unit of an infinitely divisible commodity among agents reporting a number between zero and one. Nash, Pareto optimal Nash, and strong equilibria are analyzed for the games induced by our mechanism. We show that when the mechanism is anonymous, monotonic, standard and order-preserving, the Pareto optimal Nash and strong equilibria coincide and assign Pareto optimal allocations that are characterized by so-called maximal coalitions: non-involved agents prefer getting zero over an equal coalition share, whereas for agents in the coalition the opposite holds.