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Linking the BOPC growth model with foreign debt dynamics to the goods and labour markets: A BOP-IXSM-Okun model

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Abstract. We link the BOPC growth model to the goods market, foreign debt dynamics and Okun's law. A new condition for getting the Thirlwall effect of world GDP growth on domestic growth is that investment and exports should react less than savings and imports, all as a share of GDP, to an increase in the domestic growth rate. If this condition holds, the Thirlwall effect is present for stable and unstable debt/GDP dynamics and for positive or negative reactions of the current account to domestic growth. Okun's law translates the effect on the domestic GDP growth rate to a change of the unemployment rate. In unstable models, the change of world GDP growth may turn around the debt/GDP dynamics. Estimations support the specification of the theoretical model and lead to simulations of the Thirlwall effect and interest rate shocks on output growth. In the presence of banks consortia, unstable debt dynamics are the empirically relevant case for Brazil. A crisis can be less likely according to a simple model of profit maximizing bank consortia through a jump into a steady state for the debt/GDP ratio; unstable, increasing debt/GDP processes cannot be ruled out and may lead to crises unless the empirics of the stability conditions gets more favourable and leads the country-bank model into a stable steady state. Keywords: Balance-of-payments constrained growth; foreign debt dynamics. JEL code: F43, O11, O41.

1. Introduction

Traditional Keynesian models determine output in the markets for goods and money. Neoclassical models add a labour market and a production function and determine output from the supply side. Balance-of-payments constrained (BOPC) models determine output through a trade balance equation often extended by some exogenous variables also belonging to the balance of payments (Thirlwall 1979, 1983, 2011). The purpose of this paper is to integrate these approaches in a way that preserves the results of the BOPC approach.[2]

Thirlwall (1982) and McCombie (1985) noticed that, after Harrod (1933) there are orthodox Keynesian income-expenditure models of the goods market without balance-of-payments constraint and that there are balance-of-payments constrained models without goods market

[1] I am grateful to Danilo Spinola for inspiring discussions and useful comments.

[2] The BOPC approach has been extended by unit costs determining domestic prices in the presence of productivity effects from Verdoorn's law. The result is a model with two driving horses, productivity enhancement and world GDP growth (Blecker 2021a, b). It has this result in common with its neoclassical counterpart, the two-gap models in the tradition of Bardhan and Lewis (1970) based on imported capital goods. In our paper we ignore the emphasis on imported capital goods, which is a crucial mechanism of the model by Bardhan and Lewis (1970). Moreno-Brid (1998-99) emphasizes the importance of imports for imported capital goods when specifying adjustment processes. At this moment there is no need to improve models in regard to these aspects.

model. The recent literature has mostly continued the tradition of BOP constraint without non-trade parts of the goods market. Extensions of the BOPC approach to include foreign debt dynamics have been analyzed by way of imposing constant trade/GDP ratios. Moreno-Brid (1998-99, 2003) imposed a constant ratio of current account deficit to GDP. Barbosa-Filho (2001) has analyzed debt dynamics under the assumptions of a constant capital-inflow/GDP ratio, and constant export and import to GDP ratios. Bhering et al. (2019) add to this a discussion of dynamic changes in import/GDP ratios and prefer looking at a constant debt/export ratio rather than debt/GDP ratio. Our approach will derive all these constant ratios as an endogenous result from a model that integrates the BOPC line of thought with debt dynamics for the cases of debt stability and instability conditions. In particular, we provide a small model of banking consortia together with some evidence for Brazil and argue that banks do not necessarily exclude instability.

Given the emphasis of the BOPC approach as being Keynesian it is surprising that all its models are not explicitly linked to the goods market and also not to the labour market, an issue raised by Dutt (2002). Palley (2003) calls this an internal inconsistency. He suggests that capacity utilization adjusts imports or productivity growth and thereby balance supply and demand growth. This leads to supply growth determining long-run growth via Verdoorn's law. World GDP growth only affects capacity utilization when imports adjust, or productivity growth adjusts to world GDP growth. Moreover, there is no link to labour markets and no goods market equilibrium condition (see Ziesemer 2022). Dávila-Fernández et al. (2018) derive capital accumulation dynamics but there is not goods market (dis-) equilibrium assumption. There is an implicit assumption that output creates more employment without any need to consider the goods market, the center piece of Keynesian economics. The only exception is Dávila-Fernandez and Sordi (2019). They link the BOPC idea to a goods market equilibrium condition in a theoretical model. They impose that in the long run there is BOP equilibrium and therefore there is no debt accumulation in the long run and Thirlwall's law, $g_Y = \epsilon g_Z/\pi$,[3] rules. We deviate in this latter assumption allowing for debt dynamics with a link to BOPC, to the goods market, and to Okun's law. Thirlwall's effect turns out to hold in our model, but more in the generalized form of Keynesian multipliers than that of its special case of Thirlwall's law under these generalizations.

The neo-(neo)-classical labour market thinking has generated models to explain the long-run rate of unemployment. In the Walrasian model it is zero, but in bargaining models, search unemployment models or efficiency wage models and combinations of them the outcome is a long-run rate of unemployment (Pissarides 1998; Shapiro and Stiglitz 1984; Kronenberg 2010). However, in slumps and crises there is more unemployment and Okun's law is able to reflect this. We will therefore use Okun's law to link unemployment to the goods market.

In section 2 we will link the balance of payments equation to the goods market equation and Okun's law. We confirm the standard stability condition that interest rates need to be smaller than growth rates and derive an additional multiplier condition for Thirlwall's effect to hold; comparative static results from terms of trade and interest rate changes are derived. A crucial assumption for the model construction is that investment, savings, imports, and exports are all

[3] g_Y, g_Z are the growth rates of domestic and world GDP, ϵ is the income elasticity of exports, and π is the income elasticity of imports.

formulated as share of GDP and that these shares depend on the growth rates of domestic or/and foreign GDP, terms of trade and interest rates. In section 3 we present evidence for crucial properties of this type a model formulation using data for Brazil and explain it in terms of model development under the influence of time-series econometrics and empirical macroeconomics, which are sometimes even used as synonyms. Section 4 uses the empirical model with autoregressive, weakly exogenous terms of trade to run simulations confirming the Thirlwall effect on domestic output growth, as well as showing effects from shocks on terms of trade and interest rate. Section 5 further discusses the results in comparison to the literature. Section 6 sets up a small model of interest-setting bank consortia steering the economy in order to show that banks may allow debt dynamics to be stable or unstable and that jumping into a steady state is a possibility in case of instability, but loan pushing with crises cannot be excluded. Section 7 summarizes and concludes.

2. A BOP-IXSM-Okun model

The goods market equilibrium condition for open economies shown in equation (1) requires that the difference between investment, i, and savings, s, equals that of imports, m, and exports, x, of goods and services, all expressed as share of GDP. Both sides would be equivalent formulations for new foreign debt if savings and the current account would include in addition net factor income from abroad (net primary income in WDI definition) and net current transfers from abroad (net secondary income). Alternatively, they could be cancelled on both sides leading to a different definition of savings. We consider the goods market equilibrium and therefore use savings and the current account without transfers and factor income from abroad.

$$i(r,p,g_Y) - s(r,p,g_Y) = m(r,p,g_Y) - px(p,g_Y,g_Z) \tag{1}$$

In (1), r is the real interest rate, p is the terms of trade index (dimension free), which can be set to unity as long as prices are constant, g_Y is the growth rate of domestic GDP, and g_Z is the exogenous growth rate of world GDP. Having the interest rate in the import function is different from the BOPC literature, but in line with the theory in Turnovsky (1977), chapter 9, and the evidence of Fair (2018); imports consist of consumption and investment goods and they typically depend on interest rates. The export function includes the growth rate of the GDP because some economists would not see it as a demand function but rather argue that there are market segments in which high supply (growth) would simply lead to more exports. If this is not true, its impact is zero. In the theoretical model of this section, we will assume for simplicity and in line with the BOPC literature that not only world GDP growth but also terms of trade and the interest rate are exogenous. Debt dynamics is written as equation (2) with d-dot as the change of the debt/GDP ratio in a period of time *dt*.

$$\dot{d} = m(r,p,g_Y) - px(p,g_Y,g_Z) + (r - g_Y)d \tag{2}$$

The stationary line for the BOP equation (2) for no change in the *d=D/Y* ratio, $\dot{d} = 0$, in g-d plane has slope

$$\frac{\partial d}{\partial g_Y} = -\frac{m'-px'-d}{r-g_Y}$$

Assuming $r - g_Y < (>)0$, and $m' - px' - d < (>)0$, leads to four cases shown in Table 1.

Table 1: Four cases for goods market equilibrium and debt dynamics.

Current account-growth reaction	Stable debt dynamics $r - g_Y < 0$	Unstable debt dynamics $r - g_Y > 0$
$m' - px' - d < 0$	Case 1	Case 2
$m' - px' - d > 0$	Case 3	Case 4

Focusing on the debtor case of $d > 0$, on the $\dot{d} = 0$ line, stability implies $m - px > 0$ and instability implies $m - px < 0$.

We deal with cases 1 and 2 in Figure 1. In panel (a) for case 1, the BOP line is negatively sloped. We draw the slope as being more negative for high d and low g_Y because the denominator of the slope formula is smaller there. The goods-market-equilibrium condition is independent of d and therefore it is vertical in g-d plane. Both lines are drawn in figure 1, panel (a). In case 2, shown in panel (b) of Figure 1, the BOP line is upward sloping and flatter for lower d and lower g_Y.

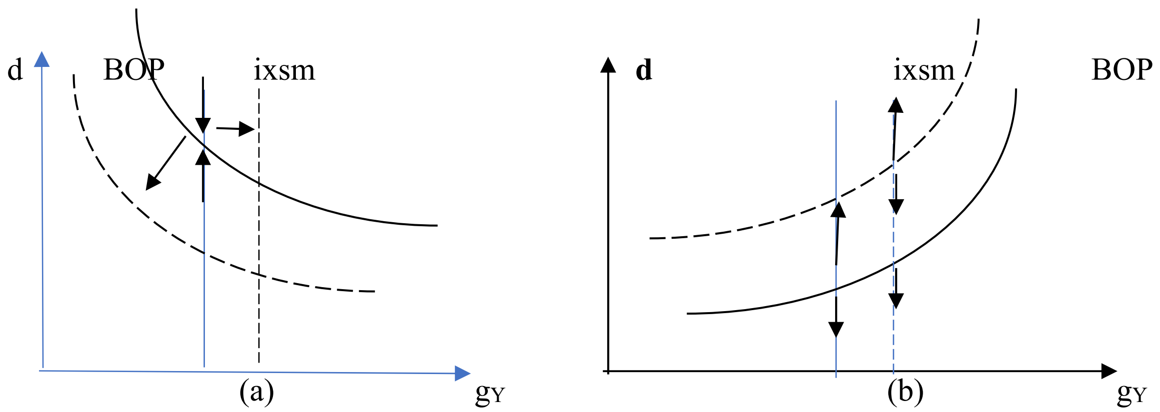

Figure 1 The Thirlwall effect in the stable case 1 and the unstable case 2 of the BOP-IXSM model, both with $m' - px' - d < 0$: Higher world GDP growth leads to higher domestic growth in both cases and lower (higher) steady-state debt/GDP ratio in case 1 (2).

In early Keynesian models (see Dornbusch 1980) and the BOPC literature there is a wide-spread view that output reacts to goods market disequilibrium, or to current account disequilibrium, when investment, savings and foreign debt are dropped (Nishi 2019; Alonso and Garcimartín 1998). Here we take this to its extreme form of an infinitely quick move of the growth rate ensuring that we are always on the goods-market equilibrium line. In the stable case 1, drawn in Figure 1, panel (a), for d above the BOP line, d is falling and moving on the goods market-equilibrium line, *ixsm*, to the intersection point. In the unstable case, drawn in Panel (b), d is increasing above the BOP line and falling below the BOP line. Vertical arrows along the *ixsm* line indicate the debt/GDP movements on the goods market equilibrium line.

The Thirlwall idea can now be analyzed by checking the effect of higher g_Z. We can see that higher g_Z, leading to a higher x, requires a higher g_Y if *i'-s'- m'+ px'< 0* in the goods-market-equilibrium equation (1), where a prime indicates the derivative with respect to g_Y (with the special case *x' = 0* if domestic growth has no impact on exports), implying that the *ixsm* line shifts to the right in both panels of Figure 1. From the BOP equation (2) we can see that higher g_Z requires a lower debt/GDP ratio d if the stability condition, $r < g_Y$, does holds. The BOP line shifts down in panel (a) and up in panel (b). In panel (a) of Figure 1 the *ixsm* shift enhances g_Y and both shifts decrease the debt/GDP ratio. In panel (b) for case 2, the *ixsm* line shifts to the right under the same condition and increases g_Y. Because of r > g_Y the BOP line shifts up. If this shift is strong enough, the direction of an unstable upward movement of the debt/GDP ratio d is turned from upward moving to downward moving. This is a new second Thirlwall effect for the unstable model of case 2. Such growth out of the debt problems was one of the hopes after the 1982 debt crisis.

The stable case 3 and the unstable case 4 are dealt with in Figure 2. Vertical arrows indicate again movements of the debt/GDP ratio, *d,* on the *ixsm* line.

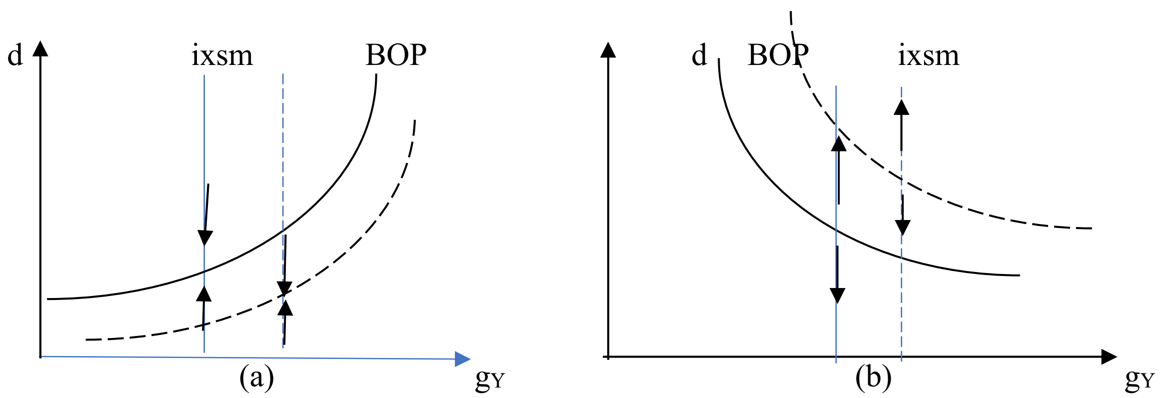

Figure 2 Thirlwall's effect in the stable case 3 in panel (a) and the unstable case 4 in panel (b), both with $m' - px' - d > 0$ in the BOP-IXSM model: Higher world GDP growth leads to higher domestic growth; effects on steady-state d depend on the size of the shifts of the stationary lines BOP and ixsm.

A change of g_Z requires higher g_Y under the assumption *i'-s'-m'+px'< 0,* implying a shift of the *ixsm* line to the right in both panels of Figure 2. Under the stability condition for case 3 in panel (a), the BOP line requires a lower value of d and therefore a shift of the BOP line down and to the right. The domestic growth rate increases, again confirming the Thirlwall effect, but the steady-state debt ratio may go up or down. Under instability of case 4 in panel (b), the *ixsm* line shifts again to the right under the condition *i'-s'-m'+px'< 0*, and the BOP curve must shift up and to the right because of the instability condition. Again, an unstable upward movement of the debt/GDP ratio may be turned around into a downward movement.

Most importantly, the Thirlwall effect holds in all four cases if *i'-s'-m'+px'< 0.* Otherwise the opposite effect would hold. Because of the evidence in the literature the opposite effect is mostly an implausible result.[4] Moreover, if output growth reacts positively to excess demand, *i'-s'-m'+px'< 0* is also a partial stability condition for the goods market because higher growth rates reduce excess demand.

The model can be extended by adding Okun's law. The unemployment rate is a function of its own lag and the domestic growth rate.

$$u = \gamma u_{-1} + \beta g_Y + \alpha; \quad \beta < 0, \gamma < 1 \tag{3}$$

Graphs of Figure 1 and 2 can be extended below the horizontal axis to have *u* on the vertical axis. The Thirlwall effect then changes the rate of unemployment. The long-run version is

$$\text{u} = \left(\frac{\beta}{1-\gamma} g_Y + \frac{\alpha}{1-\gamma}\right) \tag{3'}$$

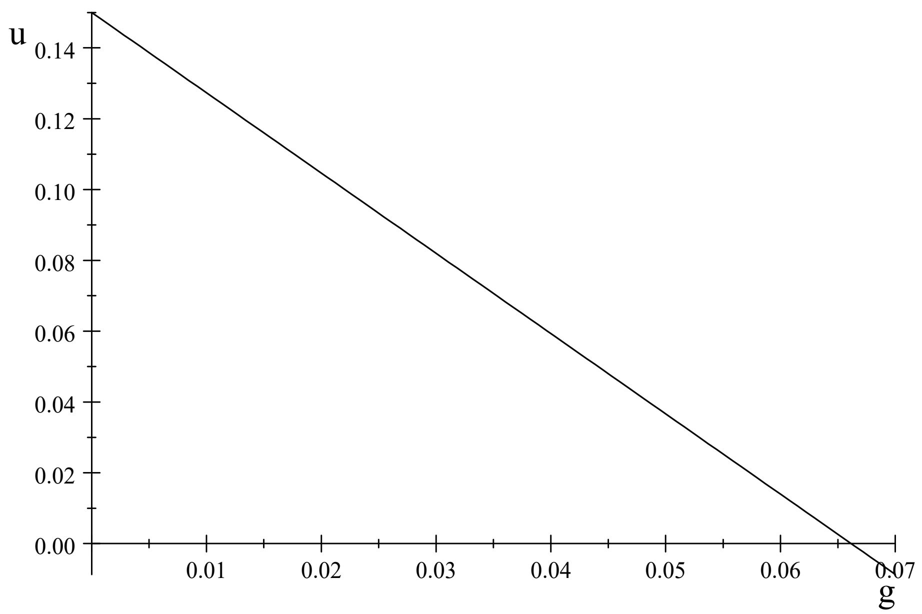

Figure 3a Okun's law for the long run for $\alpha = 0.0129, \beta = -0.193, \gamma = 0.915$.

[4] Zarembka (1972) has an interesting model example for the case of this opposite effect though. Export agriculture for banana's competes for land with domestic agriculture for vegetables, and an export boom drives up the prices for land and, as a consequence, prices for vegetables. This decreases growth rates of real wages in terms of vegetable prices. Of course, the example depends on having endogenous terms of trade, and is more relevant for countries where agriculture is still a large sector and export agriculture competes with domestically consumed goods. The scarcity of land constitutes an international version of the problem in Ricardo's growth model. An export tax can solve the problem of vegetable consuming wage earners. Similar combinations of export supply and export demand appear in Razmi (2016) and Massot and Merga (2022).

In an estimate for Brazil shown in equation (20) below we find $\alpha = 0.0129, \beta = -0.193, \gamma = 0.915$.[5] This leads to a slope of (-2.267) and an intercept of 0.15 drawn in Figure 3a.

Figure 3a shows Okun's law for the long run. With alternative hypothetical values of g_Y = 0.04, 0.03 or 0.02 from Figure 1 and 2 in all cases, we get the corresponding long-run unemployment rates of 5.93%, 8.2% and 10.47%.

In Figure 3b Okun's law is drawn for alternative growth rates of g_Y = 0.04, 0.03 or 0.02 in u-u(-1) plane; for the lowest growth rate we get the highest line according to (3). For any domestic growth rate g_Y obtained from one of the cases 1-4 of Figures 1 and 2, the unemployment rate then moves to the intersection of its Okun line with the 45-degree line. With goods market disequilibrium no such simple presentation is possible. A larger empirical model with goods market disequilibrium is shown in section 3.

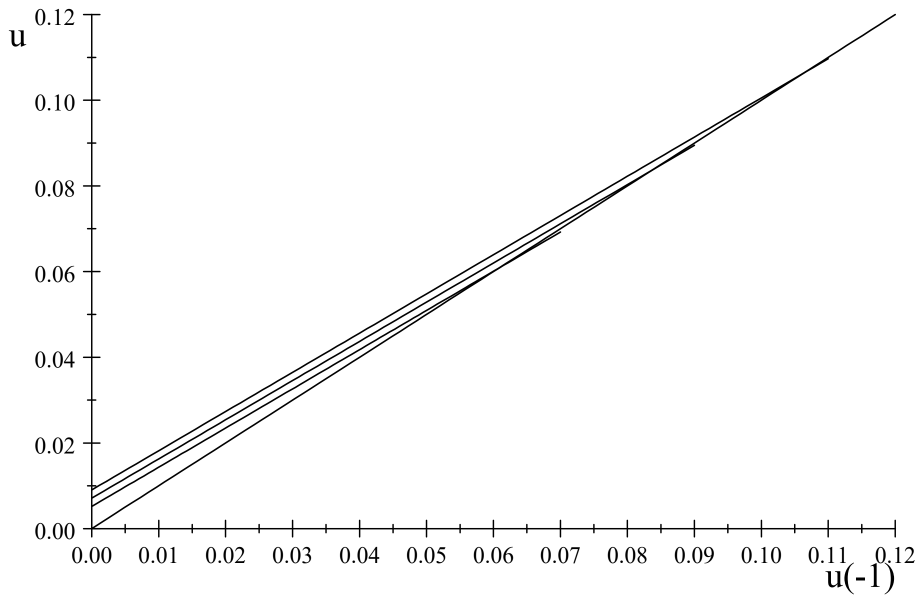

Figure 3b Okun's law for alternative rates of growth g_Y = 0.04, 0.03, 0.02 percent and the 45-degree line through the origin. Higher growth rates lead to lower intercepts and lower equilibrium unemployment rates.

When world income growth would be higher, we would get the Thirlwall effect of higher values of GDP growth in Figure 3a from Figure 1 or 2 (panel (a) or (b)), leading to lower values of long- run unemployment in Figure 3a, and shifting down the Okun-law line in Figure 3b, where the labour market moves to a new equilibrium unemployment rate. As we assume to be always in goods-market equilibrium, labour markets are slower in adjustment than goods market by construct but this is in line with Dornbusch's (1980) view on relative speeds of adjustment, where only asset markets are quicker than goods markets. Giblin (1930), cited by Thirlwall (1982) as one of his forerunners in ideas, spends a lot of space in his article on falling wages by x% leading to falling prices by (2x/3)% and hopefully more exports and employment. Wage adjustments slow down the effects in Okun's law and are

[5] León-Ledesma and Thirlwall (2002) and Muysken et al. (2015) show results for several developed countries.

included implicitly in the growth rate and intercept when showing us the changes of the unemployment rate.

Once steady-state values of growth rates are reached, imports, exports, and the trade deficit or surplus as share of GDP are also constant as assumed by Moreno-Brid (1998-99, 2003), and Barbosa-Filho (2001). With a constant debt/GDP ratio, *D/Y*, reached in the case of stability and a constant export/GDP ratio, *X/Y*, the debt/export ratio is also constant as it should according to the intuition of Bhering et al. (2019). All the plausible assumptions imposed by these authors have been derived here to be constant for the stable cases under the same conditions that support Thirlwall's result. Mathematically we could replace g_Y by the level of *Y* in the theoretical model above. But a constant value of *Y* would imply that world GDP, *Z*, must also be constant and would exclude growth although the whole spirit of BOPC growth does not allow for this by definition.

In order to get a formula that is comparable to Thirlwalls' law, $g_Y = \epsilon g_Z/\pi$, we differentiate (1) and (2) according to g_Y, *d*, and g_Z. In matrix form with x_Z as the derivation of *x* with respect to g_Z this results in

$$\begin{pmatrix} i' - s' - m' + px', & 0 \\ -m' + px' + d, & -r + g_Y \end{pmatrix} \begin{pmatrix} \partial g_Y \\ \partial d \end{pmatrix} = \begin{pmatrix} -x_Z \\ -x_Z \end{pmatrix} \partial g_Z \tag{4}$$

Cramer's rule then leads to effects for domestic growth and debt/GDP:

$$\frac{\partial g_Y}{\partial g_Z} = \frac{-x_Z}{i'-s'-m'+px'} > 0 \; for \; x_Z > 0, if \; i' - s' - m' + x' < 0; \tag{5}$$

$$\frac{\partial d}{\partial g_Z} = \frac{(i'-s'-m'+px')(-x_Z)-(-m'+px'+d)(-x_Z)}{(i'-s'-m'+px')(-r+g_Y)} = \frac{(i'-s'-d)(-x_Z)}{(i'-s'-m'+px')(-r+g_Y)} \tag{6}$$

Equation (5) generalizes Harrod's trade multiplier, the multiplier in Kennedy and Thirlwall (1979a) for exogenous exports and neglect of debt,[6] and Thirlwall's law, which would be the special case $\frac{\partial g_Y}{\partial g_Z} = \frac{x_Z}{m'}$ (obtained after dropping *i*, *s*, and *x'*) under our specification of functions, and is typically derived under the assumption of balanced trade or exogenous debt flows, which are endogenous here. The condition $-\frac{1}{i'-s'-m'-px'} > 0$ stated above is actually a necessary and sufficient condition for a positive trade multiplier effect. It is sometimes called supermultiplier in connection with the names of Hicks (McCombie 1985) or Sraffa (Nomaler et al. 2021).[7] If the denominator is negative, then a higher *i'* makes the multiplier larger. The period 1976-1982 showed a fall of world GDP growth from 5% to 0.4%.

The first equation in (6) shows the effect on debt in case of stability, $-r + g_Y > 0$. Under stability and a positive multiplier in (5), a sufficient condition for debt reduction is $-m'+x'+d > 0$, meaning that the current account surplus must react positively to the domestic growth rate. We can cancel $m'+px'$ in the middle part of (6) to obtain the last fraction. We get debt reduction if $i'- s'- d < 0$. If debt/GDP reduction through g_Y and savings react more strongly to domestic growth than investment, the debt/GDP ratios fall. But if investment reacts more

[6] Kennedy and Thirlwall (1979a) do not assume a balanced current account or equality of investment and savings and they ignore the implications of foreign debt or asset accumulation. Kennedy and Thirlwall (1979b) notice the debt flow but not debt accumulation and interest payments on debt incurred in the past.

[7] Vernengo (2016) associates the Sraffian supermultiplier with models distinguishing between savings of workers and capitalists.

strongly to domestic growth than savings and debt reduction through growth, debt/GDP ratios increase in case of stability.

For the development of the terms of trade there are two narratives, depending on the country samples chosen, when taking not too small samples (Ziesemer 2014). One type of studies finds a more or less continuous fall of a half percent per year. The other finds constant terms of trade with three downward jumps in 1921, 1938 and 1982. The story of the jumps allows us to keep the model simple and treat terms of trade effects again in the comparative static manner. Differentiating (1) and (2) by g_Y, d, and p we get

$$\begin{pmatrix} i' - s' - m' + px', & 0 \\ -m' + px' + d, & -r + g_Y \end{pmatrix} \begin{pmatrix} \partial g_Y \\ \partial d \end{pmatrix} = \begin{pmatrix} -(i_p - s_p - m_p + px_p + x) \\ m_p - x_p - x \end{pmatrix} \partial p \tag{7}$$

Using Cramer's rule again we get he effect of terms of trade on domestic growth and debt:

$$\frac{\partial g_Y}{\partial p} = \frac{-(i_p - s_p - m_p + px_p + x)}{i' - s' - m' + px'} > (<) 0 \tag{8}$$

if $\left(i_p - s_p - m_p + x(px_p/x) + 1)\right) > (<) 0$ for $i' - s' - m' + px' < 0$.

$$\frac{\partial d}{\partial p} = \frac{(i' - s' - m' + px')(m_p - x_p - x) - (-m' + px' + d)(-(i_p - s_p - m_p + x_p + x))}{(i' - s' - m' + px')(-r + g_Y)} \tag{9}$$

If the trade effects are stronger and the Marshall-Lerner condition holds, $\left(i_p - s_p - m_p + x(px_p/x) + 1)\right) < 0$, lower terms of trade as in 1982 for Brazil and many other countries lead to higher growth rates, for example, because lower terms of trade lead to more exports and more investment, and a disequilibrium adjustment. Below we show that empirically we have $s_p, m_p > 0$[8], exports are price inelastic in the short run but elastic in the long run, and $i_p > 0$, but very small. Because of the simplicity of the model keeping terms of trade exogenous, this ignores for the sake of simplicity that under the narrative for a continuous terms of trade fall, terms of trade might be driven by the domestic output as supply proxy and foreign income as demand force in levels, $p(Z, Y)$. A corresponding relation in terms of growth rates for all three variables is $g_p(g_Z, g_Y)$. Both equations are used with a positive effect of the first and negative effect of the second argument in the empirical model of Ziesemer (2022); see Ziesemer (1995a) for a theoretical model. Then, a fall in the terms of trade is not per se good or bad but rather it is crucial whether it occurs through an increase in supply growth or a sluggish demand growth as it happened to occur in 1982.[9]

For (9), a falling debt/GDP ratio when terms of trade fall, requires the sufficient condition $(-m'+px'+d) > 0$ (domestic growth leading to a reduction of the current account deficit), in

[8] The price effects for imports, and investment can be associated with effects on imported consumption goods as in Turnovsky (1977), chapter 9, and imported investment goods in Bardhan and Lewis (1970). Positive effects of terms of trade on savings are larger than those on income, which are in line with basic trade theory.

[9] Terms of trade growth fell in Brazil 1978-1983 where the early start came from an oil price increases. Brazil's growth was negative 1981 and 1983. World GDP had low growth only 1982. The exact causality seems to go from oil prices and anti-inflation policies of the US first to Brazil's GDP, then to the world economy and from there again to Brazil's GDP. This therefore is a combination of exogenous and endogenous terms of trade development.

case of stability, $(-r+g_Y)>0$, the multiplier condition $(i'-s'-m'+px')<0$, the Marshall-Lerner condition $\left(m_p-x_p-x\right)>0$, and the goods market condition $\left(i_p-s_p-m_p+x_p+x\right)<0$. Of course, if other conditions hold, debt may increase when the terms of trade fall.

The fall of world GDP and of the terms of trade for many countries did go together with an increase in world market interest rates driving up domestic ones. Differentiating (1) and (2) by g_Y, d, and r we get

$$\begin{pmatrix} i'-s'-m'+px', & 0 \\ -m'+px'+d, & -r+g_Y \end{pmatrix}\begin{pmatrix} \partial g_Y \\ \partial d \end{pmatrix}=\begin{pmatrix} -(i_r-s_r-m_r) \\ m_r \end{pmatrix}\partial r \tag{10}$$

Using Cramer's rule again we get the effect of interest rates on domestic growth and debt:

$$\frac{\partial g_Y}{\partial r}=\frac{-(i_r-s_r-m_r)}{i'-s'-m'+px'}>(<)0 \text{ if } (i_r-s_r-m_r)>(<)0 \text{ for } i'-s'-m'+px'<0. \tag{11}$$

$$\frac{\partial d}{\partial r}=\frac{(i'-s'-m'+px')(m_r)-(-m'+px'+d)(-(i_r-s_r-m_r))}{(i'-s'-m'+px')(-r+g_Y)} \tag{12}$$

Growth will be reduced by an increase in interest rates if $i_r<0$ dominates in (11) under the multiplier assumption $i'-s'-m'+px'<0$. For (12), a lower debt/GDP ratio will be obtained under the sufficient conditions (i) $-m'+x'+d>0$ (domestic growth leading to a reduction of the current account deficit), (ii) in case of stability, $(-r+g_Y)>0$, (iii) the multiplier condition $(i'-s'-m'+px')<0$, (iv) the import-interest relations $m_r<0$, and (v) the goods market condition $(i_r-s_r-m_r)<0$. Of course, if other conditions hold, debt may increase when the interest rate increases. All these comparative static results have been derived under the somewhat optimistic assumption of stability, because for instability comparative static reasoning is of limited value.

3. The specification problem for the estimation of share functions: data, estimation, and results for Brazil

The analysis above is made possible by way of specifying the functions *i*, *s*, *m*, and *x* in a way that they can be constant in a steady state with constant values for the debt/GDP ratio, growth rate of GDP, interest rate, and terms of trade. Having these variables as shares of GDP is what most papers do, but specifying the shares as functions of the GDP *growth rate* is not what other papers do and therefore we have to provide evidence for this. In the early 1980s, macroeconomic textbooks such as Turnovsky (1977) or articles such as McCombie (1985) used to specify functions as linear in levels, especially in orthodox Keynesian models. An investment function would be written as $I = aY + bR$, with R as interest rate. In modern chapters of advanced books like Dornbusch (1980), lowercase letters were used, defined as logs of the levels in more recent linear models (see also Thirlwall 1983; McCombie 1985; Blecker 2021a, b). This probably happened under the impression of the success of the log-log approach in empirical econometrics besides pragmatic advantages. Reformulating the log-log approach into levels results in functions like the typical import function in the BOPC literature, M= $p^{\eta}Y^{\rho}$, where the exponents are income and price elasticities. In the econometric estimations, including the many VAR and vector-error correction models in the

BOPC literature,[10] the linear log-log form is extended to include lags of all variables, not only the lagged dependent variable. The econometric reason to test for lags of all variables in an equation is that estimated expectations and adjustment processes have led to lags in the models (Pesaran 2015). Moreover, to be sure about problems from unit roots and cointegration it is recommended to include lags of all variables if statistically significant (Maddala and Kim 1998). Use of these assumptions in a traditional simultaneous equation model (SEM) with instrumental variables, which is still the leading method for SEM estimation (Wooldridge 2013), minimize the difference with a VAR or VECM approach. In those approaches it is a habit not to drop insignificant lags. They are made for low numbers of variables: 2 to 8 in SVARs in Kilian and Lütkepohl (2017) and five in the VECMs of Jusélius (2006) with a maximum of ten (Pesaran 2015). When more variables are required because of relevant interactions between variables, traditional simultaneous equations models are still the standard (Wooldridge 2013; Pesaran 2015). In log-log form with lags, the above equations may include, for example, *log(Y)* and *log(Y(-1))* and perhaps higher lags. If these variables have coefficients with opposite signs and about equal size, we actually have growth rates in the equations. The test for our model specification therefore is showing opposite signs and about equal size of coefficients or directly the statistical significance of growth rates.

Using data for Brazil from World Development Indicators we denote imports, exports, investment, savings (not including net factor income and transfers from abroad), and GDP as *M*, *X*, *GFCF*, *S,* and *Y,* and measure them all in constant local currency units. *U* is the unemployment rate, where gaps have been filled interpolating from Okun's law. Terms of trade, *p*, are exports of goods and services as capacity to import divided by exports in constant local currency units; *p* (2010) =1, corresponding to the zero intercept of (19) below. 'log' indicates the natural logarithm. *D* indicates a difference. Residuals are denoted as *v* without index for the equation. In case of autocorrelation of order *p*, $v = av_{-p} + \epsilon$, we append av_{-p} to the corresponding equation without indication of the equation number. The estimation method is the Generalized Method of Moments with heteroscedasticity and autocorrelation consistency for system (13)-(21) below. The estimation period is 1965-2019; 2020 is excluded to avoid end-of-sample bias from COVID pandemics. Instrumental variables and number of parameters per equation and econometric details are shown in an appendix. We get the following estimation results with p-values in parentheses or not shown if all p = 0.0000.

d(LOG(M/Y)) = -1.31 - 0.966LOG(M(-1)/Y(-1)) + 1.488LOG(P) + 0.465D(LOG(Y(-1))) –
(0.0000) (0.0000) (0.0000) (0.0000)

0.497LOG(1+R) + 0.252LOG(M(-4)/Y(-4)) + 0.36v_{-1} -0.424v_{-4} (13)
(0.0001) (0.0000) (0.0000) (0.0000)

log(X/Y) = -0.159 + 0.93log(X(-1)/Y(-1)) - 0.135LOG(P(-1)) +1.948d(LOG(Z)) –
(0.0) (0.0000) (0.0015) (0.0045)

[10] See Ziesemer (2022) for a list of references.

$$1.295d(LOG(Z(-1))) - 1.417d(\log(Y)) + 0.537d(\log(Y(-1))) - 0.2941v_{-2} \quad (14)$$
(0.0006) (0.0000) (0.023) (0.0007)

$$LOG(GFCF/Y) = -0.438 + 0.754LOG(GFCF(-1)/Y(-1)) - 0.183LOG(P(-1)) + 0.35LOG(P(-2))$$
$$+1.877D(LOG(Y)) + 0.884D(LOG(Y(-1))) - 0.099LOG(1+R(-1)) - 0.517v_{-5} \quad (15)$$

$$LOG(S/Y) = -0.675 + 0.76LOG(S(-1)/Y(-1)) + 0.108LOG(P(-1)) + 0.335LOG(P(-2)) +$$
$$3.14D(LOG(Y)) + 1.063LOG(1+R) - 0.233LOG(1+R(-1)) + 0.22v_{-3} - 0.451v_{-4} \quad (16)$$

The export/GDP ratio has a small price elasticity of -0.135 and its long-run value is -0.135/(1- 0.93) = -1.93, which is in the inelastic range in the short run but in the elastic range in the long run. Moreover, terms of trade increase imports in (13), investment in (15), and savings via an income effect in (16), which enhances savings more than income. Falling import prices increase investment, probably because that is an incentive for importing capital goods. This may point to a role for imported capital goods. Imported capital goods could be treated separately from other imports and domestic capital goods, which is beyond the scope of this paper and done theoretically in Bardhan and Lewis (1970) and Dutt (2002), and empirically in Ziesemer (2022).

Interest rates do play a role in savings and investment equations (16) and (15), and also in the import equation (13) because imported goods are either used for consumption or investment (Fair 2018).

GDP growth rates have positive effects on the ratios of imports, investment, exports[11] and savings (%GDP). The specification with interest rates, terms of trade, and GDP growth rates instead of levels of domestic GDP suggested in the theoretical part of the paper works quite well in equations (13)-(16). Ignoring the details of the specifications, (13) to (16) could therefore be abbreviated intuitively as *m(m(-1); r,p,g), x(x(-1); r,p,g; gz), i(i(-1); r,p,g), s(s(-1); r,p,g)*. In the long run, the lagged variables may be equal to the non-lagged ones leading to the expressions in (1). In this empirical perspective, the theoretical model therefore is a steady-state model as most BOPC growth models (Nishi 2019)[12] provided terms of trade, interest rates and GDP growth rates get constant over time. This type of models should be

[11] Some authors consider exports as supply function rather than demand function for times where production is higher than demand. Razmi (2016) and Massot and Merga (2022) replace income in the export function by capital thus also emphasizing supply. This requires an assumption of the possibility of supply segmentation for domestic goods and price differentiation. Massot and Merga (2022) show for Argentina that the export supply assumption alone does not predict growth well. We use the argument here in combination with export demand.

[12] Massot and Merga (2022) also use out-of-steady-state reasoning. Their emphasis on billing in dollars draws on the fact that nominal and real exchange rates differ from each other, real (effective) exchange rates differ from PPP for several years and from our terms of trade definition. In steady-state analyses these should all be very similar.

used for empirical work only with caution taking into account this long-run property. Conceptually, nee possibility would be to use the long-run versions of the functions *m(r,p,g), x(r,p,g; gz), i(r,p,g), s(r,p,g)* and the goods market equilibrium condition as cointegrating equations in a VECM provided enough data are available, which is not the case for Brazil. Having a long-run framework is in line with the literature on BOPC growth. Transitional dynamics and cyclicality are included empirically in (13)-(16).

Another crucial assumption regarding the working of the theoretical model is that the growth rate of the GDP jumps to its equilibrium value ensuring that (1) holds at any moment in time. Of course, a slow movement is more realistic. Our estimation finds the following growth-disequilibrium multiplier through excess demand with, in addition, growth rates depending negatively on the previous GDP level, and world GDP. A time trend is not included because it is highly insignificant, with a small coefficient and the wrong sign for an interpretation as technical change. Moreover, growth rates are reduced by the (expected) unemployment rate in line with Weitzman (1982). Similar to the theoretical model by Nishi (2019), this is a feedback from the labour market to the goods market. Finally, the growth rate of investment helps explaining growth.

D(LOG(Y)) = -1.94 + 0.773(GFCF - S + PX - M)/Y) - 0.35LOG(Y(-1)) + 0.381log(Z) +
(0.0013) (0.000) (0.0000) (0.0000)

0.305D(LOG(GFCF)) -4.63U(-1)^2 + 0.296v$_{-5}$ (17)
(0.0000) (0.0093) (0.0334)

In case of goods market equilibrium, constant unemployment and growth rates, differentiation of (17) yields Thirlwall's law *dLOG(Y(-1)) = (0.381/0.35)dlog(Z)*. Domestic growth is *1.09dlogZ*, nine percent larger than world GDP growth. We do not have an explicit solution for $g_Y(r,p,g_Z)$. We assume in (17) that *r* and *p* enter only through the multiplier terms whereas world GDP growth goes also into the long-run solution.

To keep the model simple, we model real interest rates, *r*, via an autoregressive process assuming it is given from the world market and an excess demand term with investment minus savings ignoring net factor income and transfers from abroad.

LOG(1+R) = 0.0733 + 0.894LOG(1+R(-1)) - 0.422LOG(1+R(-2)) + 0.277LOG(1+R(-3))

+ 0.641(-S(-2)/Y(-2) + GFCF (-2)/Y(-2)) (18)

For the terms of trade we assume that there is an autoregressive process for which we find the following regression result:

LOG(P) = -0.005 + 0.952LOG(P(-1)) - 0.102v$_{-3}$ - 0.175v$_{-5}$ (19)
(0.355) (0.0000) (0.2) (0.0055)

The plausible role of the growth rate of output in savings, investment, export, and imports as a share of GDP estimated in equations (13)-(16) supports the specification of these functions in equations (1) and (2). They allow to have the GDP growth rate g_Y, estimated in (17), on the horizontal axis of Figure 1, 2 and 3a, and linking Okun's law to it in the u-g_Y plane. The estimated version of Okun's law is as follows.

U = 0.0129 + 0.915U(-1) - 0.193D(LOG(Y)) (20)
(0.0016) (0.0000) (0.0145)

Adding population growth and the rate of technical change turns out to be statistically insignificant. A growth rate that would be higher by 0.01 would decrease the unemployment rate by 0.00193 in the short run and by a factor 11.7 times 0.00193, 2.267 percentage points in the long run. The dynamic process of world GDP can be expressed as an autoregressive process as follows.

LOG(Z) = 4.49 + 1.123LOG(Z(-1)) - 0.522LOG(Z(-2)) + 0.252LOG(Z(-3)) + 0.0042t (21)
(0.0000) (0.0000) (0.0003) (0.0088) (0.0000)

(21) allows to calculate a long-run growth rate for the world GDP of about 2.86 percent.

What a stylized parsimonious model like (1)-(3) does not take into account is the dynamics from lagged dependent variables as they appear in (13)-(21). Long-run elasticities are mostly much larger than short-run elasticities, by a factor 14 for exports and only marginally for imports, a factor three for savings, and a factor 4 for investment, all taken as a share of GDP. High coefficients of the lagged dependent variable have a strong impact in these calculations because their deviation from unity is used to divide the regression coefficient of the domestic and the world income growth rate, the terms of trade, and the interest rate in order to get their long-term effects. Imports, exports, terms of trade, unemployment and world income have near unit roots, for all but imports this is below 0.95 though.

It is a matter of research questions to have smaller or larger models. The empirical equations (13)-(21) will be used below as simulation model exploiting the dynamics for an analysis that is realistic in regard to time.

Possibilities for extensions of the model are as follows. Interest rates in (18) could be made dependent on deposit rates and foreign debt, possibly leading to foreign debt crises (Ziesemer 2022). The literature on domestically caused credit cycles recently summarized by Neves and Semmler (2022) could also be linked to the interest rate. Growth rates in (17) could be extended to include Verdoorn's law, R&D and human capital, as well as alternative approaches to technical change (Dávila-Fernández 2020), and population and resource growth (Sasaki et al. 2022). From imports in (13) and investments in (15) we could separate imported capital goods. The implication of such extensions is that equations and data for deposit rates, foreign debt, R&D, human capital, imported capital goods, carriers of technical change, drivers of population growth and resource use are needed. Ziesemer (2022) does this with 22 variables and equations and emphasis on the issue of imported capital goods. Here we keep the model as close as possible to the theoretical model of (1)-(3).

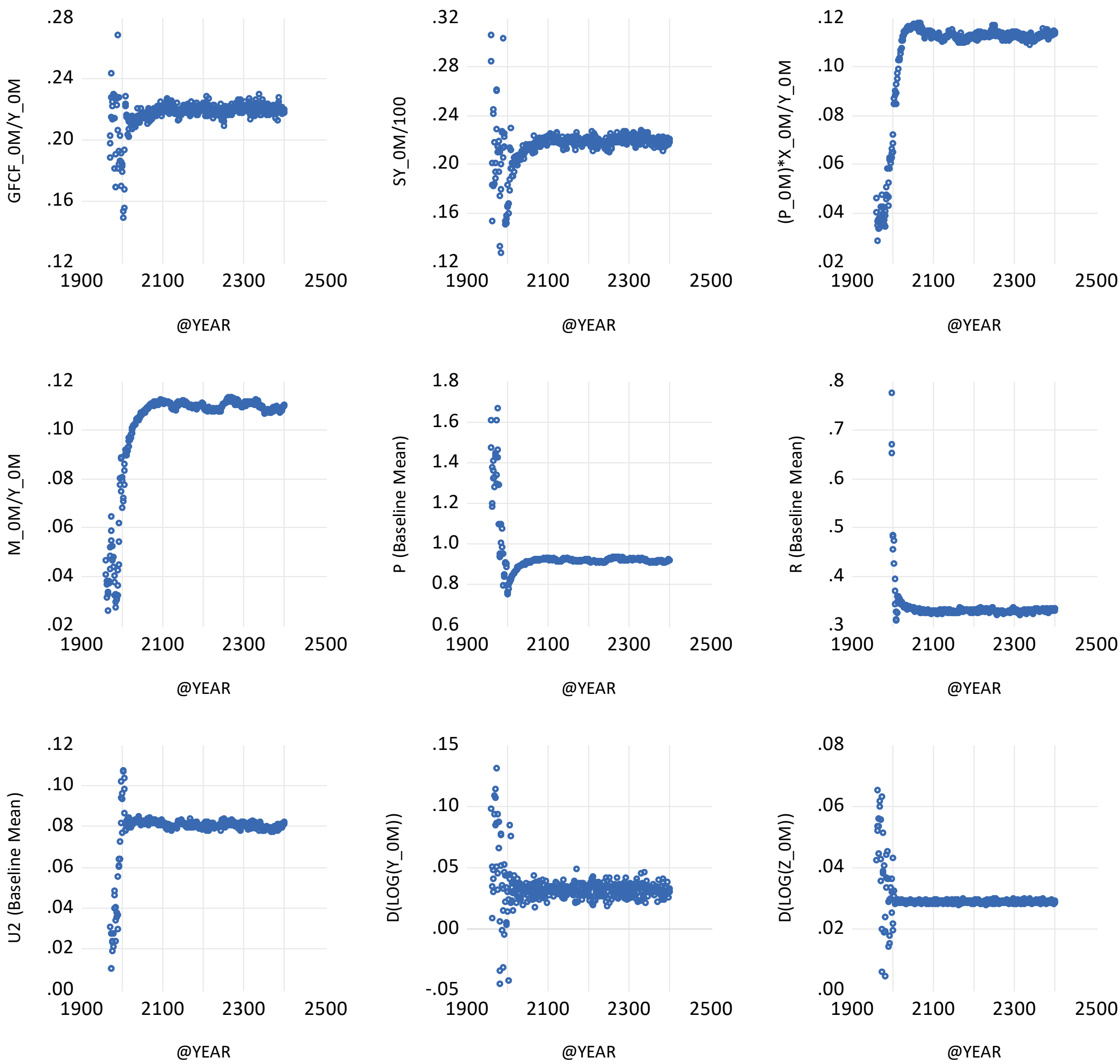

Figure 4 The dynamic, stochastic baseline simulation of equations (13)-(21) from 1000 repetitions shows stable values for all variables. '0M' on the vertical axis indicates baseline mean.

4. Baseline simulation and shocks

Stability of the empirical model is shown through a long-run baseline simulation for the period 2003-2400 in Figure 4. Figure 5 shows the goods market disequilibrium values (excluding planned changes in inventories) which are a regressor in equation (17). In the data range they are of a larger absolute size because they include shocks from oil price changes and the 1981-1983 debt crisis, Asian crisis, ICT boom and bust, and the recent Brazilian crisis; similar but currently unexpected events may still come up in the period after 2020. These results with convergence to stable stochastic values crucially depend on having modelled the terms of trade as an autoregressive process. In the following we show results for a shock on world GDP growth, terms of trade and interest rate. Also with these shocks the model can be solved until 2400. However, we also want to admit that the disequilibrium term does not return completely to zero as in Figure 5 for the baseline simulation, although growth rates have an impact on investment, savings, imports, and exports. The stabilization through output growth adjustment is not perfect. After a terms of trade shock, the disequilibrium in the goods market is between -3 and -4% of GDP. For the other shocks it is similar to Figure 5.

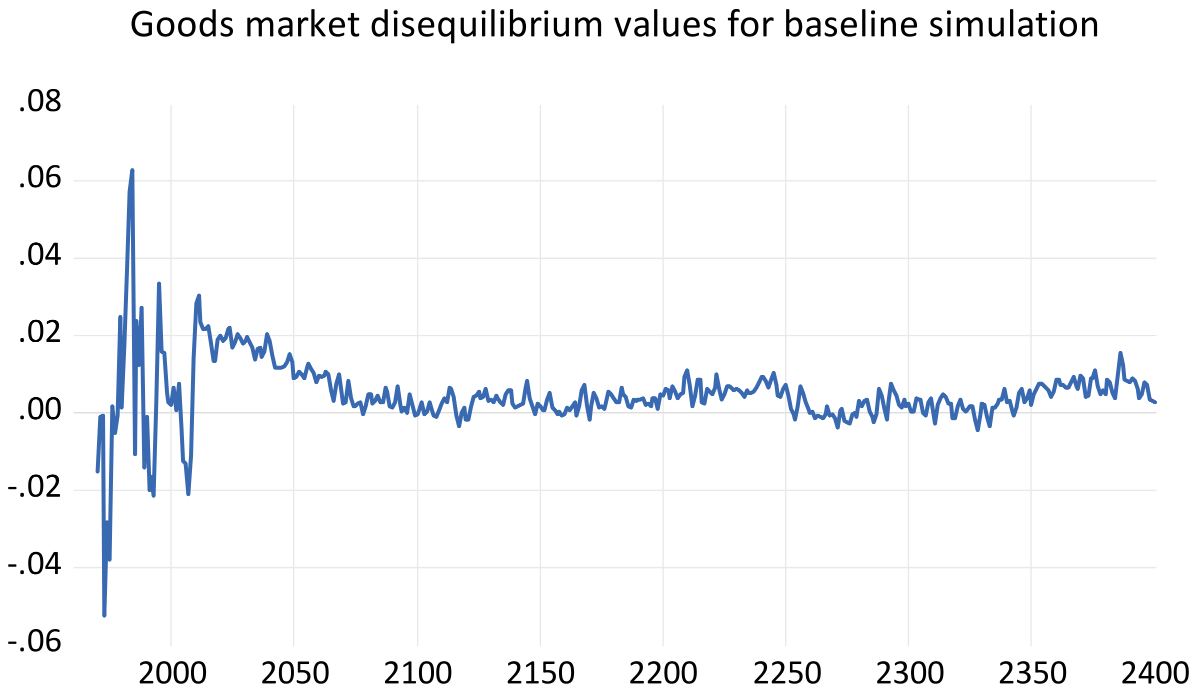

Figure 5 Goods market disequilibrium values are between (-0.053, 0.063) in our data and in an even smaller range in our simulation for the period 2020-2400.

4.1 A world GDP shock: The Thirlwall effect

Equations (13)-(21) are nine equations for nine variables measured on the right-hand side of Figure 6, 7, and 8. We add a shock of 0.005 on the intercept of equation (21) for the period until 2040 for which we indicate the size of the effects. All effects are shown in Figure 6. World income increases by up to 5390 billion dollars (lower line, left scale) but its growth rate, on which the intercept has no impact, returns to baseline. Exports at constant prices are higher by up to 45.6 billion real[13]. Terms of trade do not react as we have imposed an autoregressive process. Output is enhanced in a statistically significant way by 357 billion real; the effect works through the long-term effect of world GDP in (21) and through the multiplier.

The fall in the unemployment rate becomes statistically insignificant in 2023. Positive effects on investment are statistically significant until 2010. Investment increases up to 73.8 billion

[13] 1$ = 1.8 real in 2010; 1 real = 0.2 $ in 2020.

real, but this is just statistically insignificant after 2010. Imports increase statistically significantly by 37.1 billion real. The savings ratio and interest rates are back to baseline

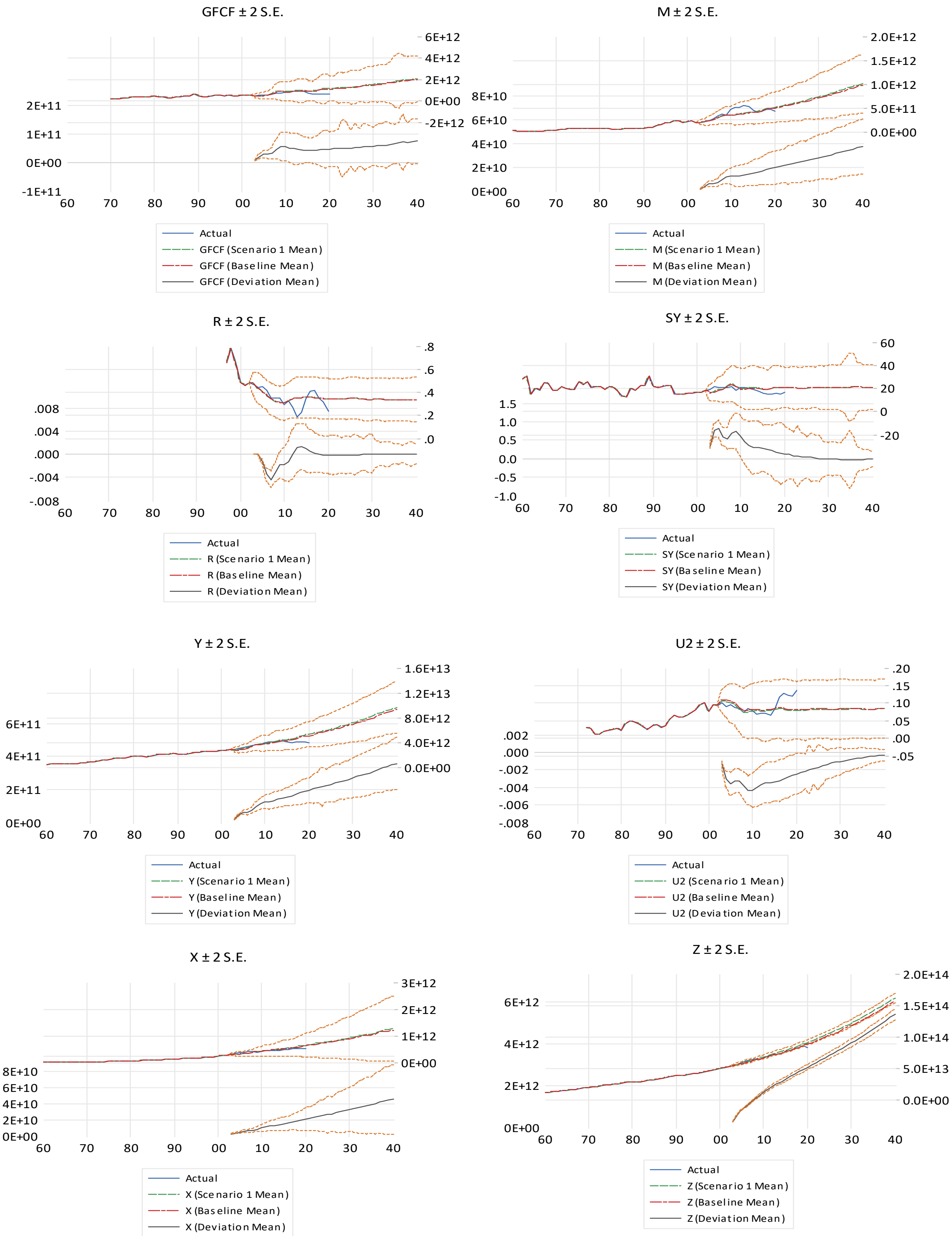

Figure 6: Baseline simulation and permanent shock on world GDP (0.005 increase of the intercept of (21)) until 2040. Baseline and shock scenario with confidence intervals shown in upper set of lines measured on the right-hand axes. Difference between baseline and shock scenario in the lower set of lines.

when increases of savings are not larger than those of investment anymore. The increase of exports is larger than that of imports and therefore the trade balance contributes to the reduction of foreign debt increases. These effects correspond to those of equations (5) and (6) for the comparative static effects in the theoretical model and for all the other variables in the model.

Figure 7 The effects of a terms of trade shock.

4.2 A terms of trade shock and its consequences

Increasing the intercept of the terms of trade equation by 0.005, a half percentage point again, we get the results of Figure 7. The increase in the terms of trade runs up to 0.082, more than eight percent beyond baseline. This decreases exports and investment and increases imports and savings, and leads to excess supply in the goods market. However, the effects on savings and investment are statistically insignificant. The negative disequilibrium multiplier reduces output (growth), and the increase of savings and falling investment reduces interest rates. The effects on falling interest rates are included as feedback effects in the simulation values: savings are reduced through them while investment and imports are enhanced. Unemployment rates increase by four tenths of a percent, 0.004, through growth reduction from the terms of trade shock. The fall of the terms of trade for Brazil 1977-1982 and for many countries around 1982 had the opposite effects because of a negative shock. However, the real change was a combination from oil shock, interest shock, and the subsequent world growth shock.

From here, it is possible to imagine what a permanent fall in the terms of trade does. When giving up the autoregressive process (19) for p and using a log-log specification for *P(Y,Z)*, it can be shown that we have falling terms of trade through (in absolute terms) a higher growth rate of Brazil multiplied by a negative coefficient compared to the growth rate of the world GDP multiplied by a slightly smaller positive coefficient. Export values would increase because in the long-run the value of exports is price elastic; investment, savings and imports (%GDP) fall. As output changes increase while shares of investment, savings and imports fall, we can conclude that export shares dominate the multiplier and increase domestic growth. As this would go more into the direction of the Prebisch-Singer literature rather than a Keynesian extension of the BOPC growth literature we do not present this extension here.

4.3 Interest shocks and their consequences

A positive change of the intercept of the interest rate equation by 0.01, similar to that around 1980, increases savings and decreases investment, both statistically insignificant though. Exports and imports are both decreasing statistically significant for a couple of years, but imports more so. The disequilibrium multiplier decreases output and unemployment increases statistically significantly for five or six years, which reduces exports even further. These results are shown in Figure 8.

5. Discussion

To get the Thirlwall effect when BOPC growth is merged with the goods market equilibrium, we need the partial stability assumption $i' - s' - m' + px' < 0$ for a positive trade multiplier. As i and x represent the demand side and s and m the flows of the supply side, the central assumption means that the supply side must respond more strongly to a higher GDP growth rate than the demand side to get Thirlwall's effect.

Debt dynamics are stable only if the stability condition $r < g_Y$ holds. As income elasticities of imports and exports have an impact on the current account, they must also have an indirect impact on the difference between investment and savings and capital flows. Effects of the exogenous variables world GDP growth, terms of trade, and interest rates all have a direct impact on the on i, x, s, and m and from there on the domestic growth rate, which feeds back to investment, savings, exports, and imports. This connects the four goods market items to each other.

Once the equilibrium value of $I/Y = i^*$ is endogenously determined, the factor supply could be added as in Dávila-Fernández et al. (2018), where i^* follows as equilibrium value of a capacity adjustment equation and deviations from i^* are assumed to be a function of capacity

Figure 8 The effects of a permanent interest rate increase.

utilization. Assuming $Y = AK$ as from a limitational production function,[14] and $\dot{K} \equiv I - \delta K$, they get $\dot{K} = i^{*}AK - \delta K$ as capital accumulation. In addition, in Oreiro (2016) capacity utilization and exchange rates adjust all variables to follow Verdoorn's law and the natural rate.

For a given labour force L we also get $Y = B(1\text{-}u)L$ from a limitational production function, with u obtained from (3). The limitational production function then determines the output level and $\dot{K} = i^{*}Y - \delta K$ drives the capital stock. Again, "capital stock always adjusts itself to the growth rate of real output that is given by BoP restrictions" (Oreiro 2016). The level of Y coming from a production function must be compatible with g_Y from (1). The complicated mechanisms of capacity utilization in Dávila-Fernández et al. (2018), Oreiro (2016), and Palley (2003) may help matching supply and demand side in terms of levels in the short and medium run (together with the lags in the empirical model and inventories in an extended version of the empirical model). Of course, this is not the major purpose of this paper.

Thirlwall (1979, 1983) defends the trade part of our model and Leon-Ledesma and Thirlwall (2000, 2002) defend Okun's law, but both separately so. It is a contribution of this paper that we bring them together by way of making the role of the goods market explicit in the BOP-IXSM-Okun model.

A theoretical model that is similar to ours is the IS-LM-BP model. We have replaced the LM part by exogenous and constant real interest and the exchange rate by exogenous terms of trade. We have added foreign debt dynamics linking up to the sustainability discussion by Moreno-Brid (1998-99), Barbosa-Filho (2001) and Bhering et al. (2019).

More complicated models with formulas similar to Thirlwall's law are those of Dutt (2002) for North-South relations and Bardhan and Lewis (1970) for developing countries, which can be connected to vertical or horizontal labour supply functions and perfect and imperfect international capital movements (Ziesemer 1995a,b, 1998). In Bardhan-Lewis models, *endogenous* terms of trade changes allow for more or less imported capital goods, and therefore price elasticities and elasticities of production of capital enter the long-run growth formula of the models besides world income and the income elasticity. Income elasticities of export demand appear also in the long-run growth formula for the terms of trade, indicating that price and non-price competitiveness are closely related. More generally, if all goods have different income and price elasticities of demand and different rates of technical change, terms of trade will also change in the long run. Massot and Merga (2022) stick to the small country assumption but endogenize the real exchange rate by emphasizing that billing takes place in foreign currencies, which helps in using the BOPC model successfully for Argentina. In this paper, we focus on the extension of Harrod's trade multiplier and Thirlwall's law as its dynamic analogue and therefore ignore other aspects of the more complicated models such as different ways to endogenize the terms of trade.

[14] *A* can be the product of capital productivity and capacity utilization. The dynamics of capacity utilization and unemployment rate open the link to business cycle research. Neoclassical economists can get a *Y/K* ratio from the marginal productivity condition of a CES function.

A major difference of our extension to the goods market is that without arbitrary assumptions on debt dynamics the Thirlwall effect comes about as a change in the whole goods market, and not only in the current account. This becomes visible through having exports and imports not only in the balance of payments, but also in the goods market equation. If we would drop investment and saving from the goods market equation (1), and debt and interest from the balance of payments (2), the goods market equilibrium condition and the balance of payments are the same, *M-X =0*. This is also implicit in equations (1)-(3) in Thirlwall (1982) and Thirlwall and Hussain (1982) where they reconstruct Harrod's trade multiplier. Therefore, it is methodologically somewhat arbitrary to emphasize the role of the balance of payments instead of the special case of the goods-market equilibrium condition. In other words, the introduction of capital inflows into the BOPC model reflects mainly[15] an assumption of an exogenous difference between investment and savings, both of which Harrod (1933) suppressed when deriving the trade multiplier. Our model re-introduces investment and savings, and, as they do not have to be balanced, it integrates foreign debt dynamics with interest payments coming in systematically.

The empirical model extends the theoretical one to allow for lagged dependent variables. It shows that in a formulation of investment, savings, exports and imports as a share of GDP these ratios are driven by domestic growth rates, interest rates, terms of trade, and world GDP. The disequilibrium multiplier enlarges the growth rate in case of excess demand. Shocks on world GDP growth rates, terms of trade, and interest rates have the expected effects and confirm that the model is empirically plausible in all investigated effects.

The empirical model leaves the debt dynamics implicit. The reason is that saving and the current account as used for debt dynamics are different from those of the goods market. They both have to include also net factor income and transfers from abroad. Including them and explaining them would make the empirical model much larger in terms of the number of variables and stochastic equations. Therefore we have abstained from this. In the case of Brazil our simulation in Figure 4 shows that investment roughly equal savings and exports roughly equal imports. In the data, interest payments are larger than net labour income and transfers from abroad. This leads to foreign debt accumulation in the case of Brazil. Equation (2) approximately is $\dot{d} = (r - g_Y)d$, and $r > g_Y$ in the data makes the dynamics unstable.

Unstable models of foreign debt dynamics, which we have not excluded so far call for some thoughts on who will get into trouble. Some authors have stated only that debt growth cannot go on forever and imposed exogenous restrictions avoiding the problem in the model without saying who is going to ensure this. Nomaler et al. (2021) assume that imports increase (decrease) whenever there is a trade surplus (deficit). Cohen (1991) assumes that creditors will limit deficits and debt accumulation. In crisis times we see that they do not impose these limitations. In contrast, in August 1981 it was Mexico, the debtor rather than the creditor, asking for renegotiations on the debt contracts. Who comes under pressure first and reacts

[15] To get this exactly, savings must include (unlike the goods market variable) net factor income and net transfers from abroad.

first in order to limit unstable debt processes is an open future-research question to which we make a small contribution linked to our model in the next section.

6. Do banks ensure stability of debt accumulation?

6.1 A simple model of banking consortia in debtor countries

In the 1982 debt crisis countries did negotiate their credit terms with bank consortia, who therefore have a monopolistic position on the credit market. The research question of this section is whether bank consortia will ensure the stability of the debt dynamics in equation (2) and avoid the instability cases of panel (b) of Figure 1 and 2. We assume that a bank consortium maximizes the discounted sum of expected future interest net revenues taking into account goods market equilibrium (1) and balance of payments equation (2) after management and owners of the banks and the consortium have agreed on a discount rate ρ. The choice of the interest rate and the implied growth rate from the goods market equilibrium condition (1) through the bank consortium then is steering the debt dynamics (2), and the banks consider the static and dynamic consequences of their interest setting, in particular that the growth rate g will react. The probability of getting the interest revenue *rD*, is assumed to be a function of the interest/GDP ratio, $\pi(rd)$; the higher the debt service the lower the probability that the country will pay, $\pi'<0$. In particular, the probability of paying decreases if debt is running up as in panel (b) of Figure 1 and 2. The higher the banks set the interest rate the higher the debt service obligation and bank revenues, the less likely the stability condition is fulfilled, and also the lower the probability that they will get the payment. This may provide an incentive to limit instability. With r_d as exogenous deposit rate paid for savings, we have the generalized Hamiltonian

$$H = \pi(rd)rD - r_dD + \lambda[m-px+(r-g_Y)d]+\mu(i-s-m+px)$$

The first-order conditions for the derivation with respect to the interest rate and the reacting growth rate are

$$H_r = \pi D\left(\frac{\pi' dr}{\pi}+1\right) + \lambda(m_r + d) + \mu(i_r - s_r - m_r) = 0 \quad (22)$$

$$H_g = \lambda(m' - px' - d) + \mu(i' - s' - m' + px') = 0 \quad (23)$$

The dynamic first-order condition for debt is

$$\dot{\lambda} - \rho\lambda = -H_D = -\left[\pi r\left(\frac{\pi' r}{\pi}\frac{D}{Y}+1\right) - r_d + \lambda\frac{r-g_Y}{Y}\right] \quad (24)$$

The transversality conditions require

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t) = 0, \text{ implying } -\rho + \hat{\lambda} < 0 \text{ (at least for a late phase)} \quad (25)$$

Solving (23) for μ we get

$$\mu = \frac{-\lambda(m' - px' - d)}{(i' - s' - m' + px')} \tag{23'}$$

Using (23') in (22) we get

$$\pi D\left(\frac{\pi' dr}{\pi} + 1\right) + \lambda(m_r + d) + \frac{-\lambda(m' - px' - d)}{(i' - s' - m' + px')}(i_r - s_r - m_r) = 0 \tag{22'}$$

(24) can be rewritten as

$$\frac{\dot{\lambda}}{\lambda} - \rho = -\frac{H_D}{\lambda} = -\frac{\pi r\left(\frac{\pi' rD}{\pi\ Y} + 1\right) - r_d}{\lambda} - \frac{r - g_Y}{Y} \tag{24'}$$

We assume that marginal revenue from increases in *r* and *D* is positive in (22) and (24) and at least as large as the deposit rate. It follows from the implication in (25) that the left-hand side of (24') is negative.

6.2 The case of unstable debt dynamics

If $\lambda < 0$ (discussed below), the first term on the right-hand side of (24') is positive and therefore the right-hand side can only be negative if $r > g_Y$, implying instability. In this case banks earn more if they set *r* high enough to allow for the instability, get high revenues through high *r*. In the unstable case when *d* is growing this implies that bank consortia accept a decreasing probability of getting the revenues and prefer compensating for this through higher interest rates.

Under which condition do we have $\lambda < 0$? With positive marginal revenue of higher interest rates, the first term in (22'), the sum of the other terms must be negative. $\lambda < 0$ then requires

$$(m_r + d) + \frac{-(m' - px' - d)}{(i' - s' - m' + px')}(i_r - s_r - m_r) > 0 \tag{26}$$

With a positive multiplier, $\frac{-1}{(i' - s' - m' + px')} > 0$, and a negative interest effect on excess demand on the goods market, $(i_r - s_r - m_r) < 0$,[16] sufficient conditions are $(m' - px' - d) < 0$ and $(m_r + d) > 0$. The first condition holds if higher growth rates improve the current account/GDP ratio, and the second condition is $d > -m_r$: The debt/GDP ratio is high relative to the impact of the interest rate *r* on the import/GDP ratio, *m,* implying that interest increases do not stabilize the current account and debt dynamics. The second condition essentially means that the impact of the interest rate on imports is weak relative to the impact of the interest rate on interest payments *rd,* which banks are earning, or the other way around, interest rate decreases would stabilize the economy only through less revenue for banks with stabilization weakened by higher imports. In sum, if growth cures the current account (%GDP) and interest rate increases do not, banks prefer to set $r > g_Y$. Expected revenues increase through interest increases although stability will not be achieved and increasing

[16] This drives output growth down in Figure 8 of the empirical model via the disequilibrium multiplier effect in (17).

interest rates do not stabilize the debt dynamics. As usual in macroeconomic models, the interest-debt effect may cause instability, and here we have shown that profit-maximizing bank behaviour does not outweigh it (i) if excess demand reacts negatively to interest increases, (ii) the current account deficit is reduced through growth rates, and (iii) imports react weakly to interest rate changes (sufficient).

If $\lambda > 0$, and (26) does not hold we have no indication regarding $r >(<) g_Y$.

Empirically, for the data of Brazil we find a negative correlation between the current account/GDP ratio and growth in Figure 9. Growth rates stabilize the current account/GDP. This implies that the first sufficient condition for $r > g_Y$ is fulfilled for Brazil.

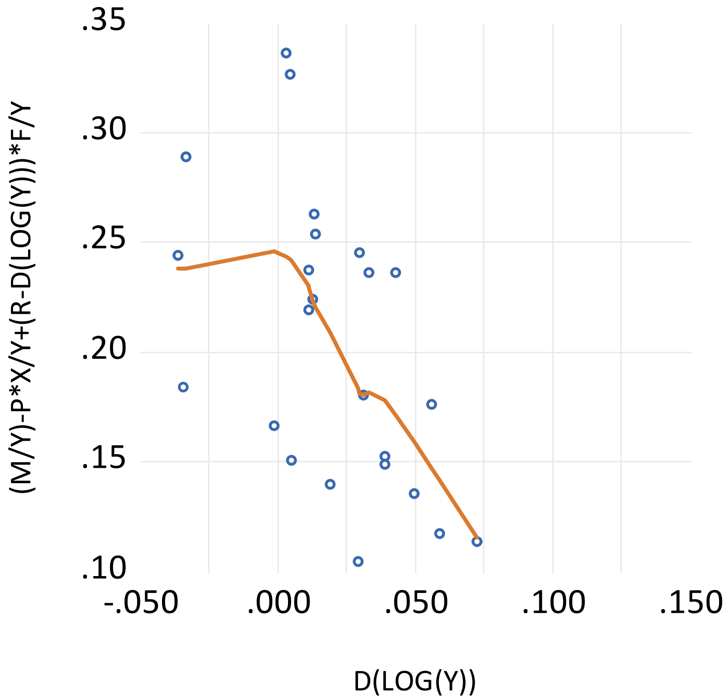

Figure 9: The correlation for Brazil's current-account/GDP ratio with GDP growth rates is negative for positive growth rates (nearest neighbour fit with bandwidth 0.6).

Moreover, the interest elasticity of imports is -0.492 according to (13). This leads to $d > -m_r = 0.492m/r$ as second condition for $r > g_Y$. Filling in data for Brazil shown in Figure 10, we find again that this condition holds for Brazil. Interest rate increases (decreases) do not (de)stabilize the current account/GDP ratio. But they would reduce (enhance) the growth rate according to (11) and the growth rate's stabilizing effect on the current account and debt dynamics. The two effects imply that it does not pay to reduce the interest rate below the growth rate to stabilize the economy, because (i) the direct effect of interest reduction is that revenues are decreasing and the current account/GDP ratio improves in rd which is mitigated through m_r, in spite of the indirect effect that growth rates increase and improve the current account/GDP ratio. As interest reduction and growth rate enhancement are two stabilizing benefits leading to a higher probability of getting revenues, the crucial conclusion is that this benefit from stabilization does not outweigh the reduced revenues from interest reductions under the two sufficient conditions.

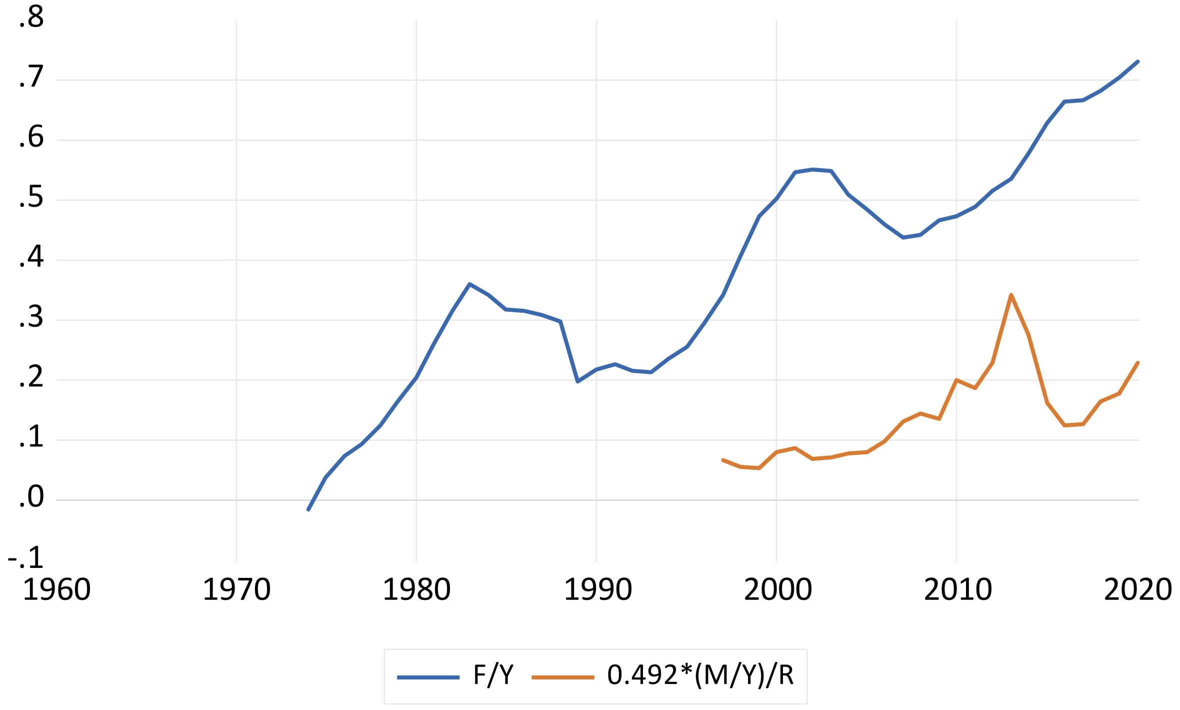

Figure 10 Interest rates do not stabilize the current account in the period 1997-2020 because d≡F/Y is larger than 0.492m/r, the absolute value of the impact of interest on imports.

Both sufficient conditions for $r > g_Y$ are fulfilled for Brazil during the period 1997-2020: higher growth rates and lower interest stabilize the current account (%GDP) under these conditions. The impact of interest rates on revenues may be worth more than trying to increase the probability of getting the payment as shown by the example of Brazil. In terms of Table 1 it follows that case 2 is empirically relevant for Brazil. If the economy is not in the steady-state of a constant debt/GDP ratio, the latter develops in an unstable manner under these conditions. Under conditions of instability, the only alternative to instability is being in the steady state.

6.3 Steady state requirements

Necessary conditions for the existence of a steady state, defined as all variables having constant growth rates, can be obtained from considering (22)-(24). Probabilities must have a zero growth rate. This requires constant *rd*. Partial derivatives of *m*, *x*, *s* and *i*, must be constant. For constant elasticities as in the empirical version of the model, this implies, for example, from (13) that $m_r = -0.492m/r$, which in turn requires constant *m* and *r*, and analogously constant *i*, *s*, and *x,* as well as *p* and g_Y. (22) then requires constant $\frac{\lambda}{D}, \frac{\mu}{D}$. (23) also, requires constant *p* and λ/μ. With constant *r*, we must have constant *d*, implying from the previous statements that

$$g_Y = \hat{Y} = \hat{D} = \hat{\mu} = \hat{\lambda} = \hat{M} = \hat{X} = \hat{I} = \hat{S}$$

These conditions imply $\hat{m} = \hat{x} = \hat{s} = \hat{\imath} = 0$.

If such a steady state exists, constant and positive d implies $(r\text{-}g_Y)d = px\text{-}m$ > (<) 0, or $d^* = \frac{px-m}{(r-g_Y)} > 0$. A trade surplus is ensured that allows for interest payments under an expansion of debt in proportion to GDP keeping the debt ratio d constant. This implies that the current account is in deficit. We can exclude the case d* < 0, because the banks then would have negative net earnings and our model is not relevant for this case; if countries succeed in having negative debt, they can avoid the monopoly power of bank consortia. The interest rate may be smaller or larger than the growth rate, implying that stability and instability are both possible through a positive or negative non-interest current account, but the economy may be in the steady state by assumption.

From (24) and (24') with $(\frac{\dot{\lambda}}{\lambda} - \rho)\lambda \geq 0$, because of the transversality condition and λ < 0, a constant right-hand side of (24) requires a constant left-hand side, which can only be achieved for constant $\frac{\dot{\lambda}}{\lambda} - \rho < 0$ which is compatible with the transversality condition.

(24) then leads to

$$-\left[\pi r\left(\frac{\pi' r}{\pi}\frac{D}{Y} + 1\right) - r_d + \lambda\frac{r-g_Y}{Y}\right] > 0,$$

or

$$\left[\pi r\left(\frac{\pi' r}{\pi}\frac{D}{Y} + 1\right) - r_d\right] < -\lambda\frac{r-g_Y}{Y} \tag{27}$$

The difference of marginal revenue and marginal costs of additional debt is positive by assumption and has an upper ceiling on the right-hand side of the inequality.

6.4 Unstable debt dynamics or steady state?

As the conditions for the case $\lambda<0$ have empirical support, and debt instability conditions hold, the only alternative to instability is for banks to jump into the steady state.

If banks choose to move the economy to a steady state current profits every period are

$\pi(r^*d^*)r^*d^*Y - r_d d^*Y$

with Y growing at the rate g_Y^* leading to profits with the same growth rate.

If banks choose a higher interest rate than that of a steady state, the growth rate gets lower, as shown in equation (11). The debt instability condition $r\text{-}g > 0$ gets even worse and so does the current account through the growth rate reduction. The economy runs into increasing debt, the probability $\pi(rd)$ of getting the debt service is going to move towards zero, and a debt crisis becomes almost certain. If the discount rate is very high, short term profits may be valued high enough to run this risk. This possibility cannot be excluded. In the case of loan pushing, defined as urging or forcing debtor to take large, risky amounts of credit as observed before the 1982 debt crisis incentives for bankers to sell credit were set in this direction (Ziesemer 1997). Moreover, the textbook wisdom is that financial crises are preceded by excessive accumulation of foreign debt (Perkins et al. 2013). If, in contrast, the banks are very cautious as after the 1982 debt crisis, they may set initial values below d^* and the economy may run down debt/GDP ratios *d* until profits become zero. Under profit maximization this is clearly an inferior banking strategy, and credit rationing or jump into a steady state are more likely and hard to distinguish from each other. Anyway, our model of this section shows that there is no guarantee and it is even empirically unlikely that banks put interest rates below growth rates to ensure stability. There is also no guarantee that banks jump into a steady state. The probability for a crisis through not paying is always positive and crises therefore cannot be excluded anyway and bank behaviour may increase the probability. Countries can avoid this only through their own policy choices.

7. Summary, conclusion, and suggestions for further research

The model specification of functions for imports, exports, savings, and investment as a share of GDP depending on the growth rates of domestic GDP allows us to connect the trade balance of BOPC models to that of the goods market and from there to Okun's law and debt dynamics. This allows for the Keynesian flavour of higher world GDP growth not only increasing domestic GDP growth but also reducing unemployment with explicit consideration of the goods market.

To get the Thirlwall effect, we have derived three important results for four constellations of debt/GDP (in-) stability and current-account reactions to domestic growth rate changes. First, all four cases require a reaction of the goods market to domestic GDP growth rates that is stronger on the supply side variables than on the demand variables, $i' - s' - m' + px' < 0$. Second, the Thirlwall effect works for stable and unstable debt dynamics, r <(>)g_Y. Third, the Thirlwall effect works for positive and negative effects of domestic growth rates on the current account, $m' - px' - d < (>) 0$.

Imported capital goods are not modeled explicitly in our Keynesian modelling approach. Several authors in the literature point out that imported capital goods are an important fact (Bardhan and Lewis 1970; Moreno-Brid 1998-99). It is an important suggestion for further research on Keynesian models to take this fact into account. Ziesemer (2022) deals with theory and empirics of this.

The empirical testing of the crucial condition $i' - s' - m' + px' < 0$ is country specific, but not the subject of this paper. Its analysis is another suggestion for further research. For Brazil, savings and imports react more strongly to growth than investment does and therefore the condition holds.

The estimation of dynamic equations for *i*, *s*, *m*, and *x*, as well as growth and Okun's law, provides some support for the empirical plausibility of the model, at least for the case of Brazil. Positive shocks on the level or growth rate[17] of the world GDP enhance the GDP level or growth rate of the Brazilian economy. Positive shocks on interest rates and weakly exogenous terms of trade decrease Brazil's output.

For the debt dynamics we cannot exclude the possibility of steady states or unstable debt dynamics with explosive debt under a high discount rate. If banks try to jump into a steady state at some point in time, marginal revenue from additional debt is larger than marginal costs by a factor proportional to the debt (in)stability condition.

Finally, growth rates of world GDP are falling over time in the data. The fall of growth rates for many countries may be driven by that and not only by falling productivity growth rates, which are emphasized much more in the growth literature.

Endogenizing the terms of trade can be shown to lead to non-constant shares of investment, savings, imports, and exports as a share of GDP. If domestic growth rates are larger than those of the world economy terms of trade will fall. This important aspect will lead to structural change and requires its own paper.

References

Alonso, José A., and Carlos Garcimartín (1998) A New Approach to Balance-of-Payments Constraint: Some Empirical Evidence. Journal of Post Keynesian Economics, Winter, 1998-1999, Vol. 21, No. 2, 259-282.

Barbosa-Filho, Nelson H. (2001) The balance-of-payments constraint: from balanced trade to sustainable debt. BNL Quarterly Review 219, 381-400.

Bardhan, P.K. and S. Lewis (1970), Models of Growth with Imported Inputs, Economica, November, p.373-85.

Bhering, G., Serrano, F., & Freitas, F. (2019). Thirlwall's law, external debt sustainability, and the balance-of-payments constrained level and growth rates of output. Review of Keynesian Economics, 7(4), 486–497.

Blecker, R.A. (2021a). Thirlwall's law is not a tautology, but some empirical tests of it nearly are. Review of Keynesian Economics, 9(2), 175–203.

Blecker, Robert A. (2021b) New advances and controversies in the framework of balance-of-payments-constrained growth. Journal of Economic Surveys, 1–39.

Cohen, Daniel, Private Lending to Sovereign States: A Theoretical Autopsy", The MIT Press, Cambridge, Massachusetts, London, England, 1991.

[17] This would require adding, for example, 0.001t instead of 0.005 to the intercept. This is equivalent to changing the time trend in (21) from 0.0042t to 0.0052t. The long-run world GDP growth rate then goes from 2.86% to 3.54%. In line with the long-run result *dLOG(Y(-1)) = (0.381/0.35)dlog(Z)* derived above from (17), the long-run domestic growth rate then goes from *(0.381/0.35)*0.0286=0.0311* to *(0.381/0.35)*0.0354= 0.0385*.

Dávila-Fernández, Marwil J. (2020). Alternative approaches to technological change in a small open economy. Journal of Evolutionary Economics 30: 279–317.

Dávila-Fernández, M., & Oreiro, J. (2021). A song of ice and fire: Competitiveness in an export-led growing economy. MPRA Paper No. 109821.

Dávila-Fernández, Marwil J., José L. Oreiro, Mario W. Dávila Dávila (2018) Endogenizing non-price competitiveness in a BoPC growth model with capital accumulation. Structural Change and Economic Dynamics 44 77–87.

Dávila-Fernández, M. J., & Sordi, S. (2019). Distributive cycles and endogenous technical change in a BoPC growth model. Economic Modelling, 77, 216-233.

Dornbusch, Rudiger W. (1980) Open economy macroeconomics. New York, N.Y. : Basic Books.

Dutt, A.K. (2002) Thirlwall's law and uneven development. Journal of Post Keynesian Economics 24(3): 367-390.

Fair, Ray C., 2018, Macroeconometric Modeling: 2018, fairmodel.econ.yale.edu/mmm2/mm2018.pdf.

GIBLIN, L F (1930) AUSTRALIA, 1930. AN INAUGURAL LECTURE. Melbourne University Press.

Harrod, R. (1933), International Economics, Cambridge University Press.

Jusélius, Katarina. 2006. *The Cointegrated VAR Model: Methodology and Applications*. Oxford: Oxford University Press.

Kennedy, Charles, and A. P. Thirlwall (1979a) Import Penetration, Export Performance and Harrod's Trade Multiplier. Oxford Economic Papers, New Series, Vol. 31, No. 2, pp. 303-323.

Kennedy, C., and Thirlwall, A. P. (1979b), The Input-Output Formulation of the Foreign Trade Multiplier, Australian Economic Papers, June, 173-180.

Kilian, Lutz, and Helmut Lütkepohl. 2017. Structural Vector Autoregressive Analysis. Cambridge: Cambridge University Press.

Kronenberg, T. (2010). Energy conservation, unemployment and the direction of technical change. Portuguese Economic Journal, 9(1), 1-17.

Leon-Ledesma, M., & Thirlwall, A. P. (2000). Is the natural rate of growth exogenous? PSL Quarterly Review, 53(215).

León-Ledesma, M.A. and A.P. Thirlwall (2002), "The endogeneity of the natural rate of growth", Cambridge Journal of Economics 26, 441-459.

Maddala, G.S. and I.-M. Kim, 1998, Unit Roots, Cointegration and Structural Change, Cambridge University Press.

Massot, Juan Miguel & Román David Merga (2022) A Balance-of-Payments-Constrained Growth Model for a Small Commodity Exporting Country: Argentina Between 1971 and 2016, International Review of Applied Economics, 36:4, 562-586.

McCombie, J. S. (1985). Economic growth, the Harrod foreign trade multiplier and the Hicks' Super-Multiplier. Applied Economics, 17(1), 55-72.

Moreno-Brid, Juan Carlos (1998-1999) On Capital Flows and the Balance-of-Payments-Constrained Growth Model. Journal of Post Keynesian Economics , Vol. 21, No. 2, pp. 283-298.

Moreno-Brid, J. C. (2003). Capital flows, interest payments and the balance-of-payments constrained growth model: A theoretical and empirical analysis. Metroeconomica, 54(2-3), 346-365.

Muysken, Joan, Ehsan Vallizadeh, Thomas Ziesemer (2015) Migration, unemployment, and skill downgrading. The Scandinavian Journal of Economics 117 (2), 403-451.

Neves, J. P. B., & Semmler, W. (2022). Credit, output and financial stress: A non-linear LVSTAR application to Brazil. Metroeconomica 73: 900–923.

Nishi, H. (2019). Balance-of-payments-constrained cyclical growth with distributive class conflicts and productivity dynamics. Metroeconomica, 70(4), 620-640.

Nomaler, Ö., Spinola, D., & Verspagen, B. (2021). Demand-led industrialisation policy in a dual-sector small balance of payments constrained economy. UNU-MERIT WP 2021-038.

Oreiro, J. L. (2016). Inconsistency and over-determination in balance-of-payments-constrained growth models: a note. Review of Keynesian Economics, 4(2), 193-200.

Palley, Thomas I. (2003) Pitfalls in the Theory of Growth: An application to the balance of payments constrained growth model, Review of Political Economy, 15:1, 75-84.

Perkins, D.H., S. Radelet, D.L. Lindauer and S.A. Block, 2013, Economics of Development, 7th ed., Norton.

Pesaran, M. H. (2015). Time series and panel data econometrics. Oxford University Press. Oxford University Press, Oxford.

Pissarides, C. A. (1998). The impact of employment tax cuts on unemployment and wages; the role of unemployment benefits and tax structure. European Economic Review, 42(1), 155-183.

Razmi, A. 2016. "Correctly Analyzing the Balance-of-payments Constraint on Growth." Cambridge Journal of Economics 40 (6): 1581–1608.

Sasaki, Hiroaki and Fukatani, Noriki and Imai, Daisuke and Kamanaka, Yusuke (2022). Sustainable Economic Growth in an Economy with Exhaustible Resources and a Declining Population under the Balance-of-Payments Constraint. MPRA Paper No. 113559.

Shapiro, C., & Stiglitz, J. E. (1984). Equilibrium unemployment as a worker discipline device. The American Economic Review, 74(3), 433-444.

Thirlwall, A. P. (1979). The balance of payments constraint as an explanation of the international growth rate differences. PSL Quarterly Review, 32.

Thirlwall, A. P. (1982), The Harrod Trade Multiplier and the Importance of Export-Led Growth, Pakistan Journal of Applied Economics, Vol. 1. No. 1, Summer.

Thirlwall, A. P. (1983) Foreign Trade Elasticities in Centre-Periphery Models of Growth and Development. Banca Nazionale del Lavoro. Quarterly Review, 249-261.

Thirlwall, A. P. (2011) Growth and Development. Chapter 16, pp. 561-573.

Thirlwall A. P., Hussain M. N. (1982): The balance of payments constraint, capital flows and growth rates differences between developing countries. Oxford Economic Papers, 34, pp. 498–509.

Turnovsky, S.J. (1977) Macroeconomic analysis and stabilization policy. Cambridge, Cambridge University Press.

Vernengo, Matias (2016) Kaldorian and Sraffian supermultipliers: a clarification. http://nakedkeynesianism.blogspot.com/2016/02/kaldorian-and-sraffian-supermultipliers.html. Wednesday, February 10.

Weitzman, M. L. (1982): 'Increasing returns and the foundations of unemployment theory', Economic Journal, 92, December, pp. 787–804.

Wooldridge, J. M. 2013. Introductory Econometrics: A Modern Approach. 5th ed. Mason, USA: South-Western, Cengage Learning.

Zarembka, P. (1972), Towards a Theory of Economic Development, San Francisco.

Ziesemer, Thomas H.W. (1995a). Economic Development and Endogenous Terms-of-Trade Determination: Review and Reinterpretation of the Prebisch-Singer Thesis. Unctad Review, 17-33. Also, UNCTAD Discussion Papers No. 87, July 1994.

Ziesemer, T. (1995b),"Growth with Imported Capital Goods, Limited Export Demand and Foreign Debt", Journal of Macroeconomics, Vol. 17, No.1, 31-53.

Ziesemer, T. (1997). From loan pushing to credit rationing: a brief note on interest shocks in a model by Basu. Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft, 569-578.

Ziesemer, T. (1998) A Prebisch-Singer Growth Model and the Debt Crises, in: DEVELOPMENT ECONOMICS and POLICY, edited by David Sapsford and John-ren Chen, Macmillan 1998, 300-317.

Ziesemer, Thomas H.W. (2014). Country terms of trade: Trends, unit roots, over-differencing, endogeneity, time dummies, and heterogeneity. International Review of Applied Economics, 28(6), 767-796.

Ziesemer, Thomas H.W. (2022) Semi-endogenous growth in a non-Walrasian DSEM for Brazil: Estimation and simulation of changes in foreign income, human capital, R&D, and terms of trade. UNU-MERIT WP 2022-013.

Appendix Instrumental variables

We add lagged regressors to instruments for equations with autoregressive terms.

Eq. (13): 8 parameters, 13 instruments are: Instruments: LOG(M(-2)/Y(-2)), D(LOG(Y(-1))), LOG(1+R(-1)), LOG(P(-1)), LOG(M(-4)/Y(-4)), C, D(LOG(M(-1)/Y(-1))), LOG(Y(-2))-LOG(Y(-3)), LOG(M(-5)/Y(-5)), LOG(Y(-5))-LOG(Y(-6)), LOG(P(-4)), LOG(1+R(-4)), LOG(M(-8)/Y(-8)); obs: 19; adj. R-sq: 0.66; DW: 2.13.

Eq. (14): 8 parameters, 11 instruments are : LOG(X(-2)/Y(-2)), LOG(P(-1)), D(LOG(Z)), D(LOG(Z(-1))), D(LOG(Y(-1))), D(LOG(Y(-2))), C, LOG(X(-3)/Y(-3)), LOG(P(-3)), LOG(Z(-2))-LOG(Z(-3)), LOG(Z(-3))-LOG(Z(-4)), LOG(Y(-3))-LOG(Y(-4)); obs: 55; adj. R-sq: 0.986; DW: 2.036.

Eq. (15): 8 parameters, 14 instruments are: LOG(GFCF(-2)/Y(-2)), LOG(P(-1)), LOG(P(-2)), D(LOG(Y(-1))), D(LOG(Y(-2))), LOG(1+R(-2)), C, LOG(GFCF(-5)/Y(-5)), LOG(GFCF(-6)/Y(-6)), LOG(P(-6)), LOG(P(-7)), LOG(Y(-5))-LOG(Y(-6)), LOG(Y(-6))-LOG(Y(-7)), LOG(1+R(-6)); obs: 17; adj. R-sq: 0.965; DW: 2.65.

Eq. (16): 9 parameters, 18 instruments are: LOG(S(-2)/Y(-2)), D(LOG(Y)), LOG(P(-1)), LOG(P(-2)), LOG(1+R(-1)), LOG(1+R(-2)), C, LOG(S(-4)/Y(-4)), LOG(S(-5)/Y(-5)), LOG(P(-5)), LOG(P(-6)), LOG(Y(-4))-LOG(Y(-5)), LOG(1+R(-4)), LOG(1+R(-5)), LOG(S(-3)/Y(-3)), LOG(P(-4)), LOG(Y(-3))-LOG(Y(-4)), LOG(1+R(-3)); obs: 18; all p = 0.0000; adj. R-sq: 0.952; DW: 2.43.

Eq. (17): 7 parameters, 6 instruments are: Instruments: C -SY(-1)/100+(GFCF(-1)+P(-1)*X(-1)-M(-1))/Y(-1) LOG(Y(-2)) $(U2(-1))^2$ D(LOG(GFCF(-1))) LOG(Z) D(LOG(Y(-5))) (-1/100)*SY(-5)+(GFCF(-5)-M(-5)+P(-5)*X(-5))/Y(-5) LOG(Y(-6)) LOG(GFCF(-5))-LOG(GFCF(-6)) U2(-6)^2 LOG(Z(-5)); obs: 42; adj. R-sq: 0.43; DW: 1.857.

Eq. (18): 5 parameters, 5 instruments are: LOG(1+R(-2)), LOG(1+R(-3)), LOG(1+R(-4)), -S(-2)/Y(-2) + (GFCF(-2))/Y(-2), C; obs: 19; adj. R-sq: 0.71; DW: 1.4.

Eq. (19): 4 parameters, 6 instruments are: C, LOG(P(-2)), LOG(P(-3)), LOG(P(-4)), LOG(P(-5)), LOG(P(-6)); obs: 54; adj. R-sq: 0.876; DW: 1.84.
Eq. (20): 3 parameters, 3 instruments are U(-1) D(LOG(Y(-1))) C; obs: 47; adj. R-sq: 0.928; DW: 2.09.

Eq. (21): 5 parameters, 5 instruments are C LOG(Z(-1)) LOG(Z(-2)) LOG(Z(-3)) @TREND; obs: 57; adj. R-sq: 0.999; DW: 1.966.

Kernel: Bartlett; Bandwidth: Variable Newey-West (3); No prewhitening, Iterate coefficients after one-step weighting matrix. Convergence achieved after 1 weight matrix, 53 total coefficient iterations. Total number of instruments: L= 77; total number of parameters: K= 57; L-K=20. P(J = 0.202616, 20) = 1. The J-statistic is low because of overidentifying constraints coming from autoregressive processes each adding only the autocorrelation coefficient to the estimated parameters but all lagged variables serving as instruments for themselves without adding parameters. Otherwise, we have one instrument per regressor.

Some of the Durbin-Watson statistics are too far away from 2. An extension of the model with other variables is possible to improve on this (Ziesemer 2022). Here we have kept the model close to the theoretical model.

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